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SPECTROSCOPY OF MESONS CONTAINING LIGHT QUARKS (u,d,s) OR GLUONS

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ABSTRACT

An overview will be given about the increase of experimental knowledge on mesons which are built by light quarks. This includes quite naturally also those mesonic states containing gluons and those with an exotic internal quark structure like $q\bar{q}-q\bar{q}$.

The resulting mesonic 'zoo' will be compared with predictions from actual phenomenological ansatzes like potential models, bag model, QCD sum rules and also more general approaches like QCD lattice gauge theories.

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1. INTRODUCTION

Our knowledge of particle physics has made considerable progress within the last ten years. Whilst already in the sixties the introduction of quarks was extremely useful to classify the particle zoo and to predict new particles, questions concerning the interactions between these new objects remained unanswered.

The attempts for an answer were inspired by the success of Quantum Electrodynamics (QED): ground and excited states -- energy spectra -- of for instance atoms -- composed of "point-like" constituents with charge e -- were explained by the movement of these constituents under the action of a coulombic potential. Quantum field theory describes 'interaction' by 'photon-exchange'. The concept allows exact prediction of electromagnetic interaction of charged particles up to the highest energies and momentum transfers (i.e. smallest structure sizes) nowadays possible. Due to its success attempts were made to transfer this concept of a quantum field theory to strong interactions of hadrons, the interactions of hadrons between each other and, more basically, the interaction of the hadronic constituents, the quarks. The result, Quantum Chromodynamics (QCD), exhibits many parallels to QED but also severe differences: again point-like constituents, the quarks, exist with the colour charge e_c and hadronic masses are explained by bound states formed by the quarks moving under the regime of the corresponding potential. But in contrast to QED with a charge singlet e^- therefore one neutral massless exchange particle, the photon, QCD works with a colour-charge triplet leading to eight colour-charged massless exchange particles, the gluons. This causes "large-distance-corrections" to a "small distance coulomb-like" potential: unlike QED where the surrounding cloud of a bare charge -- consisting of photons, partially dissociated into charged fermion pairs -- leads to a decreasing coupling constant with increasing distance, the net charge of the gluons dictates a behaviour just the other way around -- asymptotic freedom at small distances, and confinement -- increasing coupling constant -- at large distances.

The examination of the resulting theoretical predictions concerning the basic interactions of the quarks and their materialization into observable final states was (and is) a major exercise to be solved in experimental high-energy physics. As a result QCD is well accepted to be the theory

of the dynamics of subnuclear structures and nuclear forces might be understood -- cum grano salis -- as "van der Waal's forces" of QCD.

This cannot mean that the degree of understanding is sufficient to predict bound states of $q\bar{q}$ pairs or even their lifetimes or branching ratios with a comparable quality to that of, for instance, the hydrogen atom under the regime of QED and the Coulomb potential: no complete knowledge of the effective potential between the quarks and its distance dependence exists. Nevertheless, the predictive power of several different models looks encouraging -- as will be explained later. All models assume for "large" distances an attractive force between the quarks -- with an increasing strength for increasing distances -- caused by gluon colour.

This could explain why even with the most powerful accelerators no free quark nor no net colour charge could be released from its confining prison. Only their "fragments" of "normal" colour neutral particles appear in the detectors. But their behaviour proves the quark hypothesis in so far as the fragments of one quark remember the original quark by aligning to its axis (Fig. 1c). The quantitative description of this so-called "jet-phenomenon" and its energy dependence show that QCD might be treated in the framework of a perturbation theory like QED, at least at high energies.

Discovery and analysis of these hard (pulling quark and antiquark apart) processes are a merit of storage ring experiments with corresponding high energies^{1,2)}. Deep inelastic scattering experiments with comparable c.m.s. energies have proven the real existence of quarks in nucleons and determined their momentum distributions.

The study of such hard processes was the main aim of experimental high-energy physics in the seventies. But also the "small distance" spectroscopic component moved forward with the discovery of heavy quarks and the connected spectroscopy.

Up to 1974 the hadronic world was built by the (up, down, strange) quarks and their antiparticles. Then the discovery of the ψ ^{3,4)} (i.e.

charmed quarks) and in 1977 of the Υ^5 (bottom quarks) and their excited states allowed to study quantitatively the behaviour of heavy $q\bar{q}$ systems in bound states. The heavier the quarks, i.e. the smaller the ratio of "kinetic to total energy" and therefore the distance between q and $\bar{q}$, the bigger became the similarity of these states to their QED analogue, positronium, i.e. the similarity of the $q\bar{q}$ potential and Coulomb's law. For positronium the ratio $X_{e^+e^-} = \text{binding energy/mass energy}$ turns out to be $\approx 10^{-5}$; at the Υ mass this ratio has (after a suitable choice of the quark masses) a value of $X_{b\bar{b}} = 0.1$. This value increases for charmonium to $X_{c\bar{c}} = 0.2$ and for light quarks a rough guess yields $X_{u\bar{u}} \approx 0.6-0.8$ assuming constituent quark masses (see Section 2.2.1).

In this picture of a $q\bar{q}$ potential with a short range coulombic part and a long range confining part one can easily imagine: the lighter the quarks are the more ground states $q\bar{q}$ systems and even more excitations fall into the regime of confinement. For u, d, s states a nonrelativistic treatment is no longer justified and the QED-like "one gluon" exchange as a description for the interaction is no longer valid.

Nevertheless, an understanding of light quarks as building blocks of our world is needed, the knowledge of their spectroscopy is essential to evaluate the potential especially in the non-coulombic regions. But here new severe problems arise and complicate "normal" hydrogen-atom-like spectroscopy for $q\bar{q}$ systems:

Intrinsic problems

Bound states of light quarks are expected to cover the mass range from 140 MeV (m_π) to ~ 2500 MeV — most of them having decay widths in the order of 100 MeV.

- So insufficient resolution of experiments and analysis procedures can cause confusion in disentangling pure $q\bar{q}$ states of a certain configuration of spin and relative angular momentum and thus J^{PC} .

- Moreover the corresponding wave functions of observed 'bumps' could be superpositions of contributions from different origin, for instance mixtures of different angular momentum states like in the deuteron.

QCD -- specific problems

- $q\bar{q}$ states could have gluonic admixtures or form bound $q\bar{q}g$ states.
- The net colour charge of gluons gives rise to the possibility of having bound states of pure glue (glueballs).
- Colour neutral exotic quark molecules like $(q\bar{q}q\bar{q})$ are predicted.

All of these are expected (see Sections 3, 4, 5) in the same mass range as conventional states and so create additional classification problems.

The main aim of this report is to review recent experimental results on the spectroscopy of light quarks. Experimental facts concerning:

- discovery and classification of states with different J^{PC} (L,S,...);
- decay modes and lifetimes, will be presented.

This includes quite naturally an experimental status report on the recent knowledge of the "QCD intrinsic" additional states mentioned above. Even though a detailed overview of the corresponding "theoretical" increase in knowledge is beyond the scope of this article -- the reader is referred to Refs. 6 and 7 for instance, the different ways to achieve prediction capability will be mentioned and basic ideas as well as main results and their accordance or discrepancy to experimental knowledge will be briefly sketched.

The article is arranged as follows: Section 2 deals with $q\bar{q}$ objects. After a short description with respect to the criteria of the standard quark model and spectroscopic terminology the experimental knowledge concerning these objects is reviewed underlining especially new results, for instance, on the search for radial excitations.

In the comparison of the resulting scenario with predictions a special emphasis is given to phenomenological potential models. This is because this approach illuminates the analogy between QED and QCD and allows for a 'walk' starting from positronium via heavy (non relativistic) $q\bar{q}$ systems

to those composed of light (relativistic) quarks. Other approaches -- Bag models and QCD sum rules -- will be briefly mentioned referring the reader to more specific reviews.

This restriction is even more true for a fourth source of predictions, the explicit calculation of the QCD-dominated behaviour of quarks and gluons on a finite space time grid. These lattice gauge theories might overcome a common disadvantage of the other methods, the dependance on arbitrary parameters.

Section 3 deals with glueballs. After some basic remarks on glueball spectroscopy and an (incomplete) collection of predictions obtained by the same four methods mentioned above the experimental situation concerning glueballs will be presented. A 'warrant of caption' of all nowadays actual candidates will be given.

Sections 4,5, contain hermaphrodites ($q\bar{q}g$, ...) and baryonia ($qq\bar{q}q$), respectively.

The abundance of material concerning the spectroscopy of light quarks requires restriction. So $q\bar{q}$ states, glueballs, $q\bar{q}g$ states or baryonia will be subject of discussion but not baryons or dibaryons. These objects and their spectroscopy are excellently reviewed in Refs. 8,9,10, respectively.

2. $q\bar{q}$ SYSTEMS

2.1 Classification of $q\bar{q}$ systems

The fundamental triplet of flavour SU(3) (u, d, s) is multiplied with the anti-triplet ($\bar{u}, \bar{d}, \bar{s}$) yielding nine different combinations: ($u\bar{u}, u\bar{d}, u\bar{s}, d\bar{d}, d\bar{u}, d\bar{s}, s\bar{s}, s\bar{u}, s\bar{d}$). According to the rules to combine fundamental representations in the framework of SU(3) one gets: $3 \otimes 3 = 1 \oplus 8$. Whilst the six edges are unique the splitting of the three ($I_3 = S = 0$) combinations might be seen most easily by considering the subgroup SU(2). Here (see Ref. 11, Section 2.2.1.3 for a proof) the isodoublet ($\begin{smallmatrix} u \\ d \end{smallmatrix}$) transforms identically to ($\begin{smallmatrix} \bar{d} \\ -\bar{u} \end{smallmatrix}$) under isorotations. The transition

$$\begin{pmatrix} u \\ d \end{pmatrix} \xrightarrow{G} \begin{pmatrix} \bar{d} \\ -\bar{u} \end{pmatrix} ; \quad \begin{pmatrix} \bar{u} \\ \bar{d} \end{pmatrix} \xrightarrow{G} \begin{pmatrix} d \\ -u \end{pmatrix} \quad (1)$$

is described by the G parity, i.e. charge conjugation C times rotation around I_2 in isospin space:

$$G = C \cdot e^{i\pi I_2} \quad I_2 = \frac{1}{2} \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} . \quad (2)$$

$\begin{pmatrix} u \\ d \end{pmatrix} \otimes \begin{pmatrix} \bar{d} \\ -\bar{u} \end{pmatrix}$ yields three antisymmetric states with respect to G and one symmetric:

$$A: (u\bar{d}, -\bar{u}d, \frac{1}{\sqrt{2}} |u\bar{u}-d\bar{d}\rangle) ; \quad S: \frac{1}{\sqrt{2}} |u\bar{u}+d\bar{d}\rangle .$$

If, for instance, one identifies the isotriplet with the $\pi^{\pm,0}$ mesons and introduces the third s quark with $I(s, \bar{s}) = 0$, the new states could only be constructed from the G symmetric isosinglet state: the symmetric $q\bar{q}$ combination is after proper normalization:

$$\frac{1}{\sqrt{3}} |u\bar{u} + d\bar{d} + s\bar{s}\rangle .$$

It is identified with the heavier of two candidates with same spin-parity as the π 's: the $\eta'(958)$. Orthonormality fixes the SU(3) wave function of the third object -- the $\eta(549)$:

$$|\eta\rangle = \frac{1}{\sqrt{N}} (|u\bar{u}\rangle + |d\bar{d}\rangle + a|s\bar{s}\rangle)$$

$$\langle\eta|\eta\rangle = 1, \quad \langle\eta|\eta'\rangle = 0 \rightarrow N = 6, \quad a = -2 .$$

The larger strange quark mass causes at least partly the mass difference between π , η and η' . (See Section 2.2.2). The physical particles might be directly related to the SU(3) wave functions as is summarized in Table 1.

The physical states functions of corresponding " η " and " η' " in other multiplets could appear to be linear combinations of the corresponding SU(3) states. In case of ideal mixing one $I = S = 0$ state turns out to be purely strange, the other purely made from u and d only. This holds at least approximately for the vector mesons where one isosinglet, the $\omega(783)$, has about the same mass as the π^0 partner, the $\rho(765)$. The other isosinglet, the $\phi(1019)$, is heavier and nearly purely $s\bar{s}$. These physical states might be described by their SU(3) states and a mixing angle θ :

$$\phi = \phi_1 \sin \theta - \phi_8 \cos \theta = \frac{1}{\sqrt{3}} |u\bar{u}+d\bar{d}+s\bar{s}\rangle \sin \theta - \frac{1}{\sqrt{6}} |u\bar{u}+d\bar{d}-2s\bar{s}\rangle \cos \theta$$

$$\omega = \phi_1 \cos \theta + \phi_8 \sin \theta .$$

Ideal mixing:

$$\phi \stackrel{!}{=} s\bar{s} = |u\bar{u} + d\bar{d}\rangle \left(\frac{\sin \theta}{\sqrt{3}} - \frac{\cos \theta}{\sqrt{6}} \right) + |s\bar{s}\rangle \left(\frac{\sin \theta}{\sqrt{3}} + \frac{2}{\sqrt{6}} \cos \theta \right) \rightarrow \theta = 35^\circ .$$

Radiative decays: especially the M1 transitions $^3S_1 \rightarrow ^1S_0 + \gamma$, see Fig. 46 for nomenclature, are well suited to test the mixing conditions, for example for vector mesons.

Because of the necessary spin rearrangement these decays directly measure the magnetic moments of the quarks involved:

$$\vec{\mu}_q = g \cdot \mu_q \cdot \vec{S} \quad g \approx 2 \quad \text{for} \quad |\vec{S}| = \frac{1}{2} \quad \mu_q = \frac{e_q \cdot h}{2\pi m_q \cdot c}$$

$$\mu_k = \frac{e \cdot h}{2\pi m_p \cdot c} = 3.152 \times 10^{-8} \frac{\text{eV}}{\text{T}}$$

$$\chi_{\text{prot}}\left(\frac{1}{2}, +\frac{1}{2}\right) = \frac{2}{3} [\chi_{uu}(1,1) \cdot \chi_d\left(\frac{1}{2}, -\frac{1}{2}\right)] - \frac{1}{3} [\chi_{uu}(1,0) \cdot \chi_d\left(\frac{1}{2}, +\frac{1}{2}\right)]$$

i.e.

$$\mu_p = \frac{4}{3} \mu_u - \frac{1}{3} \mu_d = 2.79 \mu_k$$

$$\mu_n = -\frac{1}{3} \mu_u + \frac{4}{3} \mu_d = -1.86 \mu_k .$$

Moreover:

$$\mu_\Lambda = \mu_s \approx 0.6 \mu_d .$$

Assuming $m_u = m_d$ one has $\mu_u = -2\mu_d$ and hence $\mu_n/\mu_p = -2/3$ in excellent agreement with experiment: $\mu_n/\mu_p = -0.68$.

Having determined μ_u, μ_d, μ_s and taking the SU(3) wave functions as quoted before the matrix elements for the decay can be explicitly calculated¹¹⁻¹⁴). For instance, one expects in case of ideal mixing in the 3S_1 multiplet:

$$M(\phi \rightarrow \pi\gamma):M(\omega \rightarrow \pi\gamma):M(\rho \rightarrow \pi\gamma) = 0:3:1$$

and measures for the corresponding widths

$$\Gamma(\phi \rightarrow \pi\gamma):\Gamma(\omega \rightarrow \pi\gamma):\Gamma(\rho \rightarrow \pi\gamma) = 0.08 \pm 0.03:12.1 \pm 1.4:1$$

i.e. a somewhat low ρ width and some deviation from ideal mixing. See Refs. 15 and 16 for an experimental review and Refs. 17 to 19 for recent measurement of radiative decays of vector mesons.

Summing over the three spin alignments of the $\omega(783)$ meson one gets an absolute prediction for the $(\omega \rightarrow \pi\gamma)$ decay matrix element:

$$M^2(\omega \rightarrow \pi^0 \gamma) = \frac{2}{3} \mu^2 \quad \text{for } \mu = \frac{e \cdot \hbar}{2\pi m_q \cdot c}$$

(the factor 2/3 is due to the only two helicities of a real photon ± 1). Measuring $\Gamma(\omega \rightarrow \pi\gamma) \approx 0.86$ MeV the constituent quark mass $m_u = m_d$ can be calculated to yield: $\mu \approx \mu_p$, i.e. $m_u = m_d \approx m_p/3 \approx 300-400$ MeV.

A sensible analyzer of the mixing situation is the ratio of the radiative widths of charged and neutral K^* . This ratio can be easily related to the magnetic moments of the quarks involved

$$\begin{aligned} \alpha &= \frac{\Gamma(K^{*0} \rightarrow K^0 \gamma)}{\Gamma(K^{*+} \rightarrow K^+ \gamma)} = \left(\frac{\mu_s + \mu_d}{\mu_s + \mu_u} \right)^2 \quad (\text{Ref. 11}) \\ &= 4 \text{ for } \mu_s = \mu_d \text{ [ideal SU(3)]} \\ &= 0.25 \text{ for } m_s \rightarrow \infty \\ &= 2 \text{ for } m_s \sim 400 \text{ MeV} \end{aligned}$$

$$m_{u,d} = 300 \text{ MeV}$$

and has been recently measured^{17,23}) to a value

$$\alpha = 2.29 \pm 0.29 .$$

See Ref. 23 for a more detailed calculation of α and possible discrepancies between experiment and too naive expectations. Reference 421 discusses the comparably high analytic power of meson decays into two photons.

After this excursion further useful rules for the quotation of quantum numbers of mesons = $q\bar{q}$ systems shall be given. The spherical harmonics behave under a 180° rotation around the y axis, like

$$Y_l^0 \xrightarrow{R} (-1)^l Y_l^0 .$$

From this and the intrinsic parities of fermions and antifermions it follows

$$P(q\bar{q}) = (-1)^{l+1} . \quad (3)$$

A corresponding formula for the C parity might be derived from positronium. Its total wave function is a product of space, spin and charge contributions and behaves therefore under exchange of particles:

$$\alpha = (-1)^l \times (-1)^{S+1} \times C .$$

The "two-photon" decay of an $l = 0$ state requires

$$J = S = 0 , \quad C = +1 .$$

The "three-photon" decay of an $l = 0$ state requires

$$J = S = 1, \quad C = -1 .$$

This is equivalent to

$$C = (-1)^{L+S} . \quad (4)$$

Applying a 180° rotation to a $\chi(I, I_3=0)$ state in the isospin space. [see Eq. (2)] one gets

$$\chi(I, I_3=0) \xrightarrow{R} (-1)^I \chi(I, I_3=0) .$$

Invariance of strong interactions under isorotations allows a generalization to all members of an isomultiplet

$$G|\chi\rangle = C \cdot (-1)^I |\chi\rangle$$

$$G = C \cdot (-1)^I, \text{ respectively, } G = (-1)^{L+S+I}. \quad (5)$$

The two-photon decay of the π^0 requires $C(\pi^0) = +1$, i.e. $G(\pi^0) = -1$. Defining $C|\pi^\pm\rangle = -C|\pi^\mp\rangle$ (Ref. 130, Appendix E) this can be generalized to

$$G(\pi) = -1.$$

G parity is a multiplicative quantum number

$$G(n\pi) = (-1)^n \quad (5a)$$

and is defined only if I is an integer number.

The spectroscopic notation for a $q\bar{q}$ system $^{2S+1}L_J$, fixes J^{PC} . (Anti)-quarks have spin 1/2, $q\bar{q}$ therefore can only couple to $S = 0$ or 1 with $C \cdot P = -$ or $+$, respectively.

The two possible multiplets with $L = 0$ are:

the pseudoscalar mesons: $J^{PC} = 0^{-+}; {}^1S_0$;

the vector mesons: $J^{PC} = 1^{--}; {}^3S_1$.

Table 2 summarizes the quantum numbers of $L = 1, 2$ multiplets. A graphic representation is given in Figs. 3a-d (P waves) and Figs. 3e-h (D waves). Uncertainties in these figures as well as candidates for free places in the D and F multiplets will be discussed in Section 2.2 in more detail.

The natural parity sequence is formed by multiplets with $P = (-1)^J$, i.e. $0^+, 1^-, 2^+, \dots$, the unnatural sequence has $P = (-1)^{J+1}$.

States with quantum numbers $0^{--}, 0^{+-}, 1^{-+}, 2^{+-}, 3^{-+}, \dots$, cannot be formed by a quark and an antiquark system only. Discovery of particles with these quantum numbers is a very conclusive argument for the exotic nature of this state (glueballs, meiktons, ...). This statement is weakened by models like Bag models (see Section 2.3.2) due to the introduction of an additional parameter.

Adding other flavours the dimension of the graphic representation (Figs. 1,3) would be increased by one per flavour, i.e. consideration of charm would lead to three-dimensional arrays.

Colour is unimportant in this context, all three colours contributing with same weight in "painting" the constituents.

2.2 Spectroscopy of light quarks -- experimental situation

In view of the abundance of data for the well-known $L = 0$ families only the search for radial excitations will be reviewed. For ($L > 0$) multiplets, the emphasis lies on recent discoveries of missing objects or progress in knowledge concerning ambiguous candidates.

2.2.1 (1^{--}) mesons: 3S_1

Vector mesons have the same quantum numbers as the photon, they are therefore easily produced either in (e^+e^-) annihilation or in real photoproduction.

Very useful for the description of both production mechanisms is the so-called Vector Dominance Model^{20,247)}. Simply expressed, VDM describes vector meson production by the dissociation of the photon into a vector meson with an individual coupling constant -- equivalently dissociation into a $q\bar{q}$ pair of corresponding charge and coupling depending (in first approximation) only on the quark charge.

Consequences of VDM for both production types are:

- i) $e^+e^- \rightarrow V$. $\sigma(e^+e^- \rightarrow V)$ is simply calculated by the product of the usual $1/s$ dependence of lepton pair production and a Breit-Wigner resonance shape²¹⁾

$$\sigma(e^+e^- \rightarrow V) = \frac{3\pi}{s} \frac{\Gamma_{ee} \cdot \Gamma}{(m_V - \sqrt{s})^2 + \frac{\Gamma^2}{4}}$$

and with $m_V = \sqrt{s}$

$$\Gamma_{ee} = \frac{m_V^2 \Gamma}{12\pi} \sigma_{\text{peak}}$$

or by explicit integration

$$\Gamma_{ee} = \frac{m_V^2}{6\pi^2} \int \sigma_{ee \rightarrow V \rightarrow \text{channel}} \cdot \frac{1}{\Gamma_{\text{channel}}} . \quad (6)$$

The connection between Γ_{ee} and the coupling is calculated by explicit evaluation of the corresponding diagram shown above²²⁾

$$\Gamma_{ee} = \frac{\alpha^2}{12} \frac{4\pi}{\gamma_V^2} m_V . \quad (7)$$

Table 3 shows Eq. (7) for some vector mesons.

The "van Royen-Weisskopf" formula relates the leptonic width of a vector meson and the wave function of the corresponding $q\bar{q}$ system at the origin, expressing the point-like coupling $\gamma \rightarrow q\bar{q}$ ²³⁾

$$\Gamma_{ee} = \frac{16\pi\alpha^2 \cdot 3Q^2}{3m_V^2} |\psi_{q\bar{q}}(0)|^2 . \quad (8)$$

The m_V^{-2} dependence results from the $q^{-4} = m_V^{-4}$ dependence of the photon propagator and a m_V^2 dependence from two-body phase space. A factor α per γ -coupling as well as a colour factor 3 are taken into account. Q is the effective charge of the quarks involved. Reference 402 quotes relativistic corrections for Eq. (8) and justifies the replacement of m_V by $2m_q$ -- especially relevant for radial excitations.

Assuming flavour independence of the $q\bar{q}$ potential and roughly equal vector meson masses (ρ, ω, ϕ) one expects in Eq. (8):

$$\frac{|\psi_\rho(0)|^2}{m_\rho^2} = \frac{|\psi_\omega(0)|^2}{m_\omega^2} = \frac{|\psi_\phi(0)|^2}{m_\phi^2} = \text{const.}$$

and therefore $\Gamma_{ee}(\rho, \omega, \phi) \sim Q^2$:

$$\Gamma_{ee}(\rho) = \left[\frac{1}{\sqrt{2}} \left(\frac{2}{3} + \frac{1}{3} \right) \right]^2 ; \quad \Gamma_{ee}(\omega) = \left[\frac{1}{\sqrt{2}} \left(\frac{2}{3} - \frac{1}{3} \right) \right]^2 ; \quad \Gamma_{ee}(\phi) = \left(-\frac{1}{3} \right)^2$$

$$\Gamma_{ee}(\rho) : \Gamma_{ee}(\omega) : \Gamma_{ee}(\phi) = 9 : 1 : 2$$

and one finds indeed by experiment

$$= 8.8 \pm 2.6 : 1 : 1.7 \pm 0.4 .$$

ii) $\gamma p \rightarrow V p$. Here, the VDM picture suggests that the vector meson -- stemming from the photon -- is scattered on the target and becomes real by the corresponding four momentum transfer. If 'initial state' and 'final state' vector mesons are identical (this assumption is not necessarily true: 'off diagonal contributions', especially relevant for radial excitations), vector meson photoproduction behaves like elastic vector meson nucleon scattering and has therefore the properties of a diffractive process:

- (nearly) energy independent cross-section;
- helicity transfer in the s channel from γ to V (SCHC) with well defined decay angular distributions (see Ref. 303 for small corrections);
- steep fall of cross-section with respect to four momentum transfer t ($\gamma \rightarrow V$): $d\sigma/dt \sim e^{-b \cdot |t|}$ [$b \sim R^2$; R radius of γN interaction²⁵¹⁾].

Being quantitative, $d\sigma/dt$ ($\gamma N \rightarrow V N$) is determined by the product of coupling and the corresponding hadronic cross-section²⁴⁷⁾

$$\frac{d\sigma}{dt} (\gamma N \rightarrow V N) = \alpha \frac{\pi}{\gamma_V^2} \frac{d\sigma}{dt} (V N \rightarrow V N) . \quad (9)$$

This modifies after application of the optical theorem (assuming a purely imaginary scattering amplitude)

$$\frac{d\sigma}{dt} (\gamma N \rightarrow VN)_{t=0} = \frac{\alpha}{16} \frac{1}{\gamma_V^2} \sigma_{\text{tot}}^2(VN) \quad (10)$$

or

$$\sigma(\gamma N \rightarrow VN) = \frac{\alpha}{16b} \frac{1}{\gamma_V^2} \sigma_{\text{tot}}^2(VN) . \quad (10')$$

$\sigma_{\text{tot}}(VN)$ -- the total hadronic cross-section, which would be measured with a beam of stable vector mesons -- might be estimated by the vector meson quark content. For instance:

$$\sigma_{\text{tot}}(\phi, N) = \sqrt{[\sigma_{\text{tot}}^2(K^+N) + \sigma_{\text{tot}}^2(K^-N) - \sigma_{\text{tot}}^2(\pi^-N)]} . \quad (10'')$$

With this parametrization photoproduction of ϕ mesons is quantitatively well described by Eq. (10') over the whole γ energy range nowadays available (Fig. 4).

Putting Eqs. (6), (7), and (10') together one gets

$$\sigma(e^+e^- \rightarrow V \rightarrow \text{channel}) = \frac{32\pi^3 \alpha \cdot b}{m_V} \frac{1}{\sigma_{\text{tot}}^2(VN)} \sigma(\gamma N \rightarrow VN \rightarrow \text{channel } N) . \quad (11)$$

This formula allows in the framework of VDM a direct comparison between both vector meson production mechanisms i) and ii). Using Eq. (11) for well known objects quark models might be tested. For instance

$$\sigma_{\text{tot}}(\rho) = \frac{1}{2} [\sigma_{\text{tot}}(\pi^+p) + \sigma_{\text{tot}}(\pi^-p)] = 27 \text{ mb } (p_\pi = 10 \text{ GeV}/c)$$

and from Eq. (11)

$$\sigma_{\text{tot}}(\rho) = \left(\frac{32\pi^3 \cdot 8}{137 \cdot 0.765} \frac{10^{-29}}{0.18 \cdot 10^{-30}} \right)^{1/2} = 65 \text{ GeV}^{-2} = 26 \text{ mb} .$$

On the other hand, formula (11) allows a prediction for (e^+e^-) annihilation by inserting the photoproduction result and vice versa. It can serve as a cross-check, if one neglects 'off diagonal production mechanisms' in photoproduction.

Compared to heavy quark spectroscopy two facts imply additional problems in the light quark domain:

- Typical widths for light quark mesons are larger than those for heavy quark mesons, mixing can occur between members of different families with same J^{PC} : $(^3S_1) \leftrightarrow ^3D_1$.
- Presence of concurring non-resonant mechanisms (threshold effects) could fake candidates or distort existing ones by interference effects.

This led to the following experimental situation for the search on radially excited vector mesons: in the $I = 1/2$ sector two resonant structures in 1^- amplitudes in a partial wave analysis of the reaction $K^-p \rightarrow K^0\pi^+\pi^-n$ ^{24,277)} at 1410 and 1790 MeV could be interpreted as members of $(^3S_1)$ and 3D_1 family, respectively.

Searching for their ($I = 0,1$) partners plenty of candidates exist seen by one experiment but not confirmed by others (especially not by the corresponding complementary one: $e^+e^- \leftrightarrow \gamma p$). Typical examples are published by Refs. 25 to 27.

Even today, 1986, this puzzle is not solved completely:

The ρ family

The ground state, the $\rho(765)$, is well known. It decays mainly into $\pi^+\pi^-$. Radial excitations are expected to decay into pionic channels too and G parity requires the number of π 's to be even.

Assuming a flavour independent $q\bar{q}$ potential one could simply transfer the mass difference $M(2S-1S)$ (nomenclature see Fig. 47) found to be ~ 500 MeV for both $c\bar{c}$ and $b\bar{b}$ systems to light quarks and look for a radially excited ρ at $765 + 500 = 1265$ MeV: in Ref. 28a a possible first observation of a corresponding object was reported.

A deviation of the π form factor from a properly chosen ρ shape²⁸⁾ in the reaction $e^+e^- \rightarrow \pi^+\pi^-$ was explained by the existence of a $\rho'(1250)$. See Fig. 7b for recent remeasurements.

The ρ contribution to electromagnetic nucleon form factors shows a bump-dip structure which could be due to a radial recurrence of the ρ at 1250 MeV³²⁴⁾.

Photoproduction experiments²⁹⁻³¹⁾ found in the reaction $\gamma p \rightarrow \omega \pi^0 p \rightarrow \pi^+\pi^-\pi^0\pi^0 p$ an enhancement at 1250 MeV too. But the necessary J^P determination could either not be done at all³⁰⁾ or only on the premises of an 's-channel-helicity-conserving' production mechanism²⁹⁾.

A recent analysis³²⁾ -- with higher statistics and independent of such assumptions -- will be presented.

The analysis is based on ~8000 events of the reaction $\gamma p \rightarrow \pi^+\pi^-\pi^0\pi^0 p$, generated with a tagged γ beam of $20 < E_\gamma < 70$ GeV. The final state pions are measured by a magnetic spectrometer ($\pi^\pm$) and a photon detector (π^0) guaranteeing the elasticity of the selected events.

The (4π) mass spectrum (Fig. 5) exhibits two resonant structures. The heavier one is identified to be the $\rho'(1600)$ (see below). It disappears completely demanding a 3π substate forming an $\omega(783)$ -- dashed area in Fig. 5 -- whilst the first structure couples to $\omega\pi^0$ obviously quite strongly. A fit gives a mass of 1250 MeV and a FWHM of 200 MeV over non-resonant background of about 20%.

The J^P analysis takes account of $0^-, 1^-, 1^+$ contributions with a P wave between ω and π^0 for negative and a S or D wave for positive parity.

The decay angular distribution $I(\Omega, \Omega_H)$ depends on polar and azimuthal angles $\cos \theta, \phi$ of the object decaying into $\omega\pi^0$ and the angles $\cos \theta_H, \phi_H$ describing the ω decay into $\pi^+\pi^-\pi^0$ in properly chosen coordinate systems. A strong $\sin^2 \theta_H$ contribution suggests, in analogy to direct photoproduction of ω mesons, an ω helicity of ± 1 ,

whereas the decay $0^- \rightarrow \omega(1^-) + \pi^0$ (0^-) would require an ω helicity of 0 (Ref. 34, formula 4.31).

Also other projections give information for J^P determination: a strong $\cos 2\theta_H$ component of the ϕ_H distribution argues for either SCHC production of 1^- or non-SCHC production of 1^+ . The 'sin ϕ shape' of the ϕ distribution indicates interference of ($\omega\pi^0$) helicities $\pm 1 \leftrightarrow 0$ proving the presence of non-SCHC mechanisms.

For a detailed analysis the decay angular distribution was evaluated in terms of moments $H_{(1mLM)}$

$$I(\Omega, \Omega_H) \sim \sum_{\substack{1m \\ LM}} H_{(1mLM)} D_{Mm}^{*L1}(\Omega, \Omega_H)$$

(where D_{Mm}^{L1} are Wigner functions).

Allowing for 0^- , 1^- , 1^+ contributions to the spin parity of the object, the system is completely determined by 25 independent functions $H_{(1mLM)}$; these fix not only the relative intensities of 0^- , 1^- , 1^+ but also other process details like spin density matrix elements, decay amplitudes or the D/S wave ratio in case of 1^+ : all in all 17 free parameters. The 25 moments are evaluated by

$$H_{(1mLM)} = \sum \text{Acc}^{-1} D_{Mm}^{L1}(\Omega, \Omega_H) ;$$

(Acc = acceptance of experimental apparatus).

A fit of the free parameters to the result was done in three steps to reduce the number of free parameters from 17 to 15.

1. Step: $0^-, 1^-, 1^+$; Result: N_{0^-} small, $N_{1^+} \approx N_{1^-}$ within large errors.

2. Step: $N_{0^-} = 0$; Result: $N_{1^+} > N_{1^-}$ at 1.2 GeV, 1^- consistent with SCHC.

3. Step: $1^- = \text{SCHC}$; Result: see Fig. 6.

The resonance structure at 1250 MeV is due to the $B(1235)$, the D/S ratio of 0.25 at 1250 is consistent with the value already known for the B^{3s} and the 1^- intensity exhibits no peaks between 0.8 GeV and 1.8 GeV.

Due to this extensive analysis the existence of a $\rho'(1250)$ in the decay channel $\pi^+\pi^-\pi^0\pi^0$ is not completely disproved but made highly improbable.

The mass distribution of the reaction $e^+e^- \rightarrow \pi^+\pi^-\pi^0\pi^0$ is shown in Fig. 7a. It contains no prominent structures at 1250 MeV and could be explained by contributions of the $\rho'(1600)$ and a tail of $\rho(765)$ both decaying into $\omega\pi^0$ (see dashed and full curves in Fig. 7a, respectively).

Moreover, this distribution has the disadvantage to be a collection of points from different experiments ($m_{4\pi} < 1.4$: OLYA, $m_{4\pi} > 1.4$ GeV: FRASCATI, see Ref. 35 for references). Therefore, normalization problems might cause steps. This could be also true for very recent data from the reaction $e^+e^- \rightarrow \pi^+\pi^-$ collected from OLYA/Novosibirsk (Ref. 403) ($\sqrt{s} < 1.4$ GeV) and DM2/DCI (Ref. 404) ($1.4 < \sqrt{s} < 2.1$ GeV: Fig. 7b. Martinovic⁴⁰⁵) interprets these data by two resonances: $m_1 = 1217$ MeV, $\Gamma_1 = 380$ MeV, $m_2 = 1531$ MeV, $\Gamma_2 = 500$ MeV. Another interpretation is discussed in context with the $\rho(1600)$.

In the reactions

$$\gamma p \rightarrow \pi^+\pi^-p \quad (\text{Ref. 36})$$

$$\gamma p \rightarrow \pi^+\pi^-\pi^+\pi^-p \quad (\text{Ref. 37})$$

$$e^+e^- \rightarrow \pi^+\pi^-\pi^+\pi^- \quad (\text{Ref. 38})$$

no evidence for a $\rho'(1250)$ is found. See also Refs. 48 and 50 on the search for a $\rho'(1250)$ in the reaction $\gamma p \rightarrow \eta\pi^+\pi^-$; for instance, a bump around 1200 MeV is seen in the $1^- \eta\rho$ wave" in a "Zemach" analysis of photo-induced $\eta\pi\pi$ systems. But the observed structure can also well be explained by a decay of the $\rho(770)$ tail into $\eta\pi\pi$ ⁴⁸⁾.

There is no doubt on the existence of the $\rho'(1600)$ though mass and width from different analyses are quoted very different: see Fig. 8 for a review.

It is difficult to fix parameters of such a broad resonance because it might be distorted by channel-specific background mechanisms: a confusion between a radial ρ excitation (3S_1)' and a member of the 3D_1 family (J^{PC} also 1^{--}) cannot be excluded -- some experiments^{39,40)} even observe -- with low statistics -- a double structure in the 1400 to 1700 MeV region.

In addition, for example, in photoproduction [$\gamma p \rightarrow \rho'(1600)p \rightarrow 4\pi^\pm p$] non-resonant contributions like $(\rho\pi\pi)$ or (4π) phase space or a non-resonant $A_1\pi$ threshold enhancement^{41,42)} could broaden the structure. This would explain why in three experiments analysing the reaction $\gamma p \rightarrow \rho'(1600)p \rightarrow \pi^+\pi^-p$ ⁴³⁻⁴⁵⁾ the observed resonance shows a width of 250 MeV, distinctly smaller than the result of $4\pi^\pm$ (see Fig. 8).

Also the observed energy dependence of $\sigma(\gamma p \rightarrow \rho'(1600) \rightarrow 4\pi^\pm p)$ (Table 4) points to the interpretation that the $\rho(1600)$ is not a simple resonance produced in a 'naive' "on-diagonal photoproduction mechanism".

The analysis of the second bump (see Fig. 5) in the total mass spectrum of $\gamma p \rightarrow \pi^+\pi^-\pi^0\pi^0 p$ ⁴⁶⁾ favours a $J^P = 1^-$ assignment and therefore identification with the $\rho'(1600)$. A nice by-product of this analysis is an answer to the question whether the object decays via $\rho^0\pi\pi$ or $A_1\pi$, where " $\pi\pi$ " stands for a $I^G = 0^+ \pi\pi$ S wave and " $A_1\pi$ " for an $I^G = 1^- \rho\pi$ S wave (see the following sections). The channel $\pi^+\pi^-\pi^+\pi^-$ could only show the fact of the presence of a $\rho(765)$ in the final state but could not distinguish between these possibilities.

Only in the first case also in $\pi^+\pi^-\pi^0\pi^0$ neutral ρ 's should appear in the final state:

$\rho(1600)$ does not decay into $A_1^0\pi^0$ but decays into $A_1^\pm\pi^\mp$, i.e. $\rho^\pm\pi^0\pi^\mp$.

The experimental result⁴⁶⁾

$$\rho^0(1600) \rightarrow \frac{\rho^0 \pi^0 \pi^0}{\rho^\pm \pi^\mp \pi^0} < 0.1$$

favours the second possibility.

Having measured the $\rho'(1600)$ photoproduction in one experimental set-up in three different channels^{37,43,46)} and knowing the upper limit in another possible decay mode:

$$\sigma[\gamma p \rightarrow \rho'(1600)p] \rightarrow \eta \rho^0 p < 0.028 \text{ } \mu\text{b}^{48)}$$

one is able to quote the total photoproduction cross-section to be:

$$\sigma[\gamma p \rightarrow \rho'(1600)p] = 1.3 \pm 0.4 \text{ } \mu\text{b} .$$

This value is not accounting for a possible contribution of diffractively photoproduced g mesons⁴⁸⁾ (see Section 2.2.7):

$$\gamma p \rightarrow gp \sim 300 \text{ nb} .$$

Moreover, the photoproduced $\rho'(1600)$ is assumed to be a pure (3S_1)'-state: see below. With the partial leptonic width from the reaction $e^+e^- \rightarrow \rho(1600) \rightarrow 4\pi^\pm$

$$B \cdot \Gamma_{ee} = 2.75 \text{ keV}^{38)}$$

one gets a total leptonic width of 6.6 keV: $1.3/0.54 \times 2.75 = 6.6$. The ratio $\Gamma_{ee}(\rho')/\Gamma_{ee}(\rho) = 1$ has to be compared with the one measured for ψ/ψ' and Y/Y' :

$$\frac{\Gamma_{ee}(\psi')}{\Gamma_{ee}(\psi)} = \frac{1.72 \text{ keV}}{4.67 \text{ keV}} = 0.37 ; \quad \frac{\Gamma_{ee}(Y')}{\Gamma_{ee}(Y)} = \frac{0.51 \text{ keV}}{1.35 \text{ keV}} = 0.38 .$$

This fits qualitatively quite well into the picture of potential models [see Eq. (8) with the acc. remarks and Table 5 in Section 2.3] with a coulombic and a linear part for the $q\bar{q}$ potential. For a linear potential all leptonic widths of the radial excitations should be equal, the deviation for the Y and ψ could be explained by the influence of the coulombic part to the (heavy) quarks. This influence is missing for the light quarks forming the $\rho'(1600)$.

The leptonic width measures [via $|\psi(0)|^2$] directly the 'size' of a vector meson -- as does implicitly the slope b of the t distribution $d\sigma/dt \sim e^{-bt}$. If $\Gamma_{ee}(\rho) = \Gamma_{ee}(\rho')$ then $b(\rho) = b(\rho')$. Indeed, $b(\rho) \sim 8 \text{ GeV}^{-2}$, $b(\rho') = 5-7 \text{ GeV}^{-2}$ to be compared with $b(\phi) \sim 5 \text{ GeV}^{-2}$, $b(\psi) \sim 3 \text{ GeV}^{-2}$ (Ref. 31).

Quantitative conclusions are nevertheless dangerous because of model-uncertainties as well as experimental ones. For instance Refs. 50,51 quote a value of $B \cdot \Gamma_{ee} [\rho'(1600) \rightarrow 4\pi]$ 30% lower than Ref. 38. A careful measurement of $\Gamma_{ee}(\rho')$ or $\Gamma_{\mu\mu}(\rho')$ is essential for disentangling (3S_1)' and 3D_1 contributions from the $\rho'(1600)$ bump and thus urgently needed.

These two results on Γ_{ee} lead according to Eq. (11) to a hadronic cross-section of 13 mb or 16 mb, i.e. only 50% of that valid for the $\rho(765)$ in contradiction to expectations of a simple additive quark model. Possible explanations are:

α) Reference 52 describes a possible q^2 dependence of the product $(\sigma_{\text{tot}} \cdot 1/\gamma_V^2)$ by modification of Eq. (9) to

$$\frac{d\sigma}{dt} (\gamma N \rightarrow VN) = \lambda^2(q^2) \frac{\alpha \cdot \pi}{\gamma_V^2} \frac{d\sigma}{dt} (VN \rightarrow VN) \quad (9')$$

and determines λ by unitarity relations for the reactions $\gamma p \rightarrow Vp$ and $Vp \rightarrow Vp$:

$$\lambda(q^2 = m_\phi^2) = 0.9$$

(this value was implicitly taken into account for the calculation of the VDM prediction in Fig. 4 by the choice of parametrization of σ_{tot})

$$\lambda(q^2 = m_\psi^2) = 0.7 .$$

Therefore, a λ value of 0.7-0.8 for the $\rho(1600)$ should be expected, explaining well the 50% reduction of $\sigma_{\text{tot}}(\rho')$ with respect to σ_{tot} and should therefore be applied also for corresponding ω' and ϕ' calculation.

β) radial excitations of vector mesons are photoproduced by 'off diagonal terms' with the ground state as incoming meson: the copious elastic photo-production of B,H mesons for instance (see Section 2.2.3, especially the schematic production diagram) can be used as an argument for the relevance of this production type also for radial excitations. Equation (11) is then no longer applicable.

Different production types in photo and e^+e^- production could lead to different interference patterns between the participating resonances. Taking for instance the ρ , a ρ' at 1450 MeV - 3S_1 - and a third resonance at 1700 MeV - 3D_1 - the disagreeing shapes $e^+e^- \rightarrow \pi^+\pi^-$ (Fig. 7b) and $\gamma p \rightarrow \pi^+\pi^-p$ (see Refs. 18,276 for instance) can be nicely explained: they are caused by a "+-+" and a "++-" interference of the three states, respectively⁴⁰⁶). Indeed Ref. 276 finds an opposite sign between ρ and ρ' production amplitudes analysing photoinduced $\pi^+\pi^-$ systems.

It is almost impossible to draw firm quantitative conclusions from the existing data especially from the poor quality of knowledge concerning Γ_{ee} , i.e. $|\psi(0)|^2$. But a three resonance scenario with interference mechanisms as sketched is probably not far away from the truth:

- disagreeing e^+e^- and γp data can be "unified";
- a reason why 3D_1 states are unobserved so far ($I = 1,0$) is easily explained;
- the first radial excitation of the ρ is expected at ~ 1430 MeV, thus $m(\rho') - m_\rho > m(\psi') - m(Y') - m(Y)$ as expected from naive potential models;
- excellent accordance with more sophisticated calculations (Table 13).

Quite naturally all these statements are valid also for:

The ω, ϕ family

The negative G parity of the ω family requires an odd number of π 's for a pionic decay. Also the ϕ family -- mainly decaying into kaonic states KK, K^*K -- could be observed in $3\pi, 5\pi, \dots$ final states, either by OZI violating decays according to Fig. 9 or by an intrinsic presence of u and d quarks in the ϕ 's due to a small deviation from ideal mixing.

The recent search for radially excited ω 's and ϕ 's -- mainly performed at the DCI storage ring and (with real photons) at the Ω spectrometer (WA4, WA57) (see Refs. 38, 53, and 54, 55 for reviews) -- was concentrated on the following channels:

- i) $\pi^+\pi^-\pi^0$
- ii) $\pi^+\pi^-\pi^+\pi^-\pi^0$
- iii) $K\bar{K}$
- iv) $K^*K, K^*K\pi$.

Though both experiments observe narrow ($\Gamma \sim 100$ MeV) structures in the mass range between 1.65 GeV and 1.75 GeV the results are partially controversial. In particular, a positive identification of an object by both types of experiments in the same channel is missing and so is the possibility to determine via Eq. (11) $\sigma_{\text{tot}}(Vp)$ and thus the quark content, i.e. the membership to the ω' or ϕ' family. An important role in this search play the two reactions

$$e^+e^- \rightarrow \omega\pi^+\pi^- \rightarrow \pi^+\pi^-\pi^+\pi^-\pi^0$$

and

$$e^+e^- \rightarrow K^*K \rightarrow K_S^0 K^\pm \pi^\mp$$

analysed with the magnetic spectrometer DM1 at DCI. Figures 10a, 10d show the corresponding distributions both peaking at 1.65 GeV with a width of 140 MeV and 190 MeV, respectively. The full line in Fig. 10 corresponds to a global fit of an interference mechanism to all analysed channels (also $K_S^0 K_L^0, K^+ K^-$).

$\omega\pi\pi$ should be -- in analogy to $\rho'(1600) \rightarrow \rho\pi\pi$ -- the main decay mode of an ω' . But the leptonic width $B \cdot \Gamma_{ee}$ of $140 \text{ eV} \pm 46$ -- corrected for $\omega\pi^0\pi^0$ under assumption of an $\pi^0\pi^0 S$ wave -- is too small for an ω' interpretation. Moreover the observed $\omega\pi\pi$ width is considerably lower than the theoretical prediction arguing for an ω' width of same size as the $\rho'(1600)$ width⁵⁶⁾. So these arguments make a ϕ' interpretation of the $\omega\pi\pi$ object more plausible.

The analysis of $e^+e^- \rightarrow K^* K \rightarrow K_S^0 K^{\pm} \pi^{\mp}$ leads to a $B \cdot \Gamma_{ee}$ of 600 eV (corrected for unobserved $K^* K$ channels), i.e. a large width for an OZI allowed decay $\phi' \rightarrow K^* K$. Moreover, the authors state the fact of dominance of the $K^{*0} K^0$ decay mode over $K^{*+} K^-$: Fig. 11. This could be explained by an interference of $K^* K$ final states from $\rho'(1600)$ and ϕ' as shown in Fig. 12.

Correcting for this effect $\Gamma_{ee}(\phi') \times B(K^* K)$ has a value of 1.2 keV. Assuming a contribution of 150 eV each for 3π , 5π decay mode and another 10% from $K^+ K^-$ one gets a total leptonic width $\Gamma_{ee}(\phi')$ of 1.65 keV, larger than $\Gamma_{ee}(\phi) = 1.3$ keV.

This points against a pure ϕ' classification. The authors favour an interference model of ρ' , ω' , ϕ' which assumes the same parameters for ω' and ρ' and a small but non-vanishing u,d admixture of the ϕ' . Within this model the observed $\omega\pi\pi$ signal is due to a positive interference of ω' and ϕ' . The model prediction for $B(\omega\pi\pi) \cdot \Gamma_{ee}(\phi')$ is < 50 eV and therefore smaller than the observed one; for $B(K^* K) \times \Gamma_{ee}(\phi')$ the model predicts 500 eV resulting to a total leptonic width:

$$\Gamma_{ee}(\phi') = 0.7 \text{ keV}, \text{ i.e. } \frac{\Gamma_{ee}(\phi')}{\Gamma_{ee}(\phi)} = 0.54.$$

Given

$$\frac{\Gamma_{ee}[\rho'(1600)]}{\Gamma_{ee}[\rho(765)]} = 1 \quad \text{and} \quad \frac{\Gamma(\psi')}{\Gamma(\psi)} = 0.37$$

this value seems somewhat low. But the model fits all analysed channels of the DCI-DM1 experiment in a satisfying manner; full lines in Figs. 10a-10d.

Reference 57 reports on the discovery of a narrow resonance also at 1.65 GeV in the reaction $e^+e^- \rightarrow \pi^+\pi^-\pi^0$ (Fig. 13a). A later result from the DM2 group⁵⁸⁾ (with an experimental set-up now supplemented by a photon detector) declares this result to be wrong but publishes a $\sigma(e^+e^- \rightarrow \pi^+\pi^-\pi^0)$ distribution not being inconsistent with a (broader) resonance hypothesis at about the same mass (Fig. 13b).

Resonance or not, in both cases the partial leptonic width $B(3\pi) \cdot \Gamma_{ee}$ is small and an incorporation into the scheme presented is well possible.

This is not the case for an object at 1.67 GeV, 160 MeV wide found with high significance (totally 6σ) by the WA4 and WA57 experiments^{59,60} in the reaction $\gamma p \rightarrow \pi^+ \pi^- \pi^0 p$ [Figs. 14a⁵⁹ and 14b⁶⁰]. Both experiments measure the events completely -- tagging system for the beam photon, magnetic spectrometer for charged particles and photon detector for π^0 's. The production cross-section of the object -- quoted by counting the events in the peak -- is 100 ± 20 nb and therefore one order of magnitude larger than one would expect for a ϕ' . This expectation is based on two arguments:

- Eq. (11) quoted with the parameters of Ref. 57 for $e^+ e^- \rightarrow \pi^+ \pi^- \pi^0$ leads to $\sigma(\gamma p \rightarrow \omega/\phi p \rightarrow \pi^+ \pi^- \pi^0 p) = 10$ nb.
- The absolute number of ϕ' decaying into $\pi^+ \pi^- \pi^0$ would then be larger than or equal to the number of ϕ in the same channel: $\sigma(\gamma p \rightarrow \phi p \rightarrow \pi^+ \pi^- \pi^0 p) = 90 \pm 5 \pm 25$ nb measured by the same experiment⁶¹.

The ratio

$$\frac{\sigma(\gamma p \rightarrow V' p)}{\sigma(\gamma p \rightarrow V p)} = \frac{\sigma(\gamma p \rightarrow \rho' p)}{\sigma(\gamma p \rightarrow \rho p)} = \frac{1}{9}$$

rather suggests a classification of the object as ω' (with major decay into $\pi^+ \pi^- \pi^0$) because $\sigma(\gamma p \rightarrow \omega p) = 1010 \pm 15 \pm 290$ nb⁶¹.

A J^P analysis was performed by maximum likelihood fits to the Dalitz plots^{*)} of different 3π mass bins. The result (Fig. 15) shows a maximum of the 1^- intensity at 1.67 GeV. This is important insofar that there exists a $\omega(1670, J^{PC} = 3^{--})$ at about the same mass which is expected to be photoproduced with a cross-section in the order of $1/9 [\sigma(\gamma p \rightarrow g(1690)p]$, i.e. 0 (40 nb). Because of the similarity of 3^- and 1^- Dalitz plot distributions one could speculate, that a part of the bump is due to 'off diagonally' produced ω 's with $J^{PC} = 3^{--}$.

Allowing only for 1^- contributions and demanding consistency with the DCI data a maximum constructive interference is required between the signal

*) After checking the fact of a decay via $\rho\pi$ the $\rho\pi$ decay amplitudes were constructed with the so-called Zemach tensor formalism⁶²).

and the 1^- content of the 3π background ($\sim 25\%$, determined by the same J^P analysis): such an assumption leads to a fit to the 3π mass distribution as indicated by the dotted line in Fig. 14b and a cross-section of

$$\sigma(\gamma p \rightarrow \omega' p \rightarrow \pi^+ \pi^- \pi^0 p) = 15 \pm 2 \text{ nb} .$$

There are no narrow resonances found in the reaction $pp \rightarrow \omega \pi^+ \pi^- p$ (Refs. 67,68). A broad enhancement at ~ 1900 MeV found in the $B\pi$ substates of this reaction should be mentioned in the context of radially excited ω 's; see Fig. 62⁴¹³.

Again somewhat artificial interference mechanisms are necessary to shift down the real mass of two independent observations by the WA4 and WA57 experiments^{63,64} by about 100 MeV to the DM1 favoured value: both experiments (see Figs. 16a,b) find in the reaction $\gamma p \rightarrow K^+ K^- p$ a resonance at 1.75 GeV, 80 MeV wide. The K's were identified with a Cerenkov counter; the cross-section $\sigma(\gamma p \rightarrow \phi' p \rightarrow K^+ K^- p)$ -- quoted with or without such interference mechanism -- is of the order of 10 nb and suggests therefore a classification of the object to be a radially excited ϕ , though a spin-parity analysis is not possible due to low statistics: confusion with a 3^{--} state seems unlikely, see Section 2.2.7.

Again transferring the ratio

$$\frac{\sigma(\gamma p \rightarrow \phi' p)}{\sigma(\gamma p \rightarrow \rho p)} = \frac{1}{9}$$

to the ϕ family a total cross-section of about 40 nb is expected -- with a main decay into $K^* K$.

But the search for a ϕ' of comparable width in the $K^* K$ decay in the photo-induced reactions $\gamma p \rightarrow K^+ K^- \pi^0 p$ and $\gamma p \rightarrow K^0 K^\pm \pi^\mp p$ was negative, the upper limit for the total $K^* K$ decay mode is 15 nb (90% C.L.)⁶⁵.

In Figs. 17a,b both mass distributions are shown, the shaded area corresponding to events having a $K^*(890)$ in a $K\pi$ subsystem.

The discrepancy between the WA57 result⁶⁵ and the corresponding DCI DM1 result (see above) is expressed not only by the non-presence of resonances with comparable widths here but also by the fact of $\sim$ equal

strength of K^{0*} and $K^{*\pm}$ signals in the photo-induced $K_S^0 K^{*\pm} \pi^\mp$ final state: $16 \pm 4\%$, $19 \pm 4\%$, respectively. The interference mechanism presented above is therefore not observed. The ratio

$$\frac{\sigma(\gamma p \rightarrow K^{*0} K p \rightarrow K_S^0 K^{*\pm} \pi^\mp p)}{\sigma(\gamma p \rightarrow K^{*0} K p \rightarrow K^+ K^- \pi^0 p)}$$

-- expected from Isospin Clebsch Gordan for an $I = 0,1$ object to have a value of 4 -- is measured to be 3 ± 0.6 . The direct comparison between DCI-DM1 and WA57 in the channel $K^{*0} K^\pm$ quoted according to Eq. (11) with $\sigma_{\text{tot}}(\phi) = 9 \text{ mb}$ -- shows a discrepancy hardly to explain with interference effects: full line in Fig. 18.

It is mysterious why the ϕ' couples in the photoproduction data at least two or three times less than expected to $K^* K$; perhaps it decays with unobservably small branching ratios into $(KK + n\pi)$ systems or it prefers the S wave way via Q mesons (see Figs. 3a,c) instead of the P wave way via $K^* K$.

The search for a $\phi'(1750)$ in photoinduced $K^+ K^- \pi^+ \pi^-$ systems was unsuccessful; a broad ϕ' at 1950 MeV, 200 MeV wide -- observed in $K^+ K^- \pi^+ \pi^-$ by Ref. 66 can be explained by a threshold enhancement⁴⁷⁾.

In summary, one can review the actual status: one experiment (DM1) observes a $\phi'(1650)$ decaying mainly via $K^* K$ with a small $\omega\pi\pi$ decay mode; the latter is enhanced by a constructive interference with a broad $\omega'(1600)$. Photoproduction data show in two independent experiments a 'narrow' ω' at 1670 MeV with a major decay into $\pi^+ \pi^- \pi^0$ and a ϕ' at 1750 MeV decaying into $K^+ K^-$. It seems necessary to await independent confirmation also for the e^+e^- data. It is therefore too early to calculate whether the different patterns in photoproduction and e^+e^- data is due to similar mechanisms than those described for the $\rho'(1600)$.

Knowledge of vector mesons and their radial excitations delivers a very important key for the understanding of light quark interactions: In order to solve unsatisfying present situations partly caused by an unknown role played by the D waves, further experiments are urgently needed. This

could be done in particular at storage rings with luminosities higher than nowadays available and detectors completely identifying all final state particles.

High precision experiments on $\Gamma_{ee}, \Gamma_{\mu\mu}$ could, via Eq. (8) due to different behaviour of the wave function at the origin, be used to distinguish between S and D waves. But for instance

$$\frac{\Gamma_{ee}(\rho_D)}{\Gamma_{ee}(\rho_S)} = \frac{\psi_D^2(0)}{\psi_S^2(0)} = 0$$

is valid only in a non-relativistic limit, in the relativistic case the second derivative of $\psi_{S,D}$ has to be accounted, leading to

$$\frac{\Gamma_{ee}(\rho_D)}{\Gamma_{ee}(\rho_S)} = \left(\frac{0.17}{0.37}\right)^2 = 20\% \text{ (Ref. 280, see also Ref. 402) .}$$

2.2.2 The 0^- mesons and their radial excitations

Vector mesons are (nearly) ideally mixed:

$$\omega \sim \frac{1}{\sqrt{2}} |u\bar{u} + d\bar{d}\rangle, \quad \phi \sim |s\bar{s}\rangle, \quad m_\omega = m_\rho, \quad m_\phi \approx 2m_K^* - m_\rho. \quad (11)$$

The expectation value of the Hamiltonian for $|q\bar{q}\rangle$ wave functions $\langle q\bar{q}|H|q\bar{q}\rangle$ includes contributions from the quark masses and additional terms, for instance, depending on the spin forces between q and $\bar{q}$:

$$\pi : \langle u\bar{d}|H|u\bar{d}\rangle = M_0 + m_u + m_d \approx M_0 + 2m_u$$

$$\eta_8 : \quad = M_0 + \frac{2}{3} m_u + \frac{4}{3} m_s.$$

This leads immediately to the linear Gellman-Okubo mass formula for pure SU(3) states:

$$3m_{\eta_8} = 4m_K - m_\pi, \quad \text{i.e. } m_{\eta_8} = 613 \text{ MeV}. \quad (12a)$$

It can be derived²⁴⁵⁾ (see Ref. 69 for empirical arguments) that Eq. (12a) works better with squared masses:

$$3m_{\eta_8}^2 = 4m_K^2 - m_\pi^2, \quad \text{i.e. } m_{\eta_8} = 571 \text{ MeV}. \quad (12b)$$

Making the ansatz

$$\eta = \cos \theta \eta_8 + \sin \theta \eta_1 \quad (12c)$$

$$\eta' = -\sin \theta \eta_8 + \cos \theta \eta_1$$

one derives for the mixing angle

$$\tan^2 \theta = \frac{4m_K^2 - m_\pi^2 - 3m_{\eta_1}^2}{3m_{\eta_1}^2 - 4m_K^2 + m_\pi^2}, \text{ i.e. } \theta \approx 10^\circ. \quad (12d)$$

The analogue for Eq. (12b) and the η_1 :

$$3m_{\eta_1}^2 = 2m_K^2 + m_\pi^2$$

gives a η_1 mass far away from the physical η' mass. The real mass could be explained by additional contributions of annihilation terms $q\bar{q} \rightarrow n\bar{g} \rightarrow q\bar{q}$ to the η' state. Remember that C parity requires $n = 3$ for 3S_1 instead of $n = 2$ for 1S_0 ; moreover this effect is expected to decrease for increasing masses as might be anticipated from asymptotic freedom of QCD, thus playing a less important role for vector mesons than for pseudo-scalars.

Reference 69 describes this additional interaction by the introduction of a term $\lambda = 1/3 \langle q\bar{q} | H | q\bar{q} \rangle$, caused by gluon fields, coupling to flavour singlets and strongly OZI violating.

Setting $m_\pi = 0$, a mass formula for both isoscalars can then be deduced.

$$m_{\eta'}^2 / m_\eta^2 = \frac{1}{2} \left[\lambda + 2m_K^2 + \sqrt{\lambda^2 - \frac{4}{3} \lambda m_K^2 + 4m_K^4} \right]. \quad (12e)$$

For λ large compared to m_K^2 the sign of the $\eta \eta'$ mass splitting is well described. Fixing $m_\eta^2 + m_{\eta'}^2 = 1.22 \text{ GeV}^2$ yields $\lambda \approx 0.73 \text{ GeV}^2$, $m_\eta = 582 \text{ MeV}$, $m_{\eta'} = 937 \text{ MeV}$.

θ might be quoted analogue to [Eq. (12d)]

$$\tan^2 \theta = \frac{m_\pi^2 + 2m_K^2 - 3m_{\eta'}^2 + 3\lambda}{3m_{\eta'}^2 - m_\pi^2 - 2m_K^2 - 3\lambda}, \quad \text{i.e. } \theta = 10^\circ.$$

This cannot be understood as an exact calculation but as an illustration that a large gluonic field energy contribution is able to raise the η' mass to a much higher value than the η mass.

Decays of pseudoscalars involving photons are well suited to measure θ . In summary, all radiative decay measurements yield values $\theta \sim 15^\circ$. For instance

$$\frac{\Gamma(\eta \rightarrow \gamma\gamma)}{\Gamma(\pi^0 \rightarrow \gamma\gamma)} = 20 (\cos \theta + 2\sqrt{2} \sin \theta)^2 \quad (\text{Ref. 11})$$

leads with the values of (Ref. 16) to $\theta \sim 10^\circ$. A recent redetermination of $\Gamma(\eta \rightarrow \gamma\gamma) = 0.53 \pm 0.08 \text{ keV}$ (Ref. 69g) lifts θ to a value around 20° (Ref. 69b). See Ref. 421 for more discussions on two-photon decays of pseudoscalars.

The precise determination of θ and the amount of gluonic admixture to η, η' has been intensively discussed in the literature. Recent publications are Refs. 69a, 69c. Reference 69a argues for a pseudoscalar $|gg\rangle$ state leading to the known $\eta' = 1/\sqrt{12} |uu+dd+ss-3gg\rangle$ and a new $\eta'' = 1/\sqrt{3} |uu+dd+ss+gg\rangle$ at 2.3 GeV.

The Mark III group (Ref. 69d) have used their data on radiative ψ decays into η and η' together with other results on η, η' decays to perform a global fit to the ansatz proposed by Rosner (Ref. 69e):

$$|\eta\rangle = X_\eta \cdot \left| \frac{u\bar{u} + d\bar{d}}{\sqrt{2}} \right\rangle + Y_\eta |s\bar{s}\rangle + Z_\eta |\text{gluon or else}\rangle$$

$$|\eta'\rangle = X_{\eta'} \cdot \quad " \quad + Y_{\eta'} \cdot \quad " \quad + Z_{\eta'} \cdot \quad "$$

$$X^2 + Y^2 + Z^2 = 1.$$

Their result is

$$|X_\eta| = 0.63 \pm 0.06, \quad |Y_\eta| = 0.83 \pm 0.13 \quad (\text{see Fig. 19})$$

$$|X_{\eta'}| = 0.36 \pm 0.05, \quad |Y_{\eta'}| = 0.72 \pm 0.12 \quad " \quad "$$

So the η is nearly purely 'quark made', whilst the η' has a 35% gluonic admixture. Reference 69f argues against the neglect of second order OZI violating processes in the Mark III analysis: these are taken into account in a more general approach (Ref. 69h) to incorporate all relevant experimental information (including hadronic A_2 decays into $\eta\pi$) into a global fit. The result is basically consistent with Mark III except the quantity $|X_{\eta}| = 0.5 \pm 0.04$.

See also the remark in Ref. 418 concerning the experimental values used in the Mark III analysis.

If pseudoscalar states are made by $|q\bar{q}\rangle$ only, the quarks have to be in a relative S wave, so the states can only be radial excitations of the 1S_0 family. But the unknown amount of gluonic contributions (expected to decrease for increasing mass) makes mass predictions, especially for $I = S = 0$ states in $(^1S_0)'$ difficult.

Two experiments^{70,71)} find a broad enhancement of the $0^- (\epsilon\pi)$ S wave in a partial wave analysis of diffractively produced $(\pi^-\pi^-\pi^+)$ systems (ϵ stands for a $\pi\pi$ S wave, i.e. $J^P = 0^+$):

$$m_{\pi'} = 1270 \text{ MeV} \quad \Gamma_{\pi'} = \sim 580 \text{ MeV (Fig. 20)} .$$

Besides this significant signal Ref. 71 finds a hint for a π'' at about 1800 MeV.

A radial excitation of the K meson is observed by Ref. 72 in the ϵK decay mode: $K'(1460)$; $\Gamma_{K'} = 260 \text{ MeV}$. References 73 and 74 confirm this result by a PWA of the reaction $K^{\pm}p \rightarrow K^{\pm}\pi^-\pi^+p$ and detect moreover the $K^*(890)\pi$ and ρK decay mode (Figs. 21a-c). The curves in Fig. 21 correspond to a sum of a Breit-Wigner term and non-resonant background.

It should be mentioned that in the analysis of Ref. 77 there is no need for such an object.

The mass of a radial excitation of the $\eta(549)$ plays a key role for many potential models: $m_{\eta_R} = m_{\pi'}$ would point to ideal mixing in $(^1S_0)'$ and a more or less negligible influence of the gluonic admixtures. This

result would therefore be unexpected in the frame of actual potential models (first version of Ref. 168) quoting η_R masses in the vicinity of ~ 1400 MeV thus an η_R participating in the so-called E/i puzzle (see Sections 2.2.4 and 3). Nevertheless a somewhat weak hint for an η_R at 1275 MeV ⁷⁵⁾ - seen in a partial wave analysis of $\eta\pi\pi$ system from the reaction $\pi^- p \rightarrow \eta\pi^+\pi^- n$ in the $0^- \delta\pi$ S wave (Fig. 22a) and even weaker in the $0^- \eta\pi$ S wave (Fig. 22b) could recently be fully confirmed by data of the same reaction taken at the KEK Laboratory ⁷⁶⁾. The intensity of the same two amplitudes show bumps at ~ 1.28 GeV, the latter with a considerable background (Figs. 22c,d), respectively. Refs. 75,76 use identical analysis programs, it would be desirable to prove that this important result is independent from the analysis technique. The η_R at 1275 MeV would point to ideal mixing and therefore to an η'_R at ~ 1600 MeV not yet discovered. Mark III results ⁴⁰⁷⁾ on J/ψ decays into a 1^{--} state (photon or vector meson) and an $\eta\pi\pi$ system show bumps at $m_{\eta\pi\pi} = 1.28$ GeV not inconsistent with having $J^{PC} = 0^{-+}$. Moreover one of two structures at 1.6, 1.9 GeV, observed only in $\eta\pi\pi$ systems from radiative J/ψ decays (see Fig. 56) could be due to a radially excited η' .

2.2.3 The 1P_1 family, $J^{PC} = 1^+$

The isotriplet member is the already mentioned B_0^+ meson at 1232.5 MeV, 137 MeV wide. It was discovered in 1963 in its neutral mode decaying into $\omega\pi^0$ ⁷⁸⁾. Mass and width are average values from 15 experiments ¹⁶⁾. The name B stands for Buddha because of the quite large width with respect to the $\omega\pi$ threshold -- so far the only established decay mode. The spin parity analysis -- evaluated with a moment method especially developed for this channel by Chung ³³⁾ -- assures J^P to be 1^+ and a $\omega\pi^0$ D wave/ $\omega\pi^0$ S wave ratio of 0.3.

The fact of elastic photoproduction in the reaction

$$\gamma p \rightarrow B p \rightarrow \omega\pi^0 p \rightarrow \pi^+\pi^-\pi^0\pi^0 p \quad (\text{Refs. 31,32})$$

fixes $C = -1$ (see also Section 2.2.1, Figs. 5,6). It was speculated in Section 2.2.2 that nearly equal cross-sections for elastic B and $\rho'(1600)$ production is an argument for a similar production mechanism as shown in the diagram for the B's, i.e. an 'off diagonal' VDM contribution if 'B' is replaced by " $\rho'(1600)$ ".

The energy dependence of the cross-section for this reaction

$$\sigma(E_Y) = (0.86 \pm 0.27) \cdot \left(\frac{39}{E_Y}\right)^{0.6 \pm 0.2} (\mu\text{b}) ; \quad 20 \leq E_Y \leq 70 \text{ GeV (Ref. 32)}$$

turns out to be valid even in the "long lever arm" extrapolation down to $E_Y = 4 \text{ GeV}$ ³¹. The inclusive photoproduction of $B^\pm$ ($\gamma p \rightarrow B^\pm X$), see Fig. 23, corresponds to a high cross-section

$$\sigma(\gamma p \rightarrow B^\pm X \rightarrow \omega \pi^\pm X) = 400 \text{ nb} ; \quad 20 \leq E_Y \leq 70 \text{ GeV}$$

proving a strong γ -B coupling. This fact was used in searching for the B's in decay modes other than $\omega\pi$: an analysis of the reaction $\gamma p \rightarrow \eta \pi \pi p$ shows a 80 MeV wide resonance at 1.28 GeV with possible J^P assignments $1^-, 1^+, 2^+$ ³⁰). Believing $J^P = 1^+$ leads to a branching ratio $B \rightarrow \eta\rho$ of about 10%, and a D/S ratio in this channel much larger than that measured in the $\omega\pi^0$ channel (see above). Applying an alternative analysis technique to the same data the observed bump could well be attributed to a tail of the $\rho(765)$ decaying into $\eta\rho$ making a 1^+ classification unlikely⁴⁸).

One $I = 0$ state is the H (1190, $\Gamma = 320 \text{ MeV}$), first observed at SLAC³¹) in the reaction $\pi^- p \rightarrow \pi^+ \pi^- \pi^0 n$: Figure 24a shows the intensity of the $I = 0$, $J^P = 1^+$ $\rho\pi$ S wave for "no spin flip" on the nucleon side, Fig. 24b, the same for spin flip. Reference 15 declares the significant signals in both amplitudes as "convincing evidence".

The strong coupling of photons to $J^P = 1^+$ recently mentioned is also confirmed by the spin-parity analysis of elastically photoproduced $(\pi^+ \pi^- \pi^0)$ systems already presented on the ω' search (Section 2.2.2). The 1^+ intensity peaks not very far from $\sim 1200 \text{ MeV}$ (see Fig. 24c). The cross-section of the bump interpreted as the H meson is quoted to

$$\sigma(\gamma p \rightarrow H p \rightarrow \rho \pi p) \approx 300 \text{ nb} \quad (\text{Ref. 61}) .$$

The search for the so far undiscovered singlet H' could cause the following speculation: on the one hand equal mass of B and H suggest ideal mixing and therefore

$$m_{H'} = 2m_{Q_B} - m_B \approx 1500 \text{ MeV} .$$

On the other hand for ideal mixing (see vector mesons) one expects a ratio of the photon coupling to the three neutral multiplet members of 9:1:2 changing to 3:1:0 for SU(3) mixing. Assuming (nearly) equality of hadronic cross-sections this ratio should be transferred to photoproduction cross-sections³²⁾.

One observes

$$\frac{\sigma(\gamma p \rightarrow B p)}{\sigma(\gamma p \rightarrow H p)} \approx 3 ,$$

if $\omega\pi^0$, $\rho\pi^0$ are the main decays of B, H, respectively. The search in photo-induced $K^* K$ channels⁶⁵⁾ -- most probable decay modes of an H' -- has failed down to upper limits of a few nb. This could either be a hint for SU(3) mixing or due to the fact that the H is below $K^* K$ threshold.

For the isodoublet members of 1P_1 Q_B holds a peculiarity: they could be mixed with Q_A mesons of 3P_1 ($1^{+-} \leftrightarrow 1^{++}$). This is because these objects are no G, C eigenstates, the corresponding B and A_1 mesons cannot mix. The isospin analog SU(2) symmetry U spin or V spin, is not very strong due to the fact of the heavier s quark and therefore the SU(3) mass breaking; so the corresponding U, V spin induced "G" parities G_u, G_v are not conserved.

Indeed, experiments⁷⁴⁾ observe two objects Q_1 and Q_2 at different masses, both decaying into $K\pi\pi$ final states. This is surprising because of the equality of B and A_1 meson masses.

$$Q_1 = 1.27 \pm 0.01 \text{ GeV} \quad \Gamma_{Q_1} = 90 \pm 8 \text{ MeV}$$

$$Q_2 = 1.41 \pm 0.025 \text{ GeV} \quad \Gamma_{Q_2} = 195 \pm 25 \text{ MeV} .$$

The total $K\pi\pi$ mass spectrum of the reaction $K^-p \rightarrow K^- \pi^- \pi^+ p$ shows a significant double structure (and a contribution from the $L(1770)$, see below):

Fig. 25a. A detailed PWA proves both resonances to have $J^P = 1^+$ and detects moreover different decay modes: Fig. 26.

The full lines correspond to fits with two Breit-Wigner resonances and a coherent "Deck-type" background (see remark in, Section 2.2.4). Clearly the fits show different decay modes.

Essentially, the following holds

$$\Gamma(Q_1 \rightarrow \rho K) \gg \Gamma(Q_1 \rightarrow K^* \pi)$$

$$\Gamma(Q_2 \rightarrow K^* \pi) \gg \Gamma(Q_2 \rightarrow K \rho) .$$

With these inputs, Ref. 69 makes a mixing ansatz to evaluate the G_u, G_v eigenstates Q_A, Q_B

$$Q_1 = \cos \phi Q_A + \sin \phi Q_B$$

$$Q_2 = -\sin \phi Q_A + \cos \phi Q_B .$$

If one fixes the suppressed channels to zero

$$\Gamma(Q_1 \rightarrow K^* \pi) = \Gamma(Q_2 \rightarrow K \rho) = 0$$

then

$$\cos \phi = \sin \phi \quad \text{and} \quad \phi = 45^\circ \quad \text{and} \quad m_{Q_A} = m_{Q_B} = \frac{1}{2} (m_1 + m_2) = 1340 \text{ MeV} .$$

A detailed calculation with the real branching ratios and the corresponding U,V spin Clebsch-Gordans and decay matrix elements determined from the decays of the corresponding family members yields

$$m_{Q_A} = 1.31 \pm 0.015 \quad m_{Q_B} = 1.37 \pm 0.02 \quad (\text{Ref. 74}) .$$

2.2.4 The 3P_1 family: $J^{PC} = 1^{++}$

The $I = 1/2$ state Q_A was presented in the previous chapter, the index A hints to the isotriplet, the A_1 meson. Not the existence of the A_1 but determination of mass and width was the topic of discussions

lasting nearly 20 years (!). In 1964 Deck⁴²⁾ pointed out that (3π) resonances found in diffractive reactions like $\pi^+ p \rightarrow \pi^+ \pi^- \pi^- p$ have to be separated from non-resonant threshold enhancements.

The question whether the A_1 is intrinsically narrow and only broadened by the mechanism shown in Fig. 27 or has itself a larger width of $\Gamma = 300$ MeV has recently been answered in favour of the second hypothesis:

$$\pi^- p \rightarrow \pi^- \pi^+ \pi^- p \Rightarrow m_{A_1} = 1280 \pm 30 \quad \Gamma = 300 \pm 30 \text{ MeV (Ref. 70)}$$

$$\pi^- p \rightarrow \pi^+ \pi^- \pi^0 n \Rightarrow m_{A_1} = 1240 \pm 80 \quad \Gamma = 280-480 \text{ MeV (Ref.81) .}$$

Figures 28a-d show the result from Ref. 70. For larger four momentum transfers Deck-type contributions disappear and the A_1 is shifted to the value given above. A coherent implementation of such background leads to the dashed curves according to Ref. 86.

Recent photoproduction data observing in the reaction $\gamma p \rightarrow 2\pi^+ 2\pi^- \pi^0 p$ a Deck-free final state A_1 demand 200 MeV as a lower limit for the A_1 width⁸⁷⁾.

It is not yet clear, whether the " A_1 " mass of 1050 MeV, observed in the a priori Deck-free reaction $K^- p \rightarrow \Sigma^- \pi^+ \pi^- \pi^-$ ⁸⁸⁾ has suffered from other background mechanisms [of multi-Regge type⁷⁴⁾] or one has found there a new (perhaps exotic) particle with $J^P = 1^{+15}$.

A nice by-product of an accurate determination of A_1 parameters would provide upper limits for the ν_τ mass from the decay $\tau \rightarrow A_1 \nu_\tau$: for $m_\nu = 0$ or 500 MeV the peak mass of a 300 MeV wide A_1 would decrease by 50 MeV.

One of the two ($I = 0$) 3P_1 members is certainly the D(1285), mainly decaying via $\delta\pi$, therefore $\eta\pi\pi$ ($\sim 50\%$), but confusion with $\eta_R(1275)$ is possible. Other decay modes are $KK\pi$ (10%) or $\rho\pi\pi$ (40%). See Ref. 288 for unusual features of D production in pp collisions probably due to gluonic interactions in the collision.

The small coupling to kaonic final states and the mass equality to A_1 point to ideal mixing. The second $I = S = 0$ state is therefore expected at a mass of about 1400 MeV.

A possible candidate is the $E(1430)$ --- discovered already 20 years ago⁹⁰⁾. But different experiments observing bumps at ~ 1400 MeV in different reactions drew different conclusions concerning J^{PC} : 0^{-+} or 1^{++} , see Ref. 69 for a review for these 'early years' of the E.

In 1980 in radiative ψ decays ($e^+e^- \rightarrow \psi \rightarrow \gamma K^+ K^- \pi^0$) an object "iota" was observed at 1440 MeV decaying into $K^+ K^- \pi^0$ ⁹³⁾. This "i" is a well established 0^- object¹⁶⁾ and therefore certainly not a member of the 1P_1 family. Whether it has a conventional qq content or is mainly made by gluons will be discussed in Section 3.

The existence of the "i" has changed the viewpoint on many open experimental questions.

For instance, the discovery of the E ⁹⁰⁾ was probably the discovery of the "i" because there is a "formidable angular momentum barrier" in $p\bar{p} \rightarrow E(1^{++})\pi^+\pi^-$ ⁹⁹⁾ and on the other hand the dominant $p\bar{p}$ state (for annihilation at rest) has $J^{PC} = 1^{--}$ (3S_1) as the ψ ¹⁵⁾. Note also, that the experiments, who assign the E as a 1^{++} state, observe it together with a $D(1285)$ --- contrary to $p\bar{p}$ at rest or radiative ψ decays.

The relevant question therefore has changed from:

- "Is the bump at 1420 MeV a 0^{-+} or a 1^{++} object"?
- to:
- "Is there a 1^{++} object and 0^{-+} objects in the 1400-1460 MeV region"?

This latter question cannot be answered in an unambiguous way: the experimental situation can be summarized by two (contradicting) scenarios.

Answer 'yes' scenario: an analysis of 3000 events of the reaction $\pi^- (3.9 \text{ GeV}) p \rightarrow K_S^0 K^\pm \pi n$ in a CERN bubble chamber experiment⁹⁹⁾ shows clear D and E signals in the $(KK\pi)$ mass distributions (Fig. 29a).

Analysing the E isospin $I = 0$ was determined by the non-observation of corresponding charged objects in the $(K_S^0 K^- \pi^0)$ and $(K_S^0 K^0 \pi^-)$ mass distributions. The J^P analysis was performed by maximum likelihood fits to different $(KK\pi)$ mass bins, where the decay amplitudes are constructed as two-body decays $(K^* K, \delta\pi)$ according to the formalism of Zemach⁶²⁾. Figure 30a shows the 1^+ intensity versus the $(KK\pi)$ mass. A peak at the E mass is clearly visible. The C parity results from the sign of interference between $K^{*0} K^0$ and $K^{*\pm} K^\mp$. A positive sign and therefore positive C parity gives a better fit to the mass distribution for instance of the charged K^* (see Fig. 30b).

This result was fully confirmed by applying the same analysis technique to centrally produced E's of the reaction π^+/p (85 GeV) $p \rightarrow (\pi^+/p) (K_S^0 K^\pm \pi^\mp) p$ ^{91,92)}. With improved statistics (see Fig. 29b) this analysis reaches a clear answer in favour of a 1^{++} solution of the E peak. See, for instance, the predictions of 0^{-+} and 1^{++} hypothesis for the $M^2(KK)$ distribution. The E is moreover found to decay only via $K^* K$ and not via $\delta\pi$ in either $KK\pi$ or $\eta\pi\pi$ (see Section 2.2.6 for the δ , see also the discussion on the "i" in Section 3 showing a strong $\delta\pi$ decay mode).

From this it is obvious to classify the E(1430) as the missing member of the $1P_1$ family.

The 'answer no' scenario: A high statistics Dalitz plot analysis of the reaction π^- (8 GeV) $p \rightarrow K^+ K^0 \pi^- n$ finds the E(1420) to be mostly ($\geq 70\%$) a 0^{-+} state decaying predominantly via $\delta\pi$, with a non-vanishing $K^* K$ decay mode and interference between both⁹⁴⁾. (See Fig. 30d for the acceptance corrected intensity of the 0^{-+} wave with combined $\delta\pi$ and $K^* K$ decay mode.) This result is entirely confirmed by replacing the 8 GeV π^- beam by 6.6 GeV antiprotons (Ref. 94a).

So in this scenario it might well be that E and "i" are identical [even though recent measurements point to a somewhat higher "i" mass (see Section 3): in this case one would have two pseudoscalar states]. As can be seen in Fig. 22c, there might be confirmation of this scenario by⁷⁶⁾ observing a bump at the E mass in the $0^- (\delta\pi)$ wave.

Nevertheless, it is impossible to judge which scenario is correct and which has suffered from improper accountance of acceptances or insufficient number of partial waves involved.

See also Ref. 333 arguing for an effective potential $V_{K\bar{K}}$ between the kaons in $K\bar{K}\pi$ final states whose neglect might fake $K\bar{K}$ distribution for 0^{-+} to look like 1^{++} .

Attempts to use other criteria for solving the E/i puzzle are also not fully conclusive:

- The quite high production cross-section in $\pi^- p$ interactions points against a pure $s\bar{s}$ object⁸⁹⁾ as well as the fact that Ref. 95 observes D's in $K^- p$ interactions but no E's.
- An SU(3) fit (with a too small A_1 mass, see above) yields a D-E mixing angle inconsistent with the Gellman-Okubo relation (Eq. 11)⁸⁹⁾. But a recent analysis from the 'Lepton F' experiment at Serpukhov²⁸²⁾ determines the deviation of the E mixing angle from ideal mixing to be $< 13^\circ$ and so favours a $(s\bar{s})$ interpretation of the E, i.e. the normal classification.

One might hope for a complete spin parity analysis of a clear E bump observed at 1420 MeV in the $(\eta\pi^0\pi^0)$ decay channel at the GAMS spectrometer³⁴⁴⁾.

An alternative candidate for the 3P_1 family could be the $D'(1526 \text{ MeV}, \Gamma = 107 \text{ MeV})$ observed (with poor statistics) in the reaction $K^- (4.2 \text{ GeV}) p \rightarrow K_S^0 K^+ \pi^- \Lambda$ ⁹⁶⁾. Its mass value points away from ideal mixing towards more or less SU(3) like wave functions (see Eq. 11).

2.2.5 The 3P_2 family $J^{PC} = 2^{++}$

Analogous to the strong relation between vector mesons and (single) photons these so-called tensor mesons are most easily produced in two-photon processes: $\gamma\gamma$ scattering as observed in storage rings in the reaction

$$e^+e^- \rightarrow e^+e^- + \gamma\gamma.$$

All members of this family are identified beyond any doubt and should be mentioned only briefly.

$I = 0$: $f(1270,179)$ decaying into 2π (83%), $\eta\pi$ (3%)

$f'(1515,75)$ " " $K\bar{K}$ dominant

$I = \frac{1}{2}$: $K^*(1434,100)$ " " $K\pi$ (50%), $K\pi\pi$ (see Fig. 31, Ref. 97)

$I = 1$: $A_2(1318,110)$ " " $\rho\pi$ (70%), $\eta\pi$ (14%), $\omega\pi\pi$ (10%) .

Due to low statistics older analyses claimed for a double structure in the (3π) mass distribution, so-called split A_2 , see for example mass distribution shown in Ref. 98. This is clearly wrong: Figure 32 shows the corresponding distribution taken with the JADE detector at PETRA. See also Ref. 312.

Equation (11) predicts nearly ideal mixing: $|\theta - \theta_{\text{ideal}}| = 7^\circ$. This is confirmed⁹⁹⁾ by measuring the 2γ decay width of the f and f' , a direct analyser of the quark charges involved:

$$\Gamma(f', f \rightarrow \gamma\gamma) \sim |\langle q^2 \rangle|^2$$

for $SU(3)$ mixing one expects:

$$\frac{\Gamma(f \rightarrow \gamma\gamma)}{\Gamma(f' \rightarrow \gamma\gamma)} = \frac{\frac{1}{6}(\frac{4}{9} + \frac{1}{9} - \frac{2}{9})^2}{\frac{1}{3}(\frac{4}{9} + \frac{1}{9} + \frac{1}{9})^2} = 0.125$$

whereas ideal mixing predicts 12.5⁴²¹⁾.

Measurements yield, accounting for a phase-space factor $(m_f/m_{f'})^3$, a ratio of

$$\frac{2.85 \pm 0.26 \text{ keV}}{0.11 \pm 0.05 \text{ keV}} \times \left(\frac{1.27}{1.52}\right)^3 \approx 15 .$$

From here one quotes $0.3^\circ \leq |\theta - \theta_{\text{ideal}}| \leq 9.6^\circ$. Figure 33 shows the f' decay into K^+K^- ¹⁰⁰⁾. Reference 382 reports on the discovery of a radial

excitation of the f at ~ 1800 MeV. This is important in the context of another 2^{++} object, the $\theta(1700)$, which is the most actual glueball candidate and will be discussed in Section 3.

2.2.6 3P_0 family: $J^{PC} = 0^{++}$

The last of four P wave multiplets is less well understood, especially as the widths of possible members are very different. This caused speculations, that quark molecules¹⁰²⁾, glueballs¹⁰³⁾ or (KK) bound states¹⁰⁴⁾ are wrongly classified as members of this family.

Candidates for membership are:

- the δ (983 MeV, $\Gamma = 54$ MeV) isotriplet. It decays mainly into $\eta\pi$ and also, though very close to threshold, into $K\bar{K}$ ¹⁰⁴⁾. This opening of the $K\bar{K}$ channels just in the centre of the δ mass could broaden the true width by "cusp effects" compared to the 54 MeV measured in the $\eta\pi$ channel¹⁰⁵⁾. The quantum numbers of the δ are determined by the decay of D (1285) into $\delta^+\pi^-$, $\delta^-\pi^+$, $\delta^0\pi^0$ to be $I^G = 1^-$ (therefore $\delta \not\rightarrow 2\pi$ but $\delta \rightarrow \eta\pi$); $J^P = 0^+$ and hence $C = +1$ follows by non-observation of $\rho\pi$ decays. A surprisingly copious $\delta^\pm$ production is reported from photo-induced inclusive $\eta\pi^\pm$ systems with $x_F > 0.5$: see Fig. 34 for $m_{\eta\pi^\pm}$ from $\gamma p \rightarrow \eta\pi^\pm + X$ ¹⁰⁶⁾. This leads to a large photoproduction cross-section

$$\sigma(\gamma p \rightarrow \delta^\pm X \rightarrow \eta\pi^\pm X) \simeq 400 \pm 100 \text{ nb}$$

and a production dynamics in terms of Feynman x (x_F) and p_T qualitatively and quantitatively very similar to inclusive photoproduction of charged $B^\pm$ mesons. Before interpreting this, the list of 0^{++} objects will be continued.

- S^* (975, $\Gamma = 33$ MeV) decaying into $\pi\pi$ (75%) and KK (25%). Most information on the $(\pi\pi)$ S wave S^* comes from interference effects between $(\pi\pi)$ S waves and $(\pi\pi)$ P waves (ρ 's)¹⁰⁶⁻¹⁰⁸⁾ and from direct production in $\pi^- p \rightarrow \pi\pi n$ ^{109,110)}: $I^G = 0^+$, $J^{PC} = 0^{++}$. Mark II data¹¹¹⁾ in the inclusive $(\pi^+\pi^-)$ mass distributions from ψ decays exhibit after proper background $\sim$ parametrization and $\sim$ subtraction, a significant S^* signal: Fig. 35a. A bump observed in centrally produced $\pi^+\pi^-$ pairs in the reaction $\pi^+ (85 \text{ GeV}) p \rightarrow \pi_{\text{fast}}^+ (\pi^+\pi^-) p$ is most likely interpreted as being

due to the S^* : see Fig. 35b¹¹²⁾. A strong S^* signal is reported⁴⁰⁷⁾ very recently from Mark III in $(\pi^+\pi^-)$ mass distribution recoiling against a ϕ in hadronic ψ decays.

Reference 113 reports on an indication of a radial excitation of the S^* at 1770 MeV.

- ϵ (~ 1300 , $\Gamma = 200-1400$ MeV (!), $I^G = 0^+$) decaying into $\pi\pi$ ($\sim 90\%$) and KK ($\sim 10\%$). The Particle Data Group lists 174 publications concerning this object. Though its existence seems undoubtedly proven -- see for example the two-meson S wave intensities from $\pi^-p \rightarrow K_S^0 K_S^0 n$ ²⁹⁰⁾, $\pi^- \rightarrow \pi^+\pi^-n$ (Ref. 291), respectively: Fig. 36 a,b -- accurate fixing of mass and width of this very broad ($\pi\pi$) S wave seems difficult. S waves have many contributors, last but not least phase space. Cusp effects of the $\pi\pi$, KK , and $\eta\eta$ thresholds could distort a Breit-Wigner shape. The actual knowledge on the ϵ and on the question whether there is another broad scalar object at ~ 600 MeV (so called up-down ambiguity) is extensively discussed by²⁸⁵⁾ implementing also recent experimental results from $\pi^+\pi^-$ scattering amplitudes²⁸³⁾ and from 2γ physics²⁸⁴⁾. Very recently^{112a)} an ϵ signal ($m_\epsilon = 1220 \pm 40$ MeV, $\Gamma_\epsilon = 320 \pm 40$ MeV) was observed in the spin 0 wave of π -induced $\eta\eta$ pairs: $\text{Br}(\epsilon \rightarrow \eta\eta) \sim 0.02$. See also, the result of a coupled channel analysis $pp \rightarrow pp(\pi^+\pi^-)(K\bar{K})$ of ISR data¹¹⁷⁾ arguing for at least three and probably four resonances in the S^*, ϵ domain.

- κ kappa (1350 , $\Gamma = 250$ MeV) decaying into $K\pi$. Figure 37 shows the argand-diagram of the elastic $(K^-\pi^+)$ S partial wave extracted from 43,000 events of the reaction $K^-p \rightarrow K^-\pi^+n$ at the SLAC LASS spectrometer¹¹⁴⁾: the full line leaves the unitarity circle which is repaired by subtracting the $I = 3/2$ contribution well known up to 1.6 GeV: dotted line. Both curves show a 90° phase transition, therefore a resonance at about 1400 MeV and moreover an indication of a radial excitation κ' at 1800 MeV. Though this general behaviour confirms a couple of previous experiments¹⁶⁾, mass determinations of different analyses are quite different: from 1278 MeV¹¹⁵⁾ to 1425 MeV¹¹⁶⁾, with an average at 1350 MeV. Again cusp effects, for example, at the $K\eta'$ threshold could influence the intrinsic κ parameters.

- $G(1592, \Gamma = 210 \text{ MeV})$ dominantly decaying into $\eta\eta$. This object, discovered in the reaction $\pi^- p \rightarrow Gp \rightarrow \eta np$ at Serpukhov²³⁰⁾, see also Ref. 112a, is -- according to the authors -- a very good glueball candidate. It will be therefore discussed in Section 3 in more detail.

Even neglecting this object difficulties are obvious to build a multiplet from the candidates presented.

The isotriplet δ couples strongly to s quarks ($\eta, K\bar{K}$), the less heavy $I = S = 0$ state S^* to $K\bar{K}$. On the other hand the heavier $I = S = 0$ state, the ϵ , prefers pionic channels though the mass splitting suggests ideal mixing and therefore pure strange quark content.

Reference 69 suggests this scenario:

ϵ, κ and perhaps $(S^*)'$ (1770) form together with an isotriplet to be discovered (see Ref. 118 for a candidate) the scalar multiplet 3P_0 .

δ and S^* are regarded as $q\bar{q}$ - $q\bar{q}$ molecules fitting into a multiplet scheme as shown in Fig. 63. It predicts for its non-strange members quark contents like those of a π meson + a $s\bar{s}$. The latter explains the strong coupling of δ and S^* to η 's and K 's. Due to the probably large width (otherwise they would have been discovered) of the other members of the multiplet, δ and S^* should also have an intrinsic large width. A possible narrowing mechanism could be OZI suppression of the decay into u and d quarks caused by the large s quark content and mass values below "fall apart" decays.

Another consideration declares δ, S^*, ϵ, K to be unitarized remnants of an ideally mixed conventional 1P_0 family with a non-vanishing 4 quark contamination -- the latter forming virtual "two meson bound states". Physical masses are influenced by hadronic self-energy effects and mix with each other due to threshold effects. A mass dependant mixing angle explains the funny decay pattern¹¹⁹⁾. See Ref. 272 for objections against this viewpoint.

Recent publications about the content of scalar mesons have been: Ref. 318 in favour of δ, κ being conventional $q\bar{q}$ states, Ref. 272 in favour of δ 's being broad $4q$ states, Refs. 331, 343 in favour of S^* having a large gluonic component or Refs. 168, 332 interpreting the δ as $K\bar{K}$ bound state. Especially the last picture has the nice feature to explain a mystery of the $i(1440)$: its dominant decay mode (see Section 3) is $KK\pi$, with KK mass peaking near threshold thus suggesting a $\delta\pi$ decay. Why is there no i (from ψ radiative decays) appearing in the $\eta\pi\pi$ channel? The answer^{333, 334} is that the potential $V_{K\bar{K}}$ binding two K 's to a δ distorts the $i \rightarrow KK\pi$ decay (occurring either as a three body decay or more likely as a K^*K decay) insofar that the $K\bar{K}$ mass is pulled towards the threshold. See also Ref. 339 for mechanisms to explain "the i/δ mystery".

There have been two recent experimental contributions to the discussion on δ 's nature.

- One stems from photoproduction of $\delta^\pm$. Being a $4q$ state the δ should couple strongly to two pseudoscalars in a relative S wave and only weakly to vector mesons³³⁵. The surprisingly high cross-section of inclusive δ production with real photons shows that the δ couples strongly to the incident vector meson. Describing its productions dynamics in terms of a quark recombination model²⁵⁰ where the $q\bar{q}$ state δ is formed by one quark of the vector meson and another from somewhere else p_T and x_F distributions are very similar to those of known $q\bar{q}$ objects ($B^\pm$) and are well reproduced by the model predictions.

- It seems unclear, how the 'two-photon width' $\Gamma_{\gamma\gamma}(\delta)$ can be used for a decision. Theoretical expectations disagree: some authors argue for a large width if $\delta = q\bar{q}$: [4.8 keV⁴²¹] and a small width if $\delta = 4q$ [0.27 keV³³⁶] or $K\bar{K}$ molecule [0.6 keV³³²].

In contrast others expect this quantity to be small if $\delta = q\bar{q}$: $\Gamma_{\gamma\gamma}(\delta) < 0.4 \text{ keV}^{311}$. See Ref. 332 for arguments against this calculation. Reference 337 argues that these calculations depend on the choice of input parameters and are thus not very distinctive: $2q \sim$ and $4q \sim$ ansatzes lead to $\Gamma_{\gamma\gamma}(\delta)$ of the order of 1 keV.

A recent experimental determination yields³³⁸⁾

$$\Gamma_{\gamma\gamma}(\delta) \times \text{Br}(\delta \rightarrow \eta\pi) = 0.19 \pm 0.07^{+0.1}_{-0.07} \text{ keV}$$

$$\Gamma_{\gamma\gamma}(A_2) = 1.14 \pm 0.2 \pm 0.26 \text{ keV}$$

(see $\Gamma_{\gamma\gamma}(f, f')$ in Section 2.2.5).

So, from an experimentalists point of view, the experimental value of $\Gamma_{\gamma\gamma}(\delta)$ looks somewhat low in comparison to $q\bar{q}$ hypothesis calculation. On the other hand, its experimental determination relies on $\Gamma_{\text{tot}}(\delta) = 54 \text{ MeV}$. If the real width of a $\delta (= q\bar{q})$ is larger (but has suffered from mechanisms as described above) the experimental value would become larger and could thus not be used as an argument in favour of an exotic nature³³⁷⁾. The reader is referred also to a very recent overview on 2γ decays of scalar mesons⁴²¹⁾. Here it is emphasized that despite the absolute value of $\Gamma(\delta)$ also the quantity $\Gamma_{\gamma\gamma}(\delta)/\Gamma_{\gamma\gamma}(S^*)$ can be used as an analyzer for the internal structure of the objects involved.

In trying to draw a conclusion, it seems quite plausible that the δ has both a conventional and an exotic component; the question is who dominates. One could expect more insight by measuring the A dependence of δ production on different targets, for example, in photoproduction: ($q\bar{q}$) states and four quark molecules should show up different mean free path lengths and thus a different shadowing behaviour. Preliminary unpublished results focussing to a different production cross-section of the photoinduced $\eta\pi$ systems mentioned above in the H_2 target and in a scintillation counter behind the target show no abnormal behaviour of δ 's compared to other $\eta\pi$ systems⁴¹⁴⁾.

2.2.7 $q\bar{q}$ systems with $L > 1$

Whilst for the ($L = 0$ or 1) $q\bar{q}$ systems the SU(3) multiplets are well-known, this knowledge is much poorer for $L = 2, 3, \dots$. Some of ($L > 1$) multiplets are Regge recurrences (i.e. particles with same P, C, G, but a spin increased by 2) of families already presented:

$$^3D_3 \rightarrow ^3S_1, \quad ^3F_4 \rightarrow ^3P_2$$

and could be detected not only directly in the s channel but also due to their action as t channel exchange particles.

The ($L > 1$) multiplets whose members lie on the four (degenerated -- see Fig. 38) "leading trajectories" ρ , ω , A_2 , f are quite well-known: the 3D_3 multiplet contains as isotriplet the g (1690, $\Gamma = 200$ MeV), and as one isosinglet the ω (1670, $\Gamma = 166$ MeV). Both have $J^{PC} = 3^{--}$. G parity requires the g to decay into an even number of π 's, ω into an odd number. Reference 120 observes the g in the reaction

$$\pi^- (12 \text{ GeV}) p \rightarrow \pi^- \pi^0 p: \text{ Fig. 39a.}$$

(The broad shoulder beside ρ^- and g^- at 2.4 GeV is due to s channel resonances and disappears for a corresponding cut). A moments analysis shows a significant peak of the spin-3 sensitive t_6^0 moment at 1690 MeV whilst t_6^1 , t_6^2 are compatible with 0: g production by unnatural (π) exchange with helicity 0: Fig. 39b. The same authors have observed other decay channels: $\pi^- \pi^- \pi^0 \pi^+$ especially $\omega^- \pi^-$ (Fig. 40a) and $A_2^- \pi^0$ (Fig. 40b).

See Ref. 48 for a very recent observation of a diffractively photoproduced bump in the 3^- ($A_2 \pi$) wave decaying into $\eta \pi \pi$: almost probably the g meson. If one interprets this bump as being purely due to the g , the photoproduction cross-section turns out to be surprisingly high:

$$\sigma[\gamma p \rightarrow g(1690)p] \approx 300 \text{ nb.}$$

Spin and parity of the $\omega(1670)$ were determined to be 3^- by Ref. 121 in a PWA of the 3π system generated in the reaction

$$\pi^+ (7 \text{ GeV}) p \rightarrow \pi^+ \pi^- \pi^0 \Delta^{++}.$$

Figure 41a gives the 3π mass distribution: the 0 ($3^- F \rho \pi$) wave clearly contributes [beside 1 ($2^+ D \rho \pi$) wave: the A_2 meson] to the resonance at 1670 MeV. The branching mode $\omega(1670) \rightarrow B_{\frac{1}{2}}^+ \pi$ is observed by Ref. 122: Fig. 41b.

The $K^*(1780)$ stands for the corresponding isodoublet of 3D_3 , the intensity of the partial wave of the decay in $K^* \pi$ ($3^- \rightarrow 1^- + 0^-$, F wave)

is shown in Fig. 42a ¹²³⁾. Also a PWA of $K\pi$ systems by the same group (LASS spectrometer at SLAC) yields an Argand diagram of the F wave with a resonance at ~ 1700 MeV, $\Gamma = 300$ MeV ($\delta = 90^\circ$, elasticity $\sim 16\%$): see Fig. 42b ¹¹⁴⁾.

The next higher leading resonance with $J^P = 4^+$ seems to appear at 2070 MeV, $\Gamma = 250$ MeV, elasticity 7%: Fig. 42c, see also Ref. 97.

For an ideally mixed 3D_3 family one expects the second $I = S = 0$ state at $2 \times 1.780 - 1.690 = 1.870$ MeV. Indeed one observes in K^+K^- systems produced in the reaction $K^-(18.5 \text{ GeV})p \rightarrow K^-K^+ (\Lambda/\Sigma^0)$ a bump at about this mass in the overall mass distribution: Fig. 43a ³¹¹⁾. A PWA proves the dominant F wave nature of this bump and thus $J^P = 3^-$. A fit to the F wave intensity yields

$$m = 1.87^{+0.03}_{-0.02} \quad \Gamma = 0.16^{+0.09}_{-0.05} \text{ GeV}.$$

$$\underline{{}^1D_2: J^{PC} = 2^{-+}}$$

The isotriplet A_3 is detected ^{120,124)}: the $1(2^-Sf\pi)$ wave of the $\pi^+\pi^-\pi^-$ system peaks at 1680 MeV, $\Gamma = 250$ MeV (Fig. 43b). A candidate for the isodoublet is the L (1770), see Fig. 25. It could well be that future experiments with higher sensitivity are able to split this broad object into two resonances. Then a mixing mechanism analogous to $Q_1(1^{+-}) \leftrightarrow Q_2(1^{++})$ is possible (see Section 2.2.2). Other members are unknown.

$$\underline{{}^3D_2: J^{PC} = 2^{--}}$$

Complete lack of information (but see 1D_2 for the isodoublet).

$$\underline{{}^3D_1: J^{PC} = 1^{--}}$$

The possibility, that observed candidates for radially excited vector mesons are in fact D wave states or a mixture of S and D wave was mentioned in Section 2.2.1 as well as criteria to distinguish between these states. See, for example, the discussion concerning the large $\rho'(1600)$ width and also the funny ω' , ϕ' pattern observed in photoproduction on the one hand and at e^+e^- rings on the other. Also speculative is the classification of $\pi^+\pi^-$ resonances above 2 GeV observed in pp annihilation as 3D_1 members: $\rho'(2150)$ ¹⁶⁾.

So, in light quark spectroscopy there are no safe 3D_1 candidates with $I = 0$ or 1 . A recent observation of a bump at 1790 MeV of the $1/2(1^-P\rho K)$ amplitude in a PWA of the $(K\pi\pi)$ systems obtained in the reaction $K^- (11 \text{ GeV})p \rightarrow K^0\pi^+\pi^-n$ could well be the $I = 1/2$ member of the 3D_1 family²⁷⁷. In charmonium spectroscopy candidates for 3D_1 cc states exist: $\psi'(3770)$.

$$\underline{{}^3F_2} : J^{PC} = 2^{++}$$

Analogous relations could exist between the 3F mesons (no candidates known) and the tensor mesons 3P_2 both with $J^{PC} = 2^{++}$. No information exists concerning 3F_3 and 1F_3 , but see discussion concerning the $\xi(2220)$ in Section 3 and Ref. 422 for a 3F_3 isotriplet candidate, the $A(2050)$.

$$\underline{{}^3F_4} : J^{PC} = 4^{++}$$

The isodoublet was already shown in Fig. 42c, this result was confirmed by Ref. 340. The isotriplet could be the $\delta(2040)$ as seen by Ref. 363a. Known beyond any doubt is the isosinglet $h(2040)$, $\Gamma = 150 \text{ MeV}$, $J^{PC} = 4^{++}$, recurrence of the $2^{++}f$ meson, decaying to $\pi\pi$ and KK .

A recent analysis of the reaction

$$\pi^- (70 \text{ GeV})p \rightarrow \pi\pi n \quad (\text{Ref. 125})$$

shows beside the h meson a further recurrence: the $r(2510)$ with $J^{PC} = 6^{++}$, i.e. 3H_6 (Fig. 44a). The significance of this object is improved by the following procedure. Developing the decay angular distribution in the Gottfried-Jackson system (resonance rest system with beam as z axis) with respect to Legendre polynomials P_L it turns out that $[P_4(\cos \theta_{GJ})]^2$ has a minimum at $\cos \theta_{GJ} \approx 0.8$ whilst $[P_6(\cos \theta_{GJ})]^2$ has a maximum. A corresponding cut enhances the spin-6 ratio remarkably (Fig. 44b).

At the end of the walk through the mesonic zoo, formed by $q\bar{q}$ pairs, a list of objects will be given whose experimental evidence needs confirmation before a classification is possible.

$$\rho\rho(2100) \text{ (Ref. 128): } (\gamma\gamma \rightarrow \pi^+\pi^-\pi^+\pi^-)$$

$$g_s(1240) \text{ (Refs. 273,313): } (\pi^-p \rightarrow K_S^0 K_S^0 n)$$

$$C(1490,165) \text{ (Ref. 279): } (\pi^-p \rightarrow Cn \rightarrow \phi\pi^0 n)$$

$$X(1629,82) \text{ (Ref. 342): } [\pi^+/p \ p \rightarrow \pi^+/p \ (K^+K^-)p]$$

2.3 Models for description of $q\bar{q}$ system

As explained in the preceding section the concept of quarks gives an excellent tool to classify meson spectroscopy in a qualitative way. Some aspects of quantitative attempts to predict masses, widths, and branching ratios will be regarded in this section beginning with potential models. Though these models seem to be somewhat naive they appear to have a surprisingly big predictive power. But it should be stated here that the real quality of any ansatz is measured not only by its predictions of masses, widths (lifetimes) and branching ratios -- delivering important information on either the real content of an object or also on the underlying dynamical structure -- but also on the independence from random parameters. It will turn out that this requirement is fulfilled rather poorly by these concepts, thus calling for the necessity of a development of a more satisfactory theory within the framework of QCD.

2.3.1 Potential models; the QED analogue positronium

Positronium is a hydrogen atom with protons replaced by positrons. Rydberg's constant changes therefore according to

$$R = R_X \left(\frac{m_1}{m_1 + m_2} \right)$$

$$R(e^+e^-) = \frac{R_X}{2}, \quad R_X = \frac{m_e \cdot e^4}{8\epsilon_0^2 h^3 c} = 1.097 \times 10^7 \text{ m}^{-1}. \quad (13a)$$

The solution of the Schrödinger equation for a Coulombic potential $V(r) \sim -\alpha/r$ leads to a relation for the energy eigenvalues analogous to the H atom

$$E_n = - \frac{6.8}{n^2} \text{ eV}. \quad (13b)$$

Relativistic corrections and spin-orbit interactions cause fine splitting

$$\Delta E_{FS} = |E_n| \cdot \frac{\alpha^2}{n} \left(\frac{3}{4n} - \frac{1}{j+1/2} \right). \quad (14)$$

For the $n = 2$ level of positronium the degenerate $S_{1/2}$, $P_{1/2}$ states are lowered from 1.7 eV by 2.6×10^{-5} eV, $P_{3/2}$ by 0.53×10^{-5} eV.

Interactions between spins means interaction between magnetic moments of rotating charges. The magnetic moment of the positron is 2000 times larger than that of the proton, therefore the role of hyperfine structure becomes more important. This is also valid in the case of replacing electrons by quarks:

$$\Delta E_{HF} \sim a \cdot \vec{S}_1 \cdot \vec{S}_2 \cdot Z^2, \quad a > 0$$

therefore for $L = 0$

$$^3S_1: \vec{S}_1 \cdot \vec{S}_2 = \frac{1}{2} (\langle J \rangle^2 - S_1^2 - S_2^2) = \frac{1}{2} [j(j+1) - S_1(S_1+1) - S_2(S_2+1)] = + \frac{1}{4}$$

$$^1S_0: \quad " \quad = \quad " \quad = \frac{1}{2} (0 - \frac{6}{4}) = - \frac{3}{4},$$

i.e. $m(^3S_1) > m(^1S_0)$: for example $m_\psi > m_{\eta_c}$, $m_Y > m_{\eta_b}$, $m_\rho > m_\pi$.

So the $S_{1/2}$ state mentioned above splits to 3S_1 , 1S_0 ; analogously $P_{1/2}$, $P_{3/2}$ to 1P_1 , 3P_0 , 3P_1 , 3P_2 . The resulting ($n=2$) spectrum of positronium is shown in Fig. 45 and (simplified) in Fig. 46c. The comparison between $b\bar{b}$, $c\bar{c}$ and e^+e^- in Figs. 46a-c, respectively, suggests qualitative analogy between heavy quark states bound by $q\bar{q}$ interactions and positronium bound by electromagnetic interactions: the signs of the P wave splitting agree even though absolute numbers are orders of magnitude away from each other:

$n = 2$: for e^+e^- , $c\bar{c}$, $b\bar{b}$

$$m(^3P_0) < m(^3P_1) < m(^3P_2) < m(^3S_1).$$

The separation of main quantum number levels gives an idea for the order of magnitude of the coupling constant α_s of QCD: the fine structure constant α is replaced by $4/3 \alpha_s$. The term $4/3$ stems from the mediation of the interaction between q and $\bar{q}$ in colour singlet by the 8 gluons, i.e. 8 generators of the colour SU(3): $\lambda_i/2$

$$\langle q\bar{q} \rangle_s \left| \frac{\frac{\lambda_i}{2}(q) \frac{\lambda_i}{2}(\bar{q})}{4\pi r} \right| \langle q\bar{q} \rangle_s, \quad i = 1, \dots, 8$$

and

$$|(q\bar{q})_s\rangle = \frac{1}{\sqrt{3}} |(q\bar{q})_{\text{RED}} + (q\bar{q})_{\text{BLUE}} + (q\bar{q})_{\text{GREEN}}\rangle$$

therefore (taking only contributions of λ_s multiplied by 8)

$$(\alpha_s = \frac{g^2}{4\pi r}): \quad V(q\bar{q}) = - \frac{8}{2\sqrt{3}\cdot\sqrt{3}} \alpha_s \cdot \frac{1}{r} = - \frac{4}{3} \alpha_s \cdot \frac{1}{r} . \quad (15)$$

From Eqs. (12) and (13) follows with $m_c = 1.5 \text{ GeV}$, $m_b = 5 \text{ GeV}$

$$\frac{\Delta E_{e^+e^-}}{\Delta E_{c\bar{c}}} = \frac{(2^3S_1 - 1^3S_1)_{e^+e^-}}{(2^3S_1 - 1^3S_1)_{c\bar{c}}} = \frac{5.2}{589 \cdot 10^6} = \frac{m_e \alpha_s^2}{m_c \frac{16}{9} \alpha_s^2} \rightarrow \alpha_s = 1.07$$

$$\frac{\Delta E_{e^+e^-}}{\Delta E_{b\bar{b}}} = \frac{(2^3S_1 - 1^3S_1)_{e^+e^-}}{(2^3S_1 - 1^3S_1)_{b\bar{b}}} = \frac{5.2}{561 \cdot 10^6} = \frac{m_e \alpha_s^2}{m_b \frac{16}{9} \alpha_s^2} \rightarrow \alpha_s = 0.54 .$$

The decrease of the coupling constant with increasing quark mass is correctly described. The absolute numbers for α_s are about a factor of three too large thus proving that the assumption of a purely "Coulombic" potential between b and $\bar{b}$ and even more so between c and $\bar{c}$ is not justified. These systems are still too weakly bound to be dominated by the short-range Coulombic part of the potential only: see Fig. 48 for graphic illustration.

On the walk from e^+e^- to potential models for light quarks also those for heavy quarks should be regarded to pick up useful analogies.

Potential models for heavy quarks

The Coulomb potential predicts (nearly) degenerated S and P states for $n = 2$ for instance. A three-dimensional harmonical oscillator [$V(r) \sim r^2$] has energy eigenvalues according to

$$E_N = (N + \frac{3}{2}) \hbar\omega \quad (16)$$

leading to a term scheme as shown in Fig. 47. The nomenclature is chosen in analogy to a Coulomb potential even though many authors assign states on dashed lines in Fig. 47 as 2S, 2P, 2D,

Two important features of Fig. 47 are

- radial excitations are equidistant;

- nP is located at $\frac{(n-1)S + nS}{2}$.

The first feature is nearly fulfilled for $c\bar{c}$, $b\bar{b}$ systems (see Table 7) [the mis-ordered $\psi'(3767, J^{PC} = 1^{--})$ is interpreted to be a 3D_1 state¹²³⁾]. But the P levels of $(c\bar{c})$ are not sited at the arithmetic average and lie between the Coulombic prediction ($nP \approx nS$) and the oscillator prediction. Therefore one interpolates for $V_{q\bar{q}}(r) \sim r^x$

$$-1 \text{ (Coulomb)} \leq x \leq +2 \text{ (oscillator)} .$$

At larger distances the assumption of a linear potential " $x = 1$: $V(r) = k \cdot r$ " is suggested; the value of k might be simply estimated from a typical mass of 1 GeV, a typical distance of 1 fm = 5 GeV^{-1} leading to an energy density $k = 0.2 \text{ GeV}^2$. See Reference 132 for a more detailed calculation leading to

$$k = 0.18 \text{ GeV}^2 \approx \frac{1 \text{ GeV}}{\text{fm}} \text{ [for a value of } \alpha = \frac{L}{M^2} \sim 0.9 \text{ GeV}^{-2} \text{ (see Fig. 38)]} .$$

The fact should be stated that the ratio

$$B = \frac{\text{binding energy}^{*)}}{\text{constituent mass}} \left[\approx \frac{0.6}{3} \text{ for } c\bar{c} \text{ } (m_c \approx 1.5 \text{ GeV}) , \approx \frac{1.2}{10} \text{ for } b\bar{b} \text{ } (m_b \approx 5 \text{ GeV}) \right]$$

might be small enough for a non-relativistic treatment but is still very large compared to e^+e^- ¹²⁹⁾:

$$B = \frac{13.6}{1.022} \times 10^{-6} .$$

Many ansatzes for potential models assume a Coulombic term for the short-distance range and a linear term for the long-ranged part:

$$V(r) = -\frac{4}{3} \alpha_s \frac{1}{r} + k \cdot r + c . \quad (17)$$

*) Defined as $m(\text{open flavour threshold}) - m(^3S_1)$.

The heavier the quarks the more Coulomb-like the potential (see again Fig. 48).

In general, the non-relativistic solution of the Schrödinger equation for potentials $V(r) = k \cdot r^x$ predicts a behaviour according to Table 5 (Refs. 7,49). Summarizing Table 5 one has ($\mu = m_q/2$):

$$\begin{aligned} \Delta E &\sim \mu^{-x/x+2} \\ |\psi(0)|^2 &\sim \mu^{3/x+2} \\ \langle r^v \rangle &\sim \mu^{-v/x+2} \quad v = 1, 2 \end{aligned} \quad (18)$$

Most of the calculations of ($c\bar{c}$) and ($b\bar{b}$) spectroscopy are based on potentials of Table 5 or combinations of them. If an exact solution is not possible (e.g. for states with $L \geq 1$ within a linear potential) one tries approximation procedures by treating the Coulomb term as a small perturbation¹³⁴⁾. Another way -- very successfully used for the description of baryon spectroscopy¹³⁵⁾ -- is to solve an oscillator potential and to adjust the oscillator strength to the ground state of the potential (Eq. 17).

Table 6 lists typical potential ansatzes. Reference 136 classifies these attempts into two groups:

- QCD inspired models (Refs. 137-139, 140-142). The short range behaviour is described -- accounting for higher orders (Refs. 138,141) -- by asymptotic freedom:

$$\alpha_s(r) = \frac{12\pi}{25 \log \left(\frac{1}{r^2 \Lambda_{\overline{MS}}^2} \right)} \quad (19)$$

This "weakening" of the Coulomb singularity at $r = 0$ becomes important already for $b\bar{b}$ systems: see Eq. (22). Note the pole in Eq. (19) for $r = r_{\text{conf}} = 1/\Lambda_{\overline{MS}} \sim 3 \text{ GeV}^{-2} = 0.7 \text{ fm}$.

At larger distances either a linear part is added appropriately^{140,142)} or an analytic interpolation in momentum space is used^{138,141)}.

- Phenomenological models¹⁴³⁻¹⁴⁵). As can be seen from Table 5 (nearly) equal mass differences ($2^3S_1 - 1^3S_1$) for $c\bar{c}$ and $b\bar{b}$, respectively, point to logarithmic potentials. A slight decrease of the $b\bar{b}$ distances causes, according to Ref. 145, a choice of x slightly different from 0: $x = 0.1$.

A fit to the experimental data yields

$$V(r) = -8.064 + 6.87 r^{0.1}$$

for an optimal description.

Figure 48 shows different potentials with respect to r . An approximate position of different $q\bar{q}$ states is also indicated quoted analogously to $\langle v^2 \rangle$ in Table 5.

Reference 69 comments on the different models in Fig. 48 by saying: "They include wildly different analytic forms which would imply wildly different behaviours -- if extrapolated to the long and short range regimes. But the important point is that for the region measured by the $b\bar{b}$ and $c\bar{c}$ spectra they are virtually indistinguishable". This statement encourages to detect and measure ($t\bar{t}$) systems as well as to understand the light quark regime.

So, one is not surprised that predictions based on different models do not differ very much; see for instance in Table 7 predictions for $c\bar{c}$ and $b\bar{b}$ spectroscopy from Refs. 141,145. On the other hand, it is certainly remarkable that quarks under the regime of QCD obey such simple rules. Indeed the treatment of QCD in the frame of lattice gauge theories shows encouraging first results on the $q\bar{q}$ potential looking very similar to those in Fig. 48 (see Section 2.3.4).

The next step includes the spin dependence of potentials: $V = V_0 + V_{\text{spin}}$. Reference 146 quotes V_{spin} in leading order to depend on four different contributions: V_{Thomas} , $V_{\text{spin orbit}}$, V_{tensor} , $V_{\text{spin-spin}}$. In case of a Coulomb potential these four contributions turn out to be proportional to

$$-\frac{\alpha}{r}, -\frac{\alpha}{r}, \frac{3\alpha}{r^3}, v^2(-\frac{\alpha}{r}), \text{ respectively.}$$

For positronium one gets in lowest order (according to Breit):

$$V_{\text{spin}} = \frac{3}{2} \frac{\vec{L}(\vec{S}_1 + \vec{S}_2)}{m^2} \cdot \frac{\alpha}{r^3} + \frac{\alpha}{m^2 r^3} [3(\vec{S}_1 \cdot \vec{r})(\vec{S}_2 \cdot \vec{r}) - \vec{S}_1 \cdot \vec{S}_2] + \frac{8\pi\alpha}{3} \frac{\vec{S}_1 \cdot \vec{S}_2}{m^2} \delta(\vec{r}) \quad (20)$$

Passing to QCD in lowest order (one gluon exchange) one simply replaces α by $4/3 \alpha_s$. But as shown by Eichten and Feinberg¹⁴⁶⁾ in QCD the splitting of V into V_o and V_{spin} is not valid: the long-range non-Coulombic part of V_o influences V_{Thomas} leading to an unrealistic one gluon exchange assumption. Therefore Eq. (20) changes with

$$V_{\text{Thomas}} \sim V_o = V_{\text{short}} + V_{\text{long}} = -\frac{4}{3} \frac{\alpha_s}{r} + kr$$

$$V_{\text{spin orbit}} \sim -\frac{4}{3} \alpha_s \cdot \frac{1}{r}$$

$$V_{\text{tensor}} \sim \frac{4\alpha_s}{r^3}$$

$$V_{\text{spin spin}} \sim -\frac{4}{3} \alpha_s v^2 \left(\frac{1}{r}\right)$$

into

$$V_{\text{spin}} = -\frac{1}{2m^2} \vec{L} \cdot \vec{S} \cdot \frac{\vec{k}}{r} + \frac{2}{m^2} \frac{\alpha_s}{r^3} \vec{L} \cdot \vec{S} + \frac{4}{3} \frac{1}{m^2} \frac{\alpha_s}{r^3} [3(\vec{S}_1 \cdot \vec{r})(\vec{S}_2 \cdot \vec{r}) - (\vec{S}_1 \cdot \vec{S}_2)] + \frac{32\pi}{9} \frac{1}{m^2} \cdot \alpha_s (\vec{S}_1 \cdot \vec{S}_2) \cdot \delta(\vec{r}) \quad (21)$$

Calculating Eq. (21) from (20) one gets a result with a "+" sign in front of the first term. Gromes³⁴⁵⁾ has shown that the assumptions in Ref. 146 leading to the "+" sign were incorrect and has thus solved a long-standing question on spin-dependent quark forces: therefore "these forces are essentially understood"; Eq. (21) is identical to the result obtained by assuming a long range scalar + a short range vector exchange³⁴⁶⁾.

Taking only the "hyperfine contribution" ($\sim \vec{S}_1 \cdot \vec{S}_2$) from Eq. (21) and a linear potential, Ref. 347 quotes $m_\psi - m_{\eta_c} = 120$ MeV in excellent agreement with the experimental observation: 120 ± 6 MeV. The so far unobserved η_b (pseudoscalar $b\bar{b}$ state) is expected³⁰⁷⁾ at

$$\Delta m(Y - \eta_b) = \frac{8}{9} \frac{\Gamma_{ee}(Y)}{e^2 \alpha^2} \alpha_s(m_b) \left[1 + \frac{5.6 \alpha_s(m_b)}{\pi} \right] . \quad (22)$$

$\Gamma_{ee}(Y)$ is calculated via Eq. (7): 1.07 ± 0.25 keV¹⁴¹⁾,

exp. value: 1.24 ± 0.5 keV.

Equations (22) and (19) could be used to determine $\Lambda_{\overline{MS}}$:

$$\Delta m(Y - \eta_b) = 27/35/42 \text{ MeV} \quad \text{for} \quad \Lambda_{\overline{MS}} = 100/200/300 \text{ MeV} .$$

Moreover, one can show³⁴⁷⁾ that with a linear potential the quantity

$$m^2(^3S_1) - m^2(^1S_0)$$

should be independent from the quark flavour (despite a running α_s). Looking for $I \neq 0$ states (to remain unbiased from mixing mechanisms or gluonium admixtures) one indeed observes:

$$m_\rho^2 - m_\pi^2 = 0.574 , \quad m_D^{2*} - m_D^2 = 0.546 , \quad m_B^{2*} - m_B^2 = 0.551 .$$

On the other hand, $V_{L \cdot S}$, V_{tensor} cause the splitting of P waves:

$$\rho = \frac{m_{1^{++}} - m_{1^0}}{m_{1^{++}} - m_{0^{++}}} \approx 0.47/0.66 \text{ (exp. for } c\bar{c}/b\bar{b})$$

$$= 2 (V_{L \cdot S} \text{ only})$$

$$= -0.4 (V_{\text{tensor}} \text{ only}) .$$

The value of ρ for a $c\bar{c}$ system can be used to argue that the long-ranged part of the potential is not a vector: in this case -- independently from the form of the potential -- it can be shown that $\rho \geq 0.8$ ⁴⁹⁾. On the other hand a scalar "long range" contribution has no tensor term and a $L \cdot S$

term with different sign to a L·S term from a Coulomb (vector) potential. Thus these cancel partly and lower therefore ρ by increasing the weight of the (Coulombic) tensor part. The Coulomb (vector) part only leads to a (L·S/tensor) ratio of $6^{348)}$ and thus³⁴⁹⁾ to $\rho = 0.8$.

Table 8 contains predictions from three authors¹⁴⁶⁻¹⁴⁸⁾ of the fine structure of the P states compared to experimental results: quite good agreement is found. See Refs. 350, 351 for a complete set of experimental values for the bottomium states compared with several models; in Ref. 367 an illustrative graphic presentation of this comparison is given.

Different contributions to V_{spin} have different (quark) mass dependence. This can be used for a prediction of hyperfine splitting of mesons consisting of one u,d and one c,b quark:

$$\frac{m_b}{m_c} = \frac{m_D^* - m_D}{m_B^* - m_B} \quad \text{or} \quad m_B^* - m_B \approx 50 \text{ MeV} .$$

As mentioned in the introduction models should be judged not only by the quality of their mass predictions but also by that of widths and branching ratios.

The leptonic width Γ_{ee} (see Table 3) measures via (Eq. 8) directly $|\psi(0)|^2$. A linear potential, for instance, shows no dependence of $|\psi_n(0)|^2$ from n: therefore $\Gamma_{ee}(\psi') = \Gamma_{ee}(\psi)$. The experimental result $\Gamma_{ee}(\psi') \approx 2.37 \Gamma_{ee}(\psi)$ proves the presence of short range Coulombic contributions for the $c\bar{c}$ potential, as mentioned already in the context of $\rho'(1600)$.

Also the decay width of ortho- and para-positronium is proportional to $|\psi(0)|^2$ (with the first suppressed by α). This can be transferred directly leading to:

$$\Gamma(n_c \rightarrow \gamma\gamma) = \frac{64\pi \alpha_s^2}{27 m_c^2} |\psi(0)|^2 .$$

Pure hadronic decays of $^1S_0 / ^3S_1$ states should occur via 2/3 gluons. Therefore one expects

$$\frac{\Gamma(\psi/\Upsilon \rightarrow \text{hadrons})}{\Gamma(\eta_c/\eta_b \rightarrow \text{hadrons}) \text{ or } \Gamma(\psi/\Upsilon \rightarrow e^+e^-)} \sim \frac{\alpha_s^3}{\alpha_s^2 \text{ or } \alpha^2}$$

One determines from here $\alpha_s(m_\psi) \sim 0.2$ from $\psi \rightarrow e^+e^-$. [This result cannot be understood as a precise measurement, its determination relies on non-relativistic calculation and only lowest order contributions: see Ref. 367 for the actual status of extracting $\alpha_s(m_{c\bar{c}}, m_{b\bar{b}})$ or $\Lambda_{\overline{MS}}$ from measurements of hyperfine, L·S, or tensor contributions to V_{spin} .]

As explained already in Section 2.1 radiative widths ($c\bar{c} \rightarrow c\bar{c} \rightarrow \gamma$) are important indicators testing quark and potential models.

After appropriate modification of formulae concerning multipole radiation of atoms⁴⁹⁾ (see Ref. 352 for a more precise calculation) one gets in lowest order:

Electric dipole transitions

$$\Gamma(^3S_1 + ^3P_J + \gamma) = \frac{(2J+1)4\alpha k^2 q_c^2}{27 m_c^2} |I_1(^3S_1, ^3P_J)|^2 \quad (23)$$

k is the γ momentum; please note that the coherence requirement of atomic physics $k \cdot R \ll 1$ is often fulfilled only quite poorly:

$$\psi' \rightarrow \eta_c + \gamma \quad (856) \text{ MeV} \rightarrow k \cdot R = 0.86 \times 0.6 ,$$

see Fig. 48.

$I(^3S_1, ^3P_J)$ is the overlap integral of radial wave functions:

$$I_n = \int_0^\infty r^{n+2} R_s(r) \cdot R_p(r) dr .$$

Magnetic dipole transitions

$$\Gamma(^3S_1 \rightarrow ^1S_0 + \gamma) = \frac{4\alpha q_c^2 k^3}{3m_c^2} |I_0(^3S_1, ^1S_0)|^2 . \quad (24)$$

Neglecting contributions from the overlap integral one expects for the decay according to Eq. (23)

$$c' \rightarrow {}^3P_J + \gamma, \quad \text{a width-ratio proportional to } (2J+1)k^3.$$

Measurements at the Crystal Ball yield^{131,149)} $[\Gamma_J := \Gamma(\psi' \rightarrow {}^3P_J + \gamma)]$

$$\frac{\Gamma_0}{273^3} = (0.99) \frac{\Gamma_1}{3 \cdot (178)^3} = (1.31 \pm 0.1) \frac{\Gamma_2}{5 \cdot (132)^3}.$$

Absolute predictions for $\Gamma_0, \Gamma_1, \Gamma_2$ with a standard potential (Eq. 17) are 50, 45, 29 keV, respectively, therefore larger than the experimental values¹⁶⁾: 18, 17, 16 keV.

Improved models (Refs. 150, 154, 155) lead to better agreement by accounting for relativistic corrections: $\Gamma_{0,1,2} = 16, 23, 22$ keV (Ref. 150).

Also $(b\bar{b})$ systems obey Eq. (23) within the errors: $\Upsilon'(10020) \rightarrow \chi_b + \gamma$

$$\frac{\Gamma_0 = 1.2}{(161)^3} = \frac{\Gamma_1 = 2.0}{3 \cdot (131)^3} = \frac{\Gamma_2 = 2.1}{5 \cdot (109)^3}.$$

[The values of $\Gamma_{0,1,2}$ (keV) and a collection of potential models being in good agreement with these experimental results can be found in Ref. 350].

From Eq. (24) follows for M1 transitions (with similar assumptions):

$$\frac{\Gamma(\psi' \rightarrow \eta_c' + \gamma)}{\Gamma(\psi \rightarrow \eta_c + \gamma)} = \frac{92^3}{117^3} = \frac{1}{2}.$$

But one finds a ratio of

$$\frac{0.43 \leq \Gamma(\psi' \rightarrow \eta_c' + \gamma) \leq 2.8 \text{ keV}}{0.4 \leq \Gamma(\psi \rightarrow \eta_c + \gamma) \leq 0.8 \text{ keV}},$$

therefore probably no good agreement. Absolute predictions from Eq. (24) are even worse, probably the assumption for the singlet overlap integrals

$$[I_0(\psi, \eta_c) = I_0(\psi', \eta_c') = 1 \quad \text{but} \quad I_0(\psi, \eta_c') = 0]$$

are wrong. The experimental value $\Gamma_{\text{exp}}(\psi' \rightarrow \eta_c \gamma)$ is $\sim 0.6 \pm 0.1$ keV (Ref. 422) and therefore definitely not 0. Recent attempts¹⁵²⁾ realizing this possibility obtain satisfying improvements.

Despite such (minor) difficulties, predictions of potential models for $(b\bar{b})$ and $(c\bar{c})$ spectroscopy turn out to work extraordinarily well. It would be very helpful to discover the 1P_1 states of both quark contents [see Ref. 293 for a (statistically weak) candidate]: $^1P_1(c\bar{c})$ at 3525 MeV. Its mass compared to the centre of gravity of 3P_J is sensitive to the hyperfine part and thus measures the very short range part of the potential: see next section. Because of $C = -1$ it cannot be observed in radiative decays of ψ' . Also observation of further $D(L=2)$ objects is needed. The already found $\psi(3770)$ lies above $D\bar{D}$ threshold and is therefore quite broad (25 MeV). 1D_2 and 3D_2 states cannot decay into $D\bar{D}$ by parity reasons and are (probably) positioned below $D\bar{D}^*$ threshold and should therefore be narrow and thus useful for spectroscopic analysis.

The discovery of $(t\bar{t})$ and its spectroscopy will almost certainly decide which model has chosen the right parametrization. Table 9 shows $t\bar{t}$ predictions by Ref. 141 as function of m_t and $\Lambda_{\overline{MS}} = 200$ MeV [see Eq. (19)]. See also Ref. 353 for an overview of toponium physics.

The way to light quarks and therefore to larger distances points away from a Coulombic behaviour and therefore from a viewpoint of simplicity into the wrong direction. Nevertheless, many of the characteristics found for heavy quarks are transferable to the light quark regime.

Potential models for light quarks

The validity of this statement is indicated by Fig. 49, a complete analogue of Fig. 46: for all multiplets ideal mixing was presumed. The resulting term scheme corresponds to our actual knowledge as reviewed in the previous section. Both $s\bar{s}$ systems (Fig. 49c) and $(u\bar{u}, d\bar{d})$ systems (Fig. 49b) show qualitative similarity to positronium (Fig. 49a).

Calculating $c\bar{c}$ and $b\bar{b}$ spectroscopy the quark masses were free parameters. In Table 7 the input quark masses from Refs. 141 and 145 differed by $\Delta m_c = 320$ MeV and $\Delta m_c = 304$ MeV.

The problem of which mass to choose becomes much more important for "light" quarks. As mentioned the masses quoted by radiative ρ , ω , ϕ decays, the constituent masses $m_u = m_d \sim 300$ -400 MeV, $m_s \sim 500$ MeV, are up to two orders of magnitude larger than to those taken for the equations of motion of QCD in the asymptotic limit $q^2 \rightarrow \infty$: current quark masses.

The latter must be small -- as can be seen by a naive interpretation of $\langle p_T \rangle$ of the jets with respect to the principal axis in the reaction

$$e^+e^- \rightarrow q\bar{q} \rightarrow 2 \text{ jets} ; \quad \text{or} \quad \gamma p \rightarrow 2 \text{ jets } p^{157}) .$$

The experimental result is $d\sigma/dp_T \sim e^{-3p_T}$, i.e. $\langle p_T \rangle = 300$ MeV. This could correspond to the momentum up to which quarks are allowed to move freely in a nucleon without being called to order by confinement. Because of $m_p = 3 \cdot 300$ MeV the rest energy contribution to the total quark energy must be small.

Thus for a spectroscopic approach a scheme might be drawn where constituent quarks are built up by a "hard core current quark" plus a coat of virtual gluons and condensed $q\bar{q}$ pairs.

Potential models (especially non-relativistic ones) need (dressed) quark masses large compared to the binding energies: whilst for $c\bar{c}$, $E_b/2m_c = 0.25$, for light quarks: $E_b/2m_u$ in the order of 1 for constituent and $\gg 1$ for current quark masses.

Alternative concepts like bag models or lattice gauge calculations supersede naive non-relativistic models.

Staying with potential models spin-spin and spin-orbit splittings become stronger than for heavy quarks due to the mass dependence in Eqs. (20,21):

$$(m_\rho - m_\pi) > m_\psi - m_{\eta_c} .$$

$V_{\text{spin-spin}}$ is not treatable within a perturbation theory. A calculation of Schnitzer (Refs. 158, 317, and 349) shows that the splitting of the P waves:

$$\rho = \frac{m_{2^{++}} - m_{1^{++}}}{m_{1^{++}} - m_{0^{++}}}$$

for a potential [Eq. (17)] with the additional assumption of "1 gluon exchange" (vector exchange) also for the long range part has to have values between 0.8 and 1.4. This was fulfilled only poorly for $c\bar{c}$: $\rho_{c\bar{c}} = 0.47$. For u,d quarks:

$$\rho = \frac{m_{A_2} - m_{A_1}}{m_{A_1} - m_\delta} = 0.2$$

proving that the assumption underlined is becoming more and more wrong.

Taking a scalar behaviour for the long range part a prediction can be made for 1P_1 states¹⁵⁹). (In this case $V_{\text{spin-spin}}$ is purely due to the Coulomb term, therefore $\sim \delta^3(r)$, and disappears for P waves.) Thus $m(^1P_1)$ can be quoted from the centre of gravity of (3P_J):

$$m(^1P_1) = \frac{5}{9} m(^3P_2) + \frac{3}{9} m(^3P_1) + \frac{1}{9} m(^3P_0) . \quad (25)$$

Equation (25) predicts the unobserved (1P_1) $_{c\bar{c}}$ at 3522 MeV^{*)}. For light quarks 1P_1 is known and Eq. (25) is surprisingly well fulfilled, if one recalls the resonance parameter uncertainties and classification problems.

$$m_B = \frac{1}{9} (5m_{A_2} \oplus 3m_{A_1} \oplus m_\delta) = 1263 \text{ (exp.: 1235 MeV)}$$

$$m_{Q_B} = \frac{1}{9} (5m_K^* (1430) \oplus 3m_{Q_A} + m_\kappa) = 1381 \text{ (exp.: 1370 MeV) .}$$

($\oplus$ is used if the meson on the left is heavier than that on the right.) That there is a + without a circle for the (s+u,d) systems, i.e. non-

*) Note that Ref. 293 reports on a hint for this object at 3525 MeV!

monotonic P wave splitting is explained by Schnitzer¹⁶⁰⁾ not to be just bad luck but a trend with increasing tendency towards P wave D mesons (c+u,d). There should hold

$$m(^3P_2)_{cu} < m(^3P_1)_{cu} < m(^3P_0)_{cu} .$$

A confirmation of this relation would be an important support for the hypothesis of a (k·r) term with a scalar exchange. The validity of Eq. (25) for light quarks could be used as an argument that the classification concerning the 0^{++} mesons was reasonable. Unfortunately these 1P_0 objects enter Eq. (25) with the smallest weight, the argument seems therefore not too compelling.

Some of the actual models shall be briefly reviewed. The number of free parameters turn out to be directly proportional to their predictive power especially where the π meson mass is concerned, a key problem of (u,d) spectroscopy. Though approaches explaining "how massless quarks can form massless pseudoscalars but massive vector mesons are just being invented" (Ref. 354), it is still necessary for an understanding of the whole variety of experimental results to use "simple mechanical models" with experimentally guided input parameters as "crutches"³⁵⁵⁾.

On the other hand, the success of the harmonic oscillator potentials describing baryon spectroscopy according to Isgur and Karl looks stimulating.

See Refs. 400, 164 for the founding work to apply potential models to light quark systems. From here several ansatzes have been developed. As already mentioned the quality of these relies not only on mass calculations [which are usually not too difficult⁴⁰¹⁾] but also on those of decays especially radiative decays.

Early work (i,ii,iiia) has in general problems to fulfill this criterion, whilst the more recent publications (iiib,iv) claim for considerable progress.

i Ono's model¹⁶²⁾ contains an optimized standard potential:

$$V(r) = V_1(r) - be^{-r/c} + V_0$$

$$V_1(r) = -\frac{4}{3} \alpha_s \frac{1}{r} \quad r < R_1 \quad R_1 = \sqrt{(4\alpha_s/3a)}$$

$$= a \cdot r \quad r > R_1 .$$

The hyperfine splitting -- accounted for in accordance to Eq. (21) -- replaces the $\delta(\vec{r})$ dependence, whose validity is restricted to the non-relativistic limit, by an appropriate smearing function

$$f(r) = \frac{1}{4\pi r_0^2 r} e^{-r/r_0} .$$

With $\alpha_s = 0.31$ fixed and 10 other parameters (e.g. $m_u = m_d = 336$ MeV, $m_s = 575$ MeV) predictions of light quark spectroscopy are made as quoted in Table 10. Whilst these predictions reproduce nature quite reasonably the model fails to predict, for instance,

$$\Gamma(K^{*0} \rightarrow K^0 \gamma) / \Gamma(K^{*+} \rightarrow K^+ \gamma) = 1.3 \text{ (model)}$$

$$= 2.29 \pm 0.29 \text{ (exp.)} .$$

Allowing α_s to increase with r leads to a high probability of condensation of additional $q\bar{q}$ pairs with quantum numbers $J^{PC} = 0^{++}$ at large distances: Quark Pair Creation model.

The original $q\bar{q}$ pair acts as spectator. The overlap integral and therefore the matrix element could suffer from interference mechanisms between ψ_A, ψ_B, ψ_C leading to node structures in the total wave function. This could disturb the monotonic relation between decay width and momentum.

Normally the following relation should hold:

$$\Gamma[\psi'''(4030) \rightarrow D^* \bar{D}^*] < \Gamma[\psi'''(4030) \rightarrow D \bar{D}] .$$

QPC claims the inequality to be valid just the other way around, in agreement with experiment. Also the small width $\Gamma[\rho'(1600) \rightarrow 2\pi]$ could be understood in terms of this mechanism. See also Refs. 260, 261 and the discussion in Section 2.2.1 on the width $\Gamma[\rho' \rightarrow \pi^+\pi^-]$.

Reference 356 explains the relevance and success of QPC by its relation to a more general approach of describing meson decays in a chromoelectric flux tube breaking model.

ii An analogous method is used by Ref. 163. Here Schrödinger's equation is solved for a standard potential:

$$V(r) = -\frac{c}{r} + a \cdot r + b \quad (m_u = m_d = 0.34, \quad m_s = 0.45 \text{ GeV}) .$$

a, b, c , are fixed by a fit to known meson masses, for instance b to the distance $m_{\rho^*} - m_{\rho}$. Predictions are summarized in Table 11.

Overlap integrals and widths are evaluated in the framework of QPC. An interesting prediction is made: radial (2^3S_1) excitations of the ϕ meson are extremely broad (~ 600 MeV) with a mean decay into K^*K . In particular, they are broader than $s\bar{s}$ mesons of the 3D_1 multiplets (J^{PC} also 1^{--}). This would again lead to the conclusion that the presented ϕ' candidates (see Section 2.2.1) are either D states or mixtures of S and D waves. It explains the experimental result observing a ϕ' in the K^+K^- channel, but failing to find it in K^*K . Reference 163 predicts for pure S and D objects:

$$\frac{\Gamma(^3D_1 \rightarrow K^+K^-)}{\Gamma(^3D_1 \rightarrow K^*K)} (1.75 \text{ GeV}) = \frac{1}{2} ; \quad \frac{\Gamma(^3S_1 \rightarrow K^+K^-)}{\Gamma(^3S_1 \rightarrow K^*K)} (1.75 \text{ GeV}) = \frac{1}{7} .$$

There are no predictions on the properties of pseudoscalar isosinglets which is a main item of the next ansatz.

iiia Another approach with a treatment of spin-spin interaction different from Ono's is used by Ref. 164. With quite small quark masses ($m_u = m_d = 240$ MeV, $m_s = 478$ MeV, totally 11 free parameters), with non-relativistic calculation and a harmonic oscillator potential + additional terms for the short-range one gluon exchange and for spin dependence, and finally with additional assumptions of mixing between radial excitation one gets the mass values of Table 12 (model I).

The authors point to the good prediction of a low π mass but it seems unclear, how this depends on the choice of parameters -- especially a so-called "overall mass eigenvalue" of $M_0 = 1.1$ GeV.

Neglect of M_0 and modification of $V_{\text{spin-spin}}$ (again 11 free parameters) leads to the "model II" predictions of Table 12: the pions become heavier but some radiative decays, for instance, the ratio

$$x = \frac{\Gamma[\eta'(958) \rightarrow \omega\gamma]}{\Gamma[\eta'(958) \rightarrow \rho\gamma]}$$

are predicted remarkably better.

iiib This early work has been considerably developed in the past years (Refs. 165,166). The predictions in particular for pseudoscalar isosinglets (η 's) are calculated with and without a gluonium state at 1.5 GeV (Table 12). See Section 3 for an exp. candidate, the $i(1400)$. The result without gluonium shows that the $q\bar{q}$ content of the isoscalars is not too far away from traditional ideas, thus leaving the 'redundant' i as a gluonium state. Predictions of radiative widths from ($^1S_0, ^3S_1$) ground states agree within a factor of 3 with nature (Ref. 166).

iv Godfrey and Isgur^{16a)} published an ambitious approach to cope with all qq spectra with encouraging results: the key ingredient is a QCD inspired potential with one gluon exchange at short distances plus a linear confinement, of flavour independent Lorentz scalar type. Crucial for the success is the account taken of relativistic effects: Schrödinger's equation is solved for a relativistic Hamiltonian operator:

$$H|\psi\rangle = (H_0 + V)\psi = E\psi$$

$$H_0 = (P_q^2 + m_q^2)^{1/2} + (P_{\bar{q}}^2 + m_q^2)^{1/2} .$$

The potential for $q\bar{q}$ systems in the non-relativistic limit is presented in a familiar way:

$$V(\vec{p}, \vec{r}) = -\frac{4}{3} \left(\frac{\alpha_s(r)}{r} + br + C \right) + H^{\text{spin-spin}} + H^{\text{spin-orbit}} + H_A ,$$

H_A accounts for annihilation of $q\bar{q}$ via many gluons and is relevant for isoscalar objects. It should be mentioned that pseudoscalar mesons require a treatment of H_A different from the other multiplets^{*}).

The transition to a relativistic formulation is made by two features:

- r is smeared out by a proper smearing function over distances of the order of the inverse quark mass.
- All coefficients contributing to $V(p,r)$ become dependent on the energy of the interacting quarks. This avoids especially for light quark masses a blowing up of hyperfine splitting. Furthermore, the divergence of $\alpha_s(r)$ for large r [$r^{-2} \sim \Lambda^2$, see Eq. (19)] is cured by a proper smearing function with a free parameter α_s^{crit} corresponding to $\alpha_s(Q^2 = 0)$ after smearing procedure.

The relativistic Schrödinger equation is then solved. The number of free parameters for predicting (u,d,s,c, and b) spectroscopy is 14, with remarkably low values for $m_{u,d} = 5$ MeV, i.e. typical current quark masses.

Results for (u,d,s) spectroscopy are given in Fig. 50 in comparison with experiments: the agreement is striking.

^{*}) This is related to the U(1) problem of QCD, where the mass of the η' is lifted from 0 by non-perturbative annihilation amplitudes having an opposite sign to the perturbative two gluon annihilation amplitude leading to a complicated mass dependence of the pseudoscalar amplitude.

In particular, this scenario prefers two sets of 1^{--} resonances between 1400 and 1900 (see Table 13). This could result to a $\rho'(1600)$ being a mixture of both ρ 's and a possible confusion between ω, ϕ from 3S_1 and 3D_1 families as discussed in Section 2.2.1. Members of the 3P_0 family have masses about 200 MeV lower than the corresponding ones of 3P_2 . The assignment of $\delta(980)$ and S^* as members of this family is difficult to implement into the resulting meson scenario due to the very narrow experimental and very broad predicted width. Though it seems unclear how the sophisticated mechanisms, presented in Section 2.2.6 as responsible for the curious pattern of 3P_0 , is reproducible by the model, the authors argue that their model is able to predict correctly the closely related S wave decays $B \rightarrow (\omega\pi)_S$ and $A_1 \rightarrow (\rho\pi)_S$ and therefore prefer an interpretation of δ and S^* as $K\bar{K}$ bound states (see remark in Section 2.2.6 concerning $\Gamma_{\gamma\gamma}(\delta)$). They propose a $(q\bar{q}) 0^{++}$ multiplet consisting of κ and $\epsilon(1300)$ already known together with an $\epsilon(1090, 850 \text{ MeV}, \Gamma_{\pi\pi} = 9 \cdot \Gamma_{KK})$ and a $\delta(1090, 400 \text{ MeV}, \Gamma_{\eta\pi} \sim 1.5 \cdot \Gamma_{KK})$ to be detected.

In general the result could be understood as a breakthrough for the purpose of linking heavy and light quark spectroscopy not obtained by qualitative changes but by relativistic corrections inversely proportional to quark mass. Figure 51 gives a further illustration of this statement. But again problems arise with predicting widths, for instance:

$$\Gamma(\eta \rightarrow \gamma\gamma) = 0.25 \text{ keV (pred.)} , \quad 0.56 \pm 0.05 \text{ keV (exp.)}$$

$$\Gamma(\eta' \rightarrow \gamma\gamma) = 1.69 \text{ keV (")} , \quad 4.3 \pm 0.3 \text{ keV (")} .$$

Solving these discrepancies will be a criterion whether this ansatz (or others) succeeds to contradict a principle objection on potential models for light quarks⁴⁹⁾:

If it justified to treat these quarks as heavy objects of a certain constituent mass and to neglect their fine structure of hard core + gluon cloud and therefore to incorporate gluonic contributions or virtual $q\bar{q}$ pairs only globally and not as active participants into the effective potential?

A complete discussion of all recently published models is beyond the scope of this article. See, for instance, Refs. 167, 275, 357, and 362. Reference 363 discusses problems to account relativistic effects, especially relevant for light quarks.

2.3.2 Bag models

As mentioned in the introduction only the main ideas of bag models will be sketched and some examples of their predictions given. For deeper studies the reader is referred to Refs. 169, 328, and 399 for instance.

A commonly used picture predicts the colour electric field lines between quarks to be forced by the vacuum into a string and leads therefore to the $(k \cdot r)$ confinement potential at large distances [$k = \text{energy/length}$, see Eq. (17)]. At small distances (in case of S waves) these strings could be deformed to spherical objects (bags).

Bag models try a phenomenological description of (almost) massless, relativistic quarks under the regime of confinement. Gluon fluctuations give rise to a positive energy density B inside a bag due to their fluctuation energy. Hadrons are understood as being 'bubbles' of perturbative vacuum of volume V immersed in the non-perturbative ground state. Inside the bag quarks and gluons interact only weakly, the Lagrangian contains a QCD type term for the inner part and a surface term $\sim B \cdot V$. In order to quantize it frequently the surface of the cavity is assumed to be static and to be determined by the pressure balance condition: static cavity approximation³⁹⁹⁾.

The energy eigenvalues of quarks in such a cavity -- uncertainty relation claims $E_q \sim 1/R$ -- are given by solution of Dirac's equation for a spherical potential with walls of infinite height:

$$E_n = \frac{2.04}{R} \cdot n$$

n = number of (anti)quarks confined in the bag. The total energy $E(R)$ is the sum of the quark energies and the volume energy due to gluon fluctuations, it is minimized with respect to R :

$$E(R) = \frac{2.04}{R} n + \frac{4}{3} \pi R^3 \cdot B \quad (26)$$

and thus from $dE/dR = 0$:

$$R = (n/2\pi B)^{1/4}, \quad m = E = \frac{8}{3} \frac{n}{R}$$

$$E = m_p \text{ and } n = 3 \rightarrow R = 1.6 \text{ fm} \rightarrow B^{1/4} = 100 \text{ MeV} \text{ or } B = 12.5 \frac{\text{MeV}}{\text{fm}^3}.$$

Comparing the thus obtained proton radius with typical nuclear dimensions it seems somewhat large but such a simple estimation should not be over-interpreted.

Introducing a zero point energy ($\sim R^{-1}$), spin-spin interaction mediated by one gluon exchange, a mass for strange quarks of $\sim 350 \text{ MeV}$ and an α_s value of 2.2 (too large) meson mass predictions can be made reaching the quality of potential models¹⁷⁰:

$$m_{\text{meson}}(R) = E_o + E_V + E_Q + E_M + E_e \quad (27)$$

where

$$E_o = -\frac{Z_o}{R} \text{ zero point energy}$$

$$E_V = \frac{4}{3} \pi R^3 \cdot B$$

$$E_Q = N_{ud} \frac{\sqrt{(x^2 + m_{ud}^2 R^2)}}{R} + N_s \frac{\sqrt{(x^2 + m_s^2 R^2)}}{R}, \quad N_q, m_q \text{ number, mass of quarks of flavour } q$$

$$E_Q + E_V = E(R) \text{ [Eq. (26)] if } m = 0, \text{ i.e. } x = 2.04,$$

x momentum in units of R^{-1}

$$E_M = a_{ud,ud} M_{ud,ud} + a_{ud,s} M_{ud,s} + a_{s,s} M_{s,s}$$

M colour magnetic interaction

$a_{q,q}$ is a number depending from spin times flavour content of a meson:

vector mesons: $\vec{\sigma}_1 \cdot \vec{\sigma}_2 = 1$: e.g. $a_{s,s}(\phi) = 2$

pions: $\vec{\sigma}_1 \cdot \vec{\sigma}_2 = -3$: e.g. $a_{ud,ud}(1) = 2 \cdot -3 = -6$

$$E_e = b \cdot \epsilon$$

E_e additional colour electric interaction term between (ud) and s quarks quantified by by.

Table 14 shows the mass predictions for (3S_1) and (1S_0) mesons after minimalization of Eq. (27) with respect to R and two sets of parameters:

line 1-6: B, Z_0 , α_s are quoted by fixing m_p , m_ρ , and m_ω . The strange quark mass is adjusted to fit the Ω^- : $m_u = m_d = 0$.

line 7-12: Identical, but in addition $m_u = m_d = 108$ MeV.

The small quark masses make the prediction of m_π easier. This does not mean that such a model, sometimes called MIT bag model, has no problems with the pseudoscalars. m_η, m_η require additional corrections not quotable within this simple ansatz. These are $q\bar{q}$ annihilations into ≥ 2 gluons as already known from potential models. m_η, m_η therefore are not quoted in Table 14.

A recent modification¹⁷¹⁾ omits the zero point energy in favour of self energy contributions $\alpha_s(R) \cdot e_i/R$ of quarks and gluons (e_i free parameters). The running of α_s is parametrized as

$$\alpha_s = \frac{2\pi n}{9} \frac{1}{\ln[1+(AR)^{-n}]} \quad n \text{ free parameter} . \quad (28)$$

Reference 170 introduces the energy density B as an "ad hoc" parameter. It might be expressed (Ref. 172) as a function of $\alpha_s(R)$ and radius and other properties of hypothetical 0^{++} glueballs which fill the QCD vacuum and have velocities larger than c and therefore negative m^2 .

Then the following equation is minimized with respect to R (this is the only one which exists, see Ref. 325 for an alternative approach):

$$m(R)^2 = E(R)^2 - P(R)^2 ; \quad P(R)^2 = \sum_{ud,s} N_{ud,s} \left(\frac{x_{ud,s}}{R} \right)^2 .$$

E corresponds to Eq. (27) with the replacement mentioned above. Only valence quarks are assumed to move, all other interactions are neglected. Table 15 summarizes the predictions; it gives $m_\pi < 0$; a lift to 140 MeV would require $m_{u,d} = 70$ MeV. Again quite a large number of parameters (quoted by fixing to ρ, p, s, Ω^- mass; e_{TE}, R_0 describe the "squeezing" gluon cloud) has to be used but now a reasonable value for m_η can be given.

$B^{1/4}$ is 225 MeV ($B = 320$ MeV/fm³). It has therefore a value favoured by QCD sum rules (Section 2.3.3). See Ref. 364 for recent developments of the MIT bag model. Reference 174 reports the attempt of estimating radial excitations of π and ρ within the framework of MIT bag model. π' is expected at 900 MeV (somewhat low), a ρ' of 1500 MeV is in accordance with experimental knowledge, a $\rho''(1900)$ has to be found. The "genealogy" of more recent bag models starting from the MIT model is nicely summarized by Ref. 363.

Cloudy bag models¹⁷⁵⁾ presume a virtual pion cloud surrounding the particle with a different bag radius than that of the particle; little (Brown) bags¹⁷⁶⁾ remove these clouds outside the particle bag. See Ref. 314 for calculations of M1 radiative transitions of mesons in a cloudy bag model.

In a static cavity approximation additional multiplets with $L(q, \bar{q}) \geq 1$ should exist. These could form also exotic states: 0^{+-} , 2^{+-} etc).

The reason is that, for instance, $q\bar{q}$ -P waves are not coupled by L-S coupling (like in potential models) but by J-J coupling.

To form a 2^{++} meson a quark in a $S_{1/2}$ cavity couples with $P_{1/2}$ anti-quark according to:

$$|q(S, \uparrow) \bar{q}(P, P_z = +1, \uparrow) \rangle .$$

C parity is not any longer $C = (-1)^{L+S}$ but there exist two eigenstates with eigenvalues:

$$\frac{1}{\sqrt{2}} [|q(S,\uparrow)\bar{q}(P,P_z = +1,\uparrow)\rangle \pm |\bar{q}(S,\uparrow)q(P,P_z = +1,\uparrow)\rangle] .$$

For all 4 P waves those with inverse C parity exist: $1^{+\pm}$, $0^{+\pm}$, $1^{+\pm}$, $2^{+\pm}$.

In a simplifying picture these additional states might be explained by the bag as a further degree of freedom. Besides oscillations of $q\bar{q}$ with respect to their common centre of mass (like in potential models) q and $\bar{q}$ could move united and form a P wave with respect to the cavity. It seems not clear whether these additional states have any physical relevance. It depends on the time scale t_v of QCD vacuum fluctuations compared to the crossing time $t_c \approx 10^{-23}$ s of a quark in a cavity. If $t_v \gg t_c$ (static cavity approximation) these states should exist, otherwise ($t_c \gg t_v$, fulfilled for heavy quarks) they are spurious.

2.3.3 QCD sum rules

In this approach one has no explicit space time model for hadrons but extracts hadronic properties from those of various current-current correlations functions.

For a perturbative treatment the condition $\Lambda r \ll 1$ has to be fulfilled [A QCD cut-off parameter is defined by Eqs. (19), (28) and stands for a distance or energy where confinement comes into operation]. For typical hadronic sizes of ~ 1 fm QCD sum rules called quite early for small Λ values (≤ 200 MeV) as was found to be realistic by recent deep inelastic lepton scattering experiments¹⁷⁷⁾.

The quite lengthy mathematical formalism for constructing sum rules is extensively documented by Ref. 178. The reader is referred also to extensive recent reviews of this topic: Refs. 178a, 178b, 424. The continuation from the short to the long range distance behaviour is not done by accounting for higher orders in α_s but by accounting for so-called "power terms" $\alpha_s(Q)/(Q)^n$. These are caused by non-vanishing vacuum expectation values of quark fields and by the gluonic field strength tensor $G_{\mu\nu}^a$.

To get measurable predictions the so-called T product of two quark currents in the limit of large $Q^2 = -q^2$ is given by the following expression:

$$i \int dx e^{iqx} T\{j_\mu^A(x), j_\nu^B(0)\} = (q_\mu q_\nu - q^2 g_{\mu\nu}) \sum_n C_n O_n \quad (29)$$

O_n are operators, C_n are the Wilson coefficients.

For example, the vector current of ρ mesons looks like:

$$i \int dx e^{iqx} T[j_\mu^\rho(x), j_\nu^\rho(0)] = (q_\mu q_\nu - q^2 g_{\mu\nu}) (C_I \cdot I + C_G \cdot O_G + C_M O_M + \dots)$$

$$i \int dx e^{iqx} \langle 0 | T[j_\mu^\rho(x), j_\nu^\rho(0)] | 0 \rangle = (q_\mu q_\nu - q^2 g_{\mu\nu}) \Pi_{QCD}^\rho(Q^2)$$

where:

$$I = 1 \text{ (0 dimensional); } C_I = - \frac{1}{8\pi^2} \left[\left(1 + \frac{\alpha_s}{\pi}\right) \ln \frac{Q^2}{\mu^2} + \frac{3(m_u^2 + m_d^2)}{Q^2} \right]$$

$$O_M = m_{q\bar{q}} \text{ (4 dimensional); } C_M O_M = \left(1 + \frac{\alpha_s}{3\pi}\right) \frac{1}{2Q^4} (m_u \bar{u}u + m_d \bar{d}d)$$

$$O_G = G_{\mu\nu}^a \cdot G_{\mu\nu}^a \text{ (4 dimensional); } C_G O_G = + \frac{\alpha_s}{24\pi Q^4} G_{\mu\nu}^a G_{\mu\nu}^a$$

resulting in

$$\Pi_{\text{QCD}}^{\rho}(Q^2) \approx -\frac{1}{8\pi^2} \ln \frac{Q^2}{\mu^2} + \frac{1}{2Q^4} \langle 0 | m_u \bar{u}u + m_d \bar{d}d | 0 \rangle + \frac{1}{24Q^4} \langle 0 | \frac{\alpha_s}{\pi} G_{\mu\nu}^a G_{\mu\nu}^a | 0 \rangle \quad (30)$$

u,d quark fields

$G_{\mu\nu}^a$ gluonic field strength tensor .

Expressing $\Pi(Q^2)$ by its imaginary part ("dispersion relation") leads to a QCD sum rule:

$$\Pi_{\text{QCD}}^{\rho}(Q^2) = \frac{1}{\pi} \int_0^{\infty} \frac{\text{Im}\Pi(s) ds}{s + Q^2} \rightarrow \frac{1}{\pi} \int_0^{\infty} e^{-s/M^2} \cdot \text{Im}\Pi(s) ds$$

$$\frac{1}{\pi M^2} \int_0^{\infty} e^{-s/M^2} \text{Im}\Pi(s) ds = h_0 + \frac{h_2}{2\sqrt{M^2}^2} + \frac{h_3}{3\sqrt{M^2}^3} + \dots$$

where the coefficients h_i can be calculated from Eq. (30). For instance:

$$h_2 \sim \langle 0 | m_u \bar{u}u + m_d \bar{d}d + \frac{\alpha_s}{12\pi} G_{\mu\nu}^a G_{\mu\nu}^a | 0 \rangle . \quad (31)$$

$\text{Im}\Pi(s)$ is directly related to a cross-section:

$$R^{I=1} = \frac{\sigma[e^+e^- \rightarrow \text{hadron (isotriplets)}]}{\sigma(e^+e^- \rightarrow \mu^+\mu^-)} \sim \text{Im}\Pi(s) .$$

The range for M^2 is now found by an optimization procedure with some similarity to a walk on a tightrope⁶⁹⁾: on the one hand M^2 has to be small to keep the integral on the left hand side determined by one resonance, a too small M^2 on the other hand causes divergence on the right hand side.

That such a walk can successfully be finished has been proven by the prediction of the η_c mass to be ~ 3 GeV at a time where the experimental value was 2.83 GeV due to a wrong determination. On the other hand the prediction of the centre of gravity for $2^3P_J(b\bar{b})$ is 9830 ± 30 MeV³⁶⁵⁾ therefore somewhat low compared to the experimental value 9900 ± 0.6 MeV and also compared to the prediction of most actual potential models (see Ref. 350, Fig. 14).

Back to the ρ mass determination with:

$$\langle 0 | m_u \bar{u}u + m_d \bar{d}d | 0 \rangle = -\frac{1}{2} f_\pi^2 m_\pi^2 = -1.7 \times 10^{-4} \text{ GeV}^4 \text{ (see Ref. 178, p. 427)}$$

and

$$\langle 0 | \frac{\alpha_s}{\pi} G_{\mu\nu}^a G_{\mu\nu}^a | 0 \rangle = 1.2 \times 10^{-2} \text{ GeV}^4 \text{ (see Ref. 178, p. 456)}$$

one gets

$$f(M^2) = \frac{2}{3} \frac{1}{M^2} \int e^{-s/M^2} R^{I=1}(s) ds = 1 + \frac{\alpha_s(M)}{\pi} + 0.1 \left(\frac{0.6 \text{ GeV}^2}{M^2} \right)^2 + \dots$$

Plotting $f(M^2)$ versus M^2 one observes after an appropriate subtraction of non-resonant continuum a maximum at $M^2 = m_\rho^2$: see Fig. 52.

Because of

$$\int e^{-s/m_\rho^2} R^{I=1}(s) ds = \frac{3}{e} \frac{\pi^2}{\gamma_\rho^2} m_\rho^2$$

from Fig. 52 also the $(\gamma\rho)$ coupling and with Eq. (6) the leptonic width of the ρ can be quoted. The result is in excellent agreement with the experimental prediction (Table 4): $\gamma_\rho^2/4\pi$ (theoretical) = 0.6.

Other predictions of QCD sum rules for light quark spectroscopy

$$1.21 \leq m_{A_1}^2 \leq 1.53 \text{ GeV}^2$$

$$m_K^* = 0.93 \text{ GeV}$$

$$m_\phi = 1.07 ; \quad \frac{\gamma_\phi^2}{4\pi} = 3.5 \text{ (experimental value: 3.5!)} (m_s = 150 \text{ MeV}) .$$

Reference 366 calculates light quark masses from QCD sum rules to $m_u = 6 \pm 1$, $m_d = 10 \pm 1$, $m_s = 192 \pm 15 \text{ MeV}$.

In summary one can state that with quite few parameters QCD inspired predictions for light quark spectroscopy look encouraging. Predictions concerning glueballs or meiktons will be mentioned in the corresponding sections.

Reference 179 reviews recent calculations for charm and bottom spectroscopy based on this method. The reader is referred also to a recent publication of the original authors of QCD sum rules²⁵¹⁾ making clear that an expectation for an "equation valid once and for ever" by such a semi-phenomenological method is a misunderstanding. See Ref. 425 for an extension of the sum rule approach: Analytic Continuation by Duality.

2.3.4 Spectroscopy with lattice gauge theories

In the sequence potential models, bag models, QCD sum rules, lattice gauge theories, the last ones are the most general approach, i.e. they reach the highest degree of independence from arbitrary parameters and therefore the highest theoretical attractiveness. Their development and therefore their predictive power has made considerable progress during the last few years.

A fundamental difference between QED and QCD is the non-abelian character of the latter.

This "non-abelian" character is essential for the explanation of fundamental properties of QCD like asymptotic freedom. Gluons have non-abelian colour charges and interact with each other. This interaction has to be taken into account in order to guarantee the gauge invariance of QCD.

Maxwell's equations, written as equation of motion of the four potential A^μ , are

$$\text{in QED: } \square A^\nu - \partial^\nu (\partial_\mu A^\mu) = j^\nu$$

$$\text{in QCD: } \square A^{\alpha\nu} - \partial^\nu (\partial_\mu A^{\alpha\mu}) = j_{\text{colour}}^{\alpha\nu}$$

with the fundamental difference of non-vanishing $j_{\text{colour}}^{\alpha\nu}$ for vanishing fields (except gauge fields): gluons self-interact also in free space.

The "running coupling constant" with values in the order of 1 at large distances inhibits the evaluation of a perturbative solution for the equations of motion. Therefore, Wilson¹⁸¹⁾ suggested in 1975 to introduce a discrete space-time lattice, where the quark fields live on the sites of the lattice, i.e. only there a probability of presence is defined and their interactions (gluons) are restricted to the connection lines.

The equations of motion on a lattice are solved numerically and the continuum limit is approximated by diminishing the lattice constant.

The basic idea can be explained on a one-dimensional (time) "lattice" and a scalar field ϕ , more extensive explanations are given in Refs. 182, 183, 269, 326, 367-370 for instance.

The transfer matrix from a lattice point r to $r + 1$ (distance a) $T(\phi_{r+1}, \phi_r)$ is determined by the Hamilton operator of the system:

$$T(\phi_{r+1}, \phi_r) = \langle \phi_{r+1} | e^{-iaH} | \phi_r \rangle = e^{iLa}$$

L is the Lagrange function, see Ref. 184 for a derivation.

The wave function at the point r, ϕ_r , is given by a linear operation on ϕ_r :

$$\psi_r(\phi_r) = \int d\phi_{r-1} T(\phi_r, \phi_{r-1}) \psi_{r-1}(\phi_{r-1}) .$$

Therefore, after a step by step evaluation:

$$\psi_r(\phi_r) = \int d\phi_{r-1} \dots d\phi_1 T(\phi_{r-1}, \phi_r) \dots T(\phi_2, \phi_1) \psi(\phi_1) .$$

The expectation value of an observable quantity $\langle A(\phi_r) \rangle$ is determined as usual by the probability distribution $|\psi_r(\phi_r)|^2$:

$$\langle A(\phi_r) \rangle = \frac{\int |\psi_r(\phi_r)|^2 A(\phi_r) d\phi_r}{\int |\psi_r(\phi_r)|^2 d\phi_r}.$$

This is modified under the assumption of

- an euclidean metric with purely imaginary time and real T values;
- a connection of the lattice end point N to the first point 1 according $\phi_{N+1} = \phi_1$ to

$$\langle A(\phi_p) \rangle = \frac{\int \prod_{r=1}^N d\phi_r T(\phi_r, \phi_{r+1}) A(\phi_p)}{\int \prod_{r=1}^N d\phi_r T(\phi_r, \phi_{r+1})}$$

and furthermore with

$$\prod_{r=1}^N T(\phi_r, \phi_{r+1}) = \prod_{r=1}^N \exp(a \cdot L \cdot r) = \exp\left(\sum_{r=1}^N a \cdot L \cdot r\right) = \exp(-S)$$

to

$$\langle A(\phi_p) \rangle = \frac{1}{Z} \cdot \left[\int \prod_{r=1}^N d\phi_r A(\phi_p) \exp(-S) \right] \quad (32a)$$

$$Z = \int \prod_{r=1}^N d\phi_r \exp(-S)$$

or analogously a correlation:

$$\langle \phi_p, \phi_{p+n} \rangle = \frac{1}{Z} \cdot \left[\int \prod_{r=1}^N d\phi_r \phi_p \phi_{p+n} \exp(-S) \right]$$

(S action, Z partition function).

Particle states are considered by studying their propagation along the lattice axes: they correspond to the eigenmodes of the transfer matrix T for $a \rightarrow 0$ and for correlation functions of operators carrying the same quantum numbers than the particle. The largest of the (real and positive) eigenvalues λ_0 is associated with the vacuum or ground state of the system. It holds that

$$1 > \lambda_m / \lambda_0 \simeq e^{-ma}$$

and λ_m / λ_0 can be quoted from the relation

$$\langle \phi_p \phi_{p+n} \rangle - \langle \phi_p \rangle \langle \phi_n \rangle \sim \left(\frac{\lambda_m}{\lambda_0} \right)^n$$

and large n yielding m in units of the lattice spacing (Refs. 182,270).

Hamilton's principle of minimal action demands a process to occur with a minimal value of S . Fluctuations inhibit the evaluation of an equilibrium state in quantum field theory: contributions of different field configurations to the expectation values have to be calculated explicitly for the solution of Eq. (32). In the first step of an application to QCD only gluons fields, no quark fields exist.

Local gauge invariance enables a redefinition of the colour base on each point of a four dimensional space-time lattice. Therefore, the "gauge scale" has to be transferred from the point n into direction $\hat{\mu}$ via so-called connections (unitary 3×3 matrices, $\det U = 1$, $U^\dagger U = U U^\dagger = 1$)

$$U_{n\hat{\mu}} = \exp \left[i g \int_n^{n+\hat{\mu}} A_\mu^c(x) \frac{\lambda_c}{2} dx \right] \sim \exp \left[i g a A_\mu^c(n) \frac{\lambda_c}{2} \right] \quad (33)$$

g coupling constant

$\frac{\lambda_c}{2}$ fundamental generators of $SU(3)_c$

A_μ^c "normal" gauge field .

A change of the "colour budget" at one point changes U ; but the trace of the product of four connections which are arranged to a unit square (so-called "plaquette") is gauge invariant: the plaquette is the simplest non-trivial combination of connection or "link variables" with this property

The action of a plaquette is proportional to the trace of the product of the U matrices in a plaquette.

The total action is quoted by summation over all plaquettes¹⁸¹⁾. Thus the relevant input for Eqs. (32a,32b) is

$$S_{\text{gluon}} = S_{\text{lat}} = \beta \cdot \sum_{\square} S_{\square} = \frac{2}{g^2} \text{Re Tr } (U_1 U_2 U_3^{\dagger} U_4^{\dagger}) \quad (34)$$

$$\beta = \frac{6}{g^2} \approx \frac{1}{\text{Temperature}} .$$

This coupling plays a role analogue to the temperature in statistical physics. Integration in Eqs. (32a,b) is not done in practice over all possible $SU(3)$ matrices U but over a randomly (Monte Carlo) selected subsample.

It is straightforward to calculate (see Ref. 7 for instance) that in the limit $a \rightarrow 0$ equations (33), (34) give the action of a continuum theory.

Glueball masses are obtained from considerations of correlations between colour neutral quantities (plaquettes) at different separations. Typical results predict for scalar glueballs (0^{++}) masses between 0.75 and 1 GeV¹⁸⁶⁾, other J^{PC} configurations roughly at a doubled mass:

$$M(G) = \frac{8}{3} \frac{g^2}{a} = 2.6 \text{ GeV} \quad a = 0.2 \text{ fm}; \quad \beta = 6 .$$

See Section 3 for more details.

Incorporation of fermions creates additional problems:

- Fermions on a lattice give rise to additional states [as can be seen from additional minima of the corresponding action in momentum space not only at $k = 0$ but also at $k = \pm\pi/a$ diverging for $a \rightarrow 0$; the cure against this fermion degeneracy can be the addition of "lifting" terms in the action (Wilson)].

- The corresponding quark fields are anticommuting therefore causing problems for the integration via straightforward Monte Carlo methods.

First attempts¹⁸⁷⁾ with a 10×5^3 lattice, $a = 0.14$ fm and a quark mass determination from π and K mass ($m_{u,d} = 4.5 \pm 0.6$ MeV, $m_s = 116 \pm \pm 7$ MeV) yield for m_ρ as a scale parameter:

$$m_\rho = 1.27 \pm 0.44 \text{ GeV}$$

$$m_\Delta = 1.37 \pm 0.6 \quad "$$

$$m_K^* = 0.89 \pm 0.07 \quad "$$

$$m_\phi = 0.99 \pm 0.05 \quad " \quad .$$

It is beyond the scope of this article to account for all calculations in this strongly growing field of physics. See Refs. 270,370 and the references therein.

For instance, encouraging results come from recent calculations on a 16×10^3 lattice yield for $\Lambda_{\text{mom}} = 0.2$ GeV¹⁸⁸⁾: $m_\pi \cdot a = 5 \times 10^{-3}$ from where $m_{u,d} = 7$ MeV can be derived; further predictions:

$$m_\rho = 730 \pm 90 \text{ MeV}$$

$$m_{A_1} = 1190 \pm 90 \text{ MeV}$$

$$m_\epsilon = 660 \pm 50 \text{ MeV}$$

$$m_n = 920 \pm 100 \text{ MeV (!)} \quad .$$

The correlation function of a rectangular path C [with corners at $(0,0)$ $(0,\vec{R})$ $(T,\vec{R})$, $(T,0)$]

$$W(C) = \langle 0 | \text{tr } U_C | 0 \rangle \text{ (Wilson loop)}$$

is related to the potential energy $V(\vec{R})$ of a $q\bar{q}$ system at this distance (Ref. 370):

$$\lim_{T \rightarrow \infty} W(C) \sim e^{-V(R) \cdot T}.$$

In a strong coupling limit (i.e. where $\beta \sim g^{-2}$ is small enough to expand the "Boltzmann factor" e^{-S} [see Eq. (34)] in powers of β ; this is in analogy to high temperature expansions in thermodynamics] it follows after application of Eq. (32) to $W(C)$ ²⁵⁸⁾

$$W(C) \sim e^{-TR/a^2 \ln g^2}, \text{ i.e.}$$

$$V(R) = \kappa \cdot R \quad \text{and} \quad \kappa = \ln g^2 / a^2.$$

Recent results quote ²⁸¹⁾:

$$\sqrt{\kappa} = (54 \pm 5) \times \Lambda^0 \text{ (GeV) for SU(2) string tension}$$

$$\sqrt{\kappa} \sim 90 \quad \times \Lambda^0 \quad \text{ " for SU(3) " " }$$

$$\Lambda^0 = \Lambda^{\text{lattice}} = \frac{1}{a} \left(\frac{8\pi^2\beta}{3} \right)^{51/121} \times e^{-4\pi^2\beta/33} = 41.75 \times \Lambda_{\overline{MS}}.$$

Also deviations from a purely linear potential (i.e. forms like $V(R) = C_1/R + C_2 \cdot R$, see Fig. 48) can be obtained within the framework of lattice calculations: Ref. 371 collects these encouraging new results from several groups using different β values: Fig. 51; see also Ref. 371a.

This result may show the link between a pure QCD calculation and phenomenological potential models and may explain the success of the latter. Consequently one tries now to investigate not only the overall potential V but also V_{spin} on a lattice. First results are given in Refs. 371, 372.

One certainly can hope for further insight by these calculations. Criteria to judge their quality are

- convergence of the theory for $a \rightarrow 0$;
- model-independent predictions not only of masses but also of the errors quoted on the masses;
- incorporation of quark dynamics including decay properties of the particles, etc. (see Ref. 327 for first results).

In conclusion Section 2 dealt with $q\bar{q}$ systems. In Section 2.2 the actual knowledge of meson spectroscopy is reviewed whilst Section 2.3 describes the most actual ansatzes to understand the resulting pattern. These four different ansatzes should not be regarded as the only existing ones. General field theoretical approaches augmenting the QCD lagrangian by non-linear chiral invariant quark interactions³⁷⁹⁾ could be mentioned here as well as the picture of skyrmions. Whether the latter picture -- successfully applicable in baryon spectroscopy³⁷⁵⁾ -- has also predictive power in the mesonic domain must be proven (Refs. 329, 376).

3. GLUEBALLS

The unambiguous identification of a bound state of gluons would be an essential proof for the validity of QCD. Such a candidate with for instance exotic quantum numbers and/or -- because of its flavour-free components -- with unusual branching ratios into conventional mesons has not been found so far beyond any doubt. Mixing of glueballs and $q\bar{q}$ systems would not be in contradiction to QCD and is thus very likely to occur. This and missing or incomplete experimental information enlarge the free room for speculations. The phenomenological background will therefore be only briefly reviewed in Section 3.1 and compared with experiments in Section 3.2. See for a review also Refs. 306, 322 and, very recently, 418.

3.1 Models to describe glueballs

Two kinds of models are used to quantify the formation of bound gluonium states under the assumption of self interaction of the gluons in QCD. In non perturbative models the number of gluons in a glueball is so large that it might be understood as a string without quarks -- a string loop.

In perturbative models glueballs are composed by 2,3 ..., gluons (see below). Both ansatzes predict rather similar mass spectra, even though absolute mass predictions should be regarded with caution.

Gluons are colour charged, they are colour octets. Glueballs (colour singlets) could be composed by two ($8 \otimes 8 = 1 \oplus \dots$) or more gluons. A constituent gluon mass (and thus glueball masses) might be expected as

$$m(\text{gluon}) = \frac{\text{colour factor } (8 \otimes 8)}{\text{colour factor } (3 \otimes 3)} \times m(\text{quark}) = \frac{9}{4} \times 300 \text{ MeV} = 700 \text{ MeV} .$$

All four theoretical ansatzes (Sections 2.3.1-2.3.4) of $q\bar{q}$ spectroscopy could also be used to predict glueball spectra. The larger colour charge of the gluons enlarges in bag/potential models the bag pressure and/or potential strength compared to $q\bar{q}$ systems. Larger colour charge and spin lead in both models to an increased influence of spin dependent effects and therefore to less predictive power.

In the bag model (Refs. 190,191) massless, i.e. purely transverse gluons are trapped in a spherical cavity. The eigenmodes of single gluons are classified by

$$\begin{aligned} \text{transverse electric: } P &= (-1)^{L+1} \\ \text{transverse magnetic: } P &= (-1)^L \\ \text{TE1: } L^P &= 1^+ \quad \text{TE2: } L^P = 2^- \\ \text{TM1: } L^P &= 1^- \end{aligned} \tag{35a}$$

In the bag model the glueball ground state is found to be $(\text{TE1})^2$ and has positive parity and -- because of symmetry of the total wave function -- even spin: $J^{PC} = 0^{++}, 2^{++}$. Analogously to Section 2.3.2 its mass is obtained by calculating the eigenvalues for a massless vector field (TE1/TM1) confined to a cavity:

$$E_n = \frac{(2.74/4.49)}{R} \times n . \quad (35b)$$

Thus a naive guess of a TM/TE glueball containing n gluons is

$$M(ng) = \frac{(2.74/4.49)}{2.04} M(nq) \text{ (see Eq. 26)}$$

$$\rightarrow m_{(TE)}^2 = 1.33 m_\rho^2 \approx 1 \text{ GeV}^2$$

$$\rightarrow m_{(TM)}^2 = 2.2 m_\rho^2 = 1.65 \text{ GeV}^2 .$$

Excited states are (C is always $+1$):

$$(TE1 TE2): J^{PC} = (1,2,3)^{-+}$$

$$(TE1 TM1): J^{PC} = (0,1,2)^{-+}$$

0^{-+} , 2^{-+} objects are expected by Ref. 192 to have masses of about $\sim 1.3 \text{ GeV}$, but this expectation is just an educated guess due to the neglect of spin effects.

$3g$ systems can have any CP configuration. A similar check for their masses yields:

$$m_{(TE)}^3 = \frac{2.74}{2.04} \cdot m_{3q} = 1.34 \cdot \frac{(m_\rho + m_\Delta)}{2} = 1400 \text{ MeV} .$$

So they are probably not very different from $(2g)$ balls.

Potential models for glueballs require "constituent gluon masses" and predict therefore additional "longitudinal" states which might¹⁹⁴⁾ or might not¹⁹⁵⁾ be relevant in reality. Mass range and quantum number assignment are not remarkably different from bag model predictions. Table 16 shows a comparison of both ansatzes.

Attempts to predict glueball masses using QCD sum rules yield considerably higher mass values¹⁹⁷⁾: (0^{++}) and (0^{-+}) states are expected at 4 GeV/c, see Ref. 69 for a possible explanation of this surprisingly high value.

As mentioned in the previous section in the framework of lattice gauge theories glueball predictions cause fewer problems than meson predictions. Results from Ref. 198 are listed in Table 17. A calculation of the 0^{++} glueball¹⁹⁶⁾ mass published under the ambitious title 'Towards definitive results' quoted $m(0^{++}) = 1150 \pm 60$ MeV for $\beta = 6$, $\Lambda_{\overline{MS}} \approx 200$ MeV. A collection of very recent glueball mass calculations ($0^{++}, 2^{++}$) is given by Ref. 418. Moreover, Ref. 199 quotes even radial excitations of glueballs on a lattice.

A comparison between bag model predictions and lattice calculations is given in Ref. 266. The authors claim that agreement between the results suggests a two or three valence gluon structure for the glueballs being substantially correct.

All these predictions offer only little support for experimentalists hunting for glueballs: states should exist between 0.5 and 2 GeV, but lacking the treatment of spin-spin effects or mixing mechanisms keeps the predictive power quite small. Widths and branching ratios are only poorly predictable as well.

For OZI allowed/suppressed decays in $SU(3)_c$ one would expect for qq states M:

$$\Gamma[M \rightarrow (q\bar{q}) + (q\bar{q})] = \frac{1}{3} / \frac{1}{27} \quad (\text{Ref. 253}) .$$

Hence the total width of glueballs $G = (gg)$ can be estimated to

$$\Gamma(G) = \sqrt{(\Gamma_{\text{allow}} \cdot \Gamma_{\text{suppr.}})} \quad (\text{Ref. 200}) .$$

This is suggested from the picture that OZI suppression might be quantified by the insertion of intermediate glueball states:

$$\langle \phi | \rho\pi \rangle = \langle \phi | G \rangle \times \langle G | \rho\pi \rangle .$$

Following this rule one would typically expect

$$\Gamma = \sqrt{(15 \text{ MeV} \cdot 150 \text{ MeV})} \approx 50 \text{ MeV} .$$

But, for instance, the violation of ideal mixing for the pseudoscalars could spoil this simple OZI expectation and lead therefore to values between a few MeV and a few hundred MeV.

Flavour-free glueballs should be characterized by flavour symmetric branching ratios (flavour democracy). But phase space as well as helicity arguments for spinless states -- analogous to the suppressed ($e\nu_e$) decay of pions -- could overthrow democracy. In the latter case, final states with kaons could be favoured.

Table 18 shows typical expected glueball decay modes²⁰²⁾.

Lipkin²⁰³⁾ lists some criteria for distinguishing $q\bar{q}$ systems from glueballs:

- Contrary to $q\bar{q}$ system glueballs should obey C parity for decays into mesons not being C eigenstates, K^*K for instance: the $f(1515, 2^{++})$ therefore decays to K^+K^- and K^*K , a corresponding glueball to K^+K^- but not to K^*K .

- OZI rule predicts for ideally mixed $q\bar{q}$ systems:

$$q = u, d: \quad q\bar{q} \not\rightarrow \phi\phi, \phi\eta$$

$$q = s: \quad q\bar{q} \not\rightarrow \omega\omega, \omega\eta, \rho\rho, \rho\pi, \pi\pi.$$

The decay of glueballs is not restricted by this:

$$C = +1 \quad G \rightarrow \phi\phi, \rho\rho, \pi\pi$$

$$C = -1 \quad G \rightarrow \omega\eta, \phi\eta, \pi\rho .$$

But in summary one has to state that theoretical predictions are not very restrictive and therefore not too helpful for a glueball identification.

3.2 Experimental situation of glueball searches

The following "empirical" criteria are often used discussing a glueball candidate.

(α) copious production in "hard" gluonic channels like radiative decays of heavy vector mesons.

(β) decay into typical final states as described above. In radiative decays or 'two photon' decays the photon can be used as a probe for the (electrical) charge of the candidate constituents. Chanowitz⁴⁰⁸⁾ accounts for this and (α) in defining the "stickiness" coefficient

$$S_x = \frac{\Gamma(\psi \rightarrow \gamma X)}{\Gamma(X \rightarrow \gamma\gamma)} \cdot \frac{PS(X \rightarrow \gamma\gamma)}{PS(\psi \rightarrow \gamma\gamma)}.$$

(PS = phase space). S should then be larger for glueballs compared to mesons.

(γ) missing possibility of implementation into a conventional qq family: either by "wrong" masses of the candidate or by the simple fact that the family is already completely known.

i(1440) (See also section 2.2.4). This object was discovered in 1980 by Mark II in radiative ψ decays in the reaction $e^+e^- \rightarrow \psi \rightarrow \gamma i \rightarrow \gamma K_s^0 K^{\pm} \pi^{\mp}$: Fig. 54a,b (Ref. 93). The spin parity was determined by the Crystal Ball Group²⁰⁴⁾ in the $K^+ K^- \pi^0$ mode (Fig. 54c) assuming only $J \leq 1$ contributions to be $J^P = 0^-$ (Fig. 54d). Recent Mark III data observe the i in all $KK\pi$ modes including $K_s^0 K_s^0 \pi$, confirm the J^P determination with far better statistics and no restriction on $J \leq 1$. After releasing the "low KK mass cut" Mark III obtains mass and width at new (larger) values:

$$m(i) = 1456 \pm 5 \pm 6 ; \quad \Gamma(i) = 95 \pm 10 \pm 15 \text{ MeV} \quad (\text{Ref. 377}) .$$

These results are in full agreement to those of the DM2 group²⁹⁶⁾.

The new i mass value is interesting insofar, that in the discussion concerning J^P of the E meson there is no discrepancy about the E mass: 1420 MeV (see the discussion in Section 2.2.3, and later in this section). Also uncontested is a considerable $K^* K$ decay of the hadroproduced "E" not being evident in i decays. Here a " δ " π decay mode seems to be favoured

where " δ " stands for a low mass K^+K^- enhancement. If " δ " is the 3P_0 member, i.e. a $q\bar{q}$ object, one might ask why the i is not observed in the " $\delta\pi \rightarrow \eta\pi\pi$ " cascade in radiative ψ decays: see Fig. 56a (Ref. 377). The bump at 1380 MeV is hardly consistent with being an i with parameters as found in the analysis described.

A possible explanation was given in Section 2.2.6. A potential binds in a final state interaction the two K 's into a " δ " (understood as a $K\bar{K}$ bound state and thus distorts the i decay modes and "pulls" the K 's even from an $i \rightarrow K^*K$ decay to a $K\bar{K}$ mass near threshold³³³). This sounds very nice, but one might ask how the $O^-(\delta\pi)$ bump at the $\sim E/i$ mass shown in Fig. 22c fits into this scheme and must await the result of a J^P determination of a corresponding bump at the E/i mass observed in $\eta\pi^0\pi^0$ ³⁴⁴.

These arguments belong to the question, "are i and E in fact the same" -- and a conclusive answer is not known. Reference 419 expresses some sympathy with a two state solution collecting all experimental knowledge as known in the spring of 1986, especially the Mark III results on iotas from hadronic ψ decays (see later). The first object $E(1420)$ (J^{PC} not known, see Section 2.2.3) would then be mainly "quarkic" ($s\bar{s}$) whilst the nature of the second will be discussed in the following: "is the $i(1460)$ a glueball?":

"Glueball criterion" (α) is clearly fulfilled; What about the others?". Observation of a radiative decay seems to point away from a 'pure' glueball interpretation: the photon couples only to electric charge. The Crystal Ball²⁰⁸ and Mark III^{201,377} observe peaks around ~ 1400 MeV in the $(\rho\gamma)$ distribution from ψ radiative decays. The Mark III group has included this possible i decay into a coupled channel analysis together with $(K\bar{K}\pi)$, $(\rho\rho)$, $(\omega\omega)$ -- all observed in radiative ψ decays (Refs. 377,378). The fit assumes two pseudoscalar Breit-Wigner amplitudes at ~ 1.5 GeV and ~ 1.8 GeV, which interfere and are coupled to all four final states, except $X(1.8)$ not coupled to $K\bar{K}\pi$. A possible η' contribution is taken into account. The intensity of the coupled channel Breit-Wigner amplitude is compared in Figs. 55a-d to the data. The i parameters as determined from the $(K\bar{K}\pi)$ channel remain unaffected, $(\rho\rho)$ and $(\omega\omega)$ distributions can also be described by the i and a new state $X(1800)$, see also Ref. 380. The shape of the $(\rho\gamma)$ distribution is not well reproduced.

Moreover the coupling ratio $g_{\gamma\rho}/g_{\rho\rho} = (3000 \pm 600)^{-1}$ as given by the overall fit is much smaller than the VDM expectation [$= 2\pi\alpha/3\gamma\rho^2 = (400)^{-1}$; 2/3 from $\rho\rho$ isospin].

The analysis of Ref. 278 reaches similar conclusions.

From this it seems questionable whether the bump in the $(\rho\gamma)$ spectrum is really due to the i or due to the $E(1420)$. A spin parity analysis of the $(\rho\gamma)$ peak has not yet been published. The three decay modes clearly connected to the i have approximate branching ratios

$$\begin{aligned} B(J/\psi \rightarrow \gamma i) \times B(i \rightarrow KK\pi) &= 5 \times 10^{-3} \\ &\times B(i \rightarrow \rho\rho) = 1.5 \times 10^{-3} \\ &\times B(i \rightarrow \omega\omega) = 0.3 \times 10^{-3} \end{aligned}$$

and thus show flavour democracy, i.e. fulfill criterion (β). These numbers also answer a possible objection from criterion (γ): whether the i can be a radially excited η' : the decay into $\psi \rightarrow \gamma i$ is the largest decay of ψ into non- $c\bar{c}$ states and therefore larger than $\psi \rightarrow \gamma \eta'$ ^{*}). This is even more true for $\psi \rightarrow \gamma \eta$, but the radial recurrence of the η , the η_R has been almost probably discovered anyhow (see Section 2.2.2). This discovery causes severe problems for models [Ref. 168 (first version), 381] which interpreted the i to be mainly a radially excited η raised from its isovector partner by 'two gluon annihilation mechanisms' already causing partly the mass splitting of the ground state pseudoscalars.

Thus all this is pointing to a glueball classification of the i (possibly mixed with an $X(1.8) = (\eta')_R$ being mainly $s\bar{s}$ ²⁰⁹). A key role for a final answer play the charge sensitive decay modes:

$\rightarrow (\gamma\gamma)$; see Refs. 294, 419 for upper limits, and:

$\rightarrow (\rho/\omega/\phi\gamma)$.

^{*}) Moreover π' , η_R , i in $(^1S_0)'$ conflict with the Gell-Mann Okubo mass formula (Ref. 209).

If the $(\gamma\rho)$ peak at ~ 1400 MeV seen in $\psi \rightarrow \gamma(\gamma\rho)$ does not belong to the iota a major glueball component seems almost likely. Reference 315 quotes decision criteria from ratios of hadronic, radiative and "two photon" decays of i by means of VDM.

Speculations of a different nature of the original iota and the $\rho\gamma$ bump were caused by the observation of a lower mass and a larger width of the latter (Table 19). It should be reminded that Mark III observes also a $\eta\pi\pi$ bump in radiative ψ decays at ~ 1380 MeV (Fig. 56a). Further input to these speculations might come from very recent (preliminary) Mark III data on hadronic ψ decays⁴⁰⁷⁾. In general, also here lower mass values, different widths and 'bump features' seem to emerge for the bump in the 1400-1450 region. See Figs. 56b,c for the reaction $e^+e^- \rightarrow \psi \rightarrow \omega KK\pi/\omega\eta\pi^+\pi^-$ and Table 19 for a summary.

→ If the bump belongs to the iota, then in the first approximation there is a non-vanishing quark content. Its presence inspires models with iota being a mixture of $q\bar{q}$ states (different flavours and radial excitations) and a considerable gluonium contribution²⁰⁶⁾ or even those models surviving without any gluonium admixture²⁰⁹⁾. Here the $q\bar{q}$ contents with very different wave functions could interfere strongly and produce effects like the nuclear giant resonance²¹¹⁾.

A nice summary is given in Ref. 319 paying attention also to the similarity between $(q\bar{q})^*$ and $(q\bar{q}g)$ spectroscopy, i.e. to the $iota = q\bar{q}g$ possibility.

As mentioned, radiative iota decays measure its $q\bar{q}$ content; if -- somewhat simplifying -- the iota is "quark made" [due to the large $(\rho\gamma)$ width probably mainly "u,d made"] one expects a mainly s made partner and thus a bump in $(\phi\gamma) \sim 300$ MeV heavier. A "gluon made" iota should show bumps in $(\rho/\omega/\phi\gamma)$ at the same position with a VDM guess for the branching ratios of 9:1:2³⁰⁹⁾.

Reference 309 reviews also quantitative predictions of radiative iota decays

$$\rightarrow i = (q\bar{q})^* : \Gamma(i \rightarrow \gamma\rho) = 100-300 \text{ keV}$$

$$\rightarrow i = \text{blueball} : \Gamma(i \rightarrow \gamma\rho) = 0(0.5-1) \text{ MeV} .$$

$$\rightarrow i = \text{hermaphrodite (see Section 4)} \Gamma(i \rightarrow \gamma\rho) = 0.7-1.5 \text{ MeV}$$

The surprising fact that a gluonium state has not only a non-vanishing radiative width but even a larger one than $(q\bar{q})^*$ is due to the explicit or implicit mixture of the gluonic part with the quarkonic part. Predictions range -- depending on the nature of the underlying structure:

$$\begin{array}{ll} \text{from } \Gamma(i \rightarrow \gamma\gamma) = 1 \text{ keV} & \text{to } \Gamma(i \rightarrow \gamma\gamma) = 3.5 \text{ MeV} \\ & \text{(Ref. 330)} \qquad \qquad \qquad \text{(Ref. 210)} \end{array}$$

$$\text{or from } \Gamma(i \rightarrow \rho\gamma) = 16 \text{ keV} \qquad \qquad \text{to } \Gamma(i \rightarrow \rho\gamma) = 10 \text{ MeV}$$

This has to be compared with nowadays experimental knowledge from Mark III, Crystal Ball, DM2 and TASSO, TPC

$$[\Gamma(i) = 95 \pm 18 \text{ MeV} , B(\psi \rightarrow \gamma i) = 7 \times 10^{-3}]$$

$$\Gamma(i \rightarrow \gamma\gamma) \leq 2.2 \times 7/5 = 3 \text{ keV (Ref. 294), } 2.6 \text{ keV (Ref. 418)}$$

$$\Gamma(i \rightarrow \rho\gamma) = 1.4 \pm 0.4 \text{ MeV (Ref. 377) .}$$

Calculating the "stickiness" from these numbers⁴⁰⁹⁾ one gets $S_i : S_{\eta} : S_{\eta'} = > 50:4:1$.

Summarizing the iota puzzle, one cannot draw firm conclusions about its nature. But one must state that there has been considerable progress in the last two years (see Ref. 132 for a review on the 1984 situation). The discovery of the η_R and the coupled channel analysis from Mark III has put new constraints to phenomenology. One might hope for a final analysis of the iota in hadronic ψ decays from Mark III for an answer to the essential problem whether there exist more than one state in the 1400-1500 MeV region.

$\Theta(1700)$. This object was first observed in 1982 by the Crystal Ball group in the reaction $e^+e^- \rightarrow \psi \rightarrow \gamma\Theta \rightarrow \gamma\eta\eta$, see Fig. 57²¹²⁾.

P and C must be even; decay angular distributions suggest J to be 2 though a 0^{++} classification cannot be completely ruled out due to a possible $f'(1515)$ contamination.

Recent Mark III data (Refs. 213, 297) confirm this observation in the K^+K^- channel (Fig. 58) and $\pi^+\pi^-$ channel:

$$m_\theta = 1720 \pm 10 \text{ MeV}$$

$$\Gamma_\theta = 130 \pm 20 \text{ MeV}.$$

The 2^{++} assignment is supported by Mark III (a corresponding analysis of the DC1-DM2 group leads to equal probabilities of 0^{++} and 2^{++}). Following the Mark III result (i.e. $J^P = 2^{++}$) the production mechanism measured in terms of helicity amplitudes A_0, A_1, A_2 turns out to be strikingly different between known $q\bar{q}$ objects f, f' ($A_2 = 0$) and θ ($A_1 = -A_0 = A_2$), see Ref. 377 for details. Reference 342 reports θ production in centrally π/p -produced K^+K^- pairs at a somewhat higher mass of 1742 MeV.

Criteria (α), (β), (γ) show the θ to be the 'almost certainly first non $q\bar{q}$ object'²¹⁶⁾ or 'the smoking gun' in the search for glueballs³²²⁾.

(α): θ is produced also in a large fraction of radiative ψ decays [$B(J/\psi \rightarrow \gamma\theta) \geq 1.6 \times 10^{-3}$ compared to $B(J/\psi \rightarrow \gamma f) \sim 7 \times 10^{-3}$]. See Ref. 409 quoting an abnormally high stickiness of the θ compared to f, f' .

(β): Decay modes are

$$K^+K^- : \eta\eta : \pi^+\pi^- \sim 3:2.4:1$$

and thus flavour democracy can be concluded especially if one accounts that the larger phase space has led to a 'spraying' of u,d decays of the θ into many light meson final states with unobservable small partial widths²¹⁵⁾.

(γ): The 3P_2 family is well known, a radial excitation of the f has also been found. Moreover, the decay modes of the θ disfavour strongly the θ being a radially excited (ud) system. Mass relations rule out a radial excitation of the f' . The production mechanism points to a different nature of θ from those of f, f' .

Conclusions about θ 's nature from criteria (β) and (γ) are a matter of discussion: see Ref. 418 for a different viewpoint.

Meikton and $4q$ classification seem unlikely because of the non-observation of related $I = 1$ or $S = 1$ states²¹³⁾.

It is not clear how very recent (unpublished) Mark III data⁴⁰⁷⁾ on hadronic θ decays agree with a glueball classification of the θ :

$$B(J/\psi \rightarrow \omega''\theta'') \sim 10^{-3} \sim 5 \times (J/\psi \rightarrow \phi''\theta'') .$$

This would conflict with flavour democracy and point to a conventional ($q\bar{q}$) structure of the θ : three gluon decays of the ψ building up an ω (non-strange, copiously) or a ϕ (strange, rarely) opposite to a θ with a corresponding quark content. The same production mechanism would lead to

$$J/\psi \rightarrow \omega f \gg J/\psi \rightarrow \omega f' \quad \text{and} \quad J/\psi \rightarrow \phi f' \gg J/\psi \rightarrow \phi f$$

as it is observed by the DM2 group⁴¹⁰⁾.

In Fig. 58 another signal was clearly visible:

$\xi(2220)$. This object has been observed by the Mark III group in the (K^+K^-) and ($K_S^0 K_S^0$) modes in radiative ψ decays (Ref. 213). Its width is consistent with the experimental resolution. With doubled statistics this result was recently confirmed by the same group³⁸³⁾. Its parameters:

$$\text{mass} = 2.23 \pm 0.006 \pm 0.014 \text{ GeV}$$

$$\text{width} = 0.026^{+0.020}_{-0.016} \pm 0.017 \text{ GeV}$$

from K^+K^- mode (4.5σ) and -- consistently within the errors -- from $K_S^0 K_S^0$ (3.6σ): $B(\psi \rightarrow \gamma \xi \rightarrow \gamma KK) = 7.3 \times 10^{-5}$. The full data sample is shown in Fig. 59: a) Mark III: K^+K^- , b) Mark III: $K_S^0 K_S^0$, c) DM2: K^+K^- , d) DM2: $K_S^0 K_S^0$ ³⁸⁴⁾. The spectra measured by DM2 seem to show a (broader) enhancement at lower mass inconsistent with being a ξ as found by Mark III:

$$\text{Mark III} : B(J/\psi \rightarrow \xi) \times B(\xi \rightarrow K\bar{K}) \sim 8.3 \times 10^{-5}$$

$$\text{DM2} : \quad \quad \quad \leq 5.5 \times 10^{-5}$$

Also the CLEO group fails to confirm the ξ (Ref. 221), but a narrow structure at 2.22 GeV observed in $\eta\eta'$ systems of the reaction π^- (38, 100 GeV) $p \rightarrow \eta\eta'X$ at the GAMS spectrometer is possibly due to the ξ state⁴¹⁶⁾.

Theoretical speculations: Technipion²¹⁹⁾, $s\bar{s}g$ hybrid²²⁰⁾, $\Lambda\Lambda$ baryonium³⁰⁴⁾. Godfrey and Isgur²⁵⁴⁾ point out that within their model¹⁶⁸⁾, the $\xi(2220)$ mass, decay width, and production cross-section is well consistent with an "ordinary" $q\bar{q}$ interpretation: $L = 3$, ss , $J^{PC} = 2^{++}$: 3F_2 . A classification as a Higgs particle is unlikely due to the large branching ratio and due to the non-observation in typical weak decay modes like $\mu^+\mu^-$ (Ref. 215). Before deciding between the presented interpretations and further speculations the existence question has to be unambiguously solved and a J^P analysis has to be performed.

The search for scalar glueballs -- expected around 1 GeV -- at the CERN ISR AFS spectrometer was negative²²³⁾ up to masses of 1.15 GeV. (See Ref. 117 in this context.) The reader is referred also to Refs. 385, 342, 279, and 273 for hadron induced glueball candidates awaiting confirmation.

Other candidates from hadronic experiments are:

$\underline{g_T(2120)}, \underline{g_T(2220)}, \underline{g_T(2360)}$. The BNC/CCNY collaboration²²⁴⁾ claims evidence for three glueballs with properties as listed in Table 20. With increased statistics the authors claim these states to be confirmed within the errors²³⁵⁾. The results come from a phase shift analysis of the $\phi\phi$ system in the reaction $\pi^-p \rightarrow \phi\phi n \rightarrow 4K^\pm n$. A possible production diagram is shown in Fig. 60a. Figure 60b shows the intensity of (2^{++}) waves.

The classification of the three states as glueballs is inspired by the consideration that double OZI suppressed $\phi\phi$ production (see Fig. 60a) is nevertheless observed with a surprisingly high cross-section:

$$\sigma_{\text{tot}}(\pi^-p \rightarrow \phi\phi n) \approx 20 \text{ nb}.$$

An experiment at the CERN Omega spectrometer [WA67²²⁷⁾] analyses inclusive $\phi\phi$ production in π^- (85 GeV) Be interactions. The inclusive mass distribution is well described by a fit with the two heavier resonances [see Table 20, the state $g_T(2120)$ might be cut by acceptance]. Also the S,D mixture of the second and the pure D wave nature of the third state can be reproduced, an analysis of angular correlations in terms of moments²²⁸⁾ leads to good description by a $J^P = 2^+$ dominance.

The question whether these resonances can be regarded as resonances at all or (in case they are) as "the discovery of glueballs"²¹⁴⁾ has been emotionally discussed in the past. Reference 216 expresses strong doubts on the interpretation of these states as resonances. Also the argument of a "high cross-section and double OZI suppression" is a target of criticism: Lipkin²²⁶⁾ argues that the crossed channel $\phi n \rightarrow \phi \pi^- p$ is not suppressed at all. Ono¹⁶²⁾ criticizes the comparison of the OZI suppression mechanisms in $\pi^- p \rightarrow \phi \phi n$ and $\pi^- p \rightarrow \phi n$ as used in the analysis. The latter diagram would require three gluons in Fig. 60a. He favours the $g_T(2120)$ to be a radial excitation of the f' .

Karl³⁸⁶⁾ claims to explain the measured cross-section in terms of the chain

$$u\bar{u} \rightarrow gg \rightarrow s\bar{s} \rightarrow \phi\phi$$

with a two gluon intermediate state as described by perturbative QCD and a sequential fragmentation $s\bar{s} \rightarrow \phi\phi$. Furthermore, he explains the observed 2^{++} quantum numbers as a natural consequence of the vector coupling of the gluons. Thus helicity $(u\bar{u}) = \pm 1$ and $\ell(gg) \geq 2$, $C(gg) = +1$ and $S(u\bar{u}) = 1$, i.e. $L(u\bar{u}) = \text{odd}$ and $P(u\bar{u}) = +1$: $2^{++}, 3^{++}, 4^{++}$. Only 2^{++} is accessible at BNL energies. See Lindenbaum³⁸⁷⁾ criticizing the criticism.

It seems unlikely that $\xi(2220)$ and $g_T(2220)$ are identical due to the huge width discrepancy. The search of the Mark III group in radiative ψ decays of the type $\psi \rightarrow \gamma\phi\phi$ was negative, but the authors admit low statistics and low acceptance in the relevant mass region: more data are urgently needed.

The objects need confirmation, not only looking in $\phi\phi$ decay modes, but also in other vector meson-vector meson modes like $\rho\rho$, $\omega\omega$ [Mark III $B(J/\psi \rightarrow \gamma g_T) \times B(g_T \rightarrow \rho\rho) < 6 \times 10^{-4}$]. This would be also extremely useful to weight a four (strange) quark hypothesis of these objects³⁸⁸):

$$(s\bar{s} - s\bar{s}) \rightarrow \phi\phi \quad s \text{ wave}$$

$$\rightarrow \rho\rho, \omega\omega.$$

G(1590). This candidate was found in Serpukhov with the hodoscope spectrometer GAMS 2000 in the reaction

$$\pi^- (38 \text{ GeV}) p \rightarrow G + X \rightarrow \eta\eta + X \quad (\text{Refs. 229, 230})$$

$$m_G = 1592 \pm 25 \text{ MeV}$$

$$\Gamma_G = 210 \pm 40.$$

Its redundancy in the 0^{++} family as well as its $\eta\eta$ decay (Table 18) point to a glueball interpretation.

Its quantum numbers are $I^G_{J^{PC}} = 0^+0^{++}$. Assuming 1) one pion exchange, 2) no nucleon spin flip, 3) $L(\eta, \eta) \leq 2$ a comparison of the decay angular distribution in the Gottfried-Jackson system with expectations for (η, η)

S and D waves yields

- a strong D wave contribution consistent with a (first time observed) $\eta\eta$ decay mode of the $f(1270)$;
- an S wave contribution with parameters listed above: Fig. 61.

According to the authors, two observations point against a quarkonic structure of this object:

$$B(G \rightarrow \pi^0 \pi^0) < 0.3 B(G \rightarrow \eta\eta)$$

$$B(G \rightarrow K\bar{K}) < 0.6 B(G \rightarrow \eta\eta).$$

But the same group has observed the $\eta\eta'$ decay of the G:

$$\frac{\Gamma(G \rightarrow \eta\eta')}{\Gamma(G \rightarrow \eta\eta)} = 2.7 \pm 0.8 \quad (\text{Ref. 298}).$$

This has to be compared with the value of 3 as predicted by Gerstein (Ref. 298a) for a scalar glueball.

The spectrometer was then moved from Serpukhov to CERN and enlarged (GAMS 2000 $\rightarrow$ GAMS 4000): the data obtained with 100 GeV pions fully confirm the quoted results (Refs. 112a, 344).

It is not clear why the G has not been seen in radiative ψ decays. This fact motivated Gerstein to change his glueball to a (4q) or meikton assignment (or their mixture)³⁸⁹. See Ref. 331 for another possibility.

Before giving weight to these interpretations one must certainly wait for confirmation of the G by other experiments, maybe in 2γ physics.

4. HERMAPHRODITES, HYBRIDS, MEIKTONS

All three names stand for mixed states consisting of quarks, anti-quarks, and gluons.

Because of the poor experimental evidence (see below) even compared to the glueball situation theoretical models will be only briefly mentioned. Many ansatzes already presented can be used also for predictions of the meikton spectroscopy.

$\rightarrow$ In the context of a chromodynamic flux tube model Ref. 390 argues for even more reliable predictions in the meikton sector compared to the gluonium sector (see Ref. 356 for an application of a similar approach to conventional mesons, which is defended in Ref. 420 against theoretical prejudices of being too naive for a correct description of a domain reigned by QCD confinement). Mass predictions are for a ρ, ω -like $q\bar{q}$ content 1900 MeV, for an ϕ like $q\bar{q}$ content 2100 MeV. Quite large widths are in this case expected except for those hybrids with exotic $J^{PC} = 0^{+-}, 1^{-+}, 2^{+-}$: $\Gamma \leq 250$ MeV. An explanation why even these states have not been observed yet is given by the string picture and by the fact that the decay mesons have relative coordinates parallel to the string; they can absorb the unit of string angular momentum only if they have very different spatial wave functions. Thus simply decay modes like $\pi\pi, \pi\eta, KK$ will be suppressed compared to experimentally difficult ones like $\pi B, KQ$, etc. In diffractive photoproduction $J^{PC} = 0^{+-}, 2^{+-}$ meiktons could be easily produced via "plucked ρ, ω, ϕ states".

Indeed a broad bump observed at 1900 MeV in $(B\pi)$ substates of photo-produced $(\omega\pi^+\pi^-)$ (Ref. 413) systems would fit into this picture: in Fig. 62a the amount of B's for different $\omega\pi^+\pi^-$ mass windows is shown. An enhanced B production it is obvious for $1.6 \leq M_{\omega\pi^+\pi^-} \leq 2.0$ GeV. In Fig. 62b the $(B\pi)$ mass distribution is presented after an appropriate peak-wings procedure. The resulting spectrum could well be due to a $\omega(1670, J^{PC} = 3^{--})$ together with a ~ 250 MeV wide meikton state as predicted by Ref. 390. Certainly alternative interpretations of the nature of such a broad enhancement are not ruled out.

→ See Ref. 262 for hybrid predictions from lattice calculations. Predictions are in the same ball park than those from Bag models.

→ "QCD sum rules are especially suited for $(q\bar{q}G)$ states because in this framework it is very simple to construct a composite operator of quarks and gluon fields with appropriate quantum numbers" (Ref. 232). With light u,d quark masses (5.5 MeV) and

$$\langle 0 | \frac{\alpha_s}{\pi} G_{\mu\nu}^a G_{\mu\nu}^a | 0 \rangle$$

as given in Section 2.3.3, reference 391 quotes Meikton masses as follows:

$$M = 1.65/2.28/3.3/3.65 \text{ GeV} \quad \text{for} \quad J^{PC} = 1^{--}/1^{+-}/0^{++}/0^{--}$$

(see also recent calculations from Ref. 412).

→ the (MIT) Bag approach is frequently used for hybrid predictions. The work goes back to Ref. 392. See Refs. 211, 231, 319, 393 for reviews. In the Bag nomenclature hybrids can be formed as:

$$|q\bar{q}TE\rangle = (0^{--} \text{ or } 1^{--}) \times 1^+: 0^{--}, 1^{--}, 1^{--}, 2^{--}$$

$$|q\bar{q}TM\rangle = (0^{--} \text{ or } 1^{--}) \times 1^-: 1^{+-}, 0^{++}, 1^{++}, 2^{++}.$$

In order to predict a corresponding spectroscopy of the nonets in a Bag model it is necessary (already in the one gluon exchange approximation) to account for contributions like:

Results of a corresponding calculation are listed in Table 21 (Ref. 231). Calculations of Barnes and Close²³³ (see Ref. 319 for a review) agree well with the range given in Table 21 and show in particular the same sequence of masses

$$m(0^{-+}) < m(1^{-+}) < m(1^{--}) < m(2^{-+}) .$$

The authors point to the fact that the pseudoscalar (πg), (ηg), ($\eta' g$) mass pattern is similar to the one predicted by various models (for instance Ref. 168) for conventional $q\bar{q}$ states.

Decay channel predictions for "Bag meiktons" do not show any peculiarities to profit from for their discovery. Assuming $\vec{J}\vec{J}$ coupling one gets:

$$|q\bar{q}g\rangle \rightarrow |q\bar{q}\rangle_{L=0} + |q\bar{q}\rangle_{L=1} \quad (\text{S wave}):$$

for instance

$$|\rho g, 1^{-+}\rangle \rightarrow \pi D, \rightarrow \pi A_1$$

or

$$|q\bar{q}g\rangle \rightarrow |q\bar{q}\rangle_{L=0} + |q\bar{q}\rangle_{L=1} \quad (\text{P wave}) ,$$

for instance

$$|\rho g, 1^{-+}\rangle \rightarrow \pi \eta, KK^* .$$

See Ref. 231 for a full list of predictions: these calculations do not suppress decay into meson pairs with equal wave function contrary to Ref. 390.

Meiktons with a TM gluon are expected to be some hundred MeV heavier than the ones with a TE gluon.

The experimentalists' life becomes more difficult due to these additional states: for instance, beside radially excited vector mesons and 3D_1 states meiktons also play a role by making things, in the original sense of the word, mixed up.

$c\bar{c}$ spectroscopy has to take $(c\bar{c}g)$ states into account too: Ref. 234 identifies $\psi(4030)$ and $\psi'(4159)$ as mixtures of radial ψ excitations and $c\bar{c}g$ meiktons. This leads to minor corrections for heavy quark spectroscopy, which could become more important in the light quark regime.

Here the experimental knowledge is almost zero, an A'_3 observed by Ref. 124 beside the A_3 at 1850 MeV could be interpreted as a $(\rho|\omega, TE)$ meikton according to Table 21. Lindenbaum's glueball candidates²²⁴⁾ as well as the D(1526) (see Section 2.2.4) have been considered to be candidates²¹⁵⁾.

A rapid change of the forward-backward asymmetry of $\eta\pi^0$ systems in the reaction $\pi^-p \rightarrow \pi^0n$ ³²¹⁾ at $m(\eta\pi^0) = 1.3$ GeV can be explained by the presence of a 1^{-+} exotic meikton in that mass region³²²⁾. A meikton candidate observed in $B\pi$ substates of $\omega\pi^+\pi^-$ states from the reaction $\gamma p \rightarrow \omega\pi^+\pi^-p$ was already mentioned (Ref. 413).

The mass region up to 2.5 GeV is already overwhelmed by bound states of light quarks, or gluons and their mixtures.

Nevertheless, there is another source contributing to this confusing puzzle.

5. $q\bar{q}$ - $g\bar{g}$ MOLECULES

Such objects could have exotic quantum numbers like $Q = 2$ or $S = +2$ and therefore be easily detected: real exotics. In contrast the quark molecules with conventional quantum numbers are called: cryptoexotics.

Beside mesonic production and decay mechanisms these objects, if above threshold, could be easily produced by or decay into nucleon-antinucleon states.

A molecule consisting of two diquarks ($qq-\bar{q}\bar{q}$) behaves -- from colour point of view -- like a ($Q\bar{Q}$) system: a diquark corresponds to an antiquark. This can be seen as follows: to make a blue quark white one either needs an antiblue antiquark or a red-green diquark.

The ($Q\bar{Q}$) triplet-antitriplet could annihilate immediately leading to states of (unobservable?) large width: true baryonium.

The broad baryonium states -- discussed in the seventies -- would fall into this category (Refs. 235,263). A one gluon exchange would change the colour balance of the ($3-\bar{3}$) system and create a ($6-\bar{6}$) system. This system cannot immediately annihilate with a condensing ($q\bar{q}$)-($3-\bar{3}$) state because of $6 \otimes 3 \neq 1$: mock baryonium.

These states therefore could be quite narrow, ≈ 10 MeV. A couple of narrow baryonia observed in the last ten years, especially the S(1936), would nicely fit into this picture (Refs. 236-242). But all these objects could not be confirmed by succeeding experiments (see Refs. 243, 244, 398 for instance). References 264 and 265 summarize the experimental knowledge as known at this time.

Various results on this topic, especially on the existence of the S(1936) have been reported in the meantime. With antiproton beams (6.1 and 8.9 GeV) at SLAC (!) ³⁰⁵⁾ claims to observe narrow (< 40 MeV)

structures in the $M(N\bar{N})$ distributions obtained in the reaction $\bar{p}N \rightarrow (\pi/p)_{\text{fast}} + X$, "whose interpretation as a resonance cannot be ruled out"

→ one at 2.02 GeV in $M(p_{\text{fast}}\bar{n})$: $\bar{p}p \rightarrow p_{\text{fast}}\bar{n} 2\pi^+\pi^-$

→ one at " " in $M(n\bar{p})$: $\bar{p}p \rightarrow \pi_{\text{fast}}^+ \bar{p}n\pi^+\pi^-$

→ one at 1.92 GeV(!): $M(p_{\text{fast}}\bar{p})$ in $\bar{p}p \rightarrow p_{\text{fast}}\bar{p}\pi^+\pi^-\pi^0$.

Accounting for an appropriate incorporation of beam attenuation in a long target a reanalysis of $p\bar{p}$ interactions originally published by Ref. 398 gave "evidence for a narrow resonance at 1937 MeV":

$$\Gamma = 2.3 \text{ MeV} \quad \text{and} \quad \Gamma \cdot \sigma_T = 107 \pm 44 \text{ MeV} \cdot \text{mb} \text{ (Ref. 395).}$$

Also Ref. 397 draws different conclusions than Ref. 244 from the same data set claiming to observe a structure at $1934 \pm 3 \text{ MeV}$, $\Gamma = 6 \pm 4 \text{ MeV}$ in elastic $p\bar{p}$ backward scattering.

Before getting too excited about a revival of the S(1936) one should also account for recent negative results³⁹⁴⁾ quoting upper limits for S production in $p\bar{p}$ annihilations with a numerical value 3 standard deviation lower than the cross-section quoted in Ref. 395.

Whilst Ref. 394 does not rule out a narrow bump-dip pattern in the mass region of 1935–1941 MeV a totally negative result is reported from Ref. 415: measurements of $\bar{p}p$ total cross-sections from 388 to 599 MeV/c in small antiproton momentum steps give 'evidence against the S meson'.

A final answer will hopefully come with the results from the CERN antiproton beam (LEAR), where high luminosities never available so far and special designed experimental devices³⁰⁴⁾ will be used to analyse the $p\bar{p}$ reactions.

Below $N\bar{N}$ threshold colour arguments lead to a prediction of a crypto-exotic ($J^{PC} = 0^{++}$) four quark nonet at about 1000 MeV with rather narrow widths. The flavour content of the members is given in Fig. 63

(Ref. 69). Exotic four quark S wave states live in $(10)_-$, $(\overline{10})_-$, or $(27)_-$ plets, constructed from $SU(3)_{\text{colour}} \times SU(2)_{\text{spin}} \times SU(3)_{\text{flavour}}$. They can decay through OZI super-allowed mechanisms leading to unobservable large widths. This is expected to hold also for four quark states with $L > 0$. Isgur and Weinstein²⁴⁵⁾ expect the multiplet presented in Fig. 63 to be the only one observable.

Jaffe³²³⁾ interprets $\delta(980)$ and $S^*(975)$ as four quark states due to their low width compared to the other 0^{++} nonet members $\kappa(1420)$, $\varepsilon(1300)$ and their copious decay in strange final states. Arguments against and in favour were given in Section 2.2.5.

The enhancements seen in the reaction $e^+e^- \rightarrow e^+e^-\rho^0\rho^0$ (Refs. 128,310) could be also due to four quark molecules (Refs. 246,316).

Whether or not bound $p\bar{p}$ states seen in the reaction $p\bar{p} \rightarrow X \rightarrow \gamma + \text{mesons}$ ²⁵⁵⁾ fall into this category will be certainly discussed in more detail if the succeeding experiments are able to confirm these states²⁵⁶⁾; see Ref. 417 for recent upper limits.

This holds also for a state at 1480 MeV produced in antiproton annihilation at rest in deuterium:

$$\bar{p}(0.6 \text{ GeV}) + d \rightarrow \pi^- + X_0(1485, \Gamma = 200 \text{ MeV}) + p \text{ (Ref. 396) .}$$

6. SUMMARY

There were two motivations to write this review.

i) An overview should be given of our experimental knowledge of the light quark and gluonic spectroscopy. A walk through the $q\bar{q}$ meson zoo shows quite good understanding of the quarks in relative S and P waves; but going to higher orbital momenta or radial excitations, gaps appear and uncertainties become bigger. Theoretical ansatzes to describe quantitatively this zoo are schematically presented putting a special weight on the (surprisingly well working) potential models but pointing to the necessity of exact understanding of all phenomena within the framework of QCD.

A direct consequence of QCD's validity should be the existence of bound states made purely of gluons or of quark-gluon mixtures or even exotic "many quark states". In this review our knowledge of these states is treated in a similar manner as the $q\bar{q}$ states. The experimental status and the theoretical models present a rather confusing puzzle of particles in the mass range from the pion mass up to ~ 2.5 GeV. The situation becomes even more complicated if one takes into account the mixing possibilities of appropriate states: $q\bar{q}$ systems of different L could as well be mixed as $q\bar{q}$ states and glueballs in many combinations.

ii) But coming to the second motivation this may not be a reason to give up but should be understood as a challenge. The quarks discussed here are the ones the world is built from.

So theorists are asked to present more precise models, to estimate mass, lifetime and decay modes so accurately that decisions between alternatives can be made:

Are mesons pure $q\bar{q}$ states or which ones have non-negligible gluon contributions?

Do glueballs appear in pure form or always in connection with quarks?

What role do meiktons play and do four-quark states really exist?

Experimentalists are encouraged to perform new experiments with improved techniques and increased statistics in order to produce the necessary input for the theoreticians.

High intensity hadron (π/K) factories are needed to improve knowledge of ($L \geq 2$) ($q\bar{q}$) systems or glueballs for instance.

Upgraded nucleon-antinucleon experiments, those being started at CERN LEAR, should answer questions on four quark states or other exotics for instance.

New lepto- and photoproduction experiments (certainly with high intensity, not necessarily with high energy, possibly with polarized beams

and targets) will yield new insight into meson quantum number determination and discover new states like radial excitations for instance.

e^+e^- storage rings with e^+ energies up to 3 GeV but high luminosity and fully equipped detectors could finally solve the ambiguities existing for radial excitations of vector mesons and will offer therefore the unique possibility to measure the spectroscopy of light quarks with quality comparable to heavy quarks.

As already said our world is built up (mainly) by light quarks; high energy physicists are in charge to find out how it is built up.

The aim of this write-up is to stimulate the enthusiasm in this direction.

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0^- -family		1^- -family	
octet	$\pi^+ = u\bar{d}\rangle$	$\rho^+ = u\bar{d}\rangle$	
	$\pi^- = d\bar{u}\rangle$	$\rho_0^- = d\bar{u}\rangle$	
	$\pi^0 = \frac{1}{\sqrt{2}} u\bar{u} - d\bar{d}\rangle$	$\rho^0 = \frac{1}{\sqrt{2}} u\bar{u} - d\bar{d}\rangle$	
	$K^0 = d\bar{s}\rangle$	$K^{*0} = d\bar{s}\rangle$	
	$K^+ = u\bar{s}\rangle$	$K^{*-} = u\bar{s}\rangle$	
	$K^- = s\bar{u}\rangle$	$K^{*+} = s\bar{u}\rangle$	
	$\bar{K}^0 = s\bar{d}\rangle$	$\bar{K}^{*0} = s\bar{d}\rangle$	
singlet	$\eta = \frac{1}{\sqrt{6}} u\bar{u} + d\bar{d} - 2s\bar{s}\rangle$	$\omega \approx \frac{1}{\sqrt{2}} u\bar{u} + d\bar{d}\rangle$	} mixed states - -
	$\eta' = \frac{1}{\sqrt{3}} u\bar{u} + d\bar{d} + s\bar{s}\rangle$	$\phi \approx s\bar{s}\rangle$	

Table 1

L	S	Name	J^{PC}	$G(I = 1)$	$G(I = 0)$
1	0	1P_1	1^{+-}	+1	-1
1	1	3P_0	0^{++}	-1	+1
1	1	3P_1	1^{++}	-1	+1
1	1	3P_2	2^{++}	-1	+1
2	0	1D_2	2^{-+}	-1	+1
2	1	3D_1	1^{--}	+1	-1
2	1	3D_2	2^{--}	+1	-1
2	1	3D_3	3^{--}	+1	-1
3	0	1F_3	3^{+-}	+1	-1
3	1	3F_2	2^{++}	-1	+1
3	1	3F_3	3^{++}	-1	+1
3	1	3F_4	4^{++}	-1	+1

Table 2

	Γ_{ee} (keV)	$\gamma_V^2/4\pi$	Γ_{ee}/Q^2
ρ	6.6	$0.5(0.6)^*$	13.2
ω	0.7	4.9	12.6
ϕ	1.3	3.5	11.7
ψ	4.6	3.0	10.3
Υ	1.1	3.6	10.0
ρ'	6.6	1.0	13.2

Table 3

Ref	/30/	/37/	/43,44,276/	/46,47/
Kanal	$4\pi^\pm$	$4\pi^\pm$	$\pi^+\pi^-$	$\pi^+\pi^-\pi^0\pi^0$
E_γ (GeV)	9.3	20-70	20-70	20-70
m_ρ (GeV)	1.43 ± 0.05	1.53 ± 0.03	1.59 ± 0.02	~ 1.65
Γ_ρ (GeV)	0.65 ± 0.1	0.4 ± 0.05	0.23 ± 0.08	~ 0.2
σ (μb)	1.6 ± 0.4	0.54 ± 0.17	0.1 ± 0.02	0.6 ± 0.2
b (GeV^{-2})	5.6 ± 0.3	7 ± 0.5	~ 6	~ 6

Table 4

	x	ΔE	$ \psi(0) ^2$ ***	$\langle r \rangle$	$\langle r^2 \rangle$	$\langle v^2 \rangle$ **
harm. Oscillator	2	$\mu^{-\frac{1}{2}}$	$\mu^{\frac{3}{4}}$	$\mu^{-\frac{1}{4}}$	$\mu^{-\frac{1}{2}}$	$\frac{Eb}{4\mu}$
linear potential	1	$\mu^{-\frac{1}{3}}$	μ	$\mu^{-\frac{1}{3}}$	$\mu^{-\frac{2}{3}}$	$\frac{Eb}{6\mu}$
Coulomb potential	-1	μ	μ^3	μ^{-1}	μ^{-2}	$\frac{Eb}{2\mu}$
logarithm. potential	0^*	const	$\mu^{\frac{3}{2}}$	$\mu^{-\frac{1}{2}}$	μ^{-1}	$\frac{\text{const}}{4\mu}$

* $x = 0$ because $\ln \frac{r}{r_0} = \lim_{\epsilon \rightarrow 0} \frac{1}{\epsilon} ((\frac{r}{r_0})^\epsilon - 1)$

** quoted from $\langle T \rangle = m_Q \langle v^2 \rangle = \frac{1}{2} \langle \vec{r} \cdot \vec{\nabla}(V(\vec{r})) \rangle$ (virial theorem)
and $\langle T \rangle + \langle V \rangle = Eb$

*** in general $|\psi(0)|^2 = (m_Q^{3/2}/4\pi^2) E_n^{1/2} \cdot \frac{dE_n}{dn}$ /402/

Table 5

V(r)	Reference
$-\frac{A}{r} + Kr + V_0$	/137/
$-\frac{4}{3} \int \frac{d^3 q}{(2\pi)^3} e^{i\vec{q} \cdot \vec{r}} \cdot \frac{1}{q^2} \cdot \frac{16\pi^2}{\ln(1 + \frac{q^2}{\Lambda^2})}$	/138/
$-\frac{4}{3} \frac{a_s}{r} + \frac{32\pi(a_s B)^{1/4}}{3} + C(a_s^3 B)^{1/4}$	/139/
$A \ln(r/r_0)$	/143/
$a + br^x \quad x \sim 0.1$	/145/

Table 6

	$c\bar{c}$			$b\bar{b}$		
1^{--} (ground state)	3095 MeV			9460 MeV		
	(Exp.)	Buchmüller	Martin	(Exp.)	Buchmüller	Martin
(Quark mass)		1480 MeV	1800 MeV		4870 MeV	5174 MeV
2S - 1S	589	600	592	561	560	560
$\Gamma_{ee}(2S)/\Gamma_{ee}(1S)$	0.46	0.45	0.40	0.43	0.44	0.41
	± 0.06					
3S - 1S	935	1020	937	890	890	900
$\Gamma_{ee}(3S)/\Gamma_{ee}(1S)$	0.16?	0.32	0.25	0.32	0.32	0.35
4S - 1S	1319?	1380	1185	1116	1160	1140
$\Gamma_{ee}(4S)/\Gamma_{ee}(1S)$				0.21	0.26	0.27

Table 7

	Exp.*	Buchmüller	Eichten Feinberg	Beavis
$c\bar{c}$	$2^{++}-1^{++}$	46 MeV	57 MeV	69 MeV
	$1^{++}-0^{++}$	95 MeV	83 MeV	71 MeV
$b\bar{b}$	$2^{++}-1^{++}$	21 MeV	19 MeV	23 MeV
	$1^{++}-0^{++}$	32 MeV	28 MeV	24 MeV
χ_b'	$2^{++}-1^{++}$	16 MeV	18 MeV	17 MeV
	$1^{++}-0^{++}$	15 MeV	21 MeV	24 MeV

*update from /16/, /350/, /351/

Table 8

m_t (GeV)	E_{1S} (GeV)	$E_{2P}-E_{1S}$ (MeV)	$E_{3P}-E_{1S}$ (MeV)	$E_{2P}-E_{1S}$ (MeV)	$E_{3D}-E_{1S}$ (MeV)	$(E_{2S}-E_{2P})/(E_{2S}-E_{1S})$
15	29.3	446	784	700	950	0.22
20	39.2	459	801	719	964	0.218
25	49.1	470	816	734	979	0.217
30	59	480	831	748	993	0.213
40	78.9	500	860	774	1023	0.205
50	98.8	521	890	800	1054	0.192
60	118.7	545	920	827	1085	0.180
70	138.6	572	951	856	1118	0.158
80	158.5	600	983	885	1152	0.140

Table 9

$u\bar{u}$	1S	2S	3S
3S_1	772 ρ (776)	1554 ρ' (1600)	2157
1S_0	132 π (135)	1296 π' (1240)	1957

$u\bar{u}$	1P	2P	3P	1D	2D
$S=1$ $J=L+1$	1266 A_2 (1318) f (1273)	1913 f^* (1799)?	2466	1541	2144
$S=1$ $J=L$	1245 A_1 (1230) D (1285)	1880	2422	1697	2253
$S=1$ $J=L-1$	1034 S (983) S^* (975)	1705 S^* (1720)	2257	1741	2272
$S=0$ $J=L$	1221 B (1230) H (1190)	1864	2413	1651	2204

$s\bar{s}$	1S	2S	3S
3S_1	1010 ϕ (1020)	1713 ϕ' (1750)	2233
1S_0	696	1568	2120

$s\bar{s}$	1P	2P	3P	1D
$S=1$ $J=L+1$	1496 f' (1515)	2047 g_T (2160)?	2512	1795
$S=1$ $J=L$	1428 E (1420)	1993	2460	1824
$S=1$ $J=L-1$	1278 g_S (1240)?	1877	2353	1807
$S=0$ $J=L$	1436	1999	2470	1808

Table 10 (continued next page, experimental values in brackets)

$\bar{u}s$	1S	2S	3S
3S_1	900 $K^*(892)$	1648	2212
1S_0	501 $K(495)$	1464	2069

$\bar{u}s$	1P	2P	1D
$S=1$ $J=L+1$	1399 $K^*(1434)$	2002	1680
$S=1$ $J=L$	1383 $Q_2(1410)$	1978	1780
$S=1$ $J=L-1$	1196	1833	1801
$S=0$ $J=L$	1305 $Q_1(1270)$	1914	1738

Table 10 (continued)

$a = 0.16, c = 1.2$		$b_u = -0.63$
		$b_{us} = -0.49$
		$b_s = -0.36$
ρ' 1.61 (1.55)		
π' 1.3 (1.24)		
ρ 0.77 (0.77)	B 1.23 (1.235)	
π 0.14 (0.14)		
$K^{*'} 1.73 (1.65?)$		
$K' 1.33 (1.45?)$		
$K^* 0.89 (0.89)$		
$K 0.49 (0.49)$	Q_B 1.40 (1.35)	
$\phi'' 2.46 (-)$		
$\phi' 1.84 (1.75)$	ϕ_D 1.97 (-)	
$\phi 1.02 (1.02)$		

Table 11:

Mass predictions /164/				/166/ [GeV]	
Particle	Model I	Model II	Experiment	without	with Gluonium
I = 1: π'	1.22	1.29	(1.300)	0.135	
π	0.16	0.22	0.135	1.1	
ρ'	1.34	1.74	1.60	1.54	
ρ	0.74	0.71	0.773	0.766	
I = $\frac{1}{2}$: K'	1.24	1.38	1.4	1.27	
K	0.48	0.35	0.496	0.513	
K'	1.39	1.84	...	1.6	
K	0.89	0.89	0.892	0.902	
I = 0: η	0.48	0.44	0.549	0.530	0.570
η_R	1.89	1.39	(1.275)	1.39	1.39
η'	1.11	0.99	0.957	1.07	1.05
η'_R	2.61	1.61	...	2.5	2.1
ω	0.76	0.74	0.783	0.783	
ω'	1.19	1.77	(1.67)	1.67	
ϕ	0.98	1.07	1.02	1.05	
ϕ'	1.34	1.89	1.75	1.69	

Table 12

	3D_1 : mass	width	$(^3S_1)'$: mass	width
ρ	1670	~ 300	1420	~ 500
ω	1670	~ 150	1420	~ 300
ϕ	1910	~ 150	1680	~ 300
K^*	1790	> 300	1560	> 350

Table 13

Particle	M_{exp}	M_{bag}	R_O	E_O	E_V	E_Q	E_M	E_e
ρ	0.77 ± 0.01	0.783	4.71	-0.390	0.196	0.868	0.110	0
K	0.892	0.928	4.65	-0.395	0.189	1.039	0.091	0.004
ω	0.783	0.783	4.71	-0.390	0.196	0.868	0.110	0
ϕ	1.019	1.068	4.61	-0.399	0.183	1.207	0.076	0
K	0.495	0.497	3.26	-0.564	0.065	1.407	-0.415	0.003
π	0.139	0.280	3.34	-0.549	0.070	1.222	-0.462	0

$$B^{1/4} = 0.145 \text{ GeV}, \quad Z_O = 1.84, \quad \alpha_s = 2.2, \quad m_s = 0.279 \text{ GeV}, \quad m_{ud} = 0 \text{ MeV},$$

ρ	0.77 ± 0.01	0.783	5.47	-0.356	0.170	0.860	0.110	0
K	0.892	0.925	5.37	-0.363	0.160	1.033	0.091	0.004
ω	0.783	0.783	5.47	-0.356	0.170	0.859	0.110	0
ϕ	1.019	1.063	5.26	-0.370	0.151	1.206	0.076	0
K	0.495	0.371	0.73	-2.688	0	5.863	-2.807	0.001
π	0.139	0.175	1.13	-0.796	0.022	3.717	-1.822	0

$$B^{1/4} = 0.125 \text{ GeV}, \quad Z_O = 1.95, \quad \alpha_s = 3.0, \quad m_s = 0.353 \text{ GeV}, \quad m_{ud} = 108 \text{ MeV}$$

Table 14

Particle	m_{exp} (GeV)	m_{bag} (GeV)	R (GeV^{-1})	a_s (R)
π	0.139	< 0	2.42	0.74
K	0.495	0.372	2.55	0.77
η	0.549	0.630	2.30	0.70
ρ	<u>0.770</u>	0.770	2.65	0.79
K	0.892	0.914	2.66	0.80
ϕ	1.019	1.057	2.70	0.81

$$\Lambda = 0.172 \text{ GeV}, \quad e_{TE} = 0.107, \quad e_q = 0.607, \quad m_s = 0.288 \text{ GeV};$$

$$B^{1/4} = 0.228 \text{ GeV}, \quad R_O = 2.73 \text{ GeV}^{-1}$$

Table 15

$m(O^{++})$	=	1100 - 1200 MeV
$m(O^{-+})$	=	1300 - 1400 MeV
$m(1^{-+})$	=	1400 - 1550 MeV
$m(2^{++})$	=	1550 - 1650 MeV
$m(2^{-+})$	=	1700 - 1800 MeV

Table 16 a

C_{TE}/C_{TM}	J^{PC}	$(TE)^2$ O^{++}	$(TE)^2$ 2^{++}	$(TE)(TM)$ O^{-+}	$(TE)(TM)$ 2^{-+}	$(TM)^2$ O^{++}	$(TM)^2$ 2^{++}
1/2		670	1750			1930	2640
1		1140	2120	1440	2300	1550	2300
2		1560	2470			1330	1940

Table 16b

$m(O^{++})$	=	740 ± 90 MeV
$m(O^{-+})$	=	1420 + 240 MeV - 170 MeV (!)
$m(2^{++})$	=	1620 ± 100 MeV
$m(1^{-+})$	=	1730 ± 220 MeV
$m(O^{--})$	=	2880 ± 300 MeV
$m(2^{--})$	=	3420 ± 300 MeV
$m(1^{+-})$	=	2980 ± 300 MeV

← exotic states

Table 17

Type	J^{PC}	Typical decays
(1) TE TE	$0^{++}; 2^{++}$	$\pi\pi; K\bar{K}; \eta\eta; \eta'\eta'; \rho\rho; \omega\omega; K^*\bar{K}^*; \phi\phi; \eta\eta'; \epsilon\epsilon$
(2) TE TM	$0^{-+}; 1^{-+}; 2^{-+}$	$\pi; \bar{K}\kappa; \eta S^*; \eta'\epsilon; \rho B; K^*Q; \dots$
(3) TM TM	$0^{++}; 2^{++}$	Same as (1)
(4) TE TE TM	$0^{-+}; 1^{--}; 3^{--}$	$\pi; \bar{K}\kappa; \eta'\epsilon; \pi\rho; K\bar{K}^*; \eta\omega; \eta'\phi; \omega f; \dots$
(5) TE TE TM	$0^{++}; 1^{++}; 2^{++}$	Same as (1) + $\pi\delta; \bar{K}\kappa; \eta S^*; \dots$
	$1^{+-}; 2^{+-}; 3^{+-}$	$\pi\rho; \bar{K}K^*; \eta\omega; \eta'\phi; \pi B; \bar{K}Q; \eta D; \dots$
(6) TE TE TM	$0^{-+}; 1^{-+}; 2^{-+}$	Same as (2)
	$1^{--}; 2^{--}; 3^{--}$	$\pi B; \bar{K}Q; \rho A_2; \bar{K}^*K^{**}; \omega f; df'; \dots$
(7) TM TM TM	$0^{++}; 1^{+-}; 3^{+-}$	Same as (1) + $\pi B; \bar{K}Q; \rho A_1; \rho A_2; \dots$

Table 18

Ref.	J/ ψ decay	Mark II	Crystal Ball	Mark III	DM2
377	$\gamma(K\bar{K}\pi)$	$m = 1440^{+10}_{-15}$ $\Gamma = 50^{+30}_{-20}$	1440^{+20}_{-15} $=55^{+20}_{-30}$	$1456 \pm 5 \pm 6$ $95 \pm 10 \pm 15$	$1460 \pm 3 \pm 8$ $150 \pm 12 \pm 15$
377	$\gamma(\gamma\rho)$	$m =$ $\Gamma =$	1390 ± 25 185^{+110}_{-80}	$1420 \pm 15 \pm 20$ $133 \pm 55 \pm 30$	1401 ± 18 174 ± 44
407	$\gamma(\eta\pi\pi)$	$m =$	1395*,**		
407 (prel.)	$\omega(K\bar{K}\pi)$	$m =$ $\Gamma =$ $(m_{K\bar{K}} \text{ "wider" comp. to corresp. rad. decay})$	$1444 \pm 5^{+10*}_{-20}$ $40^{+17}_{-13} \pm 10$		
	$\phi(K\bar{K}\pi)$	no iota seen			
	$\omega(\eta\pi\pi)$	$m =$ $\Gamma =$	$1421 \pm 8 \pm 10^{*,**}$ $45^{+32}_{-23} \pm 15$		
	$\phi\eta\pi\pi$	$m =$	1395??		

Table 19

*See Fig. 56, **: Cut on δ in $\eta\pi$ subsystems.

State	I^C	J^{PC}	Mass (MeV)	Γ (MeV)	S Wave $S = 2$	D Wave $S = 2$	D Wave $S = 0$
$g_T(2120)$	0^+	2^{++}	$2120^{+20}_{(-)-120}$	$300^{+150}_{(-)-50}$	$\sim 50\%$	$\sim 50\%$	
$g_T(2220)$	0^+	2^{++}	2220 ± 20 (2231)	200 ± 50 (133)	$\sim 50\%$	$\sim 50\%$	
$g_T(2360)$	0^+	2^{++}	2360 ± 20 (2293)	200 ± 50 (198)			$\sim 100\%$

Table 20

J^{PC}	Type	$C_{TE}/C_{TM} = \frac{1}{2}$		$C_{TE}/C_{TM} = 1$		$C_{TE}/C_{TM} = 2$	
		mass [GeV]	radius [GeV ⁻¹]	mass	radius	mass	radius
1^{--}	ρ/ω	1.64	6.10	1.83	6.35	2.02	6.56
	K^*	1.80	6.03	1.99	6.29	2.18	6.50
	ϕ	1.96	5.95	2.16	6.22	2.35	6.44
0^{-+}	ρ/ω	1.20	5.50	1.41	5.81	1.61	6.05
	K^*	1.41	5.42	1.62	5.74	1.82	5.98
	ϕ	1.61	5.34	1.82	5.67	2.03	5.91
1^{-+}	ρ/ω	1.41	5.80	1.61	6.05	1.80	6.31
	K^*	1.59	5.73	1.80	5.98	1.99	6.25
	ϕ	1.78	5.66	1.99	5.90	2.18	6.18
2^{-+}	ρ/ω	1.79	6.30	1.97	6.51	2.15	6.70
	K^*	1.94	6.24	2.13	6.45	2.31	6.65
	ϕ	2.09	6.17	2.28	6.39	2.47	6.59

Table 21

Figure Captions

- Fig. 1 Schematic comparison between QED- and QCD-graphs
- Fig. 2 SU(3) relation $3 \otimes \bar{3} = 8 \oplus 1$
- Fig. 3 a-d P wave multiplets (see the appendix for the new nomenclature
3 e-h D wave multiplets acc. to refs 422,423)
- Fig. 4 $\sigma(\gamma p \rightarrow \phi p)$ versus E_γ /271/. See /55,257/ for supplemented data points
- Fig. 5 $m_{\pi^+\pi^-\pi^0\pi^0}$ from the reaction $\gamma p \rightarrow \pi^+\pi^-\pi^0\pi^0 p$ /32/
- Fig. 6 a-c Fit results for spin-parity determination of (4π) -states in
 $\gamma p \rightarrow \pi^+\pi^-\pi^0\pi^0 p$ /32/
- Fig. 7 a $\sigma(e^+e^- \rightarrow \pi^+\pi^-\pi^0\pi^0)$ versus $\sqrt{s}$ /35+ref. incl. there/.
- Fig. 7 b $|F_\pi|^2$ from $e^+e^- \rightarrow \pi^+\pi^-$ versus $\sqrt{s}$
Full points: ref. 403, open circles: ref. 404
- Fig. 8 Masses and widths of structures reported in the four and two pion systems. The length of the bars indicate the Breit-Wigner widths
/35+ref. incl. there/
- Fig. 9 Schematic drawing of the OZI-suppressed decay $\phi \rightarrow \pi^+\pi^-\pi^0$
- Fig. 10 a-d $\sigma(e^+e^- \rightarrow \omega\pi^+\pi^-)$ versus $\sqrt{s}$ /53/ (a)
 $\sigma(e^+e^- \rightarrow K^+K^-)$ (b)
 $\sigma(e^+e^- \rightarrow K_S^0 K_L^0)$ (c)
 $\sigma(e^+e^- \rightarrow K_S^0 K^\pm \pi^\mp)$ (d)
Full lines are explained in the text
- Fig. 11 Dalitzplot of $K_S^0 K^\pm \pi^\mp$ /38/
- Fig. 12 Schematic drawing of $\rho' \leftrightarrow \phi'$ interference
- Fig. 13 a $\sigma(e^+e^- \rightarrow \pi^+\pi^-\pi^0)$ versus $\sqrt{s}$ /57/
13 b $\sigma(e^+e^- \rightarrow \pi^+\pi^-\pi^0)$ versus $\sqrt{s}$ /58/
- Fig. 14 a $m_{\pi^+\pi^-\pi^0}$ from $\gamma p \rightarrow \pi^+\pi^-\pi^0 p$ /59/
14 b $m_{\pi^+\pi^-\pi^0}$ from $\gamma p \rightarrow \pi^+\pi^-\pi^0 p$ /60/

Fig. 15 1^- intensity versus $m_{\pi^+\pi^-\pi^0}$ /60/

Fig. 16 a $m_{K^+K^-}$ from $\gamma p \rightarrow K^+K^-p$ /63/

16 b $m_{K^+K^-}$ from $\gamma p \rightarrow K^+K^-p$ /64/

Fig. 17 a $m_{K^+K^-\pi^0}$ from $\gamma p \rightarrow K^+K^-\pi^0p$ /65/

17 b $m_{K_S^0K^\pm\pi^\mp}$ from $\gamma p \rightarrow K^0K^\pm\pi^\mp p$ /65/, shaded area: 1 $K\pi$ combination compatible with a $K^*(890)$

Fig. 18 as 17b, but $K^\pm\pi^\mp$ compatible with K^{*0} , the curve is explained in the text /65/

Fig. 19 Constraints on X and Y : (a) and (b) from $J/\psi \rightarrow VP$ decays, (c) and (d) from radiative transitions. The shaded areas correspond to $\pm 1\sigma$ error on X and Y , and the shaded line in (b) corresponds to the 90%-confidence-level upper limit on $X_{\eta'}$. In (c), the two vertical bands correspond to the two possible solutions for $\Gamma(\rho^0 \rightarrow \pi\pi)$. $\eta_1(\eta_8)$ shows the quark content expected for a pure singlet η' (octet η). The 10° pseudoscalar-mixing-angle predictions are also shown /69d/.

Fig. 20 $m_{3\pi}$ from $\pi^-p \rightarrow \pi^-\pi^-\pi^+p$: $(0^-0^+\epsilon S)$ -intensity /15/

Fig. 21 Relative 0^- -intensities versus $m_{K\pi\pi}$ /74/

a) $0^-P\ 0^+(K^*\pi)$ intensity

b) $0^-P\ 0^+(\rho K)$ "

c) $0^-S\ 0^+(\epsilon K)$ "

Terminology: $J^P_{LM}{}^\eta(X_1, X_2)$

where J^P spin parity

L orbital momentum between X_1, X_2

M magnetic substate of $K\pi\pi$ -system

η naturality of parity exchange

Fig. 22 0^- intensity versus $m_{\eta\pi^+\pi^-}$

a) $0^-S\ \delta\pi$ b) $0^-S\ \epsilon\eta$ /75/

c) $0^-S\ \delta\pi$ d) $0^-S\ \epsilon\eta$ /76/

Fig. 23 $m_{\omega\pi^\pm}$ from $\gamma p \rightarrow \omega\pi^\pm X$ /79/

Fig. 24 a $(01+\rho S0^-)$ -intensity versus $\pi^+\pi^-\pi^0$ /81/, no nukleon spin flip

24 b as a, but nukleon spin flip

Terminology: IJPX₁LM_n - see fig. 21

Curves: full Breit-Wigner + background, dashed Breit-Wigner only

24 c $1^+(\rho-\pi)$ intensity versus $m_{\pi^+\pi^-\pi^0}$ from $\gamma p \rightarrow \pi^+\pi^-\pi^0 p$ /61/

Fig. 25 $m_{K\pi\pi}$ from $K^-p \rightarrow K^-\pi^-\pi^+p$ /74/

Fig. 26 a-e 1^+ intensities versus $m_{K\pi\pi}$ /74/

a) $1^+S0^+(K^*\pi)$

b) $1^+S0^+(K\rho)$

c) $1^+P0^+(\kappa\pi)$

d) $1^+P0^+(\epsilon K)$

e) $1^+D0^+(K^*\pi)$

Fig. 27 Non resonant threshold mechanisms possibly influencing A_1 -production

Fig. 28 a-d $m_{3\pi}$ from $\pi^-p \rightarrow \pi^-\pi^+\pi^-p$ /70/ for 4 different t intervals.

Full line: Breit-Wigner background, dashed line: background only (see text)

Fig. 29 a $m_{K^\pm K_S^0 \pi^\mp}$ from $\pi^-p \rightarrow K_S^0 K^\pm \pi^\mp n$ /89/

29 b as 29a /92/

Fig. 30 a 1^+ intensity versus $M_{K\bar{K}\pi}$ /89/

30 b $m_{K_S^0 \pi^\pm}^2$ from $M_{K\bar{K}\pi}$. Full / dashed lines are expectations for $C(KK\pi) = +/-$ /89/

30 c $m_{K_S^0 K^\pm}^2$ /91/

30 d $J^{PG} = 0^{--}$ waves with combined K^*K^- and $\delta\pi$ -contributions versus $m_{KK\pi}$ /94/

Fig. 31 $m_{K^-\pi^+}$ from $K^-p \rightarrow K^-\pi^+n$ /97/

Fig. 32 $m_{\pi^+\pi^-\pi^0}$ from $\gamma\gamma \rightarrow \pi^+\pi^-\pi^0$ /99/

Fig. 33 $m_{K^+K^-}$ from $\gamma\gamma \rightarrow K^+K^-$ /100/

Fig. 34 $m_{\eta\pi^\pm}$ from $\gamma p \rightarrow \eta\pi^\pm X$ /79/

- Fig. 35 a $m_{\pi^+\pi^-}$ from $e^+e^- \rightarrow \psi \rightarrow \pi^+\pi^-X$ /111/
 35 b $m_{\pi^+\pi^-}$ from $\pi^+p \rightarrow \pi_{fast}^+(\pi^+\pi^-)p$ /112/
 Fig. 36 a $(K^0\bar{K}^0)$ -S-wave intensity from $\pi^-p \rightarrow K_S^0\bar{K}_S^0n$ /290/
 36 b $(\pi^+\pi^-)$ -S-wave-intensity from $\pi^-p \rightarrow \pi^+\pi^-n$ /291/
 Fig. 37 Argand diagram of elastic $(K^-\pi^+)$ -S-wave from $K^-p \rightarrow K^-\pi^+n$ /114/
 Fig. 38 Particle spin J versus M^2 (Chew-Frautschi-plot)
 Fig. 39 a $m_{\pi^-\pi^0}$ from $\pi^-p \rightarrow \pi^-\pi^0p$ /120/
 39 b Moments t_6^0, t_6^1, t_6^2 versus $m_{\pi^-\pi^0}$ /120/
 Fig. 40 a $m_{\omega\pi^-}$ from $\pi^-p \rightarrow \omega\pi^-p$ after cut to s-channel resonances /120/
 40 b $m_{A_2\pi^0}$ from $\pi^-p \rightarrow A_2\pi^0p$ /120/
 Fig. 41 a $m_{\pi^+\pi^-\pi^0}$ from $\pi^+p \rightarrow \pi^+\pi^-\pi^0\Lambda^{++}$ /121/. Shaded area corresponds to the $O(3^-F\rho\pi)$ intensity. Notation: $I(J^P L X_1 X_2)$
 41 b $m_{\pi^+\pi^-\omega}$ from $K^-p \rightarrow \pi^+\pi^-\omega\Lambda$ /122/
 Fig. 42 a Partial wave intensity $3^- \rightarrow (K^*\pi)$ -F wave versus $m_{K\pi\pi}$ /123/
 42 b,c Argand diagram of $(K^-\pi^+)$ elastic scattering partial wave amplitudes for F- and G-waves /114/
 Fig. 43 a $m_{K^+K^-}$ from $K^-p \rightarrow K^+K^- \Lambda/\Sigma^0$ /341/
 43 b $(2^-f\pi S)$ -wave intensity versus $m_{3\pi}$ from $\pi^-p \rightarrow \pi^-\pi^+\pi^-p$ /120/
 Fig. 44 a $m_{\pi^0\pi^0}$ from $\pi^-p \rightarrow \pi^0\pi^0n$ /125/
 44 b $m_{\pi^0\pi^0}$ after cut on $\cos \theta_{GJ}$ /125/ (see text)
 Fig. 45 $n = 2$ levels of positronium /129/. The numbers in parantheses represent the $O(\alpha^4)$ (fine structure) and the $O(\alpha^5)$ (radiative corrections to the Bohr energy level (1/8 Ry)). Thus for example these corrections in 2^1P_1 are - 3536 - and - 3 MHz resp.

$$n = 1: \quad {}^3S_1 \xrightarrow{14 \mu\text{sec}} 3\gamma \quad C = -1$$

$$\quad \quad {}^1S_0 \xrightarrow{0.12 \text{ nsec}} 2\gamma \quad C = +1$$

Fig. 46 a-c Positronium levels (a) (simplified presentation, obey cut in scale) compared to charmonium (b) and bottonium (c) levels.

Nomenclature for multipol-radiation

$$x \rightarrow y + \gamma \quad \text{MJ: } P_x \cdot P_y \cdot (-1)^{\Delta J} = -1$$

$$\text{EJ: } P_x \cdot P_y \cdot (-1)^{\Delta J} = +1$$

Fig. 47 Term scheme of the three dimensional harmonic oscillator

Fig. 48 Several potentials with respect to $q-\bar{q}$ distance r /141/

Fig. 49 a-c $a \equiv \text{Fig. 47a}$ compared to u,d-levels (b) and strangeonium levels (c)

Fig. 50 Comparison of model predictions and experiment for u,d,s quarks /168/
a) isoscalars, b) isodoublets, c) isotriplets

Fig. 51 graphic illustration of the universality of meson dynamics from the Υ to the π showing the splitting of 3P_2 and 1S_0 from 3S_1 in the $b\bar{b}$, $c\bar{c}$, $s\bar{s}$, $u\bar{s}$ and $u\bar{d}$ families

Fig. 52 $f(M^2)$ versus M^2 /178/

Fig. 53 $q\bar{q}$ -potential from lattice calculations at various β -values /371/

Fig. 54 a $K_S^0 K^\pm \pi^\mp$ invariant mass distribution for events with an observed γ from $\psi \rightarrow \gamma K_S^0 K^\pm \pi^\mp$

54 b as a but without an observed γ , shaded regions are restricted by $m_{K\bar{K}} < 1050 \text{ MeV}$ /93/

54 c $K^+ K^- \pi^0$ invariant mass distribution for events consistent with the hypothesis $\psi \rightarrow \gamma K^+ K^- \pi^0$. Shaded region is restricted by $m_{K\bar{K}} < 1125 \text{ MeV}$ /204/

54 d Intensity of the $0^-(\delta\pi)$ wave versus $m_{K^+ K^- \pi^0}$ /204/

Fig. 55 Invariant mass distributions for (a) $K\bar{K}\pi$, (b) $\rho\rho$, (c) $\omega\omega$, and (d) $\gamma\rho$ (Mark III) corrected for relative efficiencies (arb. units, 0.025 GeV bins). The curves represent the results of the coupled channel analysis as described in /377, 379/

Fig. 56 a-c Invariant mass distributions from J/ψ decays /407/

a) $m_{\eta\pi^+\pi^-}$ from $J/\psi \rightarrow \gamma \eta\pi^+\pi^-$, ($\eta \rightarrow \gamma\gamma$ or $\eta \rightarrow \pi^+\pi^-\pi^0$)

$|m_{\eta\pi^\pm} - m_\delta| < 50 \text{ MeV}$ (see also /377/)

b) $m_{K\bar{K}\pi}$ from $J/\psi \rightarrow \omega K\bar{K}\pi$

c) $m_{\eta\pi^+\pi^-}$ from $J/\psi \rightarrow \omega \eta\pi^+\pi^-$; $|m_{\eta\pi^\pm} - m_\delta| < 50 \text{ MeV}$

Fig. 57 $\eta\eta$ invariant mass distribution for events consistent with $J/\psi \rightarrow \gamma\eta\eta$.

The solid curve represents a fit to the mass distribution, dashed curve represents background /212/

Fig. 58 $M_{K^+K^-}$ from Mark III with a fit including $f'(1515)$, θ and $\xi(2220)$ /213/

Fig. 59 Radiative decays of J/ψ into $K\bar{K}$ -states /384/

a) $m_{K^+K^-}$, b) $m_{K_S^0 K_S^0}$ (Mark III)

c) $m_{K^+K^-}$, d) $m_{K_S^0 K_S^0}$ (DM 2)

Fig. 60 a The reaction $\pi^- p \rightarrow \phi\phi n$ mediated by two gluon exchanges

60 b Partial wave decomposition of the $\phi\phi$ -spectrum by the BNL/CCNY collaboration showing the three g_T states /224/

Fig. 61 S-wave intensity versus $M_{\eta\eta}$ /229/

Fig. 62 a $m_{\omega\pi^\pm}$ for three different $\omega\pi^+\pi^-$ -mass-windows as indicated.

Full line: background only, dashed line: background incl. $B^\pm$ -signal

62 b Mass distribution of ' $B\pi$ '-systems after background subtraction /413/

Fig. 63 Predictions for a $qq-\bar{q}\bar{q}$ nonet with $J^{PC} = 0^{++}$ /69/

Fig. 1

Fig. 2

Fig. 3(a-d)

(see the appendix for the new nomenclature acc. to refs 422,423)

Fig. 3(e-h)

(see the appendix for the new nomenclature acc. to refs 422,423)

Fig. 4

Fig. 5:

Fig. 6

Fig. 7

Fig. 8:

Fig. 9

Fig. 10 a-d:

Fig. 11

$$g(1600) \sim d\bar{d} - u\bar{u}; \quad \phi(1650) \sim s\bar{s}$$

Fig. 12

Fig. 13a

Fig. 13b

Fig. 14a

Fig. 14b

Fig. 15:

Fig. 16a

Fig. 16b

Fig. 17 a

Fig. 17 b

Fig. 18

Fig. 19

Fig. 20:

Fig. 21

Fig. 22

Fig. 23

Fig. 24

Fig. 25

Fig. 26

Fig. 27

Fig. 28

Fig. 29

Fig. 30

Fig. 31

Fig. 32

Fig. 33:

Fig. 34

Fig. 35

Fig. 36a

Fig. 36b

Fig. 37

Fig. 38

Fig. 39 a

Fig. 39 b

Fig. 40 a

Fig. 40 b

Fig. 41

Fig. 42

Fig. 43

Fig. 44 a

Fig. 44 b

Fig. 45

Fig. 46

Fig. 49

Fig. 50

Fig. 51

Fig. 52

Fig 53

Fig 54

Fig. 55

Fig. 56

Fig. 57

Fig. 58

Fig. 59

Fig. 60 a:

Fig. 60 b

Fig 6 1

Fig 6 3

Fig. 62

APPENDIX

In the 1986⁴²²⁾ edition of the Review of Particle Properties the authors have decided to change nomenclature for mesonic states. See Ref. 423 for a justification. The correlation between the names used in this review and the new ones are (all particles not mentioned remain as they are):

	Old name	New name	J^{PC}	I^G	Section
Non strange	S(975)	f (975)	0^{++}	0^+	2.2.6
	δ (980)	a_0 (980)	0^{++}	1^-	"
	H(1190)	h_1 (1190)	1^{+-}	0^-	2.2.3
	B(1235)	b_1 (1235)	1^{+-}	1^+	"
	g_S (1240)	f_0 (1240)	0^{++}	0^+	2.2.7
	f (1270)	f_2 (1270)	2^{++}	0^+	2.2.5
	A_1 (1270)	a_1 (1270)	1^{++}	1^-	2.2.4
	η_R (1275)	η (1275)	0^{-+}	0^+	2.2.2
	D(1285)	f_1 (1285)	1^{++}	0^+	2.2.4
	ε (1300)	f_0 (1300)	0^{++}	0^+	2.2.6
	A_2 (1320)	a_2 (1320)	2^{++}	1^-	2.2.5
	E(1420)	f_1 (1420)	1^{++}	0^+	2.2.7, 3.2
	i (1440)	η (1440)	0^{-+}	0^+	"
	f' (1525)	f'_2 (1525)	2^{++}	1^-	2.2.5
	D(1530)	f_1 (1530)	1^{++}	0^+	2.2.4
	G(1590)	f_0 (1590)	0^{++}	0^+	3.2
	ω (1670)	ω_3 (1670)	3^{--}	0^-	2.2.7
	A_3 (1680)	π_2 (1680)	2^{-+}	1^-	2.2.7

./.

$\rho(1690)$	$\rho_3(1690)$	3^{--}	1^+	2.2.7
$\eta(1700)$	$X(1700)$	?	?	3.2
$\theta(1690)$	$f_2(1720)$	2^{++}	0^+	3.2
$S(1730)$	$f_0(1730)$	0^{++}	0^+	2.2.6
$f(1810)$	$f_2(1810)$	2^{++}	0^+	2.2.5
$\phi(1850)$	$\phi_J(1850)$	?	?	2.1.7
$S(1935)$	$X(1935)$	?	?	4
$h(2030)$	$f_4(2030)$	4^{++}	0^+	2.2.7
$\delta(2040)$	$a_4(2040)$	4^{++}	1^-	"
$A(2050)$	$a_3(2050)$	3^{++}	1^-	
$A(2100)$	$\pi_2(2100)$	2^{-+}	1^+	
$\epsilon(2150)$	$f_2(2150)$	2^{++}	0^+	
$\xi(2220)$	$X(2220)$	$0, 2^{++}$	0	3.2
$\xi_T(\dots)$	$f_2(\dots)$	2^{++}	0^+	3.2
$\rho(2250)$	$\rho_3(2250)$	3^{--}	1^+	2.2.7
$r(2510)$	$f_6(2510)$	6^{++}	0^+	"
$\chi(3415)$	$\chi_0(3415)$	0^{++}	0^+	2.3.1
$\chi(3510)$	$\chi_1(3510)$	1^{++}	0^+	"
$\chi(3555)$	$\chi_2(3555)$	2^{++}	0^+	"
$\chi_b(9860)$	$\chi_{b0}(9860)$	$0^{++}(?)$	?	2.3.1
$\chi_b(9895)$	$\chi_{b1}(9895)$	$1^{++}(?)$	?	"
$\chi_b(9915)$	$\chi_{b2}(9915)$	$2^{++}(?)$	?	"

analog for $\chi_b(10235, 10255, 10270)$

Strange
mesons

$Q(1280)$	$K_1(1280)$	1^+	$1/2$	2.2.3
$\kappa(1350)$	$K_0^*(1350)$	0^+	"	2.2.6
$Q(1400)$	$K_1(1400)$	1^+	"	2.2.4
$K^*(1430)$	$K_2^*(1430)$	2^+	"	2.2.5
$K(1400)$	$K(1460)$	0^-	"	2.2.2
$K_2(1580)$	$L(1580)$	2^-	"	
$K_2(1770)$	$L(1770)$	2^-	"	2.2.3
$K^*(1780)$	$K_3^*(1780)$	3^-	"	2.2.7
$K^*(2060)$	$K_4^*(2060)$	4^+	"	"