CERN-TH.4508/86

ON THE GLOBAL STABILITY OF GRAVITATIONAL LUMPS

F. Alexander Bais^{*}, César Gomez and Valery A. Rubakov[‡]

CERN - Geneva

ABSTRACT

The semi-classical stability of gravitational lumps is analyzed using cobordism theory. It turns out that not all topological numbers characterizing the lumps are conserved, in fact only the Pontryagin and Stiefel-Whitney numbers are. Some examples are given. Modifications due to the presence of fermions are briefly discussed.

^{*}) Institute for Theoretical Physics, University of Amsterdam, The Netherlands.

[‡]) Institute for Nuclear Research, Academy of Sciences of the U.S.S.R., Moscow 117 312, U.S.S.R.

1. Introduction

Topological considerations in field theory have led to a number of important developments; the existence of solitons or lumps with their interesting physical properties, the phenomena of vacuum tunnelling through instantons, the discovery of global anomalies etc. On the other hand, the ongoing search for an ultimate unification scheme including gravity, such as supergravity theories or superstrings, has confronted us with the idea that space-time may well have more than four dimensions. If this is to be true, then one would expect a rich variety of physical effects of gravitational nature which derive from the topological structure of space-time.

In this paper we address various problems related with the conceivable existence of gravitational solitons or lumps. We are not, in the first place, interested in the construction of explicit solutions to the relevant equations, but rather focus on the general question of what may render such solutions stable. A number of criteria for this to be the case will be established. Though our considerations apply equally well to the question of vacuum stability, we will focus mainly on the case of time independent lumps. We assume a Kaluza-Klein ground state $M^0 = R^3 \times K$ with infinity $M_\infty^0 = S^2 \times K$, where K is some d -dimensional compact manifold. In view of our subsequent analysis it is important to divide the gravitational lumps into two distinct classes according to their geometry at spatial infinity.

Type I: The first class of solitons consists of smooth manifolds M with the property that its infinity M_∞ is different from M_∞^0 globally. The manifold K may be such that it is possible to have a non-trivial bundle at infinity, i.e., $M_\infty = (S^2 \times K \rightarrow S^2)$ which requires the occurrence of long range gauge fields.

The prototype of this class is the Kaluza-Klein monopole^{1),2)}, in the 5-dimensional Kaluza-Klein theory, or its higher dimensional analogues³⁾. Spatial infinity of the K-K vacuum is just the (global) direct product $M_\infty^0 = S^2 \times S^1$. The monopole corresponds to the Euclidean Taub-NUT solution which at infinity tends to $M_\infty = S^3$, to be thought of as the non-trivial Hopf bundle $S^1 \times S^2 \rightarrow S^2$. Again, the crucial property of the type I lumps is that their infinity M_∞ is topologically different from the vacuum infinity M_∞^0 .

Type II: The second class of lumps consists of smooth manifolds $M \neq M^0 = R^3 \times K$ but with the same infinity $M_\infty = M_\infty^0 = S^2 \times K$. An example of such a lump is the magnetic dipole solution²⁾ in $d = 5$ Kaluza-Klein, which corresponds to the

Euclidean Kerr solution. Another example was provided by Witten⁴⁾, though not quite in the Kaluza-Klein framework. He noted that instead of K^n one may take a manifold Y^n with the same topology by with a different differentiable structure.

These can be constructed in those dimensions $n > 6$ for which exotic spheres exist.

An important property of type II lumps is that they can be compactified (topologically) by adding the point at infinity. The K-K vacuum compactifies to $S^3 \times K$ and Y^n to S^n_{exotic} . This property will be useful in our later considerations. So far we have only divided n -dimensional manifolds M into different classes according to their asymptotic behaviour, ignoring completely any notion of stability. As a matter of fact, an analysis of the criteria for (topological) stability of gravitational lumps is still lacking, this in contrast with the situation for (spontaneously broken) gauge theories⁵⁾. It is in fact on this question that this paper will provide a number of general results.

Let us consider two n -dimensional manifolds M_1 and M_2 with $M_{1\infty} = M_{2\infty}$ and compute their topological invariants. The question whether one of them can decay into the other is then basically the question whether there exists an $(n+1)$ -dimensional smooth Euclidean manifold which interpolates between them. Naively one would maybe expect that all topological invariants of M_1 and M_2 have to be equal; but this is actually not the case. A trivial example is the following: take $M_1 = S^1$ and $M_2 = S^1 + S^1$ (disjoint union) these spaces are topologically different, but nevertheless there is a smooth pair of "trousers" interpolating between them (Fig. 1). In higher dimensions the situation is similar. In the following we establish the criteria for the existence of a smooth Euclidean space-time manifold which interpolates between a given lump at $t = -\infty$ and the vacuum (or another lump) at $t = +\infty$, with the restriction that spatial infinity remains fixed. Such a configuration will have finite action because it is smooth and well behaved at infinity, and if existent it implies semi-classical instability. Whether the converse holds is less obvious, one can imagine an interpolating manifold which is not quite smooth and yet has finite action. It should be added, however, that we do not know of any example where semi-classical decay is described by such a non smooth configuration. It turns out that the question of topological stability of gravitational lumps, can be conveniently formulated mathematically as a problem in cobordism theory.

The paper is organized as follows. In the next section we give a brief exposé of some of the relevant results of cobordism theory. In Section 3 we make

the definition of stability more precise and exploit cobordism. We discuss a number of examples in the standard (4+1)-dimensional Kaluza-Klein theory. Subsequently we address the question under what conditions stable lumps can actually exist in general. In Section 4 we include fermions (i.e., spin structures) into the picture and show under what conditions they may stabilize (or suppress the appearance of) lumps.

2. Tools from Cobordism Theory

Our concept of instability as the existence of a smooth interpolating space-time between the gravitational lump and a vacuum space configuration can be set up in a more mathematical way using the concept of cobordism. As was stated before, for type II lumps we may compactify both the vacuum space and the lump. Then, the existence of an interpolating space-time just means that the disjoint union of the lump L_n and the vacuum V_n , as manifolds, is exactly the boundary of some space-time W_{n+1} :

$$L_n \cup V_n \equiv L_n + V_n = \partial W_{n+1} \quad (2.1)$$

Equation (1) defines an equivalence relation between n -dimensional manifolds which is compatible with the operation $+$ defined by the disjoint union. The equivalence classes are called cobordism classes and they form an additive group Ω_n whose zero element is the class of manifolds which bound some $(n+1)$ -dimensional manifold⁶⁾.

From a physical point of view the previous definition of cobordism is too general and disregards some structural properties which are required from a physical point of view in order for a manifold to be a candidate lump, space or space-time. We think, in particular, of orientability. For orientable manifolds the cobordism defined by Eq. (1) must be modified as follows⁶⁾:

$$L_n \cup \{-V_n\} \equiv L_n + (-V_n) = \partial W_{n+1} \quad (2.2)$$

where $-V_n$ is just defined by reversing the orientation of V_n . We also require the orientation of $L_n + (-V_n)$ to be induced by the orientation of W_{n+1} . With these conditions, Eq. (2) defines an equivalence relation between oriented manifolds and (in the same way as for the unoriented case) the set of these

equivalence classes forms a new additive group Ω_n^* : the oriented cobordism group.

The fact which makes cobordism a physically interesting concept is that it allows us to know what topological numbers, if any, we should take into account in order to consider a given space configuration to be stable. In fact, and this is new from cobordism, some topological conservation laws can be broken as a gravitational effect. Notice that this kind of effect has nothing to do with the breaking of any symmetry because topological charges are not Noether.

The problem of what topological numbers are conserved by cobordism has been solved in mathematics for quite a large class of different classes of cobordism theories. In particular for unoriented cobordism the only conserved topological numbers are the Stiefel-Whitney⁶⁾ numbers which are elements of Z_2 and are defined using the Stiefel-Whitney classes [elements of $H^i(M, Z_2)$ of the manifold]. In fact, two closed n -manifolds belong to the same cobordism class if and only if all their corresponding Stiefel-Whitney numbers are equal. For oriented cobordism which is the relevant case for physical applications we need to require both the Stiefel-Whitney numbers and the Pontryagin numbers⁶⁾ to be the same. The Pontryagin classes are defined as characteristic classes of the orthogonal frame bundle, which is real and has a structural group O_n . Therefore they depend only on the even Chern classes and are given by

$$P_i = (-1)^i C_{2i} \quad (2.3)$$

where C_{2i} are Chern classes ($C_2 = \text{tr } R \wedge R$, $C_4 = R \wedge R \wedge R \wedge R$, ...). The Pontryagin numbers for a n -dimensional manifold are given by:

$$\int_M P_{i_1} \dots P_{i_r} \quad (2.4)$$

where $i_1, i_2, \dots, i_r$ is any partition of n (i.e., $i_1 + \dots + i_r = n$). Let us now list the first few oriented cobordism groups⁷⁾

$$\begin{aligned} \Omega_0 = \mathbb{Z}, \quad \Omega_1 = \Omega_2 = \Omega_3 = 0, \quad \Omega_4 = \mathbb{Z}, \quad \Omega_5 = \mathbb{Z}_2, \quad \Omega_6 = \Omega_7 = 0 \\ \Omega_8 = \mathbb{Z} \oplus \mathbb{Z}, \quad \Omega_9 = \mathbb{Z}_2 \oplus \mathbb{Z}_2, \quad \Omega_{10} = \mathbb{Z}_2, \quad \Omega_{11} = \mathbb{Z}_2 \end{aligned} \quad (2.5)$$

All higher cobordism groups are non-zero. Notice that only in $4K$ dimensions we have $\mathbb{Z}$ -factors, which just reflects the fact that only in these dimensions we can

have non-zero Pontryagin numbers, the rest of the groups can only have torsion elements (by torsion element we mean elements of which some integral multiple vanishes). We mention that torsion in Ω_n implies that some Steifel-Whitney numbers can be different from zero in dimension n . The converse is not true, Ω_4 for instance does not contain torsion nevertheless CP_2 has second Stiefel-Whitney number different from zero.

3. Criteria for Stability

In this section we will give a precise definition of stable type II lumps in Kaluza-Klein theories, using the notion of cobordism as introduced above. We assume that the vacuum space is $R^3 \times K$ with K any compact manifold. Then we define a stable type II lump as any space manifold M_n satisfying:

$$M_n|_{\infty} \simeq R^3 \times K|_{\infty} \quad (3.1a)$$

with the condition that there is no space-time manifold W_{n+1} such that:

$$\bar{M}_n + (-(S^3 \times K)) = \partial W_{n+1} \quad (3.1b)$$

where $\bar{M}_n$ is the compactification, in the usual sense, of M_n and $S^3 \times K$ the compactification of the vacuum space $R^3 \times K$. An alternative definition would be the following. Consider as the vacuum space $B^3 \times K$ where B^3 is a 3-ball with some finite radius (this can only go through if there are no long range forces as in the case for type II lumps). The lump is any space manifold M such that

$$\partial M = \partial(B^3 \times K) = S^2 \times K \quad (3.2a)$$

with the condition that there is no space-time manifold W_{n+1} such that:

$$M \cup_g (B^3 \times K) = \partial W_{n+1} \quad (3.2b)$$

Here $\cup_g$ means the gluing together of M and $B^3 \times K$ at their boundaries. The physical meaning of (3.2b) is that of tunnelling with fixed boundary conditions. Consider a $t = -\infty$ the lump M if we keep the boundary conditions fixed at intermediate times then the decay of M into $B^3 \times K$ at time $t = +\infty$ corresponds to a space-time with a boundary $M \cup_g (B^3 \times K)$ (Fig. 2). From these definitions of lumps

and using cobordism theory we easily see that a necessary condition for the existence of lumps is just that Ω_n^* be non zero. In fact, both conditions (3.1b) and (3.2b) reduce to the statement that $\bar{M}_n + (-(S^3 \times K))$ or $M \cup_g (B^3 \times K)$ must be non-trivial elements of Ω_n^* .

Before extending our analysis to type I lumps, we first present some examples which illustrate our concept of stability for type II lumps. From the structure of the cobordism groups (2.4) we conclude that additive conserved quantum numbers of type II lumps exist only in $n+1$ dimensions with $n = 4K$. Moreover, for $n = 2, 3, 6$ and 7 no stable type II lumps exist. For the other n values ($n \neq 4K$) lumps have only multiplicative quantum numbers associated with the torsion elements of Ω_n . In particular one expects the existence of stable lumps in $(9+1)$ and $(25+1)$ dimensions of potential interest for string theories.

As was pointed out above, the stability of a type II lump is most easily determined after identifying the points at spatial infinity, so that both the vacuum and the lump become compact manifolds without boundary. Moreover, since the Pontryagin and Stiefel-Whitney numbers of the compactified vacuum $S^3 \times K$ are all zero (because $S^3 \times K$ bounds $B^4 \times K$ where B^4 is the four-dimensional ball), it suffices to determine the Pontryagin and Stiefel-Whitney numbers of the compactified lump itself to decide on its stability.

A particularly simple example of a lump is an exotic sphere $\tilde{S}^n$ in $(n+1)$ dimensions ($n \geq 7$)⁴⁾. Such a sphere is obtained by covering the sphere by too overlapping regions and connecting the two by the topologically non-trivial diffeomorphism π . $\tilde{S}^n$ and S^n are then topologically distinct (though they are homotopic), so $\tilde{S}^n$ may be viewed as a lump in the vacuum S^n (interpreted as the topological compactification of $\mathbb{R}^n$). However, as all Pontryagin and Stiefel-Whitney numbers are zero, we have to conclude that this lump is semi-classically unstable.

Let us now turn to some examples in the standard $(4+1)$ -dimensional Kaluza-Klein theory. As $\Omega_4 = \mathbb{Z}$, i.e., Ω_4 does not contain torsion, it suffices to only consider the Pontryagin number of the manifold $M \cup_g (B^3 \times K)$ (the same remark holds in 8 and 12 dimensions). This number is relatively simple to calculate because there exists an integral representation for it. In fact:

$$\int_{M \cup_g (B^3 \times K)} R \wedge R = \int_M R \wedge R - \int_{B^3 \times K} R \wedge R \quad (3.3)$$

Moreover, in four dimensions the Pontryagin number of a compact manifold without boundary is related to the Hirzebruch signature τ by

$$P_1 = 3\tau \quad (3.4)$$

It is then convenient to express the stability condition in terms of τ rather than P_1 , because the Hirzebruch signature of a manifold with boundary like M or $B^3 \times K$ is a topological invariant. So,

$$\tau(M \cup_g (B^3 \times K)) = \tau(M) - \tau(B^3 \times K) \quad (3.5)$$

and the lump is stable if and only if

$$\tau(M) \neq \tau(B^3 \times K)$$

An example of a type II lump in (4+1) dimensions is the well-known Kaluza-Klein magnetic dipole solution²⁾, the spatial part of which is just the Euclidean Kerr solution⁸⁾

$$dl^2 = \frac{1}{(r^2 - a^2 \cos^2 \theta)} \left[\Delta (dx^5 + a \sin^2 \theta d\psi)^2 + \right. \\ \left. + \sin^2 \theta (r^2 - a^2) d\psi - a dx^5 \right]^2 + (r^2 - a^2 \cos^2 \theta) \left[\frac{dr^2}{\Delta} + d\theta^2 \right] \quad (3.6)$$

where

$$\Delta = r^2 - 2mr - a^2 \geq 0, \quad 0 \leq \theta \leq \pi, \quad 0 \leq \psi \leq 2\pi, \\ 0 \leq x^5 \leq 2\pi K \quad K = \frac{\sqrt{m^2 + a^2}}{2m[m + \sqrt{m^2 + a^2}]}$$

At infinity this manifold coincides with the K-K vacuum, and as pointed out before, we consider the dipole as a manifold with boundary $S^2 \times S^1$. This lump has vanishing signature, $\tau = 0$, but it is non-trivial in the sense that its Euler characteristic is non-zero $\chi = 2^{(9), 2)}$. The vacuum manifold $B^3 \times S^1$ has, however, $\chi = \tau = 0$. Applying the stability criterion we are led to the conclusion that the dipole can decay into the vacuum, in spite of the fact that its Euler characteristic is non-vanishing.

At this point it is appropriate to make a remark which may be relevant to the compactification of the heterotic string from ten to four dimensions over some compact manifold K which could be a Calabi-Yau space or an orbifold. Because $\Omega_6 = 0$, transitions between different K_1 and K_2 are possible according to our criterion. It has been shown that for the Calabi-Yau spaces K_1 the number of

families of fundamental fermions in the effective four-dimensional unified model equals the Euler characteristic χ_i of K_i ¹⁰⁾. Consequently, this number would not be conserved in the transitions described above. In fact, as these manifolds are Ricci flat their action vanishes, and one would naively argue that the proper ground state stable with respect to tunnelling would consist of some superposition of K_i spaces. We should keep in mind though that fields other than the gravitational ones (for example fermions) could eliminate this bizarre possibility. Indeed additional constraints could arise because of the choice of spin-structure as is argued in Section 4. These arguments do not seem to apply here as we are in six dimensions and space-time fermions have the same spin structure. Another more conventional possibility is that the presence of fermionic zero-modes in the interpolating space-time which would make the tunnelling amplitude vanish. This point remains to be clarified.

The gluing procedure U_g (Fig.2) outlined before enables us also to consider the possibility of semi-classical transitions between type I lumps which have the same geometry at infinity. An interesting example is given by the Taub-NUT and the Taub-Bolt monopoles in (4+1) dimensional K-K theory compactified to $R^3 \times S^1$. The spatial metric of a Taub-NUT monopole is^{1),2)}

$$d\ell^2 = V (dr^2 + 4m(1 - \cos\theta)d\phi)^2 + \frac{1}{V} (dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2) \quad (3.7)$$

$$V = 1 + 4m/r$$

where m is related to the radius of a fifth dimension by

$$m = \frac{1}{8} R \quad (3.8)$$

The spatial metric of Taub-Bolt monopole is

$$d\ell^2 = \frac{1}{\Delta} [\Delta (dr^2 + 2N \cos\theta d\psi)^2 + \sin^2\theta (r^2 - 1) d\psi^2 + \Delta (\frac{dr^2}{\Delta} + d\theta^2)] \quad (3.9)$$

where

$$\Delta = r^2 - N^2, \quad \Delta = r^2 - \frac{5}{2}Nr + N^2, \quad N = \frac{1}{4}R$$

The Taub-NUT and Taub-Bolt solution have the same behaviour at infinity, for large r they both are the Hopf bundle $S^2 \times S^1 \rightarrow S^2$. The masses of these monopoles are respectively²⁾

$$M_{\text{Taub-Nut}} = \frac{R}{8}, \quad M_{\text{Taub-Bolt}} = \frac{5R}{32} \quad (3.10)$$

So one should ask whether Taub-Bolt can decay in Taub-NUT. To answer this question we glue together these manifolds at spatial infinity and ask whether $M_{\text{Taub-NUT}} \cup_g M_{\text{Taub-Bolt}}$ has zero Pontryagin number. Equations (3.2a-b) imply that it is sufficient to compare their signatures. One has⁹⁾

$$\tau_{\text{Taub-Nut}} = 1, \quad \tau_{\text{Taub-Bolt}} = 2 \quad (3.11)$$

hence the decay of the Taub-Bolt monopole into the Taub-NUT one is not possible. It is in principle possible, however, that Taub-Bolt decays into Taub-NUT plus some stable not yet found lump with unit Pontryagin number and zero magnetic charge.

Having presented these examples, we now address a more general question. Let n be such that $\Omega_n \neq 0$ and let us choose a particular vacuum space $R^3 \times K$. Is it always possible to construct an n -dimensional lump whose infinity coincides with that of the given vacuum, and which cannot decay into this vacuum? The answer is affirmative and we sketch the construction of the manifold. Let V^n be a compact manifold without boundary belonging to a non-trivial element of Ω_n . Consider now the connected sum of V^n and $R^3 \times K$ (step 1 in Fig. 3). The connected sum $A^n + B^n$ of two manifolds A^n and B^n is the manifold which is obtained by taking out an open ball b^n from A^n and B^n and gluing the remaining parts of A^n and B^n together along the boundary of the removed ball. For the case at hand the resulting manifold $V^n + (R^3 \times K)$ has the required behaviour at infinity. After compactifying spatial infinity (step 2 in Fig. 3) this manifold has the same Pontryagin and Stiefel-Whitney numbers as the manifold V^n we started off with¹¹⁾. Henceforth it cannot decay into the vacuum.

We conclude this section by discussing a point emphasized by Witten⁴⁾, namely that only those lumps should be considered which can be pair created from the vacuum. We claim that any type II lump can be pair created from the vacuum.

The argument runs as follows. Consider a type II lump M^n with infinity of $R^3 \times K$. After compactification we obtain a manifold V^n without boundary with certain Pontryagin and Stiefel-Whitney numbers $(P_1, \dots, P_k; W_1, \dots, W_l)$. Then there exists a manifold $\tilde{V}^n$ with numbers $(\tilde{P}_1, \dots, \tilde{P}_k; \tilde{W}_1, \dots, \tilde{W}_l)$ such that $\tilde{P}_i + P_i = \tilde{W}_a + W_a = 0$. And the manifold $V^n + \tilde{V}^n$ has vanishing Pontryagin and Stiefel-Whitney numbers and can thus be created from the vacuum.

The argument given above cannot directly be carried over to the case of type I lumps, because those have non-trivial behaviour at spatial infinity. A similar construction goes through, however, in certain cases. We illustrate this possibility by considering the creation of a monopole antimonopole pair in the (4+1) dimensional K-K theory. The monopole M corresponds to the Taub-NUT metric (3.7). An antimonopole $\bar{M}$ is obtained by replacing ϕ by $(-\phi)$ in (3.7). Clearly the solutions have equal but opposite magnetic charges. As the boundaries of both M and $\bar{M}$ are locally just $S^2 \times S^1$ we can glue M and $\bar{M}$ together at some part of the boundary to obtain a new manifold U which has the trivial boundary which is $S^2 \times S^1$ globally corresponding to zero magnetic charge. Therefore we may extend U to behave at infinity just like the vacuum. The manifold thus obtained is a monopole-antimonopole manifold, and the question is whether it can decay into the vacuum. To answer the question we have to determine the signature τ . This can be done because there exists a simple expression for τ in terms of the magnetic charge and the number of bolts^{12), 2)}. Since the monopole-antimonopole manifold has neither bolts nor magnetic charge it follows that $\tau = 0$, and therefore it can be created from the vacuum. It is indeed not obvious that the procedure just outlined will in general provide enough information about the lump-antilump manifold to prove that its Pontryagin and Stiefel-Whitney numbers are all vanishing.

4. Effects of Fermions

The concept of oriented cobordism is a good criterion of stability if we only require the space-time to be an orientable differentiable manifold, however, whenever we want to include fermions into the theory we have to require that space-time is a spin-manifold, i.e., that it admits the existence of globally defined spinors. The topological obstruction for this is given by the second Stiefel-Whitney class which has to be zero for a spin manifold. Stability in the presence of fermions requires the extension of the cobordism concept to spin manifolds, i.e., to manifolds with a given spin structure. Then two smooth closed spin manifolds V_n and M_n are in the same spin cobordism class if and only if there exists a $(n+1)$ -dimensional spin manifold W_{n+1} such that:

$$V_n + (-M_n) = \partial W_{n+1} \quad (4.1)$$

where the spin structure on $V_n + (-M_n)$ is induced by the spin structure of W_{n+1} .

The group of equivalence classes defined by (4.1) is the spin cobordism group Ω_n^{spin} ¹³⁾. This cobordism group is determined by Stiefel-Whitney classes and KO theory classes. We can actually get some non-trivial information about Ω_n^{spin} without going through the details of KO theory, by considering its relation with oriented cobordism. In fact, there exists a homomorphism $F_n: \Omega_n^{\text{spin}} \rightarrow \Omega_n^*$ which has a non-zero kernel only in dimensions $(8K+1)$ and $(8K+2)$ ¹³⁾. This result implies in particular that $\Omega_3^{\text{spin}} = \Omega_6^{\text{spin}} = \Omega_7^{\text{spin}} = 0$. On the other hand, the fact that $\text{Ker } F$ can be different from zero in $(8K+1)$ and $(8K+2)$ dimensions does imply the existence of manifolds with certain spin-structure in these dimensions with zero Stiefel-Whitney and Pontryagin numbers, which are not the boundary of any spin manifold. This is of physical interest as it implies the existence of lumps with vanishing topological numbers whose stability is solely due to the fermions. Particular examples of such objects are provided by any non-trivial element of $\text{Ker } F$. These are generated by odd projective spaces ¹³⁾. These lumps should be considered in the same spirit as the exotic spheres of Ref. 4), but now stabilized by the fermions. It is appropriate to add a word of caution concerning the application of spin cobordism. In applying spin cobordism we have tacitly assumed that all fermions in the theory are defined with the same spin structure; from a physical point of view this is not at all necessary. If we do admit spinors with different spin structures (assuming this is possible) the stability analysis may become more complicated than the one dictated by spin cobordism.

ACKNOWLEDGEMENTS

The authors would like to thank Luis Alvarez Gaumé, Viktor Berezin, Bernard Grossman, Vadim Kuzmin and Igor Tkachev for stimulating conversations. Two of us (F.A.B. and V.A.R.) would like to thank the CERN Theory Division for its hospitality.

REFERENCES

- 1) R. Sorkin, Phys. Rev. Lett. 51 (1983) 87.
- 2) D.J. Gross and M.J. Perry, Nucl. Phys. B226 (1983) 29.
- 3) F.A. Bais and P. Batenburg, Nucl. Phys. B253 (1985) 162.
- 4) E. Witten, Comm. Math. Phys. 100 (1985) 197.
- 5) S. Coleman, Proceeding Erice Summer Institute (1975).
- 6) F. Hirzebruch, Topological Methods in Algebraic Geometry, Springer Verlag, New York (1966).
- 7) J. Milnor and J. Stasheff, Characteristic Classes, Princeton University Press, Princeton (1974).
- 8) G.W. Gibbons and S.W. Hawking, Phys. Rev. D15 (1977) 2752.
- 9) T. Eguchi, P.B. Gilkey and A.J. Hanson, Physics Reports 66 (1980) 213.
- 10) P. Candelas, G.T. Horowitz, A. Strominger and E. Witten, Nucl. Phys. B258 (1985) 46.
- 11) Both Stiefel-Whitney and Pontryagin numbers are additive under the connected sum. We thank Y. Vaintrob for a proof of this statement (private communication).
- 12) G.W. Gibbons and S.W. Hawking, Comm. Math. Phys. 66 (1979) 291.
- 13) R. Stong, Notes on Cobordism Theory, Princeton University Press, Princeton (1968).

FIGURE CAPTIONS

- Fig. 1: Joining two topologically distinct manifolds M_1 and M_2 by a smooth manifold.
- Fig. 2: The equivalence between tunnelling with fixed boundary conditions and the existence of manifold with boundary $M \cup_g B^3 \times K$.
- Fig. 3: Constructing the connected sum of V^n and $R^3 \times K$ (step 1) and compactifying spatial infinity (step 2).

Figure 1.

Figure 2.

Figure 3.