ON WEAKER FORCES THAN GRAVITY

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NOTE ADDED IN PROOF

Roberto Battiston points out that, though the experiments of Refs. 19) and 20) extracted the Fourier component with the period $T_{\odot}$ of rotation of the earth relative to the sun, their runs typically lasted less than three days. The period T_M relative to the moon differs from $T_{\odot}$ by a mere 3%. Therefore a Fourier analysis based on T_M may also give significant results. If the periods were identical, two new limits could be added to Fig. 2, by substituting $d_{\odot}$ by the distance to the moon in Eq. (7), and increasing ϵ by the ratio (~ 178) of local accelerations towards the sun and the moon. At the 3σ level the lower of this limits would be slightly below the "Eötvös" dotted line, at $\lambda \sim 4 \cdot 10^9 \text{m}$.

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A B S T R A C T

I briefly review the limits on hypothetical new interactions with a range accessible to detection by laboratory and geophysical experiments, and I elaborate upon the limits at larger distances imposed by a rich body of astronomical observations. The systematic deviations correlated with baryon number that have been recently unearthed from the data of Eötvös' classical experiment, can bravely but consistently be interpreted as a new force whose range is narrowly bracketed somewhere in between Mercury's distance to the Sun and our distance to the Moon. This hypothetical long-range interaction would be five and a half orders of magnitude weaker than gravity. The new force would make like bodies attract, and would therefore not constitute an elegant explanation of why protons have not been seen to decay.

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Much attention has been recently attracted by indications¹⁾⁻⁴⁾ or claims of deviations⁵⁾ from Newtonian gravity. The usual convention is to parametrize the effects as an extra Yukawa potential of relative strength α and range $\lambda = \mu^{-1}$

$$V(r) = V_N + \Delta V(m_i, m_j) = -G \frac{m_i m_j}{r} [1 + \alpha e^{-r/\lambda}] \quad (1)$$

This consuetudinary choice conceals the possibility that α be a material-dependent constant. More than a possibility, this is presumably a necessity. Indeed, it is not known how to construct a consistent field theory wherein the exchange of particles of spin $\neq 2$ would lead to a static potential with couplings strictly proportional to mass. A natural and consistent example of a material-dependent α would correspond to the potential generated by the exchange of some new light vector particle, coupled to a linear combination of the unused conserved global charges of the standard model⁶⁾. The anomaly-free charges of that type are $B - \sum_n L_n$ and $L_n - L_m$, with B baryon number and L_n the lepton number of the n -th generation. For a given isotope of ordinary, electrically neutral, bulk matter the most general linear combination of these charges reduces to a linear combination of neutron number N and proton (or electron) number Z . Thus, for a new vectorial gauge force one expects a somewhat modified Eq. (1):

$$V = -G \frac{m_i m_j}{r} [1 + \kappa \beta_i \beta_j e^{-r/\lambda}] \quad (2a)$$

$$\beta_i \equiv (N_i \cos \theta + Z_i \sin \theta) / [m_i / 1 \text{amu}] \quad (2b)$$

where V is the potential between two atoms of atomic mass m_i, m_j , and θ, κ, λ are the parameters describing the most general new vector force of this quasi-standard type⁶⁾. A vector force between identical objects is necessarily repulsive, $\kappa < 0$ in Eq. (2a). For a force due to the exchange of particles of spin $\neq 1$, current conservation and Gauss' theorem are not valid concepts, and one cannot expect the couplings to different atoms to be precisely given by a simple charge such as $N \cos \theta + Z \sin \theta$, with θ a universal constant, but a coupling of roughly this form would not constitute a surprise; a particle with isoscalar couplings to protons and neutrons, for instance, would generate an approximate $N+Z$ coupling. For all elements but hydrogen, the "charges" β_i of Eq. (2b) are very similar functions of θ , except for a sensitive region around $\cos \theta = -\sin \theta$. A material independent new force, as in Eq. (1), may therefore be a good first approximation, that we shall adopt in our discussion when not explicitly considering the comparison between different materials.

Laboratory limits on $\alpha(\lambda)$ from refinements and variations of the classical experiment of Cavendish are shown in Fig. 1. The curves labelled (a), (b) and (c) are from Refs. 7), 8) and 9), respectively. These limits are slightly different for positive and negative α , and are shown separately in Figs. 1a and 1b. The geological limit "f" is from measurements of the acceleration of gravity as a function of depth, h , in a mine shaft¹⁰⁾. More recent geological data have been interpreted as observations¹⁾⁻⁴⁾, rather than limits, on deviations from Newton's law. These results are shown as the dashed line in Fig. 1b. Figure 1c is a redrawing, on a linear α -scale, of this interesting part of Fig. 1b. It shows the values of $\alpha(\lambda)$ that best describe the results of Holding and Tuck²⁾, gathered with special care in an Australian mine. Also shown are the characteristic sizes of the error bars^{4),6)} induced by uncertainties in the local density distribution. As discussed in much detail in Ref. 6), these systematic uncertainties invalidate the conclusion that a new repulsive (i.e., vector) force with an "intermediate" range of hundreds of metres has been convincingly unearthed. The limit on $\alpha(\lambda)$ relevant at the largest distances covered in Fig. 1 stems from a comparison of satellite and land-surface data, that I proceed to discuss along with other limits at astronomical distances.

There exists a very considerable body of knowledge on how precisely planets, moons, asteroids, artificial satellites and even pulsars follow the Keplerian laws, modified by general relativity in those cases in which an apsidal precession is observable. I have borrowed or converted the most relevant data I know of into "one standard deviation" limits on the non-Newtonian $\alpha(\lambda)$ of Eq. (1), and plotted the results in Fig. 2. I have not bothered to consider minor differences between theoretical and experimental central values nor to distinguish between the very similar cases of positive and negative α : in Fig. 2, only the $\alpha > 0$ results are shown. Most of the limits on α for $\lambda > 1$ km shown in Fig. 2 stem from comparisons of measurements of the acceleration of different falling or orbiting bodies. At an altitude h over the surface of a spherical body of mass M , radius R , and uniform density ρ , the acceleration $g(h)$ corresponding to a potential of the form Eq. (1) is such that

$$(R+h)^2 g(h) = G M f(R/\lambda, h/\lambda) \quad (3a)$$

$$f(x,y) = 1 + \frac{3}{2} \alpha (x-1 + (1+x)e^{-2x})x^{-3} (1+x+y)e^{-y} \quad (3b)$$

The uniform density approximation is abundantly justified for $\lambda \gg R$, the situation in which we shall extract the long-range limits most relevant to our

discussion. Suppose measurements of $g(h)$ are made at two significantly different mean distances, h_1 and h_2 , via observations of two probes, such as an artificial satellite and our moon. Assume that the Newtonian ($\alpha=0$) expectation of a constant $(R+h)^2 g(h)$ in Eqs. (3) is satisfied with a relative precision $1\pm\epsilon$. The corresponding limit on α is

$$|\alpha(\lambda)| \leq \epsilon / |f(R/\lambda, h_1/\lambda) - f(R/\lambda, h_2/\lambda)| \quad (4)$$

It is this kind of limit, presently discussed in more detail, that is shown in Fig. 2 as the curves labelled "Earth surface/Lageos, Moon/Lageos, Moon/Earth surface, Lunar surface gravity, Mercury fly-by and Venus fly-by".

Lageos is an artificial satellite orbiting at an average $h = 5900$ km, that has been carefully tracked for over 80 months. The results of these observations can be expressed as the value of $GM_{\oplus}$ ($\oplus$ stands for the Earth) or as a calculated, purely Newtonian, gravitational acceleration $g(0)$ at zero height over the equator of an equipotential biaxial ellipsoid "equivalent" to our planet. The results^{11),12)} are:

$$GM_{\oplus} = 398600.4342 \pm 0.002 \text{ km}^3/\text{s}^2 \quad (5a)$$

$$g(0) = 9.780324(3) \text{ m/s}^2 \quad (5b)$$

The value of $g(0)$ from direct measurements on the surface of the Earth is $g(0) = 9.780326 \text{ m/s}^2$, with an error that Rapp¹²⁾ estimates to be such that the agreement with Eq. (5b) is at the level of 0.5 ppm. The curve^{13),14)} labelled "Earth Surface/Lageos" in Figs. 1 and 2 is based on these results, Eq. (4), and a slightly more conservative $\epsilon = 10^{-6}$. The value of $GM_{\oplus}$ reported by Williams and Dickey from tracking our natural satellite¹²⁾ is $398600.444 \pm 0.010 \text{ km}^3/\text{s}^2$. This compares with the Lageos result of Eq. (5a) to a relative precision $\epsilon \approx 2.5 \times 10^{-8}$. The corresponding limit is labelled "Moon/Lageos" in Fig. 2. The limits labelled "Mercury fly-by and Venus fly-by" are taken from Ref. 15). They follow once more from Eq. (4) with h_1 = minimum altitude of the spacecraft encounter and $h_2 = 10^8$ km, a typical distance to other bodies whose perturbative effects may be used to determine $GM(\text{planet})$. These limits extend rather flatly towards large distances because, unlike the previously discussed ones, they compare results at two very different values of h . The "Lunar surface gravity" limit in Fig. 2 I have also directly borrowed from Mikkelsen and Newman¹⁵⁾.

The secular precessions of the orbits of Mars, Mercury, Icarus and the binary pulsar PSR 1913+16 agree remarkably well with the predictions of general relativity. Let $a(e)$ be the semi-major axis (excentricity) of the elliptical orbit. For the non-Newtonian acceleration of Eqs. (3), the perturbatively predicted precession per orbital round-trip is

$$\delta\omega \simeq -\pi a (df/dh)|_{h=a} \simeq -\pi \alpha \left(\frac{a}{\lambda}\right)^2 e^{-\frac{a}{\lambda}} [1 + O(\alpha, e^2)] \quad (6)$$

The values of the larger orbital axis, a , and of the precision, ϵ , to which theory and observation agree for the value of $\delta\omega/2\pi$, are reported in Table I. The corresponding limits on $\alpha(\lambda)$ are graphed in Fig. 2. The data¹⁹⁾ on PSR 1913+16 are now precise enough to determine the masses of the pulsar and its companion, m_p and m_c , from the gravitational red-shift and time dilation γ , and the period derivative $\dot{P}$ (assumed to be due to gravitational radiation). The result can be used to predict the mean rate of periastron advance, $\langle\dot{\omega}\rangle$, to ~3%, in agreement with the much better determined experimental value. To do all this, use

$$\begin{aligned} \gamma &= G^{2/3} c^{-2} e (P/2\pi)^{1/3} m_c (m_p + 2m_c) (m_p + m_c)^{-4/3} = 0.00438(12) \text{ s} \\ \dot{P} &= -(192\pi G^{5/3}/5c^5) (P/2\pi)^{-5/3} (1-e^2)^{-7/2} (1+73e^2/24+37e^4/96) m_p m_c (m_p + m_c)^{-1/3} = -2.40 \pm 0.09 \\ \langle\dot{\omega}\rangle &= 3 G^{2/3} c^{-2} (P/2\pi)^{-5/3} (1-e^2)^{-1} (m_p + m_c)^{2/3} = 4.2263(3) \text{ deg/yr [EXP.]} \end{aligned}$$

and $P = 27906.98163(2) \text{ s}$, $e = 0.617127(3)$. The uncertainty in the prediction of $\langle\dot{\omega}\rangle$ translates into $\delta\omega/2\pi \sim 3 \times 10^{-7}$, used in Table I and Fig. 2.

The last two limits reported in Fig. 2 stem from Eötvös-like experiments with the Sun as the source of the pull. Roll, Krotkov and Dicke²⁰⁾ report that aluminum and gold fall towards the Sun with the same acceleration, to a precision better than $\epsilon = 10^{-11}$. This corresponds, in the notation of Eqs. (2), to a limit

$$|\kappa \beta_{\odot} [\beta_{Al} - \beta_{Au}]| \left(1 + \frac{d_{\odot}}{\lambda}\right) e^{-d_{\odot}/\lambda} < \epsilon \quad (7)$$

where $d_{\odot}$ is our distance to the Sun. The specific limit for an assumed coupling to baryon number ($\cos\theta = \sqrt{2}/2$, $\alpha \approx \kappa/2$) is labelled "Dicke" in Fig. 2. The limits on α for hypothetical couplings to neutron or proton number ($\cos\theta = 1, 0$), not shown in the Figure, are approximately two orders of magnitude better. This reflects the fact that the baryon number and atomic mass of the stable elements are, to a very good approximation, proportional to one another. Labelled "Panov" in Fig. 2 is the $\cos\theta = \sqrt{2}/2$ limit implied by the experiment of Braginskii and Panov²¹⁾, who claim that the Sun-ward accelerations of aluminum and platinum coincide to about one part in 10^{12} . Limits on $\alpha(\lambda)$ at distances outside the range covered in Fig. 2 do not constrain hypothetical new forces to be weaker than gravity is.

Much ado about a possible "fifth force" was recently stirred by an article of Fischbach et al.⁵⁾. These authors argued that the marginal geological indications, in Fig. 1c, of a deviation from Newtonian gravity were due to the same force responsible for a minute but systematic material-dependent correction to the gravitational pull of the Earth, buried unnoticed since the turn of the century amidst the results of the classical experiment of von Eötvös and collaborators²²⁾. It has been repeatedly pointed out that the expressions of Ref. 5) imply a different magnitude and sign for the two allegedly related effects. Even more serious a criticism is the observation^{23), 6)} that in an ideal flat terrain in isostatic equilibrium, the effects of an intermediate range ($0(1m) < \lambda < R_{\oplus}$) fifth force would be invisible to Eötvös, though they could be observable in a Galileo experiment truly and directly comparing the accelerations of different falling bodies, rather than measuring torques. The point, briefly reviewed, is the following. Eötvös' torsion balance was aimed at the detection of a possible material-dependent ratio of inertial to gravitational mass. Let $\vec{g}_N$ be the local Newtonian acceleration of gravity, and $\vec{a}$ the local inertial axifugal acceleration, due to the rotation of the Earth. Let $\vec{F}_a = m_a^{(g)} \vec{g}_N + m_a^{(i)} \vec{a}$, in an obvious notation, be the total force acting on each of the two balanced substances, $a = 1, 2$. If the equivalence principle is incorrect, $\vec{F}_1$ and $\vec{F}_2$ are not parallel, and a differential torque develops on the hanging wire of the torsion balance as it is bodily rotated by 180° around its vertical axis. Figure 3 makes this point visual. Let us now accept the equivalence principle and introduce a material-dependent intermediate or short-range fifth force ($\lambda < R_{\oplus}$, the radius of the Earth). A flat terrain in isostatic equilibrium, such as the surface of a quiet sea, has its layers laid perpendicular to the local "vertical" direction $\vec{g}_N + \vec{a}$. A fifth force exponentially damped with depth and exerted by this underlying material is directed along $\vec{g}_N + \vec{a}$. To first order, it cannot exert a torque around the suspension wire of a balance, that also hangs along $\vec{g}_N + \vec{a}$. An observed material dependent torque, if due to a short-range force, would have to be attributed to local inhomogeneities. Although oral tradition has it that Eötvös himself weighed over 300 pounds, new experiments would be needed to substantiate the claim of the existence of a new intermediate or short-range force. A long-range material-dependent force ($\lambda \gtrsim R_{\oplus}$), contrarywise⁶⁾, would be detectable with an Eötvös balance, independently of the local density inhomogeneities. Except in the unlikely circumstance that the terrestrial distribution of mass and of fifth force source be very different, the new

long-range force would be directed very close to $\vec{g}_N$ and act as an apparent correction to the gravitational mass. A long-range force can therefore be discussed in the same terms used by Eötvös and collaborators to present their results: differences in the accelerations with which different materials would fall, normalized to their average acceleration, $\Delta\gamma \approx (g_1 - g_2)/g_N$.

Fischbach and collaborators⁵⁾ brought to the general attention a most intriguing correlation between the deviations from null results in Eötvös' experiment, and the baryon number difference between the materials being compared. This is shown in Fig. 4a for those pairs of materials whose exact composition is definitely not debatable (the exceptions are snakewood and tallow), and that were available to Eötvös in more than minute amounts (the exception is a speck of RaBr_2 , that he briefly borrowed from the Curies). The vertical axis in Fig. 4a is proportional to $\Delta\gamma$, the horizontal one, to differences $\Delta(B/m[\text{amu}])$ in the ratio of baryon number to atomic mass, or $\beta_1(45^\circ) - \beta_2(45^\circ)$, in the notation of Eq. (2). The figure is not quite the same as the corresponding one in Ref. 5). In plotting Fig. 4a I have followed the correct procedure advocated by Yeong Kim²⁴⁾, that is, I have used the raw data on $\Delta\gamma$, and taken into account in the calculation of $\Delta(B/m)$ the contribution of the brass containers often used to hold the different materials. Similarly, I have plotted $\Delta\gamma$ against $\Delta(Z/m[\text{amu}])$ in Fig. 4b. The results shown in Fig. 4 are listed in Table II. It is abundantly clear to the naked eye that there is a significant correlation between $\Delta\gamma$ and baryon number differences, while there is no correlation between $\Delta\gamma$ and atomic (or lepton) number. A fit to the data, linear in baryon and lepton number^{*)}, yields:

$$\Delta\gamma = (5.63 \pm 0.68) 10^{-6} \Delta\left(\frac{B}{m}\right) + (9.4 \pm 8.8) 10^{-8} \Delta\left(\frac{Z}{m}\right) \quad (8)$$

For an assumed long-range force origin of the effect, this corresponds, in the notation of Eqs. (1) and (2), to the following strength relative to gravity (between two atomic mass units) and N/Z mixing angle:

*) For a fit with an extra constant material - independent term, c , the question would arise of the sign with which each pair of materials enters the calculation (i.e., $\Delta\gamma_{12} = +c + \Delta B_{12}$ versus $\Delta\gamma_{21} = +c + \Delta B_{21}$, other pairs fixed). Unlike Fischbach et al., one must try all possibilities and quote the least significant results. A fit with a constant term is totally unjustified by theory. Moreover, Eötvös and collaborators flipped the positions of every pair of test bodies, which should preclude a systematic origin of a constant term.

$$\alpha \simeq \kappa \beta_{\otimes} \beta_i \simeq \kappa/2 \simeq 5.63 \cdot 10^{-6} \quad (9a)$$

$$t_{n\theta} = 1.016 \pm 0.125 \quad (9b)$$

This new force would have the same sign as gravity; the coupling to ordinary matter would be compatible with an N+Z form. The dashed line in Fig. 4a has the slope of the first term of Eq. (8), and a nominal and overwhelming 8σ significance. The dotted line, almost indistinguishable from the dashed one, is the best fit of the form $\Delta\gamma = a\Delta(B/m)$. R.H. Dicke has warned fifth-force enthusiasts that there is a rough correlation between inverse density and (B/m) for several of the materials employed by Eötvös, and that a density-dependent sampling of temperature gradients in Eötvös' apparatus could produce a spurious effect of the observed form. Though Eötvös tried to avoid this and other systematic effects in his surprisingly asymmetric balance by exchanging the positions of the pairs of test bodies, only time and new experiments can definitely tell.

The value of α of Eq. (9a) is shown in Fig. 4 as the dotted segment. Lo and behold, there is a very narrow window of ranges, around $\lambda \sim 4 \times 10^9$ m, for which this very long shot interpretation of Eötvös' results is not excluded by the astronomical data, a point already mentioned en passant in Ref. 6). The carrier of this attractive force would have to be a scalar or tensor particle of mass $\mu = \lambda^{-1} \simeq 5 \times 10^{-17}$ eV, coupled to an atomic mass unit of an "average" substance with a strength f , such that

$$f^2/4\pi = G(1\text{amu})^2 \alpha \simeq 3.3 \cdot 10^{-44} \quad (10)$$

A new force this feeble: five and a half orders of magnitude weaker than the gravitational attraction between two protons, or forty-one and a half orders of magnitude weaker than their electromagnetic repulsion, is not particularly constrained by theory and/or by other past laboratory experiments.

Several groups have announced their intention to resuscitate active searches for forces weaker than gravity. No doubt the experimental situation will soon be richer, and may hold surprises. In this connection, I cannot refrain from pointing out that a long-range force interpretation of the systematic baryon number effect in Eötvös' data would have been much much more attractive if the

force had been repulsive. It is not only that baryon number is one of the global charges, or combinations thereof, waiting to be gauged in the standard model. The really amusing effect of a $\mu \sim 5 \times 10^{-17}$ eV vector boson, coupled to B with a strength comparable to Eq. (10), would be to explain why protons have not been seen to decay at the originally expected rate. In first order of perturbation theory, the ratio of proton decays with and without the emission of one of the these hypothetical vector bosons would nominally be of order $(f^2/4\pi)m_p^2/\mu^2 \gg 1$ (the $\mu \rightarrow 0$ singularity²⁵⁾ is due to the production of longitudinally polarized vector particles in a process that does not conserve the charge to which they couple). Though the perturbative calculation does not make sense, it indicates that one or more of these otherwise undetectable particles would be emitted in practically every proton decay, a possibility that is excluded by the energy-momentum conservation constraints presently imposed on the proton-decay candidates. Some channels, such as $p \rightarrow e^+ +$ one single unobservable particle, could still be detectable above the unavoidable neutrino-induced background. We know so little about baryon number violation that the emission of new unobservable particles in proton decay should perhaps be entertained as a possibility, in spite of its admittedly weak motivation.

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	Mercury	Mars	Icarus	Pulsar
a (km)	7×10^7	2.5×10^8	3×10^8	$(9.1 \pm 1.2) \times 10^5$
ϵ	4×10^{-10}	9×10^{-9}	10^{-8}	$\sim 3 \times 10^{-7}$
Refs.	16), 15)	17), 15)	18), 15)	19), 15)

Table I - Semi-major axis, a, and precision, ϵ , of the comparison of experiment and post-Newtonian prediction of the orbital precession, $\delta\omega/2\pi$; from a compilation by Mikkelsen and Newman¹⁵⁾. The pulsar results are improved and updated using the data analysis of Weisberg and Taylor¹⁹⁾.

Substance Nr1	Substance Nr2	$10^8 \Delta\gamma_{12}$	$10^3 \Delta(B/m[amu])_{12}$		$10^2 \Delta(Z/m[amu])_{12}$	
			not corrected	corrected	not corrected	corrected
$Ag_2SO_4 + 2FeSO_4$	$2Ag + Fe_2(SO_4)_3$	0.0 ± 0.1	0	0	0	0
Mg-Al (Z?)	Pt	0.4 ± 0.1	~ 0.50	~ 0.5	~ 0.82	~ 0.82
Cu	Pt	0.4 ± 0.2	0.93	0.93	0.56	0.56
H ₂ O	Cu	-0.5 ± 0.1	-1.71	-0.87	1.0	0.50
$CuSO_4 \cdot 5(H_2O)$	Cu	-0.3 ± 0.1	-0.86	-0.55	0.52	0.33
29.58% $CuSO_4 \cdot 5(H_2O)$ 70.42% weight H ₂ O	Cu	-0.4 ± 0.1	-1.42	-0.87	0.85	0.51
$Mg_3Si_2O_5(OH)_4$	Cu	-0.2 ± 0.1	-0.74	-0.44	0.42	0.29

Table II - Data from Eötvös and collaborators. $\Delta\gamma$ are raw data. The corrected ΔB and ΔZ take into account the brass containers.

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FIGURE CAPTIONS

Fig. 1 Various limits on deviations from Newtonian gravity, parametrized by the extra Yukawa potential of Eq. (1). The dashed or punctulated regions are excluded.

Fig. 2 Extension of the limits of Fig. 1 to astronomical distances.

Fig. 3 Principle of Eötvös' experiment. A fifth force with a range much shorter than the radius of the earth would, barring local irregularities, be directed along $\vec{g}_N + \vec{a}$ and not exert a torque on the hanging wire.

Fig. 4 Differences of normalized gravitational accelerations for a variety of pairs of substances, as measured by Eötvös and collaborators:
a) versus differences in the ratio of baryon number to atomic mass;
b) versus differences in the ratio of atomic number to mass.

- Figure 1 -

- Figure 2 -

- Figure 3 -

- Figure 4 -