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AN INTRODUCTION TO COVARIANT SUPERSTRING FIELD THEORIES^{*)}

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A B S T R A C T

A pedagogical account is given of the recent developments in covariant (super)string field theories. After reviewing the construction of the free open bosonic string field theory, the supersymmetric case is treated and the space-time supersymmetry transformations are explained. The present status and some future problems are also summarized.

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Superstring theories¹⁾ have many enthusiastic advocates, a fair number of sceptics and sizeable antagonists. This situation stems from the fact that on the one hand string theories exhibit more and more remarkable mathematical structures and yet, on the other hand, neither the deep fundamental principle, equal in elegance and power to the principle of general covariance for general relativity, nor a reliable approximation scheme to make contact with physics has been found. Obviously, much work needs to be done before we can turn the sceptics into advocates.

There are several complementary approaches to deepen our understanding of string theories, which can be roughly divided into two groups. One category based on the first quantization includes the Polyakov's path integral formulation²⁾ and the conformal field theory approach³⁾, which are intimately connected with the theory of (super-) Riemann surfaces. Also to be included in this category is the study of non-linear σ models appropriate for curved backgrounds⁴⁾. The other category is the second-quantized or the field-theory formulation⁵⁾. Especially promising is the recently developed version in which the gauge symmetries and the Lorentz covariance are made manifest⁶⁾⁻²⁹⁾. This will be the subject of our subsequent discussions.

FREE OPEN BOSONIC STRING FIELD THEORY

Let us first recall³⁰⁾ that the conventional first-quantized formulation of a (free) string, in the so-called orthonormal gauge, consists in two physical state conditions, expressing the residual conformal invariance (with quantum anomaly). They read

$$(L_0 - \alpha_0) | \text{Phys} \rangle = 0 \quad (\text{on-shell}) , \quad (1a)$$

$$L_n | \text{phys} \rangle = 0 \quad , \quad n \gg 1 \quad (\text{gauge condition}) , \quad (1b)$$

where L_n 's are the Virasoro operators and α_0 , which is 1 for the bosonic string, is the "intercept" parameter in the dual model language.

The procedure to go to the second-quantized (or field theory) formulation is a simple extension of the usual one for a point particle. Instead of a set of one-particle wave functions $\{\psi_n(x)\}$, we take a set of one-string wave-functionals

$\{\Psi_N[x^\mu(\sigma)]\}$ and form a field

$$\bar{\Psi}[x(\sigma)] = \sum_N \bar{\Psi}_N[x(\sigma)] A_N^\dagger + \text{h.c.} , \quad (2)$$

where $A_N^\dagger$ is the creation operator for the state N . It is often convenient to represent the internal part of the motion by the oscillator Fock space and expand $\Psi[x(\sigma)]$ as

$$\bar{\Psi} = [\phi(x) + \alpha_{-1}^\mu A_\mu(x) + \alpha_{-1}^\mu \alpha_{-1}^\nu B_{\mu\nu}(x) + \dots] |0\rangle . \quad (3)$$

$\phi(x)$, $A_\mu(x)$, etc., which depend only on the centre-of-mass co-ordinate x^μ , will become field operators upon second quantization. Now the two conditions, (1a) and (1b), become

$$(L_0 - 1)\bar{\Psi} = 0 \quad \Rightarrow \quad (\Box A_\mu = 0) , \quad (4a)$$

$$L_n \bar{\Psi} = 0 \quad \Rightarrow \quad (\partial^\mu A_\mu = 0) , \quad (4b)$$

where, in parentheses, we indicated the part relevant to the familiar photon field contained in Ψ for pedagogy. The generalization of the gauge transformation, $\delta A_\mu = \partial_\mu \Lambda$, for Ψ reads

$$\delta \bar{\Psi} = \sum_{n \geq 1} L_{-n} \Lambda_n , \quad (5)$$

with Λ_n 's the gauge parameter functionals.

It is well known that the above "old" formalism is not manifestly Lorentz-covariant since, e.g., in calculating loop amplitudes, one must supplement it with the transverse projection operator³¹⁾. To achieve manifest Lorentz-covariance, one must go to the BRST formalism³²⁾ by introducing a set of Faddeev-Popov ghosts and anti-ghost oscillators, c_n and $\bar{c}_n$ respectively, which satisfy the anticommutation relations $\{c_n, \bar{c}_{-m}\} = \delta_{n,m}$, $n, m \in \mathbb{Z}$. They have the hermiticity property $c_n^\dagger = c_{-n}$, $\bar{c}_n^\dagger = \bar{c}_{-n}$. In this formalism, conditions (1a) and (1b) are replaced by the condition

$$Q |phys\rangle = 0 , \quad (6)$$

where Q is the BRST charge operator. By making the dependence on the ghost zero modes manifest, it can be written as^{7),33)}:

$$Q = c_0 K - \bar{c}_0 T_+ + Q_+ , \quad (7)$$

where K , T_+ and Q_+ are given by

$$K = L_0 - \alpha_0 + \sum_{n \geq 1} n (c_{-n} \bar{c}_n + \bar{c}_{-n} c_n) , \quad (8a)$$

$$T_+ = 2 \sum_{n \geq 1} n c_{-n} c_n , \quad (8b)$$

$$Q_+ = d + d^\dagger , \quad (8c)$$

$$d = \sum_{n \geq 1} c_{-n} L_n - \sum_{\substack{n \geq 1 \\ m \geq 1}} \left(\frac{1}{2} (m-n) c_{-m} c_{-n} \bar{c}_{m+n} + (2n+m) \bar{c}_{-m} c_{-n} c_{n+m} \right) . \quad (8d)$$

The most important property of Q is its nilpotency

$$Q^2 = 0 \quad (\Leftrightarrow Q_+^2 = K T_+) , \quad (9)$$

which holds only in 26 dimensions. It is also important to note that the presence of the Hermitian anticommuting zero modes leads to the double degeneracy of the ground state and to an unusual norm of the states³²⁾. Namely, if we define the state $|0\rangle$ by

$$\bar{c}_0 |0\rangle = 0 , \quad c_n |0\rangle = \bar{c}_n |0\rangle = 0 \quad (n \geq 1) , \quad (10)$$

then $|0\rangle$ and $c_0 |0\rangle$ are degenerate in energy measured by the kinetic operator K . Further, $\langle 0|0\rangle = \langle 0|(c_0 \bar{c}_0 + \bar{c}_0 c_0)|0\rangle = 0$, whereas the definition of the norm

$$\langle 0|c_0|0\rangle = 1 \quad (11)$$

is consistent.

Now to promote this formalism to a field theory, Siegel⁶⁾ considered a string functional $\Psi[x(\sigma), c(\sigma), \bar{c}(\sigma)]$. Expanding in the zero mode c_0 , we may write

$$\begin{cases} \bar{\Psi} = (\Psi + c_0 \phi) |0\rangle, \\ \psi = \psi(\alpha_{-n}^{\mu}, c_{-n}, \bar{c}_{-n}; x^{\mu}), \\ \phi = \phi(\alpha_{-n}^{\mu}, c_{-n}, \bar{c}_{-n}; x^{\mu}). \end{cases} \quad (12)$$

Siegel then wrote down a "gauge-fixed" free action of the form

$$\begin{aligned} S &= \frac{1}{2} \langle \bar{\Psi} | [c_0 \bar{c}_0, Q] | \bar{\Psi} \rangle \\ &= \frac{1}{2} \langle \psi, K \psi \rangle - \frac{1}{2} \langle \phi, T_+ \phi \rangle, \end{aligned} \quad (13)$$

where in the second line the zero mode algebra has been performed. Expanding as in (3) one can check for example that this contains the usual action for the photon field in the Lorentz gauge, together with the appropriate FP ghost fields. From the form of S , it is obvious that it is invariant under the second-quantized BRST transformation

$$\delta_B |\bar{\Psi}\rangle = Q |\bar{\Psi}\rangle. \quad (14)$$

The immediate question one may ask is: what is it the "gauge-fixed" version of? Surprisingly, it took some time to give the answer, namely the gauge-covariant action^{7)-11),23)}, despite its simple final form. It is given by

$$S' = \frac{1}{2} \langle \bar{\Psi}_{(0)} | Q | \bar{\Psi}_{(0)} \rangle, \quad (15)$$

where the subscript (0) means that, unlike in (13), we restrict the functional to that with ghost number zero. Equation (15) is invariant under the gauge transformation

$$\delta_{\Lambda} |\bar{\Psi}_{(0)}\rangle = Q |\Lambda_{(-1)}\rangle. \quad (16)$$

In terms of ordinary fields, this represents an infinite number of gauge symmetries. Due to the nilpotency of Q , there exists an infinite sequence of gauge invariance for gauge invariance, the beginning of which is the invariance of Eq. (16) under $\delta_{\tilde{\Lambda}} |\Lambda_{(-1)}\rangle = Q |\tilde{\Lambda}_{(-2)}\rangle$.

There are several ways of deriving (15). In the following, I shall sketch the approach by Neveu, West and the others^{10),11)}, using the language of differential calculus first developed by Banks and Peskin⁸⁾. To understand the

basic strategy, it is helpful to first look at how we can obtain the gauge-invariant equation of motion for the photon starting from the gauge-fixed expression such as (4a). Start with $\square A_\mu = 0$. Since the gauge variation of the left-hand side is not zero but a derivative, we add a term of that form to cancel it. Thus, we introduce an auxiliary field ϕ and add the "gauge part" $\partial_\mu \phi$, obtaining

$$\square A_\mu + \partial_\mu \phi = 0. \quad (17)$$

This is now invariant under $\delta A_\mu = \partial_\mu \Lambda$, $\delta \phi = -\square \Lambda$. Next consider (4b), $\partial^\mu A_\mu = 0$. This gauge non-invariant equation should also be modified and we should do so in a manner consistent with (17). Thus apply ∂^μ to (17). We get $\square(\partial^\mu A_\mu + \phi) = 0$. This tells us that the gauge invariant modification of (4b) should be

$$\partial^\mu A_\mu + \phi = 0. \quad (18)$$

[If we wish, we can eliminate ϕ from (17) and (18) to obtain a more familiar form $0 = \square A_\mu - \partial_\mu (\partial \cdot A) = \partial^\nu F_{\nu\mu}$.] Equations (17) and (18) constitute the gauge covariantization of (4a) and (4b).

We now apply this strategy to the full string functional. It turns out that this can be done efficiently by introducing the following differential form calculus^{8),13)}. Let $\{e^n\}$ and $\{e_m\}$ ($n, m \in \mathbb{Z}_+$) be sets of mutually dual bases of one-forms. We can then define an $\binom{a}{b}$ form ω as

$$\omega = \omega_{n_1 \dots n_b}^{m_1 \dots m_a} e_{m_1} \wedge \dots \wedge e_{m_a} \otimes e^{n_1} \wedge \dots \wedge e^{n_b}, \quad (19)$$

where $\omega_{n_1 \dots n_b}^{m_1 \dots m_a}$ are functionals of $x^\mu(\sigma)$ and $\wedge$ denotes the antisymmetric product. The usual string field ϕ is considered to be a $\binom{0}{0}$ form. Define now the differential operator d on a $\binom{0}{0}$ form as

$$d\psi \equiv L_n \psi e^n \quad (\text{repeated indices summed}), \quad (20)$$

with the Virasoro operators L_n acting like a derivative on ϕ . To define d on arbitrary forms, we demand the nilpotency $d^2 = 0$ and the derivation property $d(\omega \wedge \rho) = d\omega \wedge \rho + (-1)^{\deg \omega} \omega \wedge d\rho$. Then it is not difficult to check that the following actions on e^n and e_n fulfil the above requirements:

$$de^n = -\frac{1}{2} [L_k, L_\ell]^n e^k e^\ell, \quad (21a)$$

$$de_n = [L_k, L_{-n}]^{-k} e^k e_\ell. \quad (21b)$$

Here, $[L_k, L_\ell]^n = (k-\ell)\delta_{n, k+\ell}$ are the structure constants of the Virasoro algebra and (21a) and (21b) can be thought of as Cartan-Maurier equations. Note that d sends an $\binom{a}{b}$ form to an $\binom{a}{b+1}$ form. After defining the inner product of two forms in an obvious manner, we can introduce the co-differential operator D by

$$(D\omega, \varphi) \equiv (\omega, d\varphi). \quad (22)$$

D so defined is nilpotent and is a map $\binom{a}{b} \rightarrow \binom{a-1}{b}$. It is essentially a "divergence" operation. For example, on a $\binom{1}{0}$ form $\omega = \omega^m e_m$,

$$D\omega = L_{-m} \omega^m. \quad (23)$$

What corresponds to the Laplace operator in the usual differential calculus is the kinetic operator K defined by

$$\begin{cases} K\psi = (L_0 - \alpha_0)\psi & \text{on } \psi = \binom{0}{0} \text{ form,} \\ Ke^n = ne^n, \quad Ke_n = ne_n. \end{cases} \quad (24)$$

The final ingredient in the Banks-Peskin calculus is the "metric" operator $\Downarrow$. It is an anti-derivation and it is defined by

$$\Downarrow\psi = \Downarrow e^n = 0, \quad (25a)$$

$$\Downarrow e_n = ne^n \equiv \eta_{nm} e^m. \quad (25b)$$

Equation (25b) shows that it acts like a metric. The operators defined above satisfy the following mutual relations:

$$\{\Downarrow, d\} = \{\Downarrow, D\} = \{\Downarrow, K\} = 0 \quad (26a)$$

$$dD - Dd = 2K\Downarrow. \quad (26b)$$

The important relation (26b) is valid only in 26 dimensions and is a consequence of a non-trivial identity involving the Virasoro structure constants.

We are now ready to generalize the covariantization procedure explained for the photon field to the full string case. In the following, we use the notation such as ω_b^a to indicate an $\binom{a}{b}$ form. Let us start from the gauge non-invariant equation of motion (4a), which can be written as $K\psi^0 = 0$. Mimicking (17), we want to add a "gauge part" to make the equation gauge-invariant. Comparing (5) with (23), we can write this as

$$K\psi^0 + D\phi'_0 = 0 \quad (27)$$

This is indeed invariant under

$$\delta\psi^0 = D\Lambda'_0, \quad \delta\phi'_0 = -K\Lambda'_0. \quad (28)$$

Consider now the other equation (4b), which can be written as $d\psi^0 = 0$. To see how we should modify it in the manner consistent with (27), we apply d on (27). We then get

$$Kd\psi^0 + (dD - Dd)\phi'_0 + Dd\phi'_0 = 0. \quad (29)$$

Using (26b), we can rewrite it as

$$K(d\psi^0 + 2\Downarrow\phi'_0) = -Dd\phi'_0. \quad (30)$$

Note that the right-hand side, which was zero for the photon field, does not vanish in general. Because of this, instead of an equation like (18), we should add another term to compensate for it. Thus we set

$$d\psi^0 + 2\Downarrow\phi'_0 + D\psi'_1 = 0, \quad (31)$$

where ψ'_1 is an additional auxiliary functional $\binom{1}{1}$ form. Putting (31) into (30), we get $D(d\psi^0 - K\psi'_1) = 0$ so it is consistent to set

$$d\phi_0' = K\psi_1' . \quad (32)$$

Equations (27), (31) and (32) comprise the desired set of equations invariant under (28) plus $\delta\phi_1^1 = -d\Lambda_0^1$.

Now the appearance of the $\binom{1}{1}$ form ϕ_1^1 in (32) with the kinetic operator K , just like for ϕ_0^0 , suggests that there might be an extended formulation¹¹⁾ where ϕ_k^k 's ($k=0,1,2,\dots$) appear. To motivate further, apply d on (32). We see that setting $d\phi_1^1 = 0$ is consistent with (32). But this is just like the $d\phi_0^0 = 0$ equation which we gauge-covariantized. So the previous procedure may be repeated and it would introduce new fields ϕ_1^2 and ϕ_2^2 . Along these lines, one arrives at the following enlarged set of equations:

$$\begin{cases} K\psi_R^R - d\phi_{R-1}^R + D\phi_R^{R+1} = 0 \\ d\psi_R^R + 2\Downarrow\phi_R^{R+1} + D\psi_{R+1}^{R+1} = 0, \quad (R=0,1,2,\dots, \phi_{-1}^0 \equiv 0). \end{cases} \quad (33)$$

They are invariant under a larger set of gauge transformations given by

$$\begin{cases} \delta\psi_R^R = -d\Lambda_{R-1}^R + D\Lambda_R^{R+1} + 2\Downarrow\Lambda_{R-1}^{R+1} \\ \delta\phi_R^{R+1} = -K\Lambda_R^{R+1} + d\Lambda_{R-1}^{R+1} + D\Lambda_R^{R+2} \end{cases} . \quad (34)$$

At this point it is not difficult to further discover that the right-hand sides of (34) are themselves invariant under the gauge transformations

$$\begin{cases} \delta\Lambda_{R-1}^R = D\tilde{\Lambda}_{R-1}^{R+1} + 2\Downarrow\tilde{\Lambda}_{R-2}^{R+1} + d\tilde{\Lambda}_{R-2}^R \\ \delta\Lambda_R^{R+1} = K\tilde{\Lambda}_{R-1}^{R+1} - d\tilde{\Lambda}_{R-2}^{R+1} + D\tilde{\Lambda}_{R-1}^{R+2} \end{cases} , \quad (35)$$

and this state of affairs repeats ad infinitum. This is precisely the kind of situation mentioned below Eq. (16). In fact there is an isomorphism between the differential form languages and those of BRST. The correspondences are: $e_n^{\pm} \leftrightarrow c_{-n}^{\pm}$, $e_n \leftrightarrow c_{-n}^-$, $2\Downarrow \leftrightarrow T_+$, $d \leftrightarrow d$, $D \leftrightarrow d^+$, $K \leftrightarrow K$ and the value $a-b$ for $\binom{a}{b}$ form corresponds to the ghost number.

Once these correspondences are established, the equations (33) and (34) can be cast into extremely compact forms

$$Q |\bar{\Psi}_{(0)}\rangle = 0, \quad (36)$$

$$\delta |\bar{\Psi}_{(0)}\rangle = Q |\Lambda_{(-1)}\rangle, \quad (37)$$

where

$$\begin{cases} |\bar{\Psi}_{(0)}\rangle = (\psi + c_0 \phi) |0\rangle \\ \psi = \psi_0 + \sum \psi_m^n c_{-n} \bar{c}_{-m} + \dots \\ \phi = \sum \phi^n \bar{c}_{-n} + \sum \phi_{\ell}^{nm} \bar{c}_{-n} \bar{c}_{-m} c_{-\ell} + \dots \end{cases} \quad (38)$$

Equation (36) obviously follows from the action already announced in (15).

Finally, let us briefly mention the salient feature of the gauge fixing of the action (15)¹¹⁾. For definiteness, consider the Feynman-Siegel gauge where the ϕ component of $|\bar{\Psi}_{(0)}\rangle$ is set to zero. (This corresponds to $\partial^\mu A_\mu = 0$ for the photon field.) We denote this by $P_- |\bar{\Psi}_{(0)}\rangle = 0$, where P_- projects ϕ out of $|\bar{\Psi}_{(0)}\rangle$. The Faddeev-Popov (FP) procedure introduces the ghost $|\bar{\Psi}_{(-1)}\rangle$ and the anti-ghost $|\bar{\Psi}_{(+1)}\rangle$ functionals (not to be confused with the ghost co-ordinates c_n and $\bar{c}_n$) and their action $\langle \bar{\Psi}_{(+1)} | P_- Q | \bar{\Psi}_{(-1)} \rangle$. Corresponding to the gauge invariance of gauge invariance (35), this action is itself invariant under $\delta |\bar{\Psi}_{(-1)}\rangle = Q |\Lambda_{(-2)}\rangle$ and we need to fix the gauge, e.g., by $P_- |\bar{\Psi}_{(-1)}\rangle = 0$. FP procedure should again be applied and it introduces ghosts for ghosts $|\bar{\Psi}_{(2)}\rangle$ and $|\bar{\Psi}_{(+2)}\rangle$. In this manner, ghost fields with an arbitrary integer ghost number naturally appear and, combined with the original string field $|\bar{\Psi}_{(0)}\rangle$, they can be assembled into a general string field $|\bar{\Psi}\rangle = \sum_{n=-\infty}^{\infty} |\bar{\Psi}_{(n)}\rangle$. This $|\bar{\Psi}\rangle$ is precisely the one which appears in the Siegel's gauge-fixed action (13).

FREE OPEN SUPERSTRING FIELD THEORY

Once we understand how to construct the free covariant bosonic string field theory, it is essentially straightforward to generalize it to the superstring case in the so-called "spinning string" [or Neveu-Schwarz (NS)-Ramond (R)] formulation³⁴⁾, where the world-sheet superconformal invariance is manifest.

Subtleties, however, do exist and we shall point to them as we go along.

To begin, let us briefly summarize the NS-R formulation, primarily to set our notation. Aside from the usual bosonic oscillators α_n^μ and the reparametrization ghost oscillators $c_n, \bar{c}_n$, we have the following free fields which come in pairs depending upon anti-periodic (NS) or periodic (R) boundary conditions that they satisfy. The fermionic two-dimensional spinor fields will be denoted by

$$\begin{cases} \lambda_{NS}^\mu(z) = H^\mu(z) = \sum_{s \in \mathbb{Z} + \frac{1}{2}} b_s^\mu z^{-s} \\ \{b_s^\mu, b_r^\nu\} = -g^{\mu\nu} \delta_{s+r,0} \end{cases} \quad (NS) \quad (39)$$

$$\begin{cases} \lambda_R^\mu(z) = \frac{i}{\sqrt{2}} \Gamma^\mu(z) = \frac{i}{\sqrt{2}} \sum_{n \in \mathbb{Z}} \Gamma_n^\mu z^{-n} = -\sum d_n^\mu z^{-n} \\ \{d_n^\mu, d_m^\nu\} = -g^{\mu\nu} \delta_{n+m,0}, \quad (\Gamma_0^\mu = \gamma^\mu) \end{cases} \quad (R) \quad (40)$$

Fixing the gauge, à la BRST, for the supergauge part of the superconformal invariance introduces the commuting supergauge ghosts³³⁾:

$$\begin{cases} e_{NS}(z) = \sum e_r z^{-r}, \quad \bar{e}_{NS}(z) = \sum \bar{e}_r z^{-r} \\ [e_r, \bar{e}_s] = \delta_{s+r,0} \quad (r, s \in \mathbb{Z} + \frac{1}{2}) \\ e_r^+ = e_{-r}, \quad \bar{e}_r^+ = -\bar{e}_{-r} \end{cases} \quad (NS) \quad (41)$$

$$\begin{cases} e_R(z) = \sum e_n z^{-n}, \quad \bar{e}_R(z) = \sum \bar{e}_n z^{-n} \\ [e_n, \bar{e}_m] = \delta_{n+m,0} \quad (n, m \in \mathbb{Z}) \\ e_n^+ = e_{-n}, \quad \bar{e}_n^+ = -\bar{e}_{-n} \end{cases} \quad (R) \quad (42)$$

The super-Virasoro operators are given by

<u>NS sector</u>	<u>R sector</u>
$\begin{cases} L_n^{NS} = L_n^\alpha + L_n^b \\ G_r = \oint \frac{dz}{2\pi i z} G(z) z^r \\ G(z) = \lambda_{NS}^\mu(z) P_\mu(z) \end{cases}$	$\begin{cases} L_n^R = L_n^\alpha + L_n^d \\ F_m = \oint \frac{dz}{2\pi i z} F(z) z^m \\ F(z) = \lambda_R^\mu(z) P_\mu(z) \end{cases}$
$(P^\mu(z) = \sum \alpha_n^\mu z^n)$	

(43)

Using them, the physical state conditions are expressed as

$$\begin{array}{cc} \text{NS sector} & \text{R sector} \\ \left\{ \begin{array}{l} (L_0^{NS} - \frac{1}{2}) |phys\rangle = 0 \\ G_r |phys\rangle = 0, \quad r \gg \frac{1}{2} \\ L_n^{NS} |phys\rangle = 0, \quad n \gg 1 \end{array} \right. & \left\{ \begin{array}{l} F_0 |phys\rangle = 0 \\ F_n |phys\rangle = 0, \quad n \gg 1 \\ L_n^R |phys\rangle = 0, \quad n \gg 1 \end{array} \right. \end{array} \quad (44)$$

Starting from these conditions, we may repeat, with some modifications, the procedure described in the previous section for the bosonic case and develop the differential form formulation of the free superstring field theory^{14),15)}. The result in turn can again be written in terms of the BRST operator Q ³³⁾. Its expression in the NS sector is of the form Eq. (7), with the understanding that contributions from $e-\bar{e}$ oscillators and G-F gauges are included in the definitions of K , T_+ and Q_+ . In the R sector, it takes the form

$$Q = c_0 K - \bar{c}_0 T_+ + Q_+ + e_0 F - e_0^2 \bar{c}_0 - \bar{e}_0 S_+, \quad (45)$$

where F and S_+ are given by

$$F = F_0 + \sum_{n \gg 1} \frac{1}{2} n (c_{-n} \bar{e}_n - \bar{e}_{-n} c_n) - 2 \sum_{n \gg 1} (e_{-n} \bar{c}_n + \bar{c}_{-n} e_n), \quad (46)$$

$$S_+ = \sum_{n \gg 1} \frac{3}{2} n (e_{-n} c_n - c_{-n} e_n). \quad (47)$$

One can check that Q in both sectors is nilpotent if and only if the dimensionality of the space-time is 10.

Just as for the bosonic case, the equations of motion can be written as

$$Q_{NS,R} | \bar{\Psi}_{NS,R} \rangle = 0. \quad (48)$$

Making the dependence on the zero modes explicit, the string functional $| \bar{\Psi}_{NS,R} \rangle$ can be written as

$$| \bar{\Psi}_{NS} \rangle = (\bar{\Psi}_{NS} - c_0 \phi_{NS}) | 0_{NS} \rangle, \quad (49a)$$

$$|\bar{\Psi}_R\rangle = (\Psi_R + e_0 \phi_R + c_0 F \phi_R) |0_R\rangle. \quad (49b)$$

There are two remarks to be made here. First, note a rather special form of (49b), in which the commuting zero mode e_0 appears only linearly and the field ϕ_R appears twice¹⁴⁾⁻¹⁶⁾. This form follows naturally from the differential form consideration and is closed under the action of Q_R . (An alternative formulation in which all the powers of e_0 appear exists¹⁷⁾, but we shall not discuss it here.) The second has to do with the definition of the ground state $|0_R\rangle$. As for the non-zero modes, we demand, as usual, $e_m |0_R\rangle = \bar{e}_m |0_R\rangle = 0$ for $m > 1$. With respect to the zero modes e_0 and $\bar{e}_0$ we have a situation similar to, but subtler than, the bosonic case described in the previous section. If we define $\bar{e}_0 |0_R\rangle = 0$, then the Hermitian conjugate of $|0_R\rangle$, which we shall denote by $\langle \tilde{0}_R |$, is orthogonal to $|0_R\rangle$, just like in the bosonic case. The difference is that there is no simple "metric" operator, like c_0 , to insert to get a unit norm [see Eq. (11)]. Thus we define¹⁸⁾ a new bra $\langle 0_R |$ and a ket $|\tilde{0}_R\rangle$ such that

$$\begin{cases} \langle 0_R | 0_R \rangle = 1 \\ \langle 0_R | e_0 = 0, \quad \langle 0_R | \bar{e}_0 \neq 0, \end{cases} \quad (50a)$$

$$\begin{cases} \langle \tilde{0}_R | \tilde{0}_R \rangle = 1 \\ e_0 | \tilde{0}_R \rangle = 0, \quad \bar{e}_0 | \tilde{0}_R \rangle \neq 0. \end{cases} \quad (50b)$$

Need for (at least) these two sets of ground state vectors will become more evident when we discuss space-time supersymmetry^{3),18),28)}.

With this preliminary, we can now display the action for the free open superstring:

$$S = S_{NS} + S_R, \quad (51)$$

$$S_{NS} = \frac{1}{2} \langle 0_{NS} | \tilde{\Psi}_{NS}^+ Q_{NS} \Psi_{NS} | 0_{NS} \rangle, \quad (52a)$$

$$\begin{aligned} S_R &= \frac{1}{2} \langle 0_R | (-\bar{e}_0 c_0) \bar{\Psi}_R^\dagger Q_R \bar{\Psi}_R | 0_R \rangle \\ &= \frac{1}{2} \langle \Psi_R, F \Psi_R \rangle + \langle \Psi_R, Q_+ \phi_R \rangle - \frac{1}{2} \langle \phi_R, \frac{1}{2} (F T_+ + T_+ F) \phi_R \rangle. \end{aligned} \quad (52b)$$

Equation (52a) is essentially the same as the action for the bosonic string. For the R sector, we have shown, in the second line of (52b), the form after the zero mode algebra, which contains the generalization of the Majorana action, $\frac{1}{2} \langle \Psi_R, F \Psi_R \rangle$. [The insertion of the zero-modes $(-\bar{e}_0 c_0)$ in the first line is necessary in order to pick out this Majorana action.] Gauge invariances and the gauge fixing are just as in the bosonic case. It is instructive to display the massless field content. The components ψ_{NS} , ϕ_{NS} , etc., in (49), can be expanded as:

$$\begin{aligned} \psi_{NS} &= \phi + b_{-1/2}^\mu A_\mu + \dots \\ \phi_{NS} &= \bar{e}_{-1/2} \phi_{1/2} + \dots \\ \psi_R &= \lambda + \dots \\ \phi_R &= \bar{e}_{-1} \phi_1 + \dots \end{aligned} \quad (53)$$

The massless fields are the photon A_μ , its auxiliary field $\phi_{1/2}$, and the Majorana field λ . It is well known that the physical spectrum of this theory is supersymmetric if one discards roughly half of the states by the Gliozzi-Scherk-Olive (GSO) projection³⁵⁾. In our covariant framework, the GSO projector must be generalized to include ghost oscillators. In the NS sector, it is given by

$$P_{NS} = \frac{1}{2} \left[1 - (-1)^{\sum_{r \geq 1/2} (b_{-r}^\mu b_{r\mu} + \bar{e}_{-r} e_r + e_{-r} \bar{e}_r)} \right]. \quad (54)$$

It simply projects out states with odd number of half-integrally moded oscillators. For example, the tachyon ϕ in (53) is discarded by this projection, whereas the photon is kept. In the R sector, it is a generalization of a chirality projection and it reads

$$P_R = \frac{1}{2} \left[1 + \gamma^* (-1)^{\sum_{n \geq 1} (d_{-n}^\mu d_{n\mu} + \bar{e}_{-n} e_n + e_{-n} \bar{e}_n)} + e_0 \bar{e}_0 \right]. \quad (55)$$

where γ^* is the analogue of γ_5 . Under P_R , λ becomes a Majorana-Weyl spinor in ten dimensions. These projectors commute with the BRST charge so that the form of the action (52) remains intact.

Finally, let us record the massless equations of motion contained in $Q|\Psi\rangle = 0$. They read

$$p^2 A_\mu = 2 p_\mu \phi_{1/2} , \quad (56a)$$

$$p^\mu A_\mu = 2 \phi_{1/2} , \quad (56b)$$

$$\not{p} \lambda = 0 . \quad (56c)$$

FERMION EMISSION VERTEX AND SPACE-TIME SUPERSYMMETRY

Now that we have obtained the free superstring action, we look for its space-time supersymmetry. Obviously we need an operator which converts between NS and R sectors. Such an operator, which we shall denote by $W_{\text{orb}}(z)$, was constructed long ago³⁶⁾ and was used for the calculation of four-fermion scattering amplitude depicted in the Figure. Symbolically, $W_{\text{orb}}|\text{NS}\rangle = |\text{R}\rangle$ and $W_{\text{orb}}^+|\text{R}\rangle = |\text{NS}\rangle$.

Naively, one would expect $W_{\text{orb}}(z)$ to convert $\Gamma^\mu(x)$ into $H^\mu(x)$, i.e., a relation like

$$\frac{1}{i\sqrt{2}} \Gamma^\mu(x) W_{\text{orb}}(\bar{z}) = W_{\text{orb}}(\bar{z}) H^\mu(x) \gamma^* \quad (57)$$

to hold. However, since $\Gamma^\mu(x)$ and $H^\mu(x)$ have, respectively, integral and

half-integral mode expansions, such a relation cannot prevail. To make the meaning of "conversion" precise, we first note, as a motivation, the relation

$$e^{-\bar{z}L_{-1}^R} \Gamma^\mu(x) e^{\bar{z}L_{-1}^R} = \Gamma^\mu(x-\bar{z}) \left(\frac{x}{x-\bar{z}} \right)^{1/2}, \quad (58)$$

where L_{-1}^R is the Virasoro operator in the R sector. Then if we write $W_{orb}(z)$ as

$$W_{orb}(z) = e^{\bar{z}L_{-1}^R} \tilde{W}(z),$$

we expect $\tilde{W}(z)$, so defined, to satisfy

$$\frac{1}{i\sqrt{2}} \frac{\Gamma^\mu(x-\bar{z})}{\sqrt{x-\bar{z}}} \tilde{W}(z) = \tilde{W}(z) \frac{H^\mu(x)}{\sqrt{x}} \gamma^*. \quad (59)$$

Now, for $|z| > |x|$ (and actually with an additional condition $|x-z| > |z|$) both sides of (59) have an integral power expansion in x and $\tilde{W}(z)$ can be determined properly³⁶⁾. It takes the form

$$\begin{aligned} \tilde{W}(z) &= \langle 0_{NS} | \exp(A(z) + B(z)) | 0_R \rangle, \\ A(z) &= -\frac{1}{2} \sum_{r,s \geq 1/2} b_r^\mu \frac{1}{2} (\alpha_{rs} - \alpha_{sr}) b_{s\mu} \bar{z}^{-(r+s)}, \\ B(z) &= \frac{i}{\sqrt{2}} \sum_{\substack{n \geq 0 \\ r \geq 1/2}} \Gamma_{-n}^\mu \left(\begin{matrix} n-1/2 \\ r-1/2 \end{matrix} \right) b_{r\mu} (-z)^{n-r}, \end{aligned} \quad (60)$$

where

$$\alpha_{rs} = (-1)^{r+s} \frac{r}{r+s} \left(\begin{matrix} -1/2 \\ r-1/2 \end{matrix} \right) \left(\begin{matrix} -1/2 \\ s-1/2 \end{matrix} \right). \quad (61)$$

[Note that binomial coefficients appear due to the square-root cut structure in Eq. (59).]

What are of importance are the various so-called gauge densities^{37),38)} satisfied by $W_{orb}(z)$. They can be written as

$$(I) \quad L_n^R W_{orb}(z) - W_{orb}(z) L_n^{NS} = \bar{z}^n \left(z \frac{d}{dz} + \frac{5}{8} (n+1) \right) W_{orb}(z), \quad (62)$$

$$(IIa) \quad \sum_{m \geq 0} \beta_{rm} F_m W_{orb}^{(1)} = -i W_{orb}^{(1)} \gamma^* \left(G_{-r} + \sum_{s \geq 1/2} \alpha_{rs} G_s \right), \quad (63a)$$

$$(IIb) \quad (F_m + \sum_{n \neq 0} \gamma_{mn} F_n) W_{orb}(1) = -i W_{orb}(1) \delta^* \sum_{S \gg 1/2} \delta_{ms} G_S, \quad (63b)$$

where the coefficients β_{rn} , γ_{mn} , δ_{mr} are similar in form to α_{rs} . (I) simply says that $W_{orb}(z)$ is a conformal field of dimension 5/8. (IIa) and (IIb), involving an infinite number of super-Virasoro operators, look complicated, but actually it has a simple origin. They can be derived from a generic equation³⁸⁾ of the form (we have put $z = 1$ for simplicity)

$$\oint_{|x|>1} \frac{dx}{2\pi i x} f(x) \left(1 - \frac{1}{x}\right)^{1/2} i F(x) W_{orb}(1) \\ = \oint_{|x|<1} \frac{dx}{2\pi i x} f(x) \left(\frac{1-x}{x}\right)^{1/2} W_{orb}(1) G(x), \quad (64)$$

where $f(x)$ is an arbitrary function which is Laurent expandable around the origin with no other singularities. Particular choices of $f(x)$ give (IIa) and (IIb). Still more instructive is to recognize that Eq. (64) is equivalent to the operator relations

$$i F(x) W_{orb}(z) = (x-z)^{-1/2} : O(x,z) :, \quad (|x| > |z|) \quad (65a)$$

$$W_{orb}(z) G(x) = (z-x)^{-1/2} : O(x,z) :, \quad (|x| < |z|), \quad (65b)$$

where $:O(x,z):$ is a normal-ordered operator. Note that $W_{orb}(z)$ changes the cut structure and such a field is called a spin field³⁾.

Now, for the fully Lorentz-covariant description, we must supplement $W_{orb}(z)$ with the "ghost part" of the fermion emission vertex $W_{gh}(z)$. Need for such an object is apparent from the following facts: (a) since the dimension of $W_{orb}(z)$ is 5/8, not 1 required of a physical vertex, we need an operator of dimension 3/8; (b) it has been known³⁹⁾ for a long time that the transverse projection operator must be inserted in the Figure in order to prevent unphysical states from propagating. The effect of such an insertion is to change the usual bosonic propagator $\int d[dx/x] x^{L_0 - \frac{1}{2}}$ by a factor $\Delta(x)^{-1} = (1-x)^{-1/4}$ ³⁹⁾. Thus we expect that $W_{gh}(z)$ has dimension 3/8 and that it produces the desired factor $\Delta(x)^{-1}$.

Such an operator was first constructed by Friedan, Martinec and Shenker and by Knizhnik⁴⁰⁾. Although quite elegant, their construction utilizes the bosonization of the ghost number current and consequently $W_{gh}(z)$ is not expressed in terms of the ghost oscillators useful for string field theory. Recently, a concrete expression of $W_{gh}(z)$ was obtained in terms of the commuting ghost oscillators $e_n, \bar{e}_n, e_r$ and $\bar{e}_r$ 's¹⁸⁾. Before quoting the rather complicated looking form of $W_{gh}(z)$, let us briefly describe the strategy of the construction, which should make the result almost obvious. As we want the combined operator $W_{orb}(z)W_{gh}(z)$ to effect the supersymmetry transformation which is an invariance of the action (51)-(52), it is natural to seek an operator

$$W \equiv \oint \frac{dz}{2\pi i z} W(z) = \oint \frac{dz}{2\pi i z} \sqrt{z} W_{orb}(z) W_{gh}(z) P_{NS}, \quad (66)$$

such that it converts between Q_R and Q_{NS} , i.e.,

$$Q_R W = W Q_{NS} \quad (67)$$

[$\sqrt{z}$ in (66) is needed for a technical reason.]

Now focus upon a term in Q_R of the form $e_{-n} F_n$ [which is an extension of the term $c_{-n} L_n$ in (8d)]. This term should be converted by W into $e_{-r} G_r$, which is in Q_{NS} . We know from (63) how F_n 's get transformed into G_r 's by W_{orb} . Thus e_{-n} should transform, under the action of W_{gh} into e_{-n} 's "contravariantly" to F_n 's. More precisely, its conjugate $\bar{e}_n$ should transform into $\bar{e}_n$'s in exactly the same way as F_n transforms into G_r 's. In other words we expect that equations of the form (63) should hold in which F_m, G_n and $W_{orb}(1)$ are replaced respectively by $\bar{e}_m, \bar{e}_r$ and $W_{gh}(1)$. From such relations, it is easy to get the result

$$\begin{aligned} W_{gh}(z) = & \langle 0_{NS} | \exp \left(- \sum_{r,s \geq 1/2} e_r \alpha_{rs} \bar{e}_s z^{-(r+s)} \right. \\ & + i \sum_{\substack{m \geq 0 \\ r \geq 1/2}} e_r \beta_{rm} \bar{e}_{-m} z^{m-r} + i \sum_{\substack{m \geq 1 \\ s \geq 1/2}} e_{-m} \delta_{ms} \bar{e}_s \\ & \left. + \sum_{\substack{m \geq 1 \\ n \geq 0}} e_{-m} \gamma_{mn} \bar{e}_{-n} \right) | \tilde{0}_R \rangle. \end{aligned} \quad (68)$$

Note that the exponent contains the zero mode $\bar{e}_0$ and the Ramond vacuum is taken to be $|\tilde{0}_R\rangle$. This is essential in checking that $W_{gh}(z)$ above indeed has a conformal dimension $3/8$, which in turn is crucial for (67) to hold. As a further check,

one can show $\langle \tilde{0}_R | W_{gh}(y) W_{gh}^\dagger(z) | 0_R \rangle = (1-z/y)^{-1/4}$, which corresponds to the desired correction factor $\Delta(x)^{-1}$ ($x=y/z$) for the propagator of the Figure mentioned before. It is also easy to see that the GSO projectors are properly converted by W , i.e., $P_R W = W P_{NS}$.

Finally we can exhibit the supersymmetry transformation laws explicitly in terms of W . For this purpose, expand W in powers of $\bar{e}_0$ and write $W(\bar{e}_0) = W(0) + \bar{e}_0 W'(0) + \frac{1}{2} \bar{e}_0^2 W'' + \dots$. Then one can show that the following set of transformations leave the action invariant:

$$\begin{cases} \delta \psi_R = -F W(0) \psi_{NS} - W'(0) \phi_{NS} \\ \delta \phi_R = -W(0) \phi_{NS} \\ \delta \psi_{NS} = W(0)^\dagger \psi_R + W'(0)^\dagger \phi_R \\ \delta \phi_{NS} = -W(0)^\dagger F \phi_R \end{cases} \quad (69)$$

By expanding (69) into oscillator modes, one can read off, in particular, the transformation laws for the massless fields. They are

$$\begin{cases} \delta \lambda = \frac{i}{2} \not{x} \gamma_\mu \varepsilon A^\mu - i \phi_{1/2} \varepsilon \\ \delta A_\mu = -\frac{i}{\sqrt{2}} \bar{\varepsilon} \gamma_\mu \lambda \\ \delta \phi_{1/2} = 0 \end{cases} \quad (70)$$

where ε is the spinor parameter of SUSY transformations, which resides implicitly in the Ramond vacuum vector. With the use of the equations of motion (56), one can easily check that the algebra closes.

This on-shell closure was expected from the beginning by the counting of the number of bosonic and fermionic degrees of freedom. For the development of the interaction vertex it is obviously desirable to have an off-shell closure. This calls for a different formulation, which is now being actively pursued.

SUMMARY, REMARKS AND PROSPECTS

In this talk, we have given a pedagogical account for the construction and the symmetry structure of the gauge covariant open (super) string field theory actions. In all cases, it can be written in a generic form $S = \frac{1}{2} \langle \Psi | Q | \Psi \rangle$, where Q is some metric operator. This makes it clear that Q , which is originally the BRST operator for the first quantization, is the generator of the conformal (analytic) deformation in the field theory context. We have also seen that the space-time supersymmetry can be implemented in a gauge-covariant, Lorentz-covariant way. The supercovariance, i.e., the off-shell closure, is yet to be achieved.

Due to the introductory nature of this article, many other developments in this field, including the important subject of the construction of the interaction vertices, have not been covered⁴¹⁾. To partly remedy this, let us give a brief summary of the present status, which serves at the same time to display future problems.

(i) As far as the free theory is concerned, extension to closed strings is straightforward, except for one case, namely the Ramond-Ramond sector which appears in Type II superstring¹⁾. A completely satisfactory formulation, without dynamical constraints, is yet to be sought.

(ii) For the open bosonic string, two versions of interaction vertices have been constructed. One version^{19),21)}, which should be called a covariantized-light-cone version, requires four-point as well as three-point vertices and contain fictitious parameters, α_i , corresponding to the length of the strings in the light-cone field theory. At the tree level it has been checked that these parameters disappear in the on-shell amplitudes. Explicit loop-level calculations, however, have not been performed. The other version, which was proposed by Witten²³⁾ in the framework of non-commutative geometry and has subsequently been made more explicit by others²⁴⁾, needs only a cubic vertex and no fictitious α parameters. Despite the unusual way of joining the strings, the on-shell couplings have been checked to reproduce those of dual string theory^{23),24)}. Further, a general argument has been presented²⁵⁾ that to all loop-levels Feynman rules derived from this theory generate the world-sheet topologies identical to those summed over in the Polyakov's path integral formulation. A possible disadvantage of this formulation is that, unlike the other version, extension to the closed string case is not straightforward and is to be developed.

(iii) It should be mentioned that for both versions, unlike the free case, the conventional Faddeev-popov gauge-fixing procedure does not work²⁶⁾. So far, only an unsystematic procedure exists¹⁹⁾ and obviously this situation needs to be improved.

(iv) For superstring interaction, Witten²⁸⁾ has proposed a formulation, for open strings, which is similar in spirit to his bosonic interacting theory. Explicit forms of the vertices are to be worked out and the space-time supersymmetry structure of the theory should be clarified beyond his preliminary analysis.

(v) All the formulations mentioned above are implicitly tied to the flat space-time metric through the definition of the operator Q appearing in the kinetic term. Recently, taking the bosonic closed string as an example, an intriguing proposal was made of a different formulation which is free of the background geometry²⁹⁾. Starting from an action which consists only of the cubic interaction vertex, i.e., without the kinetic term, it was shown that the expansion of the string functional around any non-trivial classical solution of this theory generates a kinetic term which can be written in terms of a nilpotent operator precisely like Q . This should describe a consistent propagation of a closed string in a curved background. It is to be seen whether this conceptually appealing formulation proves practically useful for solving the compactification problem.

(vi) As was mentioned at the end of the previous section, off-shell realization of supersymmetry has not been achieved. With infinitely many auxiliary fields at hand, this might be feasible in a clever formulation. Once understood, it may lead to a yet new Green-Schwarz-type formulation¹⁾ where supersymmetry becomes manifest together with Lorentz and gauge invariances.

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