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THE CYCLIC SYMMETRIC VERTEX FOR THREE ARBITRARY NEVEU-SCHWARZ STRINGS

A. Neveu

CERN - Geneva

and

P. West

Mathematics Department, King's College
Strand, London WC2

A B S T R A C T

The generalization of the bosonic Caneschi-Schwimmer-Veneziano vertex for the scattering of three Neveu-Schwarz string states is found and its gauge identities derived. From this, one can derive by conformal mappings the corresponding covariant vertex.

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It is the purpose of this paper to give the vertex for the scattering of three Neveu-Schwarz strings. We shall find this vertex within the context of a supersymmetric dual model. In reference [1] the relation between old dual model and gauge covariant vertices was explained and employing these techniques the vertex given in this paper can be used to deduce the vertex of the gauge covariant Neveu-Schwarz theory. The techniques used in this paper are the same as in the bosonic case and they were discussed in reference [2].

Some aspects of gauge covariant superstring theory are discussed in reference [3]. We begin by discussing the concept of superconformal weight. Let us consider the superspace [4][5][6] parametrized by the complex co-ordinates (z, θ) . A summary of part of this old literature can be found in reference [8]. A superconformal transformation is one that takes

$$dL \rightarrow \Omega(z, \theta) dL \quad (1)$$

where

$$dL = dz - d\theta \theta \quad (2)$$

It is easily shown that it is of the form [4]

$$z' = g + \theta g B, \quad \theta' = g + \theta B \quad (3)$$

where

$$B = \pm \sqrt{g' - g'g}, \quad g' \equiv \frac{dg}{dz}, \quad g' \equiv \frac{dg}{dz} \quad (4)$$

We observe that the covariant derivative

$$D = \frac{\partial}{\partial \theta} + \theta \frac{\partial}{\partial z} \quad (5)$$

also scales under this transformation.

The infinitesimal conformal transformations generated by [4]

$$L_n = z^n \left(z \frac{\partial}{\partial z} + \left(\frac{n+1}{2} \right) \theta \frac{\partial}{\partial \theta} \right) \quad n = 0, \pm 1, \dots \quad (6)$$

$$G_r = z^{r+\frac{1}{2}} \left(\frac{\partial}{\partial \theta} - \theta \frac{\partial}{\partial z} \right) \quad (7)$$

satisfy the relationships [9]

$$\{ G_r, G_s \} = 2 L_{r+s} \quad (8)$$

$$[G_r, L_n] = \left(r - \frac{n}{2} \right) G_{n+r}. \quad (9)$$

In these equations, a minus sign has been inserted to take account of the appropriate action of these generators on superfields.

For completeness we record the following expressions

[9]

$$L_n = : \frac{1}{2} \sum_{m=0}^{\infty} \alpha_m^\mu \alpha_{n-m}^\mu + \frac{1}{4} \sum_{s=-\infty}^{\infty} (2s+n) b_{-s}^\mu b_{n+s}^\mu :$$

$$G_r = \sum_{n=-\infty}^{\infty} \alpha_n^\mu b_{r-n}^\mu.$$

They satisfy the above algebra provided

$$[\alpha_n^\mu, \alpha_m^\nu] = \eta^{\mu\nu} \delta_{n+m,0} ; \{b_r^\mu, b_s^\nu\} = \eta^{\mu\nu} \delta_{r+s,0}$$

For future use we note that

$$\begin{aligned} [L_n, \alpha_m^\mu] &= -m \alpha_{n+m}^\mu, & [G_r, \alpha_n^\mu] &= -n b_{r+n}^\mu \\ [G_r, b_s^\mu] &= \alpha_{r+s}^\mu, & [L_n, b_r^\mu] &= -(\frac{n}{2} + r) b_{n+r}^\mu. \end{aligned}$$

We define an operator composed out of α_n^μ and b_r^μ which is also a function of z and θ to have superconformal weight d if it satisfies the equation

$$g \phi(z, \theta) g^{-1} = \left[\frac{z}{z'}, \text{Ber} \right]^d \phi(z', \theta') \quad (10)$$

where

$$\text{Ber} = \begin{vmatrix} \frac{dz'}{dz} & \frac{d\theta'}{dz} \\ \frac{dz'}{d\theta} & \frac{d\theta'}{d\theta} \end{vmatrix} = B + \theta G' = D\theta' \quad (11)$$

and

$$g = \exp \left[\sum_{n=-\infty}^{\infty} a_n L_n + \sum_{r=-\infty}^{\infty} a_r G_r \right] \quad (12)$$

The change of co-ordinates induced by g is the same as when L_n and G_r were represented by the infinitesimal operators of equations (6) and (7). We observe that z/z' BER is none other than the Jacobian for the change in the measure

$$\int d\theta \frac{dz}{z} \quad \text{to} \quad \int d\theta' \frac{dz'}{z'}$$

For an infinitesimal transformation equation (9) reduces to

$$[L_n, \phi(z, \theta)] = z^n \left[z \frac{d}{dz} + \frac{(n+1)}{2} \theta \frac{d}{d\theta} + \left[\frac{n-1}{2} \right] d \right] \phi(z, \theta) \quad (13)$$

$$[G_r, \phi(z, \theta)] = z^{r-\frac{1}{2}} \left[z \frac{d}{d\theta} - \theta z \frac{d}{dz} - \theta \left(r - \frac{1}{2} \right) d \right] \phi(z, \theta). \quad (14)$$

We note that if

$$V = \oint_{z=0} \frac{dz}{z} d\theta \quad \phi(z, \theta) \quad (15)$$

then

$$[L_n, V] = [G_r, V] = 0 \quad (16)$$

provided ϕ has superconformal weight $d=1$. This generalizes the construction of the physical 2 point vertex used in the bosonic theory to the supersymmetric case.

The reader may verify that

$$L^\mu = Q^\mu(z) + \frac{\theta}{\sqrt{z}} \alpha^\mu(z). \quad (17)$$

where

$$Q^\mu(z) = \sum_{\substack{n=-\infty \\ n \neq 0}}^{\infty} \frac{1}{n} \alpha_n^\mu z^{-n} - p^\mu \ln z + q^\mu. \quad (18)$$

$$\alpha^\mu(z) = \sum_{r=-\infty}^{\infty} b_{-r}^\mu z^{-r}. \quad (19)$$

has superconformal weight zero while

$$G(z, \theta) = z^{3/2} \left[G(z) + \frac{\theta}{\sqrt{z}} L(z) \right] \quad (20)$$

where

$$G(z) = \sum_{r=-\infty}^{\infty} G_r z^{-r}, \quad L(z) = \sum_{n=-\infty}^{\infty} L_n z^{-n} \quad (21)$$

has superconformal weight 3.

We observe that

$$H^{\mu}(z, \theta) \equiv z D L^{\mu}(z, \theta) = \sqrt{z} \left(\mathcal{H}^{\mu}(z) + \frac{\theta}{\sqrt{z}} P^{\mu}(z) \right) \quad (22)$$

where $P^{\mu}(z) = \sum_{n=-\infty}^{\infty} \alpha_n^{\mu} z^{-n}$, has super weight $d=1$. The external factor of $\sqrt{z}$ is a result of the fact that if R has superweight d and we can write it in the form

$$R = (z)^{d/2} \left(A(z) + \frac{\theta}{\sqrt{z}} B(z) \right). \quad (23)$$

then A and B have an ordinary conformal weight of $d/2$ and $d/2+1/2$.

In order to find the Neveu-Schwarz generalization of the Caneschi-Schwimmer-Veneziano vertex, following reference [2] we now seek a superconformal transformation which belongs to the graded extension of $SU(1,1)$ that cubes to one. The cubic transformation which is a member of $SU(1,1)$ that takes 0 to 1, 1 to ∞ and ∞ to 0 is

$$z \rightarrow \frac{1}{1-z}. \quad (24)$$

This cubes to one and formed the basis for the derivation of the gauge identities of V^{CSV} in reference [2]. The new generators we now include in $SU(1,1)$ are $G_{\pm \frac{1}{2}}$. The

extensions of equation (24) which cube to 1 are

$$z_1 = \frac{1}{1-z} - \frac{\theta \textcircled{4}}{1-z}, \quad \theta_1 = \textcircled{4} - \frac{\theta}{1-z}. \quad (25)$$

and

$$z_1 = \frac{1}{1-z + \theta \textcircled{4}}, \quad \theta_1 = -\frac{\theta}{1-z} + \frac{\textcircled{4}}{1-z}. \quad (26)$$

They correspond respectively to using $(-1)^{G_{\theta G_{-\frac{1}{2}}}}$ and

$(-1)^{G_{\theta G_{\frac{1}{2}}}}$ after $e^{L_{-1}} e^{L_1}$ where $G = (-1)^{\sum_{r=\frac{1}{2}}^{\infty} b_r^\dagger b_r - 1}$. One can easily show that $(-1)^{G_{\theta G_{\mp \frac{1}{2}}}}$ before $e^{L_{-1}} e^{L_1}$ is the same as $(-1)^{G_{\theta G_{\pm \frac{1}{2}}}}$ afterwards.

The anticommuting parameter $\textcircled{4}$ will occur in the 3 string vertex which we now consider. The essential reason for this is that an N point vertex has N z's and θ 's which label the end points of the strings. However 3 of the z's and 2 of the θ 's can be fixed by the graded $SU(1,1)$ transformations leaving for the three point function still a dependence on one anticommuting coordinate. For reasons that will be discussed below we will only consider equation (26) and forget equation (25).

Following reference [2] we assume that the 3 string vertex is invariant under the action of the transformation of equation (26) applied on each of the external legs. It is then easily shown that, as for the bosonic case, the 3 Neveu-Schwarz vertex satisfies the equation

$$V \{ A R^1(z_1, \theta_1) + B R^2(z_2, \theta_2) + C R^3(z, \theta) \} = 0 \quad (27)$$

provided

$$A \left[\frac{[-1]}{(1-z)} \left(1 - \frac{\theta \otimes}{1-z} \right) \right]^d + B + C \left[\frac{z}{1-z} \left(1 - \frac{\theta \otimes}{1-z} \right) \right]^d = 0 \quad (28)$$

and R is an object of superconformal weight d . In the above z_1, θ_1 are given in terms of z, θ by equation (26) and z_2, θ_2 are related to z_1, θ_1 in the same way. The reader is referred to reference [2] for a discussion of the convergence of this formula. A particular case of this result is when R is H which has superconformal weight $d=1$. Let us consider in this special case $B = 0$ and $C = \frac{1}{z}$. We find that $A = 1$. Examining the $\theta = 0$ terms we discover the identity

$$V \left\{ \frac{H^3(z)}{\sqrt{z}} + \frac{H'(\frac{1}{1-z})}{\sqrt{1-z}} + \frac{\otimes}{1-z} P'(\frac{1}{1-z}) \right\} = 0 \quad (29)$$

for $|z| < 1, |\frac{1}{1-z}| < 1$. Letting $V = V_0 + \otimes V_1$ and taking the term proportional to $\otimes$ we find the equation

$$\frac{1}{(1-z)} V_0 P'(\frac{1}{1-z}) + V_1 \left(\frac{H^3(z)}{\sqrt{z}} + \frac{H'(\frac{1}{1-z})}{\sqrt{1-z}} \right) = 0 \quad (30)$$

Taking the term without $\otimes$ we find that

$$V_0 \left[\frac{H^3(z)}{\sqrt{z}} + \frac{H'(\frac{1}{1-z})}{\sqrt{1-z}} \right] = 0$$

We can find a similar relation for Q and H acting on V_1 and V_0 respectively by taking $R = L(z, \theta)$ and $d = 0$.

It is straightforward to obtain integral equations from equation (27). Following reference [2] we begin by integrating $CR^3(z, \theta)$ around $z = 0$ and changing the contour to include the points $z = 1$ and $z = \infty$. The final result of this calculation is

$$0 = V \left\{ \oint_{z=0} d\bar{z} d\theta \phi(z, \theta) R^3(z, \theta) + \oint_{z=1} d\bar{z} d\theta \phi(z, \theta) \cdot \left[\frac{z}{1-z} (1 - \theta \frac{\partial}{\partial z}) \right]^d R^2(z, \theta) + \oint_{z=\infty} d\bar{z} d\theta \phi(z, \theta) (-z)^d R^1(z, \theta) \right\} \quad (31)$$

Using the expression of equation (23) for R and equation (26) we find that equation (31) becomes

$$0 = V \left\{ \oint_{z=0} d\bar{z} d\theta \phi(z, \theta) z^{d/2} (A^3(z) + \frac{\theta}{\sqrt{z}} B^3(z)) + \oint_{z=0} d\bar{z}_2 d\theta \phi(-z_2) \frac{(-1)^d}{(1-z_2)(z_2)^{d-1}} \left[1 + \frac{d\theta \partial}{\partial z_2} (1-z_2) \right] \left[z_2^{d/2} A^2(z_2) - \theta \partial (1-z_2) \frac{\partial}{\partial z_2} (z_2^{d/2} A^2(z_2)) + z_2^{d/2-1/2} (\partial - \theta (1-z_2)) B_2(z_2) \right] + \oint_{z=0} d\bar{z}_1 d\theta \phi(1-z_1) \frac{(-1)^d}{z_1^d} \left\{ z_1^{d/2} A^1(z_1) - \theta \partial z_1^2 \frac{\partial}{\partial z_1} (z_1^{d/2} A^1(z_1)) + (\partial - \theta) z_1^{d/2+1/2} B^1(z_1) \right\} \right\} \quad (32)$$

where

$$z_1 = \frac{1}{1-z}, \quad z_2 = \frac{z-1}{z}. \quad (33)$$

To analyse the content of this equation we can write

$V = V_0 + \Theta V_1$ and consider terms where $\Theta = 0$ and $\Theta \neq 0$. For each of these cases we also have the choice $\phi = \phi_0(z)$ and $\phi = \Theta \phi_1(z)$. This gives four identities in all.

Let us begin by considering the case $R = H$ and so $d = 1$. The simplest identity is found by taking $\phi = \phi_0(z)$ and neglecting terms proportional to Θ . This identity involves P^μ 's acting on V_0 and is the same as one finds in the bosonic case^[1]. Consequently we must conclude that the α dependent part of V_0 is given by V^{CSV} .

Taking $\Theta = 0$ but $\phi = \Theta \phi_1(z)$ we find the identity

$$0 = V_0 \left[\oint_{z=0} \frac{dz}{z} \sqrt{z} \phi_1 \alpha e^3(z) - \oint_{z_2=0} \frac{dz_2}{z_2} \frac{\sqrt{z_2}}{1-z_2} \phi_1 \alpha e^2(z_2) + \oint_{z_1=0} \frac{dz_1}{z_1} \frac{\phi_1}{\sqrt{z_1}} \alpha e^1(z_1) \right] = 0 \quad (34)$$

This determines the dependence of V_0 on b_r^μ oscillators.

Carrying out this calculation we find that the b_r^μ dependent part of V_0 can be written as

$$\exp \sum_{t=1}^3 \sum_{r, s = \frac{1}{2}}^{\infty} b_s^{t\mu} b_r^{t+\mu} \binom{r-\frac{1}{2}}{s-\frac{1}{2}} (-1)^{s-\frac{1}{2}}.$$

Recalling the expression for V^{CSV} we easily recognize the following form for V_0 .

$$V_0 = \langle 0_{123} | \exp - \sum_{r=1}^3 \frac{1}{2\pi i} \oint_{z=0} \frac{dz}{z} \left\{ P^{\mu r}(z) Q_+^{\mu r+1} \left(\frac{1}{1-z} \right) - \mathcal{L}^{\mu r}(z) \frac{\sqrt{z}}{\sqrt{1-z}} \mathcal{L}_+^{\mu r+1} \left(\frac{1}{1-z} \right) \right\} \quad (35)$$

where

$$Q_+^{\mu}(z) = \sum_{n=1}^{\infty} \frac{1}{n} \alpha_n^{\mu} z^{-n} ; \quad \mathcal{L}_+(z) = \sum_{r=\frac{1}{2}}^{\infty} b_r z^{-r} \quad (36)$$

Taking $\phi = \phi_0$ and terms proportional to $\mathbb{O}$ in equation (32) we find the identity

$$0 = V_1 \left\{ \oint_{z=0} \frac{dz}{z} \phi_0 P^3(z) + \oint_{\bar{z}_2=0} \frac{d\bar{z}_2}{\bar{z}_2} \phi_0 P^2(\bar{z}_2) - \oint_{\bar{z}_1=0} \frac{d\bar{z}_1}{\bar{z}_1} \phi_0 P(\bar{z}_1) \right\} + V_0 \left\{ + \oint_{\bar{z}_2=0} \frac{d\bar{z}_2}{\bar{z}_2} \phi_0 \bar{z}_2 \frac{\partial}{\partial \bar{z}_2} \left(\mathcal{L}^2(\bar{z}_2) \right) - \oint_{\bar{z}_1=0} \frac{d\bar{z}_1}{\bar{z}_1} \phi_0 \bar{z}_1 \frac{\partial}{\partial \bar{z}_1} \left(\sqrt{\bar{z}_1} \mathcal{L}'(\bar{z}_1) \right) \right\} \quad (37)$$

This equation determines V_1 to be

$$V_1 = V_0 \sum_{t=1}^3 \left[\sum_{\substack{n=1 \\ r=\frac{1}{2}}}^{\infty} b_r^{\mu t} \binom{n-1}{r-\frac{1}{2}} (-1)^{r+\frac{1}{2}} \alpha_n^{\mu t+1} - \sum_{\substack{n=0 \\ r=\frac{1}{2}+n}}^{\infty} \alpha_n^{\mu t} (-1)^n \binom{r-\frac{1}{2}}{n} b_r^{\mu t+1} \right] \quad (38)$$

One finds that this result can be written in the form

$$V_1 = V_0 \sum_{r=1}^3 \frac{1}{2\pi i} \oint_{z=0} \frac{dz}{z} d\theta \left[P^r(z) \frac{1}{\sqrt{1-z}} \mathcal{L}_+^{r+1} \left(\frac{1}{1-z} \right) + \frac{\sqrt{z}}{(1-z)} \mathcal{L}^r(z) \cdot P_+^{r+1} \left(\frac{1}{1-z} \right) \right] \quad (39)$$

Examining equations (39) and (35) one realizes that V has a particularly simple form, namely

$$V = \exp -\frac{1}{2\pi i} \sum_{r=1}^3 \oint_{z=0} d\theta \frac{dz}{z} H^r(z, \theta) L^{r+1}(z_1, \theta_1) \quad (40)$$

where z_1 and θ_1 are related to z and θ by equation (26).

The actual vertex for the scattering of three Neveu-Schwarz strings is given by

$$\int d\theta V = V_1 \quad (41)$$

As a further illustration of the type of gauge identities satisfied by V we consider the action of L_n and G_r on the three legs. Taking $R_2 G(z, \theta)$ and $d = 3$ in equation (32) we find the equation

$$\begin{aligned} 0 = V \Big\{ & \oint_{z=0} \frac{dz}{z} d\theta \phi(z, \theta) z^{3/2} \left(G^3(z) + \frac{\theta}{\sqrt{z}} L^3(z) \right) \\ & - \oint_{z_2=0} \frac{dz_2}{z_2} d\theta \frac{\phi(z, \theta)}{(1-z_2)} \left(1 + 3\theta \frac{\partial}{\partial z_2} (1-z_2) \right) \left(\frac{1}{\sqrt{z_2}} G^2(z_2) \right. \\ & \left. - (1-z_2) \theta \frac{\partial}{\partial z_2} \left(\frac{3}{2} z_2^{3/2} G^2(z_2) \right) + (\theta - \theta(1-z_2)) \frac{1}{z_2} L^2(z_2) \right) \\ & + \oint_{z_1=0} \frac{dz_1}{z_1} d\theta \phi(z, \theta) (1-z_1)^2 \left\{ \frac{1}{z_1^{3/2}} G^1(z_1) - \theta \frac{\partial}{\partial z_1} \left(\frac{3}{2} z_1^{3/2} G^1(z_1) \right) \right. \\ & \left. + \left(\frac{\theta}{z_1} - \theta \right) L^1(z_1) \right\} \Big\} = 0 \quad (42) \end{aligned}$$

This equation contains four identities. Those found by taking terms not proportional to θ are

$$\begin{aligned} V_0 \Big\{ & \oint_{z=0} \frac{dz}{z} \phi L^3(z) + \oint_{z_2=0} \frac{dz_2}{z_2} \phi \frac{(1-z_2)}{z_2} L^2(z_2) \\ & + \oint_{z_1=0} \frac{dz_1}{z_1} \phi (1-z_1) L^1(z_1) \Big\} = 0 \quad (43) \end{aligned}$$

and

$$V_0 \left\{ \oint_{z=0} \frac{dz}{z} \phi G^3(z) - \oint_{\bar{z}=0} \frac{d\bar{z}}{\bar{z}} \phi \frac{\sqrt{1-\bar{z}}}{\sqrt{\bar{z}}} G^2(\bar{z}) + \oint_{\bar{z}=0} \frac{d\bar{z}}{\bar{z}} \phi (1-\bar{z})^{1/2} (-1)^{3/2} G'(\bar{z}) \right\} = 0 \quad (44)$$

We note that equation (43) is the same as satisfied by V_{CSV} .

Rather than find V_1 from V_0 by explicit computation from equation (37) we could realize that substituting

$$V_1 = V_0 \sum_{r=1}^3 (G_{\frac{1}{2}}^r + G_{-\frac{1}{2}}^r) \quad (45)$$

in equation (37) reduces this identity after use of the result

$$[G_r, P^\mu(z)] = z \frac{d}{dz} (z^r \partial e^\mu(z)) \quad (46)$$

and making the identification

$$\bar{\phi} = \left(\frac{z-1}{\sqrt{z}} \right) z \frac{d\phi}{dz}. \quad (47)$$

to the identity of equation (34) and the identity satisfied by V_0 and P^μ . Since equation (37) uniquely determines V_1 we can conclude that V_1 can also be written in the form given in equation (45). This form is particularly simple to manipulate when combined with the following simple identity derived from (43) and (44):

$$V_0 [G_{-\frac{1}{2}}^1 - G_{-\frac{1}{2}}^2] = -V_0 G_{\frac{1}{2}}^3 + V_0 G_{\frac{1}{2}}^1 \quad (48)$$

from which it follows that one can write for example.

$$V_1 = 3V_0 (G_{-\frac{1}{2}}^1 + G_{\frac{1}{2}}^3). \quad (49)$$

We note the similarity of this identity with equation (4) of reference [7]. Further simple identities can be found from (42) or directly by repeated use of (48) and (49); for example:

$$V_1 G_{\frac{1}{2}}^1 + V_1 G_{\frac{1}{2}}^3 = 3 V_0 (L_0^1 + L_0^3 - L_0^2). \quad (50)$$

$$V_1 (L_{-1}^1 + L_1^3 - L_0^1 - L_0^3 + L_0^2 - \frac{1}{2}) = 0 \quad (51)$$

It now remains to show that V does indeed reproduce the correct amplitude when taken with three arbitrary multiperipheral trees made with on-shell NS tachyons. One can proceed as follows: first, using repeatedly overlap equations like (30), one can slide one after the other all the external lines (except one) of tree number 3 across V , onto tree number 1. The calculation involves techniques very similar to those of ref [7] used to prove cyclic symmetry in the " F_2 " formalism, it is straightforward but lengthy, and we shall not give it here. Once line number 3 has thus been reduced to an on-shell NS tachyon, we must apply the appropriate twist on line 2, which is given by:

$$\Omega^{(\nu)} = e^{-L_1^\nu} (-1)^{R^\nu} e^{-i \oplus G_{\frac{1}{2}}^\nu} p^\nu \quad (52)$$

where

$$p = (-1)^{(G-1)\frac{1}{2}} \text{ and } G = (-1)^{\left(\sum_{r=\frac{1}{2}}^{\infty} b_r^+ b_r - 1\right)} \quad (53)$$

The reader may verify that

$$\Omega^2 = 1 \quad (54)$$

The object we wish to study is

$$V(1, 2) = V|0\rangle_3 \Omega^{(2)}. \quad (55)$$

The reader may verify the formulae

$$e^{-\frac{1}{2}G_{-1}^{\mu}} L_+^{\mu}(z, \theta) e^{\frac{1}{2}G_{-1}^{\mu}} = L_+^{\mu}(z + z\theta\theta, \theta + \theta z) \quad (56)$$

and

$$\begin{aligned} \langle 0 | \Omega &= \langle 1 | \exp \alpha_0^{\mu} L_+^{\mu}(1, \theta) \\ [\Omega^{(2)}]^{-1} L_+^{\mu}(z, \theta) \Omega^{(2)} &= L_+^{\mu}\left(\frac{1}{z}, \frac{\theta}{z}\right) - L_+^{\mu}(1, \theta) \end{aligned} \quad (57)$$

where

$$L_+^{\mu}(z, \theta) = \varphi_+^{\mu}(z) + \frac{\theta}{\sqrt{z}} p_+^{\mu}(z). \quad (58)$$

Consequently we find that

$$\begin{aligned} V(1, 2) &= \langle 0 | \langle 2 | \exp \left(- \sum_{n=1}^{\infty} \alpha_n^{\mu 1} \frac{\alpha_n^{\mu 2}}{n} \right. \\ &\quad \left. - \sum_{r=\frac{1}{2}}^{\infty} b_r^{\mu 1} b_r^{\mu 2} - \alpha_0^{\mu 3} (L_+^{\mu}(1, \theta) + L_+^{\mu}(1, \theta)) \right) \end{aligned} \quad (59)$$

which is the desired result.

The superconformal transformation between the super-upper-half plane and superstring was discussed in reference [10]. Making this transformation on the vertex presented in this paper will yield the corresponding gauge covariant vertex provided we also include the ghost oscillators.

In this paper, we have derived the general coupling of three physical states of the Neveu-Schwarz model. As expected from past experience with this model, the algebraic structure of the vertex, in equations (34) and (39), is very similar to that of its bosonic counterpart. In future work, we shall present the application of the techniques of this paper to the coupling of two Ramond states and one Neveu-Schwarz state. This will be found to be more complicated, as expected from the form of the old fermion emission vertex.

Note added

After this work was completed we received a preprint (Nordita - 86/8; ISSN 0106-2646) by P. Di Vecchia, R. Nakayama, J.L. Petersen and S. Sciuto which also presents the expression for V_0 in equation (35).

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