References


A.V. Bäcklund, Über Curven und Flachentransformationen. Lund Univ. Arsskr. 10, 1 (1875)

A.V. Bäcklund, Om ytor med konstant negativ krökning. Lund Univ. Arsskr. 19, 1 (1883)

References


G. Bennettin, L. Galgani, A comment on the reliability of the Toda criterion for the existence of a stochastic transition. Physica A 87, 381 (1977)


G.D. Birkhoff, Dynamical Systems (American Mathematical Society, Providence, 1927)


J.D. Cole, On a quasilinear parabolic equation occurring in aerodynamics. Q. Appl. Math. 9, 225 (1951)

P. Collet, J.P. Eckmann, Iterated Maps of the Unit Interval as Dynamical Systems (Birkhäuser, Basel, 1980)

H.C. Corben, P. Stehle, Classical Mechanics (Krieger, Melbourne, 1974)

References

P. Debye, Vortrage über die Kinetische Theorie der Materie und der Elektrizitat (Teubner, Leipzig, 1914)
R.L. Devaney, An Introduction to Chaotic Dynamical Systems (Addison-Wesley, Reading, 1989)
L.P. Eisenhart, A Treatise on the Differential Geometry of Curves and Surfaces (Ginn, Needham Heights, 1901), p. 190 (also Dover, 1960)
G. Falkovich, K. Gawedzki, M. Vergassola, Particles and fields in fluid turbulence. Rev. Mod. Phys. 73, 913 (2001)
J. Feder, Fractals (Plenum, New York, 1988)
References


J. Froyland, Introduction to Chaos and Coherence (Institute of Physics Publications, Bristol, 1992)


P. Garabedian, Partial Differential Equations (Chelsea, New York, 1984)


I.M. Gel’fand, B.M. Levitan, On the determination of a differential equation from its spectral function. Am. Math. Soc. Transl. (2) 1, 253 (1955)


H. Goldstein, C. Poole, J. Safko, Classical Mechanics, 3rd edn. (Pearson, Upper Saddle River, 2002), Chap. 10

V.V. Golubov, Lectures on Integration of the Equations of Motion of a Rigid Body About a Fixed Point (State Publishing House, Moscow, 1953)


J. Guckenheimer, P. Holmes, Nonlinear Oscillations, Dynamical Systems, and Bifurcations of Vector Fields (Springer, Berlin, 1986)


P. Hagedorn, Nonlinear Oscillations (Oxford University Press, London, 1988)

J. Hale, H. Kocák, Dynamics and Bifurcations (Springer, Berlin, 1991)


M. Hénon, C. Heiles, The applicability of the third integral of motion: some numerical experiments. Astron. J. 69, 73 (1964)


References


E. Hopf, The partial differential equation $u_t + uu_x = \mu u_{xx}$. Commun. Pure Appl. Math. 3, 201 (1950)


C.L. Ince, Ordinary Differential Equations (Dover, New York, 1956)


A. Jeffrey, T. Kakutani, Weak nonlinear dispersive waves, a discussion centered around the Korteweg-de Vries equation. SIAM Rev. 14, 582 (1972)


L. Kadanoff, Scaling laws for Ising models near $T_c$. Physics 2, 263 (1966)


A.N. Kolmogorov, The local structure of turbulence in incompressible viscous fluid at very large Reynolds numbers. C. R. Acad. Sci. USSR 30, 299 (1941)


A.N. Kolmogorov, A refinement of previous hypotheses concerning the local structure of turbulence in a viscous incompressible fluid at high Reynolds numbers. J. Fluid Mech. 13, 82 (1962)

D.J. Korteweg, G. de Vries, On the change of form of long waves advancing in a rectangular canal and on a new type of long stationary waves. Philos. Mag. (5) 39, 422 (1895)

S. Kovaleskaya, Sur le problème de la rotation d’un corps solide d’un point fixe. Acta Math. 12, 177 (1899)


References


E.N. Lorenz, Deterministic nonperiodic flow. J. Atmos. Sci. 20, 130 (1963)

S.K. Ma, Modern Theory of Critical Phenomena (Benjamin/Cummings, Redwood City, 1976)


B. Mandelbrot, The Fractal Geometry of Nature (Freeman, New York, 1983)

B. Mandelbrot, Multifractal measures, in Fractals in Geophysics, ed. by C.H. Scholz, B. Mandelbrot (Birkhäuser, Basel, 1989)


J.C. Maxwell, Matter and Motion (Society for Promoting Christian knowledge, London, 1920)


H. Poincaré, Les Methods Nouvelles de la Mecanique Celeste (Gauthier-Villars, Paris, 1892)


J.W.S. Rayleigh, On maintained vibrations. Philos. Mag. 15, 229 (1883)

J.W.S. Rayleigh, On convection currents in a horizontal layer of fluid, when the higher temperature is on the underside. Philos. Mag. 32, 529 (1916)


B.K. Shivamoggi, Multifractal aspects of the fine-scale structure of temperature fluctuations in isotropic turbulence. Physica A 221, 460 (1995a)


References

S.H. Strogatz, Nonlinear Dynamics and Chaos (Addison-Wesley, Reading, 1994)
A.A. Tsonis, Chaos (Plenum, New York, 1992)
F. Verhulst, Discrete symmetric dynamical systems at the main resonances. Philos. Trans. R. Soc. A 290, 435 (1979)


Index

**A**
- Action-angle variables, 89, 100, 110, 114, 128, 132, 134, 349
- Amplitude-frequency relation, 41
- Anharmonic lattice, 246, 247
- Area-preserving mapping, 126, 150, 155, 156, 158, 178, 182
  - tangent map, 160, 172, 351
  - twist map, 158, 159
- Arnol’d cat map, 178
- Arnol’d diffusion, 101
- Arnol’d web, 101
- Asymmetric spinning top, xviii
- Asymptotic stability, 13, 15–17, 170

**Attractor**
- basin of attraction, 101, 172, 191, 192, 201
- Liapunov exponent, 170, 171, 173, 184, 193
- Lorentz attractor, 219
- limit cycle, 170, 192, 218, 224
- Poincaré-Bendixson theorem, 192
- strange, xxiii, xxvii, 189, 192–194, 213, 224, 323

**Autonomous systems**, 9
- Auto-Bäcklund transformation, 297, 298, 314–316, 357
- Auto-correlation function, 170, 175–177, 211

**B**
- Bäcklund transformation, 294, 295, 297, 298, 312
- Baker’s transformation, 352
- Basin of attraction, xxi, xxvii, 101, 172, 191, 192, 201
- Bernoulli shift, 183, 233, 234, 352
- Bianchi’s Theorem of Permutability, 298

**Bifurcations**, xxiii, 45, 228, 237
- breaking, 63
- homoclinic, 218
- Hopf, xxiii, 48, 61, 63
- one-dimensional maps, 63, 65, 231
- period-doubling, 65, 220, 223, 227, 228, 230, 237
- saddle-node, 45, 48, 52, 53, 63, 237
- tangent, 237, 238
- transcritical, 46, 48, 50, 52–54, 63, 64

**Binary sequences**, 182
- Boussinesq equation, 353, 354

**Breakdown of integrability**, 123, 150
- global criteria, 128
- local criteria, 123
- magnetic-island overlap, 131
- resonance-overlap, 128

**Broken symmetry**, 343
- Brownian motion, xviii

**Burgers equation**, 265, 313

**C**
- Canonical perturbation theory, 107, 110, 350
- Canonical transformations, 81, 83, 92, 95
  - infinitesimal, 92–95
- Cantor’s set, 198–200, 219, 239, 323, 328
- Center, 23, 30
- Center Manifold Theorem, 16, 18, 19

**Chaos**
- chaotic advection, 187

**Chaos**
- conservative systems, xix, 149
- control, xvii, xxiv, xxvi
- chaotic advection, 187
- dissipative systems, 189, 190

Chaos (cont.)
  fractal, xxvii, 195, 207, 239
  intermittency, xxvi, 238
  Kolmogorov entropy, 170, 173, 174
  Lagrangian chaos, 187
  Liapunov exponent, xxvii, 170, 193, 194, 219, 235
  logistic map, 224, 225, 230, 232–238
  Lorenz equations, 212, 213, 221
  period doubling, xxiv, 224
  renormalization group, xxvii, 229
  stretching and folding, xxii, 169, 181, 193, 194, 219, 235
  universality, xxvii, 229, 241

Chaotic advection, 343

Chirikov map, 159

Cnoidal waves, 258, 260, 263

Cole-Hopf transformation, 265, 295, 315

Conservation laws, 246, 287–289, 342

Conservative systems, xxi, 149

Continued fractions, 119

Contractive map, 190

Correlation dimension, 207, 210, 211

Couette flow, xxiii

Critical exponents, 229, 342, 343, 345, 346

Critical phenomena, xxvi, 342, 343, 346

Critical point, 342

D

Deterministic systems, xviii, 182, 184

Diffeomorphism, 114, 151, 224

Direct-scattering problem, 271, 279, 283, 284, 355, 356

Dissipative systems, xix, xxi–xxiii, xxvii, 189, 190, 192, 193

Duffing’s equation, 15, 30, 38
  forced, 38, 42, 43

Dynamical systems, 2, 6, 174, 177, 178, 189, 191, 193, 194, 210, 238, 288, 301, 355
  Center Manifold Theorem, 16, 18
  conservative systems, xix, 149
  dissipative systems, xxi, 189, 190
  equilibrium points, 1
  Hartman-Grobman Theorem, 17, 45
  phase-plane analysis, 33
  Poincaré-Bendixon Theorem, 37
  stability, 13

E

Energy shell, 100–102

Ensemble average, 326

Equilibrium point, 6, 9
  elliptic, 23, 33
  focus, 25, 35, 46, 62, 63
  hyperbolic, 17, 29
  node, 28, 45
  supercritical, 52

Ergodic system, 178, 186

F

Fermi-Pasta-Ulam recurrence, 245

Fibonacci numbers, 122

Fixed point
  elliptic, xx, 156, 159, 161–166
  hyperbolic, xx, 156, 161–164, 166, 167, 180, 181
  parabolic, 161

Fixed-Point Theorem, 163

Fractal, xxiii, 195, 197, 202, 239, 323, 324
  box-counting method, 201
  Cantor set, xxii, 198–200, 219, 239, 323, 328
  capacity dimension, 196, 240
  correlation dimension, 207
  dimension, xxvii, 195–202, 204, 205, 219, 324, 327, 334, 351
  entropy, xxvii, 203
  generalized fractal dimension, 202–204, 207, 324, 332, 333, 339, 340, 345
  Hausdorff dimension, 196, 232, 328, 330, 331
  information dimension, 207
  Koch’s snowflake, 199, 201
  Lorenz attractor, 219
  multi-fractals, 202, 324
  non-uniform fractals, 197, 207, 208
  Sierpinski triangle, 200, 201
  turbulence, 323, 324, 327, 335, 342, 343, 345

Frequency locking, 106

Frobenius-Perron equation, 232

G

Gardner’s transformation, 288

Gelfand-Levitan-Marchenko equation, 274

Goursat problem, 275

H

Hamiltonian dynamics, 69
  action-angle variables, 89
  area-preserving mapping, 126, 150, 155, 156, 158, 178, 182
  canonical perturbation theory, 107, 110
  canonical transformations, 81, 83, 92, 95
  Hamilton-Jacobi equation, 86–89, 91
  Hamiltonian, 74, 77, 81
  Hamilton’s equations, 70, 72, 79
Hamiltonian dynamics (cont.)

Hamilton's principal function, 70
Hamilton's principle, 69
infinitesimal canonical transformations, 92, 95
Lagrange's equations, 72, 83
Lagrangian, 69–71, 73, 82
Legendre transformation, 72
Liouville’s Theorem, 79, 149, 150
Noether’s Theorem, 73
Poincaré’s Recurrence Theorem, 80
Poisson's brackets, 93–96
separable systems, 98
symplectic structure, 69
Harmonic oscillator, 22, 24, 76, 83, 88, 91, 114
Hartman-Grobman Theorem, 17
Hausdorff dimension, 169, 196, 323, 328–331, 336
Hénon map, 164
Hénon-Heiles system, 309
Heteroclinic points, 166, 169, 181
Hirota’s transformation, 265, 278
Homeomorphism, 17
Homoclinic orbits, xx, 166, 218, 264
Hopf bifurcation, xxiv, 46, 57
Horseshoe, 169, 193–195
Hysteresis, 41

I
 Implicit Function Theorem, 46, 47, 49–51, 53–55, 58
Infinitesimal canonical transformations, 92, 95
Information dimension, 207, 211
Integrable systems, 97, 99, 100, 104, 106, 107, 110, 152, 157, 166, 167, 301
breakdown of integrability, 123
canonical perturbation theory, 107, 110
invariant tori, xx, 97, 100, 102, 114, 115, 118, 128, 152, 156, 166
Kolmogorov-Arnol’d Moser Theorem, xx, 114, 118
Painlevé property, x, 301, 302
solitons, 245, 269, 284, 286, 287, 298, 299
surface of section, 151, 152, 157
Interacting solitary waves, 265, 268
Intermittency, xxvi, 238, 318, 321–324, 326–330, 334, 336, 343
Internal resonances, 116, 136, 140, 143, 144
Hénon-Heiles system, 309
higher-order resonances, 138, 143
Invariant manifold, 166
Invariant measure, 101, 179, 185, 186, 191
Invariant probability distribution, xviii, 101
Invariant tori, xix, xx, 97, 100, 102, 114, 115, 118, 128, 150, 152, 156, 161, 164, 166
winding number, 114, 152, 161, 162
Inverse-scattering problem, 271, 276, 282
Inverse-scattering transform, 246
Inverse-scattering transformation, 269, 271, 290
Inviscid dissipation of energy, 327, 335, 343, 345
Ion-acoustic waves, 246, 251
Iso-energetic non-degenerate system, 157

J
 Jump phenomena, 1, 41

K
 Kaplan-Yorke conjecture, 206
Koch’s snowflake, 199, 201
Kolmogorov entropy, 170, 173–175
Kolmogorov microscale, 326, 329, 335, 341, 343
Kolmogorov theory of local similarity of turbulence, 317
Kolmogorov-Arnol’d Moser Theorem, xx, 114, 118
Korteweg-de Vries equation, 246, 248, 251, 253, 255, 257, 260, 263, 265, 269, 270, 272, 274, 288, 289, 293, 296, 304, 315

L
 Lagrange’s equation, 73
Lagrangian chaos, 187
Landau equation, 217
Lax pair, 290, 291, 294, 312, 314, 316
Legendre transform, 203, 204, 332
Liapunov exponent, xxvii, 170, 171, 173, 175, 180, 182, 184, 193, 231, 233, 234
Liapunov function, 15
Liapunov stability, 13, 15
Limit cycle, xxi, 35–37, 46, 62, 170, 192, 218, 224
Linear difference equation, 220
Liouville equation, 295
Liouville’s Theorem, 79
Lipschitz property, 3
Logistic map, 223, 232–234, 238
Bernoulli shift, 233, 234, 352
chaos, 233, 237
intermittency, 238
invariant probability distribution, 101
Liapunov exponents, 231, 233–235
odd-period cycles, 224
period-doubling bifurcations, 220
Logistic map (cont.)
renormalization group, 229
saddle-node bifurcation, 237
stretching and folding, 235
universality, 229
Lorenz equations, 212, 213, 221
chaotic behavior, 213
homoclinic bifurcation, 218
Lorenz “map”, 220, 221
subcritical bifurcation, 52
supercritical bifurcation, 215

M
Magnetic confinement, 131
Magnetic-island overlap, 131
Manifold, 19, 20, 150, 192
center, 18–22, 45, 46, 60, 166, 215, 217
stable, 17–20, 45, 166–168, 214
unstable, 18–20, 166, 168, 214
Many-body problem, 110
Markov process, 185, 232
Master equation, 185, 232
Mean field theory, 343
Melnikov’s method, 168
Micro-canonical ensemble, 101
Miura’s transformation, 296
Mixing, 178–182, 184, 187, 201, 328
Modified Korteweg-de Vries equation, 289, 296
Multi-fractals, 202, 324
correlation dimension, 207
generalized fractal dimension, 203, 204, 324, 333, 339, 340, 345
information dimension, 207
Kaplan-Yorke conjecture, 206
singularity spectrum, 202, 204, 324, 331, 334
turbulence, 317, 318, 321, 329, 335, 345

N
N-soliton solution, 298, 299
Natural measure, 101, 172, 197, 207
Newton’s method, 191, 192
Noether’s Theorem, 73
Non-autonomous system, 37
Non-degenerate system, 107, 118
Non-integrable systems, 97, 107, 114, 167
Nonlinear differential equations, x
amplitude-frequency relation, 41
chaotic behavior, 9
hysteresis, 41
jump phenomena, 41
subcritical instability, 35, 52

subharmonic resonance, 43
supercritical equilibrium, 36
Nonlinear Schrödinger equation, 358
Normal form, 59, 66
Nyquist critical frequency, 176

O
Order parameter, 343
Osledec’s Multiplicative Ergodic Theorem, 171

P
Painlevé property, 301, 302, 305
auto-Bäcklund transformation, 314–316
Burgers equation, 313
Cole-Hopf transformation, 315
Hénon-Heiles system, 309
integrable systems, 301
Korteweg-de Vries equation, 246, 304, 315
Lax pair, 312, 314, 316
Painlevé transcendent, 303–305
partial differential equations, 312, 314
singularity analysis, 301, 305
Zakharov-Kuznetsov equation, 359
Partition function, 203
Peano’s space-filling curve, xxii
Period doubling bifurcations, xxiv, 224, 231
Phase space, xix–xxii, xxvii, 32, 76, 79, 118
average, 101, 173, 174, 201
conservative systems, 149
dissipative systems, 189, 190
reconstruction, 211
symplectic structure, 69
Phase transitions, xxvi
Phase-plane analysis, 22
Poincaré-Bendixson Theorem, 37, 192
Poincaré-Birkhoff Fixed Point Theorem, xx, 161, 162
Poincaré-Hopf Theorem, 100
Poincaré’s Recurrence Theorem, 80, 81, 149, 190
Poisson’s brackets, 96
Power spectra, 170, 176
Predictability horizon, xxii, 173

Q
Quadratically irrational number, 120
Quasi-periodic motion, xxii, 100, 164, 170, 175, 177, 180

R
Random-β model, 335
Random-walk process, 186
Rayleigh oscillator, 212
Index

Rayleigh-Bénard convection, xxv, xxvi, 213
Recurrence, 69, 80, 81, 246
Reflectionless potentials, 281, 282, 284
Renormalization group, xix, xxiii, 229, 342
Resonance, xx
  overlap, xx, 128, 129, 131, 169
  subharmonic, 43
Ruelle-Takens scenario, xxiii

S
Sampling theorem, 176
Scalar diffusion, 359
Scale invariance, 195, 199, 202, 229, 324, 325, 327, 343
Shallow water waves, 246, 248
Shannon’s information theory, 173
Sierpinski carpet, 328
Sierpinski triangle, 200, 201
Similarity transformation, 255
Simple pendulum, xxi, 31, 32, 77, 113, 118, 130, 166
Sine-Gordon equation, 357
Singular point, 302, 305
Singularity spectrum, 202, 204, 324, 331, 332, 334, 335
Solitary waves, 257, 259, 265, 268, 269
Solitons, 245, 269, 284, 286, 287, 298, 299
Spherical pendulum, 73–75
Stability
  asymptotic, 13, 15–17, 170
  Liapunov, 13, 15
  structural, 13
  subcritical, 35, 52
Stokes waves, 256
Surface of section, xx, 32, 128, 150–152, 155, 157, 158, 161, 162, 238
Symplectic structure, 69

energy dissipation, 317, 321–331, 333–335, 343
  flatness factor, 318
  fractal dimension, 324, 327, 333, 334, 345
  fractals, 327, 328, 336
  gamma distribution, 323
  generalized fractal dimension, 324, 333, 339, 340, 345
  intermittency, 318, 321–324, 326–330, 334, 336, 343
  inviscid dissipation of energy, 327, 335, 343, 345
  Kolmogorov microscale, 326, 329, 335, 341, 343
  Kolmogorov refined similarity hypothesis, 334
  Kolmogorov theory of local similarity, 317, 321
  logarithmic normal distribution, 322
  multi-fractals, 317, 324
  multi-scaling behavior, 341, 359
  probability distribution function, 325
  random-β model, 335, 337–340
  Reynolds number, 317, 318, 321, 322, 325, 326, 340, 342, 344
  Richardson cascade, 329
  Ruelle-Takens scenario, xxiii
  scale invariance, 324, 325, 327, 343
  singularity spectrum, 324, 331, 335
  skewness, 319
  statistically universal state, 317, 318
  transition to the dissipation range, 340
  universality, 342
  velocity structure function, 323, 324, 327, 332
  vorticity field, 320
Twist map, 158, 159, 162, 163

U
Universality in chaos, xix, 229
Universality in turbulence, 342

V
Van der Pol oscillator, 348

W
Wiener-Khinchin Theorem, 177

Z
Zakharov-Kuznetsov equation, 359