CALT-68-1376
DOE RESEARCH AND
DEVELOPMENT REPORT

Covariant Heterotic Strings and Odd Self-Dual Lattices*

Wolfgang Lerche
CERN, Geneva, Switzerland

Dieter Lüst**
California Institute of Technology, Pasadena, CA 91125

ABSTRACT

We investigate the implications of modular invariance for covariantly formulated heterotic strings. It is shown that modular invariant heterotic strings are characterized by odd self-dual Lorentzian lattices which include charges of the bosonized superconformal ghosts. The proof of modular invariance involves the anomaly in the ghost number current in a crucial way.

August 20, 1986

* Work supported in part by the U.S. Department of Energy under contract DEAC 03-81-ER40050.

** Supported by a scholarship of the Max Kade Foundation, New York. Address after October 1, 1986: Max-Planck-Institut für Physik and Astrophysik, Föhringer Ring 6, D-8000 München 40.

1. Introduction

The requirement of modular invariance of one loop amplitudes turned out to be a crucial ingredient for the formulation of the supersymmetric $SO(32)$ and $E_8 \times E_8$ heterotic string theories [1] – the momenta of the compactified left-moving bosons must lie on an even self-dual lattice. In the fermionic description, modular invariance forces to sum over different spin structures with well defined coefficients [2]. This is equivalent to performing appropriate $(-1)^F$ projections. By modifying the definition of the fermion number projection many new heterotic string theories have been obtained [2–4]. These include the tachyon free model with $O(16) \times O(16)$ gauge group [3,4] and theories with tachyons and/or wrong space-time statistics.

In the bosonic description, the changed boundary conditions imply that the momenta of the corresponding bosons lie on twisted, i.e., shifted lattices [4]. However the new 16-dimensional lattices are no longer self-dual, because left- and right-moving sectors are mixed in a nontrivial way. The implications of modular invariance for such twisted lattices have not been absolutely clarified.

In this letter we will show that in covariant bosonization language [5] for all heterotic string theories the momentum vectors of the left-moving compactified bosons combined with the weights of the bosonized fermions plus the charges of the bosonized superconformal ghosts of the right-moving spinning string [6] must lie on an odd self-dual lattice with Lorentzian signature. Then a modular invariant one-loop vacuum amplitude obtains and the correct averaging over different spin structures in the fermionic description is automatically ensured. We speculate that all ten-dimensional heterotic string theories as well as their toroidal compactifications are different vacua of some 14-dimensional heterotic string theory.

2. Partition Functions

In the superconformal gauge, the Lagrangian for all heterotic strings is [1]

$L = L_L + L_R$ with

$$\begin{aligned} L_L &= \frac{1}{2}(\partial_+ X_L^\mu)^2 + \frac{1}{2}(\partial_+ X_L^I)^2 + b^{++}\partial_+ c_+ \\ L_R &= \frac{1}{2}(\partial_- X_R^\mu)^2 + \frac{i}{2}\psi^\mu \not{\partial}_- \psi_\mu + b^{--}\partial_- c_- + \beta^- \partial_- \gamma, \end{aligned} \quad (1)$$

$\mu = 1, \dots, 10, I = 1, \dots, 16, \partial_+ = \partial_{\bar{z}}, \partial_- = \partial_z; b, c(\beta, \gamma)$ represent the ghosts for fixing (super) reparametrization invariance. The conformal weights of the fields are such that the central charges in the left and right Virasoro algebras add up to zero separately.

States are generated by covariant vertex operators. For details see ref. [5]. It is convenient to bosonize the fermions and spinor ghosts according to $J^{2i, 2i+1} \equiv i:\psi^{2i}\psi^{2i+1}: = \partial_- H^i, i = 1, \dots, 5$ and $j \equiv \beta\gamma = \partial_- \phi$. Then vertex operators associated with the fermionic part of theory can be represented (omitting momentum dependent factors) as

$$V_\lambda = e^{\lambda_L^I X_L^I(\bar{z})} e^{\lambda_R^A H^A(z)} e^{q\phi(z)} c_\lambda, \quad \lambda = (\lambda_L, \lambda_R, q). \quad (2)$$

The vector λ cannot be arbitrary. Since it multiplies compactified coordinates, λ belongs to some lattice so that operator products

$$V_\lambda(z) V_{\lambda'}(0) = \bar{z}^{\lambda_L \lambda'_L} z^{\lambda_R \lambda'_R - qq'} V_{\lambda+\lambda'}(0) c(\lambda, \lambda') + \dots \quad (3)$$

represent shifts on that lattice. (c_λ are appropriate cocycle factors yielding a projective representation $c_\lambda c_{\lambda'} = c_{\lambda+\lambda'} c(\lambda, \lambda')$ of the translation group.) The second requirement is that the operator product algebra is local, i.e., has not branch cuts. This means, the vectors λ belong to an integral Lorentzian lattice $\Gamma_{16,5,1}$

$$\langle \lambda, \lambda \rangle \in \mathbf{Z},$$

with metric $((-)^{16}; (+)^5, -)$.

For instance, in the $SO(32)$ heterotic string the massless states are generated by $\lambda_L = (\pm 1 \dots \pm 1 \dots)$, $\lambda_R = (\dots \pm 1 \dots)$, $q = -1$ or $\lambda_R = (\pm \frac{1}{2} \pm \frac{1}{2} \pm \frac{1}{2} \pm \frac{1}{2} \pm \frac{1}{2})$, $q = -\frac{1}{2}$ (even number of $-$ signs). More generally, operators with ghost charge $q \in \mathbf{Z}$ generate Neveu-Schwarz (NS) states, while operators with $q \in \mathbf{Z} + \frac{1}{2}$ ("spin fields") generate the Ramond (R) sector.

It was noted in [7] that in the case of the right-moving superstring $\Gamma_{0;5,1}$ is in fact odd self-dual, and it is easily checked that for the $E_8 \times E_8$, $SO(32)$ and $O(16) \times O(16)$ heterotic strings $\Gamma_{16;5,1}$ is also odd self-dual. We will prove in the following that modular invariance of the one-loop vacuum amplitude Z ,

$$Z = \int d\tau d\bar{\tau} (\text{Im } \tau)^{-2} \chi(\tau, \bar{\tau})$$

$$\chi(\tau, \bar{\tau}) = \text{Tr}(e^{-2\pi i \bar{\tau} \bar{L}_0} e^{2\pi i \tau L_0} \Phi), \quad (4)$$

for all heterotic strings and their toroidal (untwisted) compactifications generally requires a Lorentzian odd self-dual lattice. (The argumentation can easily be extended also to type II string theories [8]; in (4), Φ denotes some phase factor.)

In order to derive the partition function, we will understand that the X^μ variables are in light cone gauge, while we retain the β, γ ghost system. Accordingly, a state associated with "weight" $\lambda = (\lambda_L, \lambda_R, q)$ and oscillator number N_L, N_R has mass

$$\begin{aligned}
m^2 &= \frac{1}{2} m_L^2 + \frac{1}{2} m_R^2 \\
\frac{1}{8} m_L^2 &= \frac{1}{2} \lambda_L^2 + N_L - 1 \\
\frac{1}{8} m_R^2 &= \frac{1}{2} \lambda_R^2 - \frac{1}{2} q^2 - q + N_R - 1
\end{aligned} \tag{5}$$

The peculiar linear dependence on the ghost charge will prove essential for modular invariance. It arises due to the anholomorphic anomaly in the ghost number current [5]

$$\partial_{+j} = \frac{1}{8} Q R^{(2)} \sqrt{g}, \tag{6}$$

where $R^{(2)} \sqrt{g}$ is the two-dimensional scalar curvature density, and for the ghost system in question, $Q = 2$. The mass formula (5) follows from the anomalous operator product of the energy momentum tensor with $e^{q\phi}$ which tells that $e^{q\phi}$ has conformal weight $-\frac{1}{2} q(q + Q)$ [5].

To prove modular invariance of Z we consider for a while a specific odd self-dual lattice $\Gamma_{16;5,1}$ generated by "roots" $(\pm 1 \dots \pm 1)$ and "spinor weights" $(\pm \frac{1}{2} \dots \pm \frac{1}{2})$ (odd number of $-$ signs). This corresponds to a nonsupersymmetric heterotic string theory with gauge group $SO(32)$ and 32 tachyons. But later we will allow the dimension D of space-time to vary and consider general lattices $\Gamma_{12+d;d+1,1}$ which are certain rotations of

$$\Gamma_{12} \oplus (d + 1) \Gamma_{1,1} \tag{7}$$

where Γ_{12} is the odd 12-dimensional self-dual Euclidean lattice [9] generated by $(\pm 1 \dots \pm 1 \dots)$ and $(\pm \frac{1}{2} \dots \pm \frac{1}{2})$ (even number of $-$ signs), and $\Gamma_{1,1} = \text{diag}(1, -1)$. Therefore, we keep in the following the dependence on $d = 14 - D$ explicitly, but

understand $d = 4$ since $D = 10$ for the moment.

To get an idea of the partition function $\chi_{12+d;d+1,1}(\tau, \bar{\tau})$, consider first the lattice function of a right-moving superstring alone ($w = (\lambda_R, q)$)

$$\theta_{d+1,1}(\tau) = \sum_{w \in \Gamma_{d+1,1}} e^{2\pi i \tau (\frac{1}{2}\lambda_R^2 - \frac{1}{2}q^2 - q - 1)} e^{2\pi i q}. \quad (8)$$

We include a phase factor such that Ramond ($q = \frac{1}{2} + \mathbb{Z}$) and Neveu-Schwarz ($q \in \mathbb{Z}$) states have opposite statistics.

Clearly the sum extends over arbitrarily high ghost numbers, and thus cannot represent a physical partition function. Namely, as it is well known [5], certain states which differ by one unit of ghost charge are in fact equivalent. More specifically, vertex operators differing by integral units of q can generate the same physical state, but in vacua with different Bose ghost sea charges $|q\rangle = \exp q \phi |0\rangle$; the transition of one ghost vacuum $|q\rangle$ to another one $|q+1\rangle$ is physically irrelevant and called "picture changing." In other words, the sum in eq. (8) extends over all possible sets of equivalent "pictures".

To determine the amount of overcounting in (8), we note that the picture changing operation has the generic form (ξ is an auxiliary ghost) [5]

$$V_{q+1} = [Q_{BRS}, \xi V_q] \quad (9)$$

Since

$$[Q_{BRS}, \xi] = T_F e^\phi + \dots, \quad T_F = (\partial_- X^\mu) \psi_\mu \quad (10)$$

we can rewrite (9) using the bosonized form of ψ^μ (e_i is the i th unit vector) as

$$(\partial_- X^{\pm i})(e^{\pm H^i} e^\phi)(z) V_{w=(\lambda, q)} = (z^{\pm \lambda_i - q} \partial_- X^{\pm i}) V_{w'=(\lambda \pm e_i, q+1)}(z) + \dots \quad (11)$$

In words, picture changing bosonically amounts to adding a light-like vector

$(\pm 1 \dots, 1)$ to w and to multiplying by appropriate $\alpha^\pm$ oscillators in a way such that the conformal weights of the vertex operators are the same.

To proceed we remark that the physical light cone partition function is obtained from (8) by just putting the last two entries of w to fixed values (then the first four entries do not belong any more to an integral lattice):

$$w \rightarrow w_{l.c.} = \begin{cases} (\dots, 0, -1) & \text{NS-sector} \\ (\dots, \frac{1}{2}, -\frac{1}{2}) & \text{R-sector.} \end{cases} \quad (12)$$

This means all the other states with different two last entries are equivalent to states of the form (12), i.e., can deviate only by sums of light-like vectors. Therefore we rewrite the general weight in a convenient form

$$w = w_0 + p + \Delta, \quad (13)$$

where

$$w_0 = \begin{cases} (10000-1) & \text{NS-sector} \\ (\frac{1}{2} \frac{1}{2} \frac{1}{2} \frac{1}{2} \frac{1}{2} - \frac{1}{2}) & \text{R-sector} \end{cases}$$

and p denotes a "four-dimensional" root vector $(n_1, n_2, n_3, n_4, 0, 0)$ ($n_i \in \mathbf{Z}$, $\sum n_i = \text{even}$) such that $w_0 + p$ corresponds to $w_{l.c.}$ in (12). Δ is the sum of two light-like vectors

$$\begin{aligned} \Delta &= m\Delta_1 + n\Delta_2 \quad m, n \in \mathbf{Z} \\ \Delta_1 &+ (000011) \\ \Delta_2 &+ (000101). \end{aligned} \quad (14)$$

Substituting (13) into (8) we obtain, defining a vector $e = (000001)$

$$\begin{aligned}
\theta_{d+1,1}(\tau) &\equiv \sum_{w \in \Gamma_{d+1,1}} e^{2\pi i \tau (\frac{1}{2} \langle w, w \rangle + \langle w, e \rangle - 1)} e^{2\pi i \langle w, e \rangle} \\
&= \sum_{w_{l.c.}} e^{2\pi i \tau (\frac{1}{2} \langle w_{l.c.}, w_{l.c.} \rangle + \langle w_{l.c.}, e \rangle - 1)} e^{2\pi i \langle w_{l.c.}, e \rangle} \sum_{\Delta} e^{2\pi i \tau (\frac{1}{2} \langle \Delta, \Delta \rangle + \langle w_{l.c.} + e, \Delta \rangle)}
\end{aligned}
\tag{15}$$

Now, as can be easily checked, upon shifting the sum over Δ the last term does in fact not depend on $w_{l.c.}$. On the other hand, the first term is precisely the physical light cone partition function. Therefore, to obtain the physical partition function from (8) one has just to divide it by

$$\theta_{1,1}(\tau) = \sum_{\Delta} e^{2\pi i \tau \frac{1}{2} \langle \Delta, \Delta \rangle}, \tag{16}$$

which is the same as the partition function of the two-dimensional Lorentzian even self-dual lattice $\Gamma_{1,1}$.

To return to the full theory, we combine (8) with the left-moving part to obtain as complete partition function

$$\begin{aligned}
\chi_{12+d;d+1,1}(\tau, \bar{\tau}) &= \frac{(\text{Im } \tau)^{-6 + \frac{1}{2}d}}{\bar{\eta}^{24}(\bar{\tau}) \eta^{12}(\tau) \theta_{1,1}(\tau)} \\
&\cdot \sum_{\lambda = (\lambda_L, \lambda_R, q) \in \Gamma_{12+d;d+1,1}} e^{-2\pi i \bar{\tau} (\frac{1}{2} \lambda_L^2)} e^{2\pi i \tau (\frac{1}{2} \lambda_R^2 - \frac{1}{2} q^2 - q - \frac{1}{2})} e^{2\pi i q},
\end{aligned}
\tag{17}$$

with the Dedekind eta function

$$\eta(\tau) = e^{i\pi\tau/12} \prod_{n=1}^{\infty} (1 - e^{2\pi i n \tau}). \tag{18}$$

3. Modular invariance.

First let us show that Z is invariant under modular transformations $\tau \rightarrow \tau + 1$. Usually, character functions of odd lattices are not invariant under this transformation. However, the situation is different here due to the linear anomaly term in the mass formula. Since η^{12} changes sign under $\tau \rightarrow \tau + 1$, while $\theta_{1,1}$ remains invariant, we require that the lattice sum also changes sign. This implies, states with $q = (\text{half-})$ integer corresponding to (R) NS-states have to be associated with (odd) even points on the lattice

$$\frac{1}{2}\langle\lambda, \lambda\rangle + q \in \mathbb{Z}. \quad (19)$$

It is easily seen that all states of our lattice do obey this condition. In fact, it is closely related to the condition of invariance under σ -reparametrizations, i.e.,

$$m_L^2 = m_R^2 \quad \text{or} \quad \frac{1}{2}\langle\lambda, \lambda\rangle + q = N_L - N_R,$$

which all physical states do obey.

Now check $\tau \rightarrow -\frac{1}{\tau}$; we will follow the approach of [10]. To prove invariance, consider first again only the right moving part $\Gamma_{d+1,1}$ of our lattice, corresponding to the spinning string: $w = (\lambda_R, q)$. Defining a vector $e = (0 \cdots 0 1)$ we can write the right moving lattice part as

$$\begin{aligned} \chi_{d+1,1}(\tau) &= \eta^{-d}(\tau) \theta_{1,1}^{-1}(\tau) \sum_{w \in \Gamma_{d+1,1}} e^{2\pi i \tau (\frac{1}{2}\langle w, w \rangle + \langle w, e \rangle - \frac{1}{2})} e^{2\pi i \langle w, e \rangle} \\ &= \eta^{-d}(\tau) \theta_{1,1}^{-1}(\tau) \sum_{w_0} e^{2\pi i \tau (\frac{1}{2}\langle w_0, w_0 \rangle + \langle w_0, e \rangle - \frac{1}{2})} \\ &\quad \cdot e^{2\pi i \langle w_0, e \rangle} \theta_{\Gamma_{d+1,1}}[e + \tau(w_0 + e)/\tau]. \end{aligned} \quad (20)$$

w_0 are minimal weights with label the distinct conjugacy classes, Γ^τ denotes the

"root" lattice and

$$\theta_{\Gamma}(\nu/\tau) = \sum_{w \in \Gamma} e^{2\pi i \tau (\frac{1}{2} \langle w, w \rangle)} e^{2\pi i \langle w, \nu \rangle}. \quad (21)$$

From

$$\theta_{\Gamma_{d+1,1}}(\nu/\tau) = \frac{1}{\sqrt{Z_R}} \frac{1}{(-i\tau)^{1+\frac{1}{2}d}} e^{-\frac{i\pi}{\tau} \langle \nu, \nu \rangle} \theta_{\Gamma_{d+1,1}}(-\frac{\nu}{\tau} / -\frac{1}{\tau}), \quad (22)$$

with Γ^* denoting the dual lattice of Γ and Z_R the order of the center, we find upon resubstitution of (21, 22) into (20) and using

$$\begin{aligned} \eta^{-d}(\tau) &= (-i\tau)^{d/2} \eta^{-d}(-\frac{1}{\tau}) \\ \theta_{1,1}^{-1}(\tau) &= -i\tau \theta_{1,1}^{-1}(-\frac{1}{\tau}) \end{aligned} \quad (23)$$

that

$$\chi_{d+1,1}(\tau) = \eta^{-d}(-\frac{1}{\tau}) \theta_{1,1}^{-1}(-\frac{1}{\tau}) \sum_{w \in \Gamma^*} e^{-\frac{2\pi i}{\tau} (\frac{1}{2} \langle w, w \rangle + \langle w, e \rangle - \frac{1}{2})} e^{-2\pi i \langle w, e \rangle} S(w). \quad (24)$$

A priori, the sum over w is unrestricted, i.e., over all conjugacy classes; however, the factor

$$S(w) = \frac{1}{\sqrt{Z_R}} \sum_{w_0} e^{-2\pi i \langle w_0, w \rangle} \quad (25)$$

equals unity only for weights of a self-dual lattice [10]. Hence $\chi(\tau) = \chi(-\frac{1}{\tau})$ requires that the ghost lattice of the superstring is self-dual.

Combining now with the left-moving part, one obtains

$$\begin{aligned} \chi_{12+d;d+1,1}(\tau, \bar{\tau}) &= (\text{Im}(-\frac{1}{\tau}))^{-6+\frac{1}{2}d} \bar{\eta}^{-24}(-\frac{1}{\bar{\tau}}) \eta^{-12}(-\frac{1}{\tau}) \theta_{1,1}^{-1}(-\frac{1}{\tau}) \\ &\cdot \sum_{\lambda=(\lambda_L, \lambda_R, q)} e^{\frac{2\pi i}{\tau}(\frac{1}{2}\lambda_L^2)} e^{-\frac{2\pi i}{\tau}(\frac{1}{2}\lambda_R^2 - \frac{1}{2}q^2 - q - \frac{1}{2})} e^{-2\pi i q} S(\lambda) \\ S(\lambda) &= \frac{1}{\sqrt{Z_L} \sqrt{Z_R}} \sum_{\lambda_0} e^{-2\pi i \langle \lambda_0, \lambda \rangle}. \end{aligned} \quad (26)$$

Again, $\chi(\tau, \bar{\tau}) = \chi(-\frac{1}{\tau}, -\frac{1}{\bar{\tau}})$ follows precisely if $S(\lambda) = 1$, i.e., λ belongs to a self-dual Lorentzian lattice $\Gamma_{12+d;d+1,1}$. This is true for our specific lattice with $d = 4$ considered so far.

A crucial observation now is that all odd self-dual Lorentzian lattices of given rank are isomorphic to each other [9], i.e., they are related by Lorentz type rotations which leave the factor S invariant. However, to count the number of physically distinct modular invariant heterotic string theories one has to be careful. This is because the physical states in a given picture are characterized by a fixed ghost charge and longitudinal component (cf. (12)). The specific charge assignments are crucial for invariance of the vacuum amplitude under $\tau \rightarrow \tau + 1$. It is at the moment not clear as to what extent one is allowed to rotate the last two entries of lattice vectors without spoiling modular invariance. A subset of definitely modular invariant heterotic theories are characterized by rotations of $\Gamma_{12} \oplus (d+1)\Gamma_{1,1}$ belonging to $SO(12+d, d)$, acting on all but the last two entries of a lattice vector. To count physically distinct theories, we have to subtract the number of rotations which leave the mass formula invariant. Thus, in $D = 14 - d$ dimensions there are at least

$$N = \dim \frac{SO(12+d, d)}{SO(12+d) \times SO(d)} = d(d+12) \quad (27)$$

parameters characterizing distinct modular invariant heterotic theories. For

$D = 10$ one has $N = 64$. (It is amusing that the same number is obtained if one classifies all modular invariant sums over spin structures in the fermionic formulation, if one groups the fermions into sets of eight [11].) It would be interesting to check whether there are new tachyon-free models. We should emphasize, however, that we focussed only on one-loop modular invariance. It is conceivable that multiloop modular invariance restricts the infinity of one-loop invariant theories.

The situation is very similar to toroidal compactification of a given ten-dimensional heterotic string, as discussed in [12,13]. In $D < 10$ we obtain toroidal (untwisted) compactifications of all ten-dimensional theories. Of course, many of these have tachyons and/or fields with wrong statistics just like many in ten dimensions. Hence, it may be difficult to identify internal and space-time degrees of freedom. However, the possibility of having chiral $N = 1$ supersymmetric theories in four dimensions cannot be excluded. (For the same reasons as such theories do exist in ten dimensions.) For $10 < D \leq 14$ we obtain modular invariant partition functions not considered before.

Recently, lattice rotations applied to ten-dimensional strings have been interpreted [14] as due to vacuum expectation values of internal g_{MN} , B_{MN} and A_M fields. Similarly, we are lead to speculate that the lattices corresponding to all heterotic strings in a given dimension are related by certain vacuum expectation values. More specifically, we conjecture that all heterotic strings are (at least formally) vacuum states of a unique 14-dimensional ($d = 0$) theory. In fact, following [14] the number of independent background parameters precisely coincides with N in eq. (27).

This fourteen-dimensional theory has tachyons and fields with wrong statistics. However from the two-dimensional point of view, it may be a perfect superconformal field theory. The conformal anomalies cancel due to the

presence of the ghosts. These, of course, cannot have any more the interpretation as arising due to fixing of sheet super-reparametrization invariance.

Acknowledgments

We thank A. Kosteletzky, O. Lechtenfeld, R. Rohm, S. Samuel and S. Watamura for discussions and A. Schellekens for valuable comments. W.L. would like to express his gratitude to the California Institute of Technology for its kind hospitality during the completion of this work.

References

1. D.J. Gross, J. Harvey, E. Martinec and R. Rohm, Phys. Rev. Lett. 54 (1985) 502; Nucl. Phys. B256 (1985) 253; Nucl. Phys. B267 (1986) 75.
2. N. Seiberg and E. Witten, Spin structures in string theory, IAS preprint, February 1986.
3. L. Alvarez-Gaumé, P. Ginsparg, G. Moore and C. Vafa, Phys. Lett. 171B (1986) 155.
4. L.J. Dixon and J.A. Harvey, String theories in ten dimensions without space-time supersymmetry, Princeton preprint, February 1986.
5. D. Friedan, E. Martinec and S. Shenker, Phys. Lett. 160B (1985) 55; Nucl. Phys. B271 (1986) 93.
6. P. Ramond, Phys. Rev. D3 (1971) 2415;
A. Neveu and J.H. Schwarz, Nucl. Phys. B31 (1971) 86.
7. J. Cohn, D. Friedan, Z. Qui and S. Shenker, Covariant quantization of supersymmetric string theories: the spinor field of the Ramond-Neveu-Schwarz model, Chicago preprint EFI 85-90 (1986).
8. M.B. Green and J.H. Schwarz, Nucl. Phys. B181 (1981) 502;
M.B. Green, J.H. Schwarz and L. Brink, Nucl. Phys. B198 (1982) 474.
9. P. Goddard and D. Olive, in Vertex operators in mathematics and physics, J. Lepowsky et al., Springer-Verlag (1984).
10. A. Casher, F. Englert, H. Nicolai and A. Taormina, Phys. Lett. 162B (1985) 121.
11. Communicated by L. Alvarez-Gaumé.
12. K.S. Narain, Phys. Lett. 169B (1986) 41.

13. V. P. Nair, A. Shapere, A. Shominger and F. Wilczek, Compactification of twisted heterotic string, preprint NSF-ITP-86-58 (1986).
14. K.S. Narain, M.H. Sarmadi and E. Witten, A note on toroidal compactification of heterotic string theory, preprint (1986).