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THE ROLE OF CLASSICAL SYMMETRIES IN THE
LOW-ENERGY LIMIT OF SUPERSTRING THEORIES

Hans Peter Nilles
CERN - Geneva

ABSTRACT

Due to the appearance of certain classical symmetries in the low-energy limit of superstring theories, some relevant parameters remain undetermined. We investigate the breakdown of these symmetries in the loop expansion. This then enables us to clarify which properties of the low-energy theory are artifacts of the classical approximation. Our results are relevant for the relations between gauge coupling constants and the magnitude of the supersymmetry breakdown scale.

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There are still many problems to be solved before one can make a statement about a possible connection between superstring theories and known physics in the 100 GeV range. One question is the compactification of six spatial dimensions from $d = 10$ to $d = 4$. The simplest possibility would be a (twisted) torus compactification and this seems to be consistent with all presently known constraints from the string theory. In the resulting $d = 4$ theory, one then obtains several classical scale invariances due to the fact that the radii of the torus are not fixed at the classical level and this reflects itself in a degeneracy of the vacuum state. The statement remains true also for more fancy ways of compactification, like e.g., Calabi-Yau manifolds. An additional scale invariance comes from the fact that the gauge coupling constant is related to the vacuum expectation value of the dilaton field which again, classically, remains undetermined. At the classical level, we thus cannot understand the basic parameters of the low-energy limit of the string theories such as gauge coupling constants and the compactification scale. Only through understanding the breakdown of these classical symmetries, will we be able to answer these questions. In this note, we shall investigate the breakdown of those classical scale symmetries through quantum corrections and compare our results with results obtained through strictly classical considerations. This will then enable us to clarify which properties of the classical action might survive in the full theory and separate them from those which are just artifacts of the classical considerations. Our analysis will be at the level of perturbation theory and only at the end will we comment on non-perturbative effects. We will also restrict ourselves to the framework of $N = 1$ supergravity in $d = 4$ since extended supergravities predict the absence of chiral fermions whereas the non-supersymmetric model has to face a hierarchy problem. In addition, the $N = 1$ case has a simple structure and will allow us to make non-trivial statements. They will concern the relation between gauge coupling constants and the question of supersymmetry breakdown in the observable sector. Quantum corrections will substantially change the classical results obtained in this context.

Let us begin with the classical considerations. The relevant bosonic fields can be divided into three classes: S, T and C, where we use the notation of Refs. 1) and 2), i.e.,

$$\begin{aligned} S &= \phi^{-3/4} \exp(3\sigma) + i\theta \\ T &= \phi^{3/4} \exp(\sigma) + CC^* + i\eta \end{aligned} \tag{1}$$

are gauge singlets and the C's are those fields that transform non-trivially under the gauge group (e.g., 27 of E_6). ϕ is the dilaton field and σ is the fluctuation of the overall size of the compact manifold, i.e., $\exp(\sigma)$ defines the radius of compactification in units of the Planck length. There could be several T-fields but we shall restrict our discussion to the case of only one. Including more of them is straightforward. The same applies to the C-field. The pseudo-scalar fields (hereafter called axions) have their origin in the zero modes of the antisymmetric tensor field B_{MN} . Since B only couples through its field strength, θ and η will only have derivative couplings, at least for perturbative considerations. This has important consequences. There will be two axial U(1) symmetries corresponding to a shift of S and T by an imaginary constant³⁾. Furthermore S and T cannot appear in the superpotential which is an analytic product of superfields and does not contain derivative couplings⁴⁾. Finally, we have to consider two classical scale invariances corresponding to the vacuum degeneracy discussed earlier. In $d = 4$ they read⁵⁾⁻⁷⁾

$$\begin{array}{ll} g_{\mu\nu} \longrightarrow t^4 g_{\mu\nu} & \psi_\mu \longrightarrow t \psi_\mu \\ S \longrightarrow t^4 S & \chi \longrightarrow t^{-1} \chi \\ T \longrightarrow T & \lambda \longrightarrow t^{-1} \lambda \end{array} \quad (2)$$

with the classical action scaling like

$$\mathcal{L} \longrightarrow t^4 \mathcal{L} \quad (3)$$

and

$$\begin{array}{ll} S \longrightarrow r^{-1/2} S & \psi_\mu \longrightarrow r^{-1/4} \psi_\mu \\ T \longrightarrow r^{1/2} T & \chi \longrightarrow r^{-1/4} \chi \\ C \longrightarrow r^{1/4} C & \lambda \longrightarrow r^{-1/4} \lambda \end{array} \quad (4)$$

with

$$\mathcal{L} \longrightarrow r^{-1/2} \mathcal{L} \quad (5)$$

These four symmetries give strong constraints for the $d = 4$ $N = 1$ supergravity Lagrangian⁶⁾. The general Lagrangian including terms up to two derivatives is

defined by two functions: the gauge kinetic function $f(\phi_i)$ which is analytic in the left-handed superfields ϕ_i , and the Kähler potential

$$G(\phi_i, \phi_i^*) = K(\phi_i, \phi_i^*) + \log |W(\phi_i)|^2 \quad (6)$$

where $W(\phi_i)$ is the superpotential. We limit ourselves here to the action up to two derivatives because they are the leading terms in an expansion with respect to the slope parameter α' and the dominant terms at low energies. New terms that appear in α' perturbation theory are not expected to break the considered symmetries, which can be verified in first order. This seems, however, no longer to be true for non-perturbative effects which we shall discuss later. The easiest way²⁾ to extract f and G from an action given in component fields is given by the gauge kinetic terms

$$e_4 (\text{Re} f F_{\mu\nu} F^{\mu\nu} + \text{Im} f \epsilon^{\mu\nu\rho\sigma} F_{\mu\nu} F_{\rho\sigma}) \quad (7)$$

and the gravitino mass term

$$e_4 \exp(G/2) \bar{\psi}_\mu \gamma^{\mu\nu} \gamma_5 \psi_\nu \quad (8)$$

The transformations (2)-(5) then imply

$$\begin{aligned} \text{Re} f &\longrightarrow t^4 r^{-1/2} \text{Re} f \\ \exp(G/2) &\longrightarrow t^{-2} r^{1/4} \exp(G/2) \end{aligned} \quad (9)$$

In addition, we deduce from the axial symmetries that the imaginary parts of S and T are unphysical and should thus not appear in $\text{Re} f$ and G . This then leads to⁶⁾

$$f = S \quad (10)$$

and

$$\begin{aligned} G = & -\log(S+S^*) - n \log(T+T^* - 2|C|^2) + \\ & + \log |W|^2 + \bar{K} \left(\frac{C}{\sqrt{T+T^*}}, \frac{C^*}{\sqrt{T+T^*}} \right) \end{aligned} \quad (11)$$

with an arbitrary function $\bar{K}$. The symmetries imply that the superpotential W

only depends on the C-fields and that it is homogeneous of degree n . To fix n , symmetry considerations are not sufficient. But since W comes from the trilinear part⁸⁾ of the $d = 10$ Chern-Simons term, we know that $n = 3$. The reason for f and W to be restricted so strongly by symmetry considerations comes from the fact that these functions are holomorphic in the left-handed superfields, which is not true for G . In absence of S , (11) gives the most general G that leads to a potential with flat directions⁹⁾. Including S this statement might also be true, but needs to be clarified. For $\bar{K} = 0$, one obtains a so-called no scale model where the scalar kinetic terms exhibit $SU(1,1) \times SU(n,1)$ symmetry¹⁰⁾. These big symmetries are, however, broken by the presence of interactions and the only relevant symmetries are those discussed earlier. Examples indicate that the case $\bar{K} = 0$ corresponds to orbifold compactification. More complicated compact spaces, like e.g., Calabi-Yau spaces, will lead to non-trivial $\bar{K}$. In any case $\bar{K}$ will receive non-trivial contributions in α' perturbation theory¹¹⁾.

One could now try to extract some information from (11), keeping in mind that such an approximation would not allow us to determine the compactification scale. If we just consider the truncated theory, we treat the limit $M_c \rightarrow \infty$ where M_c is the compactification scale, inversely proportional to $R_c \sim \exp(\sigma)$. On the other hand, R_c will be determined by the vacuum expectation values of S and T . The consistency of the approximation therefore requires compatibility with $M_c \rightarrow \infty$, i.e., $T \rightarrow 0$. This is certainly true for the classical expression due to the vacuum degeneracy. It remains true even after including one-loop radiative correction. The effective potential becomes, however, unbounded from below at $T = 0$, in this truncated theory^{12),13)}. This just tells us that the massive modes will be important to understand the compactification scale. The truncated no scale model as a $d = 4$ field theory is just inconsistent and can only be regarded as an approximation to a more complete theory.

A discussion of the question of supersymmetry breakdown is also influenced by these vacuum degeneracies at the classical level. Given the gauge coupling constant, one would have SUSY breakdown by hidden sector gaugino condensation and definite predictions for the parameters in the low-energy theory⁸⁾. Since the gauge coupling constant is not determined in the classical approximation, the supersymmetry breakdown scale is undetermined¹⁴⁾. Moreover in this special case, the observable sector remains completely supersymmetric. This fact is at the basis of models with gravitino mass equal to the Planck mass and nonetheless observable sector mass splittings in the TeV range¹⁰⁾. The classical model, however, is inconsistent and one has to worry whether such things can happen. This accidental supersymmetry in the observable sector crucially depends on the

form of the f -function²⁾ and is only true for the classical result $f = S$. Any deviation from this form would lead to a transmission of SUSY breakdown to the observable sector with a scale that is not small compared to the gravitino mass. This situation is actually expected in general if one argues in the $d = 10$ theory. In principle, we could imagine compactifications, which lead to $N = 4$ supergravity in $d = 4$. For the reasons explained above we are, however, interested to obtain $N = 1$ supergravity, i.e., three of the four gravitini become massive at the Planck scale. The last one should have a smaller mass to be relevant at low energies.

We have seen that the classical symmetries prevent us from understanding some relevant parameters in the low-energy limit of the string. We thus have to understand how these symmetries are broken. While the axial symmetries are expected to survive the perturbative expansion and could possibly only be broken by non-perturbative effects, the situation for the symmetries (2)-(5) is different. Already at the classical level, the action gets multiplied by a constant indicating a breakdown at the quantum level. We can actually be more specific about the way this will happen⁷⁾ in the loop expansion since the new counterterms will behave homogeneous loop by loop. In this context, the loop expansion parameter can be identified with ST^{-1} such that the new counterterms in the n -loop effective action will scale as

$$\mathcal{L}_{n-loop} \longrightarrow t^{4(1-n)} r^{n-1/2} \mathcal{L}_{n-loop} \quad (12)$$

Thus the classical results for f and G as given in (10) and (11) will be modified in perturbation theory. For the behaviour of f and G (12) implies

$$\begin{aligned} f_n &\longrightarrow t^{4(1-n)} r^{n-1/2} f_n \\ \exp(G/2)_n &\longrightarrow t^{-2(1+2n)} r^{n+1/4} \exp(G/2)_n \end{aligned} \quad (13)$$

as can be derived from (7) and (8) using the transformation properties (2) and (4). Let us first discuss $f = \sum f_n$ with $f_0 = S$, the classical result. At one loop, $f_1 \rightarrow t^0 r^{1/2} f_1$ and we see that S cannot appear since it is the only scalar particle that transforms non-trivially under (2). f_1 has the transformation behaviour as T and C^2 possibly multiplied by a function that is inert under (2) and (4). The axial symmetry implies that the real part of f should be independent of the imaginary part of T and f should be an analytic function of the

fields. This implies $f_1 = aT + bC^2$ where a, b are constants that cannot be determined by the symmetry consideration. At higher loops, f_n scales with an inverse power of t and possible counterterms have to scale with S^{-n} . The axial symmetries do not allow such terms and we obtain a non-renormalization theorem for f beyond one loop, i.e., its final form is given by

$$f = S + \epsilon (T + \alpha C^2) \quad (14)$$

Observe that this non-renormalization theorem is not in contradiction with the running of the coupling constant. The coupling constant is not given directly by f but by its vacuum expectation value which is determined from the minimum of the potential and this receives non-analytic pieces even if f is unrenormalized.

Nonetheless f receives new contributions at one loop and this leads to significant physical consequences:

- i) Unlike the classical case, T now couples to the gauge field strength. In particular, this implies that the pseudoscalar η couples as an axion. An investigation of the Calabi-Yau compactification scheme shows furthermore^{2),15),17)} that η couples differently to hidden and observable sector and thus the axion pair θ and η could solve the strong CP problem of E_8' and QCD. Here, however, it has still to be checked whether the corresponding axial symmetries are only broken by gauge instantons.
- ii) As well as the axions, the scalars in S and T do now couple differently to the hidden and observable sector gauge fields²⁾. Since the corresponding gauge coupling constants are determined by the VEVs of S and T , they can now be different from each other, contrary to the classical case. This might have consequences for the condensation scale of E_8' and the magnitude of supersymmetry breakdown.
- iii) There exist now two axion-dilaton pairs, and this might generalize the mechanism of relaxation of the cosmological constant to the observable sector in the same way as happens in the hidden sector¹⁵⁾.
- iv) The new terms in (14) lead to an induced breakdown of supersymmetry in the observable sector once it is broken in the hidden sector, with magnitude $\epsilon m_{3/2}$. A gravitino mass of order of the Planck mass thus will not lead to traces of supersymmetry in the low-energy region.

Most of these consequences had been deduced already by inspection^{2),3),7),15)-21)} of the anomaly cancellation terms. This interpretation had been heavily rejected by arguments which are hard to understand²²⁾. The symmetry considerations presented here clarify this situation.

While f remained simple including the loop expansion this is not expected for G because of its non-analyticity. We obtain

$$\begin{aligned} G = & -\log(S+S^*) - 3\log(T+T^* - 2|C|^2) + \\ & + \log|W|^2 + 2\log\left(1 + \sum_{n=1}^{\infty} a_n \epsilon^n \left[\frac{S+S^*}{T+T^*}\right]^n\right) \\ & + \bar{K}\left(\frac{C}{\sqrt{T+T^*}}, \frac{C^*}{\sqrt{T+T^*}}\right) \end{aligned} \quad (15)$$

where ϵ is the expansion parameter and a_n are numerical coefficients. Because of its complexity, it is not clear yet what this implies. Discussed in the context of unbroken supersymmetry, one might conjecture this to be the most general G that leads to a potential with two flat directions. While this has been checked at the one-loop level¹⁸⁾, it has still to be clarified at higher loops. The inclusion of supersymmetry breakdown would change the situation. With this the value of S as well as T at the minimum could then be fixed and, if nothing else happens, this would imply that the gauge coupling constant can only be understood once supersymmetry is broken. If we could handle the complexity of (15), some of these questions might be clarified.

In any case we also have to worry about different mechanisms that could break the classical symmetries and which are of non-perturbative nature. Effects of gauge instantons are believed to be responsible for the breakdown of supersymmetry in the hidden sector^{8),14)}. QCD-instantons might be the source of the breakdown of the second axial symmetry and solve the strong CP-problem via the axion mechanism, barring some cosmological problems. Superficially the influence of such instanton contributions can be described through terms like $\exp(-S)$ or $\exp(-T)$ in the superpotential. They could even provide a mechanism for a relaxation of the cosmological constant in a similar way as the axion relaxes the θ angle, but an explicit discussion of such a mechanism would require a better understanding of the effective potential¹⁵⁾.

The influence of worldsheet non-perturbative effects could also be significant^{19),20),23),24)}. Worldsheet instantons could be responsible for the breakdown of the axial symmetries corresponding to the T-fields. This cannot happen for S since its pseudoscalar originates from the antisymmetric tensor with both indices in $d = 4$ Minkowski space. The breakdown of these axial symmetries depends strongly on special properties of the six-dimensional compact manifold²⁰⁾ and a general statement cannot be made. For some manifolds, these symmetries are already (partially) broken at the lowest order in the loop expansion. In the same way as the gauge instantons such effects might give contributions to the superpotential like $\exp(-T)$. If all the T-fields would receive such a contribution, one would expect the corresponding pseudoscalars to become heavy and no longer be candidates to solve the strong CP problem. But this depends strongly on compactification and some of the T-fields might not be affected by this mechanism. It could also happen that such contributions could destabilize the classical vacuum in an undesired way. But we have to keep in mind that such terms are not basically different from those terms that appear as a consequence of gauge instantons. Whether such terms are potentially dangerous or not can only be decided after an inspection of the scalar potential. In any case we need a destabilization of the classical vacuum degeneracy to understand the relevant parameters of the low-energy limit, and it seems to us that it has to be a combination of loop effects and worldsheet effects that could lead to progress in this direction.

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