Nuclear binding and quark confinement

G. Chanfray

Institut de Physique Nucléaire (et IN2P3) Université Claude Bernard Lyon 1, 69622 Villeurbanne Cédex, France

H. J. Pirner

Centre Européen de Recherches Nucléaire, TH Division, 1211 Geneva 23, Switzerland

(Received 16 June 1986)

We present a schematic solution of the color dielectric model for the nucleon and demonstrate how the nucleon can be bound in nuclear matter. The gluon field surrounding the bound nucleon is obtained from the linearized field equations of motion in the color dielectric. This schematic model of the baryon allows a comparison of the neutron and proton mass in the nucleus. We calculate the Nolen-Schiffer anomaly for several light nuclei.

I. INTRODUCTION

The quest for an understanding of hadrons in quantum chromodynamics (QCD) does not stop outside of nuclear physics.¹ It is an obvious question to ask whether the forces binding nucleons in nuclei can be related to the forces which confine the quarks in nucleons.

Experimentally,^{2,3} the European Muon Collaboration (EMC) effect hints that the structure function of a bound nucleon has a softer valence quark momentum distribution than the free nucleon. It is tempting⁴ to associate this spread of the quark wave function with nuclear binding. The present paper is based on this idea. It starts from the color dielectric Lagrangian,⁴⁻⁶ the form of which is well defined by QCD. The fields of this Lagrangian are quark fields, a colored vector field, and a color neutral scalar field. The latter two are related to averages over gluon configurations of the original Lagrangian. They contain the short wavelength fluctuations of the gluon system. The color neutral field χ may be associated with a 0^{++} glueball bound state. Since it renormalizes the product of gluon field tensors in the form $\chi^4 F_{\mu\nu} F^{\mu\nu}$, it can be interpreted as a color dielectric field. As the parameters of the Lagrangian are not yet directly determined from QCD, we have to fix them as much as possible from phenomenology or other calculations. The effective Lagrangian preserves gauge invariance for the new fields, which implies complicated couplings between the gauge degrees of freedom. Thus one may wonder what has been gained. The advantage of the new formulation, however, is that a classical description of the resulting field equations should be sufficient. In the subsequent work we will limit ourselves to the linearized gluon field equations. For the nucleon itself there are no contributions from the three and four gluon couplings on the classical level. In many respects our theoretical analysis is similar to the MIT-bag model with perturbative gluons.⁷ The bag boundary condition is obtained in the limit of a vanishing color dielectric field outside of the bag and $\chi = \bar{\chi}$ inside. The close similarity of this schematic model

to the MIT model has led us to study in the same approach the behavior of a nucleon bound in the nucleus. We will tacitly assume that the nucleon properties do not change drastically in the nucleus. Measurements like the mapping of the proton $3s_{1/2}$ orbital by electron scattering experiments on ^{206}Pb and ^{205}Tl (Ref. 8) provide strong evidence for a nucleonic picture of nuclei at nuclear matter density. One should note, however, that there is a 30% reduction of strength of this orbital observed. This experimental observation may point to an increase in size of the nucleon bound deeply in the nucleus. Very naturally, the question arises: How does confinement work on a nuclear scale?

We will show how the schematic color dielectric model allows the quarks to spread over a larger volume by changing the χ vacuum in the nucleus. This change is *calculable* as a function of the nuclear average density. Various other quantities change in the nucleus, when the valence quark wave function and the color dielectric field are changed. The tunneling of the quarks outside of the region with $\chi = \bar{\chi}$ into the intranucleonic vacuum produces a new source for gluons outside of the free nucleon bag. Using the linearized equations of the gluon field in the dielectricum, we can calculate the contribution of the glue to the bound nucleon self energy. Since the nuclear quark wave functions depend on the current quark mass, down and up quarks will behave differently. We will investigate the sensitivity of the neutron-proton mass difference to nuclear binding. The discrepancy⁹ between the Coulomb corrected mass differences of mirror nuclei and the free neutron to proton mass difference will be analyzed for valence nucleons.

The outline of the paper is as follows: In Sec. II we present the schematic solution of the color dielectric Lagrangian to the original QCD Lagrangian. In Sec. III we calculate the color dielectric field in the nucleus. Section IV is devoted to a solution of the gluon Green's function in the medium and the calculation of the nucleon gluon self energy. In Sec. V we give the dependence of the effective nucleon mass on nuclear density. In Sec. VI we

analyze the Nolen-Schiffer anomaly. In Sec. VII we give a summary.

II. THE COLOR DIELECTRIC MODEL

The color dielectric model^{5,6} proposes a local order parameter $\chi(\mathbf{x})$ which is related to the dielectric constant $\epsilon = \chi^4$ and can be calculated by averages over the gluon fields in a small volume L^4 :

$$\chi_L(x_0) = \frac{1}{N} \text{Tr} \mathcal{A}_L P \exp \left[i \oint_{x_0} A_\mu dx^\mu \right]. \quad (2.1)$$

Here, A_μ are the conventional gauge fields. The average of Wilson loops through x_0 is taken in the volume L^4 . The trace is performed in $SU(N)$ space and $N=3$ is the number of color. A well defined method of calculating the field configuration entering the dielectric field χ is given by the lattice regularization of QCD.^{10,12} The gluon part of the lattice action in $SU(3)$ is given as

$$S_{\text{QCD}} = \frac{6}{g^2} \sum_{\text{plaquettes}} \left(1 - \frac{1}{3} \text{Re Tr} U_{ij} U_{jk} U_{kl} U_{li} \right), \quad (2.2)$$

with the link variables

$$U_{ij} = \exp \left[i \int_{x_i}^{x_j} A_\mu dx^\mu \right]. \quad (2.3)$$

For a lattice with lattice constant a , the averaging volume of gluon configurations is chosen to be $L = 2a$ at the beginning.

From the moments

$$\langle \chi_L^n \rangle = \int dU e^{-S_{\text{QCD}}(U)} \chi^n(U) \quad (2.4)$$

and

$$\langle (\partial_\mu \chi_L)^2 \rangle = \int dU e^{-S_{\text{QCD}}(U)} (\partial_\mu \chi)^2$$

given by the integral over all link variables U_{ij} , one can reconstruct the effective action of the χ field given by a kinetic term and a fourth order polynomial potential $V(\chi)$:

$$\mathcal{L}_L(\chi) = \frac{1}{2} \sigma_V^2 (\partial_\mu \chi)^2 - V(\chi). \quad (2.5)$$

Note after the first averaging step $\langle \chi_L \rangle \neq 0$, so one has to restart the procedure by calculating the moments:

$$\langle \chi_{2L}^n \rangle = \int d\chi dU \exp \left[-S_{\text{QCD}}(\chi U) - \int \mathcal{L}_L(\chi) d^4x \right] \chi_{2L}^n \quad (2.6)$$

for a larger cell of size $(2L)^4 = (4a)^4$.

Hopefully, this procedure converges fast when $1/nL \rightarrow \Lambda_{\text{QCD}} \approx 0.2 \text{ GeV}$. The effective potential $V(\chi)$ determines pure glue properties of gauge theory like the glueball mass or the transition temperature from the hadronic to the plasma phase.¹¹ The continuum limit of the final effective action contains, besides the color dielectric neutral scalar field χ , also eight coarse grained gauge fields B_μ^i which are defined in the large L limit as

$$B_\mu(x) = \lim_{x' \rightarrow x} \partial'_\mu \mathcal{A}_L P \int_x^{x'} A_\mu dx^\mu. \quad (2.7)$$

The quarks can be included in the effective Lagrangian

using local gauge invariance. The lattice action for quarks

$$S_q = \frac{1}{2} i a^3 \sum_{i,j} \bar{\psi}_i \gamma^\mu e_\mu U_{ij} \psi_j \quad (2.8)$$

becomes after averaging, using Eqs. (2.1) and (2.5),

$$S_q = i a^3 \sum_i \bar{\psi}_i \gamma^\mu e_\mu (K^i + i a e^\lambda B_\lambda^i) \times (\psi_i + e^{K a} \partial_K \psi_i) \simeq i \int d^4x \bar{\psi}(x) \chi(x) \left[\gamma^\mu \partial_\mu + i \gamma^\mu \frac{B_\mu}{\chi} \right] \psi(x), \quad (2.9)$$

where on the last line, $K(\mathbf{x})$, a 3×3 matrix in $SU(3)$, has been approximated by $(1/N_c) \text{tr} K = \chi$. This same approximation has been made before in Eq. (2.6) because the average of a sum of unitary $SU(3)$ matrices is not a constant times a unitary matrix. We use the covariant derivative ($D_\mu = \partial_\mu + i B_\mu / \chi$) to write the total effective Lagrangian in the form

$$\mathcal{L} = \mathcal{L}(\chi) + i \chi(x) \bar{\psi} \gamma^\mu \left[\partial_\mu + i \frac{B_\mu}{\chi} \right] \psi - m_q \bar{\psi} \psi - \frac{\chi^4 F_{\mu\nu} F^{\mu\nu}}{4g^2}, \quad (2.10)$$

with

$$F_{\mu\nu} = \left[\partial_\mu + i \frac{B_\mu}{\chi} \right] \frac{B_\nu}{\chi} - \left[\partial_\nu + i \frac{B_\nu}{\chi} \right] \frac{B_\mu}{\chi}, \quad (2.11)$$

and m_q as current mass.

When the scale size L becomes large, it may be necessary to introduce effective fields which are composites of quark operators like $\pi(\mathbf{x}) = \bar{\psi} \tau \gamma_5 \psi$ and $\sigma(\mathbf{x}) = \bar{\psi}(\mathbf{x}) \psi(\mathbf{x})$. This approach has been considered in Ref. 13. The success of quenched QCD calculations for hadron spectroscopy attributes a less important role to these fields. From Eq. (2.4) we see that the effective color dielectric constant is related to χ as $\epsilon = \chi^\alpha$, with $\alpha = 4$. This form comes from the QCD action $S_{\text{QCD}}(\chi U)$ [cf. Eq. (2.2)]. The effective action $S_{\text{QCD}}(\chi U)$ sums over all plaquettes with 4-links U decorated by χ at each site. To simplify the quark dynamics one can introduce a new effective fermion field $\tilde{\psi}$ by rescaling the original field ψ ,

$$\tilde{\psi} = \sqrt{\chi} \psi, \quad (2.12)$$

which moves the χ dependence from the kinetic term in Eq. (2.10) to the mass term. Also, new gauge fields A_μ^a can be defined according to

$$A_\mu^a = \frac{1}{g} \frac{B_\mu^a}{\chi}, \quad (2.13)$$

where a is the color index and $A_\mu = \sum_a \lambda^a A_\mu^a$. As a first approximation we shall consider the A_μ 's as perturbative gluon fields dropping the nonlinear parts of the Lagrangian. Phenomenologically, we associate the lowest constituent glue ball state, with 0^{++} quantum numbers, with the χ fields in the effective Lagrangian $\mathcal{L}(\chi)$,

$$\begin{aligned}\mathcal{L}(\chi) &= \frac{1}{2}\sigma_V^2(\partial_\mu\chi)^2 - V(\chi), \\ V(\bar{\chi}) &= B, \\ V(\chi) &= \frac{1}{2}m_{\text{GB}}^2\sigma_V^2\chi^2 \text{ for } \chi \text{ small.}\end{aligned}\quad (2.14)$$

The parameter σ_V characterizes the stiffness of the χ field against spatial variations. In the following we will investigate the schematic model,⁴ which is obtained in the limit $\sigma_V \rightarrow 0$ and $m_{\text{GB}}^2\sigma_V^2 = \text{const.}$ [A first attempt of lattice calculation of parameters occurring in (2.14) is described in Ref. 27.] In this limit the surface energy of the bag is neglected. With the additional assumption that $\chi = \bar{\chi}$ inside the bag and $\chi = 0$ outside, we find a nucleon which is similar to the MIT nucleon. Numerical investigations of the color dielectric model have been made with realistic parameters of the Lagrangian $\mathcal{L}(\chi)$ in Ref. 14. In the schematic model we obtain a sharp bag cavity for the nucleon, the gluon energy of which remains well defined as long as we assume $\epsilon = \chi^\alpha$, with $\alpha < 2$. Of course, this coefficient is not the correct one obtained from the averaging over lattice configurations, but we neglected the finite mass of the glueball which would have smeared out the surface. Summarizing the approximations of the schematic model,

(a) drop non-Abelian gauge field, i.e., the colored tensor field is $G_{\mu\nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a$,

(b) drop derivative terms in $\mathcal{L}(\chi)$, i.e., there is no surface energy, contrary to Ref. 14, where numerical methods similar to Bickelboller *et al.* are used,¹⁷

(c) include the gluon fields A_μ^a perturbatively,

we treat the effective Lagrangian:

$$\begin{aligned}\mathcal{L} &= \frac{i}{2}\bar{\psi}\gamma^\mu\overleftrightarrow{\partial}_\mu\psi - g\bar{\psi}\gamma^\mu A_\mu^a \frac{\lambda^a}{2}\psi - \frac{m_q}{\chi}\bar{\psi}\psi \\ &\quad - V(\chi) - \frac{\chi^\alpha}{4}G_{\mu\nu}^a G^{a\mu\nu}, \\ G_{\mu\nu}^a &= \partial_\mu A_\nu^a - \partial_\nu A_\mu^a,\end{aligned}\quad (2.15)$$

with $\alpha < 2$, and $V(\bar{\chi}) = B$ for $\chi = \bar{\chi}$ and $V(\chi) = \frac{1}{2}m_{\text{GB}}^2\sigma_V^2\chi^2$ around $\chi = 0$. As parameters of the Lagrangian, we assume $m_q/\bar{\chi} = 30$ MeV, which is slightly larger than the value $\frac{1}{2}(m_u + m_d) = \bar{m}_q(Q^2)$ obtained from a chiral analysis at $Q^2 = 1$ GeV² and an effective QCD coupling constant $\bar{\alpha}_S = q^2/4\pi(\bar{\chi}^\alpha) = 2.6$. As B , we use $B = 34.3$ MeV/fm³, which gives a good fit to the nucleon and delta masses ($m_N = 939$ MeV and $m_\Delta = 1231$ MeV). In the following we will drop the special reference to $\bar{\chi}$ and set $\bar{\chi} = 1$, i.e., replace $m_q/\bar{\chi}$ by m_q and $\bar{\alpha}_S$ by α_S . One should note that, in the Lagrangian (2.15), the χ -quark coupling $[(m_q/\chi)\bar{\psi}\psi]$ is highly nontrivial and radically differs from the model of Friedberg and Lee. For the baryon masses, we obtain the same results as the MIT model, namely

$$\begin{bmatrix} M_N(R) \\ M_\Delta(R) \end{bmatrix} = 3\alpha_{\text{c.m.}}\omega(R) + \frac{0.35\alpha_S}{R} + \frac{4}{3}\pi R^3 B. \quad (2.16)$$

Here, $\alpha_{\text{c.m.}} = 0.8$ is the c.m. correction parameter and $\omega(R) = \Omega_0/R + m_q/[2(\Omega_0 - 1)]$.

The nucleon and delta bag radii are $R_N = 1.163$ fm and $R_\Delta = 1.28$ fm. These are obtained by minimizing Eq. (2.16) with respect to R . The rms radius equals 0.85 fm. It is useful to define an average mass $\bar{M}$ of the nucleon and delta,

$$\bar{M}(R) = 3\alpha_{\text{c.m.}}\omega(R) + \frac{4}{3}\pi R^3 B. \quad (2.17)$$

Minimizing (2.17) gives $\bar{M}(R) = 1051.8$ MeV and $\bar{R} = 1.22$ fm.

III. THE NUCLEON BOUND IN THE NUCLEAR COLOR DIELECTRIC FIELD NEGLECTING GLUON VECTOR FIELD

Let us now consider a nucleon in nuclear matter with density $\rho = (\frac{4}{3}\pi r_0^3)^{-1}$. At normal nuclear matter density, $r_0 = 1.14$ fm and $\rho = 0.16$ fm⁻³. This value has to be compared with the bag radius $R_N = 1.16$ fm that a free nucleon has in the schematic model. The similarity of the two numbers is quite astonishing. Indeed, one arrives at a very simple explanation of the saturation of nuclear matter in the following way: Consider the energy difference between a free nucleon with average mass $\bar{M}(\bar{R})$ at $\bar{R} = 1.22$ fm [Eq. (2.17)] and three quarks which are confined in a bag of size r_0 , the size available to a nucleon in nuclear matter:

$$E/N = \bar{M}(r_0) - \bar{M}(\bar{R}). \quad (3.1)$$

This energy difference is positive below nuclear matter density because of the bag pressure, which increases the energy of the bag inflated to size $r_0 > \bar{R}$ (see Fig. 1). Above nuclear matter density the kinetic energy of the quarks confined to a region $r_0 < \bar{R}$ gives a repulsive energy. Considering the schematic model defined in Eq. (2.15), we will see that a slightly modified color dielectric field in nuclear matter can lead to nuclear binding.

At low densities it is more advantageous for the nucleon to keep its free nucleon radius $R \approx R_N$ and leave part of the space without quarks, i.e., with a color dielectric medium where $\chi_N \approx 0$. The exact value of χ_N and R are determined by the nuclear density and the parameters of the effective Lagrangian of Eq. (2.15).

FIG. 1. Energy difference between a system of three quarks confined in a bag of size r_0 and a nucleon with radius $\bar{R}$ vs density ρ [Eq. (3.1)].

The Dirac equation for quarks in a nucleon surrounded by a dielectric medium χ_N has the form:

$$-i\boldsymbol{\alpha}\cdot\nabla\psi(\mathbf{r})+\beta M(\mathbf{r})\psi(\mathbf{r})=\omega\psi(\mathbf{r}),$$

with

$$M(\mathbf{r})=\frac{m_q}{\chi(\mathbf{r})}=\begin{cases} m_q, & |\mathbf{r}| < R \\ m_0=\frac{m_q}{\chi_N}, & |\mathbf{r}| > R. \end{cases} \quad (3.2)$$

Note the radius R can be different from the free nucleon radius. The height of the barrier $m_0=m_q/\chi_N$ determines how much the quark wave functions will leak out of the bag. The solutions of Eq. (3.2) have the form

$$\psi(\mathbf{r})=\frac{1}{\sqrt{4\pi}}\begin{bmatrix} u(r) \\ iv(r)\boldsymbol{\sigma}\cdot\hat{\mathbf{r}} \end{bmatrix},$$

with the radial wave functions

$$\begin{bmatrix} u(r) \\ v(r) \end{bmatrix}=N\times\begin{bmatrix} j_0(kr)\Theta(R-r)+Rj_0(kR)\frac{e^{-K(r-R)}}{r}\Theta(r-R) \\ \left[\frac{\omega-m_q}{\omega+m_q}\right]^{1/2}j_1(kr)\Theta(R-r)+\left[\frac{m_0-\omega}{m_0+\omega}\right]^{1/2}Rj_0(kR)\frac{e^{-K(r-R)}}{r}\left[1+\frac{1}{KR}\right]\Theta(r-R) \end{bmatrix}, \quad (3.3)$$

and the normalization N in such a way that

$$\int_0^\infty r^2 dr [u^2(r)+v^2(r)]=1. \quad (3.4)$$

The wave vectors $k=(\omega^2-m_q^2)^{1/2}$ and $K=(m_0^2-\omega^2)^{1/2}$ are related to the frequency ω , which is determined by the condition that the ratio of u/v is continuous at $r=R$, i.e.,

$$\left[\frac{\omega-m_q}{\omega+m_q}\right]^{1/2}\frac{j_1(kR)}{j_0(kR)}=\left[\frac{m_0-\omega}{m_0+\omega}\right]^{1/2}\left[1+\frac{1}{KR}\right]. \quad (3.5)$$

In the limit of a vanishing color dielectric constant χ_N in the nuclear medium, we recover the wave function and frequency of the free nucleon. The transcendental equation (3.5) has an approximate solution for $\chi/m_q R < 0.4$,

$$\omega=\frac{\Omega_0}{R}\left[1+\frac{m_q R}{2\Omega_0(\Omega_0-1)}-\frac{\chi_N}{2m_q R}\right]. \quad (3.6)$$

Besides the current quark mass correction of $\approx m_q/2$ for relativistic quarks, we see that the frequency of each cavity mode is lowered by the dielectric field $\chi_N \neq 0$ outside of the bag. Independently of the χ dynamics, one can estimate $\chi_N/(2m_q R)=1/(2m_0 R)$ to be of the order of 5–10 % if the change of confinement in the nuclear medium is relevant for the binding of the nucleon. This corresponds to a barrier $m_0 \approx 2-1$ GeV. We attach a rather important value to this parameter, since it indicates the color gap in nuclei. Excitations with energy $\hbar\omega > m_0$ should be unaffected by confinement; they should extend over the whole nucleus.

Starting from the single cell wave function of Eq. (3.3), we have been able to construct quark wave functions for “nucleons” on a cubic lattice with periodicity $2r_0$ (cf. the Appendix). One finds corrections to the energy ω of order

$$\frac{1}{R}\left[\frac{\chi_N}{m_q R}\right]^2 e^{-4m_0(r_0-R)},$$

which is a small correction to the single cell approximation. In fact, a crystal of nucleons is not a good approxi-

mation to nuclear matter, which requires a homogeneous baryon density. But up to now we have not been able to combine our wave functions of Eq. (3.3) with that requirement. Progress along the lines of Ref. 15 may be possible. For this work we will keep at moderate densities the picture of a single nucleon moving in a dielectric background.

In the approximation without gluons, we obtain, for the effective mass of the bound nucleon, the sum of the quark energies of Eq. (3.6) and the energy of the dielectric field χ_N ,

$$\tilde{M}_0(\chi_N, \rho, R)=E_q(\chi_N, R)+E_\chi(\chi_N, \rho, R), \quad (3.7)$$

where χ_N is actually also a function of ρ and R .

The energies E_q and E_χ are obtained from Eq. (3.6) and from the Lagrangian (2.15),

$$E_q=\frac{3\alpha_{c.m.}}{R}\omega(\chi_N, R), \quad (3.8)$$

$$E_\chi=\int_0^{r_0} d\mathbf{r} V(\chi) \\ =\frac{4}{3}\pi R^3 B + \frac{2}{3}\pi m_{GB}^2 \sigma_V^2 (r_0^3 - R^3) \chi_N^2. \quad (3.9)$$

The first term represents the repulsive bag energy. The second term adds to it the excitation energy of the vacuum in the intranucleonic region, i.e., outside of the nucleon bag with radius R and inside of the Wigner-Seitz cell with radius r_0 . By integrating the χ equation of motion in the region $R < r < r_0$, we obtain the average value χ_N , namely

$$\int_R^{r_0} d\mathbf{r} V'(\chi_N)=\int_R^{r_0} d\mathbf{r} \frac{m_q}{\chi_N^2} \sum_{i=1}^3 \langle \bar{\psi}_i(\mathbf{r}) \psi_i(\mathbf{r}) \rangle. \quad (3.10)$$

The right hand side is calculated to leading order in χ and $\eta=e^{-2m_0(r_0-R)}$. The contribution of other quarks is of order $\eta(\chi/m_q R)^2$ to $(m_q/\chi)\langle \bar{\psi}\psi \rangle$ in the lattice model. The equation for χ_N then becomes

$$\begin{aligned} & \frac{4\pi}{3} m_{\text{GB}}^2 \sigma_V^2 (r_0^3 - R^3) \chi_N^2 \\ &= \frac{3\alpha_{\text{c.m.}} \Omega_0}{2} \frac{\chi_N}{m_q R} \left[1 - \left[1 + \frac{1-R/r_0}{\Omega_0 - 1} \right] e^{-2m_0(r_0 - R)} \right]. \end{aligned} \quad (3.11)$$

Note that χ_N remains finite in the limit $R \rightarrow r_0$. The value of χ_N as a function of density $\rho(r_0)$ and R allows one to calculate the binding potential W of the nucleon,

$$W_0(\rho, R) = \tilde{M}_0(\chi_N(\rho, R), \rho, R) - \bar{M}(\bar{R}). \quad (3.12)$$

At the moment, without α_S corrections, the nucleon mass is $\bar{m}(\bar{R})$. To lowest order in $\chi/m_q R$, the rms radius of the valence quarks is given as

$$\langle r^2 \rangle^{1/2} = 0.73R \left[1 + 1.27 \frac{\chi_N}{m_q R} \right]. \quad (3.13)$$

Later we will correlate the binding of the nucleon with an increase of its effective rms radius.

IV. THE GLUON VECTOR FIELDS OF A BOUND NUCLEON

We now turn to the problem of the gluon contribution to the properties of the bound nucleon. We shall use the linearized version of the theory and treat the gluon fields as a perturbation. The source of the colored gluon fields are the quark currents with color index a ($t^a = \lambda^a/2$) and time-space components,

$$\rho_a(\mathbf{r}) = g \psi^\dagger(\mathbf{r}) t^a \psi(\mathbf{r}), \quad (4.1a)$$

$$\mathbf{J}_a(\mathbf{r}) = g \psi^\dagger(\mathbf{r}) \boldsymbol{\alpha} t^a \psi(\mathbf{r}). \quad (4.1b)$$

Here, $\psi(\mathbf{r})$ is the unperturbed wave function of the bound quarks given by (3.3) and (3.4). If the up-down mass difference is neglected, the quark wave functions of the three quarks are identical. It follows that the total quark color density vanishes because, for a color singlet, one has

$$\sum_{i=1}^3 t^a(i) |N\rangle = 0. \quad (4.2)$$

Thus the chromoelectric field does not contribute to the energy. From Noether's theorem, we can obtain the Maxwell equations satisfied by the colored vector fields $\mathbf{A}_a(\mathbf{r})$. In the static limit these equations reduce to

$$\nabla \cdot [\epsilon(\mathbf{r}) \nabla \times \mathbf{A}_a(\mathbf{r})] = 0. \quad (4.3)$$

Here, $\epsilon(\mathbf{r}) = \chi^a(\mathbf{r})$ plays the role of a color dielectric constant. In our two-phase picture $\epsilon(\mathbf{r})$ can have two values depending on $r < R$ or $r > R$:

$$\epsilon(\mathbf{r}) \equiv \chi^a(\mathbf{r}) = \Theta(R - r) + \epsilon_N \Theta(r - R). \quad (4.4)$$

$\mathbf{J}(\mathbf{r})$ is the transverse current which satisfies $\nabla \cdot \mathbf{J}_a(\mathbf{r}) = 0$.

It is particularly convenient¹⁶ to use a Coulomb gauge for which

$$\nabla \cdot [\epsilon(\mathbf{r}) \mathbf{A}_a(\mathbf{r})] = 0. \quad (4.5)$$

Equation (4.3) then becomes

$$-\nabla^2 [\epsilon(\mathbf{r}) \mathbf{A}_a(\mathbf{r})] + \nabla \times [\mathbf{A}_a(\mathbf{r}) \times \nabla \epsilon(\mathbf{r})] = 0. \quad (4.6)$$

Outside of the bag $\epsilon_N = \chi_N^\alpha$ is very small. As already pointed out in Ref. 17, α has to be chosen not too large in order to recover simply the MIT bag model when χ_N goes to zero. For that reason we will use in the following $\alpha = 1$ as originally proposed by Lee.¹⁸ This choice corrects for the oversimplification of sharp boundary conditions. It should be noted that this choice is not universally applicable, as can be seen from the flux-tube problem where $\alpha > 1$ is necessary.¹¹ The vector potential can be calculated using a Green's function method:

$$\mathbf{A}_a^i(\mathbf{r}) = \int d\mathbf{r}' G^{ij}(\mathbf{r}, \mathbf{r}') J_a^j(\mathbf{r}'). \quad (4.7)$$

The tensor Green's function $G^{ij}(\mathbf{r}, \mathbf{r}')$ is determined by solving the equation

$$-\epsilon(\mathbf{r}) \nabla_r^2 G^{ij}(\mathbf{r}, \mathbf{r}') = \delta_{\text{tr}}^{ij}(\mathbf{r} - \mathbf{r}') \quad (r \neq R). \quad (4.8)$$

on the right hand side of Eq. (4.8), we have introduced a transverse delta function defined by

$$\delta_{\text{tr}}^{ij}(\mathbf{r} - \mathbf{r}') = \int \frac{d\mathbf{k}}{(2\pi)^3} (\delta^{ij} - \hat{\mathbf{k}}^i \hat{\mathbf{k}}^j) e^{i\mathbf{k} \cdot (\mathbf{r} - \mathbf{r}')}. \quad (4.9)$$

In this way it is unnecessary to make the current transverse because the Green's function in Eq. (4.7) projects on its transverse part automatically.

Equation (4.8) has to be solved with the following boundary conditions:

$$\hat{\mathbf{r}}^i G^{ij}(\mathbf{r}, \mathbf{r}')|_{r=R} = 0, \quad (4.10a)$$

$$\hat{\mathbf{r}}^i \epsilon_{ijk} \frac{\partial}{\partial r_j} G^{kl}(\mathbf{r}, \mathbf{r}') \Big|_{r=R} \quad (\text{continuous}), \quad (4.10b)$$

$$\epsilon_{ijk} \epsilon_{klm} \hat{\mathbf{r}}_j \epsilon(\mathbf{r}) \frac{\partial}{\partial r_l} G^{mn}(\mathbf{r}, \mathbf{r}') \Big|_{r=R} \quad (\text{continuous}), \quad (4.10c)$$

where summation over repeated indices is understood. One may notice that these equations make $A_a^j(\mathbf{r})$ satisfy the gauge condition (4.5) and guarantee the continuity of the normal component of the magnetic B field and of the tangential component of the H field ($\mathbf{H} = \epsilon \mathbf{B}$).

Equation (4.8) can be solved by a multipole expansion. For this purpose we introduce the vector spherical harmonics $\mathbf{Y}_{JM}^l(\mathbf{r})$ defined according to

$$\mathbf{Y}_{JM}^l(\hat{\mathbf{r}}) = \sum_{\mu} Y_{l, M-\mu}(\hat{\mathbf{r}}) \mathbf{e}_{\mu} \langle lM - \mu; 1\mu | JM \rangle, \quad (4.11)$$

where the $\mathbf{e}_{\mu}$'s ($\mu = 0, \pm 1$) are the spherical unit vectors. The transverse delta function is expanded as

$$\begin{aligned}
\delta_{tr}^{ij}(\mathbf{r}-\mathbf{r}') = \sum_{lm} \left\{ \frac{\delta(r-r')}{r^2} [Y_{lm}^l(\hat{\mathbf{r}})]^i [Y_{lm}^l(\hat{\mathbf{r}}')]^{j*} \right. \\
+ \frac{\delta(r-r')}{r^2} \frac{l}{2l+1} [Y_{lm}^{l+1}(\hat{\mathbf{r}})]^i [Y_{lm}^{l+1}(\hat{\mathbf{r}}')]^{j*} + \frac{\delta(r-r')}{r^2} \frac{l+1}{2l+1} [Y_{lm}^{l-1}(\hat{\mathbf{r}})]^i [Y_{lm}^{l-1}(\hat{\mathbf{r}}')]^{j*} \\
+ \sqrt{l(l+1)} \left[\frac{\delta(r-r')}{r^2} \frac{1}{2l+1} - \Theta(r-r') \frac{r'^{l-1}}{r^{l+2}} \right] [Y_{lm}^{l+1}(\hat{\mathbf{r}})]^i [Y_{lm}^{l-1}(\hat{\mathbf{r}}')]^{j*} \\
+ \sqrt{l(l+1)} \left[\frac{\delta(r-r')}{r^2} \frac{1}{2l+1} - \Theta(r'-r) \frac{r^{l-1}}{r'^{l+2}} \right] [Y_{lm}^{l-1}(\hat{\mathbf{r}})]^i [Y_{lm}^{l+1}(\hat{\mathbf{r}}')]^{j*} \left. \right\}, \quad (4.12)
\end{aligned}$$

and the tensor Green's function can be searched for in a form

$$G^{ij}(\mathbf{r}, \mathbf{r}') = \sum_{lm} \left[f_l(r, r') [Y_{lm}^l(\hat{\mathbf{r}})]^i [Y_{lm}^l(\hat{\mathbf{r}}')]^{j*} + \epsilon_{ikq} \epsilon_{jnp} \frac{\partial}{\partial r_k} \frac{\partial}{\partial r'_n} \{ g_l(r, r') [Y_{lm}^l(\hat{\mathbf{r}})]^q [Y_{lm}^l(\hat{\mathbf{r}}')]^{p*} \} \right]. \quad (4.13)$$

In the case under consideration the quark current has the form

$$J_a(\mathbf{r}) = \frac{2u(r)v(r)}{4\pi} \boldsymbol{\sigma} \times \hat{\mathbf{r}} t^a, \quad (4.14)$$

so that only the $l=1$ component $f_1(r, r')$ of the transverse Green's function contributes to the corresponding vector potential. Explicitly, from (4.7), (4.13), and (4.14),

$$A_a(\mathbf{r}) = \frac{g}{4\pi} \boldsymbol{\sigma} \times \hat{\mathbf{r}} t^a A(r), \quad (4.15a)$$

with

$$A(r) = \int dr' 2r'^2 u(r') v(r') f_1(r, r'). \quad (4.15b)$$

The last thing we have to do is to calculate $f_1(r, r')$. For angular momentum $l=1$ the transverse electric part of Eq. (4.8) becomes

$$\left[-\frac{d^2}{dr^2} + \frac{2}{r^2} \right] [rf_1(r, r')] = \frac{\delta(r-r')}{\epsilon(r)r}. \quad (4.16)$$

Equation (4.10a) is automatically satisfied, and (4.10b) and (4.10c) give

$$f_1(r, r')|_{r=R} \text{ (continuous)}, \quad (4.17a)$$

$$\epsilon(r) \frac{d}{dr} [rf_1(r, r')] \Big|_{r=R} \text{ (continuous)}. \quad (4.17b)$$

These conditions have to be supplemented by a condition in r' which is obtained by integrating Eq. (4.16) around $r=r'$,

$$\frac{d}{dr} [rf_1(r, r')] \Big|_{r=r'_-} - \frac{d}{dr} [rf_1(r, r')] \Big|_{r=r'_+} = \frac{1}{r' \epsilon(r')}. \quad (4.18)$$

Using the fact that $\epsilon_N = \chi_N^a \ll 1$, one finds

$$\begin{aligned}
f_1(r, r') = \Theta(R-r) \Theta(R-r') \frac{1}{3} \left[\frac{rr'}{2R^3} + \frac{r_{<}}{r_{>}^2} \right] \\
+ \Theta(r_{>} - R) \Theta(R-r_{<}) \frac{r_{<}}{2r_{>}^2} \\
+ \Theta(r-R) \Theta(r'-R) \frac{1}{3\epsilon_N} \left[\frac{r_{<}}{r_{>}^2} - \frac{R^3}{r^2 r'^2} \right], \quad (4.19)
\end{aligned}$$

The chromomagnetic self-energy of the bound nucleon is

$$E = -\frac{1}{2} \alpha_s \lambda \int dr 2r^2 u(r) v(r) A(r), \quad (4.20a)$$

$$E = -\frac{1}{2} \alpha_s \lambda \int dr \int dr' [2r^2 u(r) v(r)] \times f_1(r, r') [2r'^2 u(r') v(r')], \quad (4.20b)$$

where λ is a color-spin matrix element:

$$\lambda = \frac{2}{3} \left\langle \sum_{i=1}^3 \sum_{j=1}^3 \sum_{a=1}^8 \boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j t_i^a t_j^a \right\rangle = \frac{8}{3}. \quad (4.21)$$

According to the MIT group, λ does not take into account magnetic quark self-energy.

It is interesting to look at the shape of the vector field outside of the bag, which constitutes a "measure" of the leakage of the gluon field in the nuclear medium. $A(r)$ can be calculated, with the result

$$\begin{aligned}
A(r) = \frac{1}{4r^2} \left[2 \int_0^R dr' r'^3 u(r') v(r') \right. \\
+ \frac{N^2 R^2 j_0^2(\omega R)}{K^2 \epsilon_N} (1 - e^{-2K(r_0-R)}) \left. \right]. \quad (4.22)
\end{aligned}$$

A quantity of more direct physical meaning is the "radial chromomagnetic energy density" from the gluon cloud of the bound nucleon:

$$\frac{dE}{dr} = r^2 u(r) v(r) A(r),$$

FIG. 2. Chromomagnetic energy density outside of the bag of radius R for various values of $\chi_N/(m_q R)$.

which has an exponential shape and completely vanishes outside of the bag when the color dielectric field vanishes. This quantity is shown in Fig. 2 for various values of χ_N .

Now we should draw the attention of the reader to the fact that we have not changed the orbitals and color structure of the bound nucleon (e.g., p -wave excitations). If we would do so, the linear approximation of the gluon field equations would produce long range van der Waals forces which cannot be avoided even in a color dielectric medium with $\chi=0$ in this linear approximation. Presumably, the correct solution of the field equations would cut off color polarization forces because two soft gluons pair to the glueball field, which has a finite mass and therefore decays exponentially.

V. THE EFFECTIVE MASS OF THE BOUND NUCLEON INCLUDING GLUON SELF-ENERGY EFFECT

Once the gluon contribution is taken into account, the mass of the bound nucleon becomes

$$\tilde{M}(\chi_N, \rho, R) = E_q(\chi_N, R) + E_\chi(\chi_N, \rho, R) + E_G(\chi_N, R), \quad (5.1)$$

where E_q is given by (3.6) and (3.8), E_χ by (3.9), and the

gluon contribution, E_G , by (4.20). Once again, χ_N actually depends on the bag radius R and the density ρ , but is modified by the presence of the gluon field (see below). This effective mass can be rewritten as

$$\tilde{M}(\chi_N, \rho, R) = M_N(R) + B_q(\chi_N, R) + B_\chi(\chi_N, \rho, R) + B_G(\chi_N, R). \quad (5.2)$$

$M_N(R)$ given by (2.16) is the mass of a “free” nucleon with radius R which is *a priori* different from the equilibrium radius R_N of the nucleon in vacuum.

The B ’s represent the medium corrections. One finds, for the quark term, an attractive contribution which is given, to leading order in $\chi/m_q R$, by

$$B_q = -\frac{3\alpha_{c.m.}\Omega_0}{R} \frac{\chi_N}{2m_q R}, \quad (5.3)$$

and a repulsive contribution due to the slight excitation of the vacuum in the intranucleonic region,

$$B_\chi = \frac{2\pi}{3} m_{GB}^2 \sigma_V^2 (r_0^3 - R^3) \chi_N^2. \quad (5.4)$$

The gluon term receives, to leading order in $\chi_N/m_q R$ and $m_q R$, two contributions:

$$B_G = B_G^i + B_G^0, \quad (5.5a)$$

$$B_G^i = \frac{0.054\alpha_S}{R} \frac{\chi_N}{m_q R}, \quad (5.5b)$$

$$B_G^0 = -\frac{\alpha_S \lambda}{8R} \left[\frac{\Omega_0}{2(\Omega_0 - 1)} \right]^2 \frac{1}{m_q R} \left[\frac{\chi_N}{m_q R} \right]^2. \quad (5.5c)$$

The first repulsive term is actually very small. The second term comes from the external current-current energy [contribution in the integral (4.20b) with both r and r' larger than R]. Let us now consider the problem of the determination of χ_N . From the Lagrangian (2.15) with $\alpha=1$, we first derive the equation of motion for the χ field. Including the gluon term, we then integrate this equation in a Wigner cell for radius r_0 outside of the bag with radius R :

$$\int_R^{r_0} d\mathbf{r} V'(\chi_N) = \int_R^{r_0} d\mathbf{r} \left[\frac{m_q}{\chi_N^2} \sum_i \langle \bar{\psi}_i(\mathbf{r}) \psi_i(\mathbf{r}) \rangle - \frac{1}{2} \sum_{i,j} \langle \mathbf{B}_{ai}(\mathbf{r}) \cdot \mathbf{B}_{aj}(\mathbf{r}') \rangle \right]. \quad (5.6)$$

Here, $\mathbf{B}_{ai}(\mathbf{r}) = \nabla \times \mathbf{A}_{ai}$ is the magnetic field with color a created by the quark number i . In principle, one should take into account the magnetic field created by all the quarks of the nucleus. But, from the lattice model we learn that one can restrict the summation over i and j to the valence quarks of the nucleon under consideration because quarks in other nucleons give a contribution of higher order in $\chi_N/(m_q R)$.

Then, to leading order in χ_N , one has

$$\frac{4\pi}{3} m_{GB}^2 \sigma_V^2 (r_0^3 - R^3) \chi_N^2 = \frac{3\alpha_{c.m.}\Omega_0}{2R} \frac{\chi_N}{m_q R} \left[1 - \eta \left[1 + \frac{r_0 - R}{r_0(\Omega_0 - 1)} \right] \right] - \frac{\alpha_S \lambda}{8R} \left[\frac{\Omega_0}{2(\Omega_0 - 1)} \right]^2 \frac{1}{m_q R} \left[\frac{\chi_N}{m_q R} \right]^2 \left[1 - \eta^2 \frac{R^2}{r_0^2} \right], \quad (5.7)$$

with

$$\eta = \exp \left[-2 \frac{m_q}{\chi_N} (r_0 - R) \right].$$

The right hand side of Eq. (5.7) contains, in addition to the expectation value of the scalar quark density [cf. Eq. (3.11)], the gluon energy which couples linearly to χ for $\epsilon = \chi$ in the Lagrangian [Eq. (2.15)].

As can be seen from (5.7), the limit of vanishing quark mass is well defined, with χ_N going to zero as m_q^2 due to a compensation between the quark and gluon source terms. Thus, the binding energy goes to zero in this limit [see Eqs. (5.3)–(5.5)].

The binding potential felt by a bound nucleon can now be calculated. This binding potential is defined according to

$$W(\rho, R) = \tilde{M}(\chi_N(\rho, R), \rho, R) - M_N. \quad (5.8)$$

A minimization with respect to R at a given value of the density ρ gives the radius $R(\rho)$ of the bound bag and the binding potential of the bound nucleon at this density ρ . The binding mechanism essentially comes from the non-vanishing value of the nuclear χ field and the resulting delocalization of the valence quarks.

Since the χ field is associated with a 0^{++} glueball state which would couple to $(q\bar{q})$ states in a complete theory, it is likely that it plays the same role as the 0^{++} σ meson in a conventional meson exchange picture. However, our approach is certainly not very good when the density is high, because overlap effects between quark wave functions of different nucleons and their gluon interaction energy have to be taken into account. These effects can be calculated using a solid state physics method and are negligible only when the parameter

$$\eta = \exp \left[-\frac{2m_q}{\chi_N} (r_0 - R) \right]$$

is much smaller than 1. This condition is unfortunately poorly satisfied around normal nuclear matter density and our results have a qualitative character at such a density. On the other hand, a detailed calculation of such interaction effects with this method would be doubtful because nuclear matter is not a crystal.

To relax the static approximation and to calculate, for instance, the effect of the short range interaction with implemented wave functions, is beyond our present capability. However, the short range repulsion presumably¹⁹ comes from the six, nine, or more quark bag formation. It is obvious that a six-quark bag is more easily formed when the effective size (identified with the rms) of the bound nucleon increases.

Following Miller *et al.*,²⁰ a six quark bag forms when the internucleon distance is smaller than a certain distance R_c . This critical distance corresponds to a configuration where the overlap volume of the two spherical bags is half of the total volume, which gives

$$R_c = \mu \langle r^2 \rangle^{1/2}, \quad \mu \simeq 0.7. \quad (5.9)$$

R_c is the range of a short-range potential, the height of

which is obtained by calculating the mass M_6^{SI} of the color singlet—six quark bags in the various channels $S=0, 1, I=0, 1$:

$$V^{SI}(r) = (M_6^{SI} - 2M_N) p^{SI} \Theta(R_c - r), \quad (5.10)$$

where the p^{SI} are projection operators on the various spin-isospin channels. Using a Fermi-gas picture gives the following contribution to the interaction energy per nucleon:

$$W_{sr} = \frac{1}{2} \left[\frac{1}{(4\pi/3)r_0^3} \right] \int d\mathbf{r} \left\{ V_D - V_E \left[\frac{3j_1(k_F r)}{k_F r} \right]^2 \right\} \times \Theta(R_c - r), \quad (5.11)$$

with

$$V_D = \frac{1}{16} (M_6^{00} + 3M_6^{10} + 3M_6^{01} + 9M_6^{11}) - 2M_N, \quad (5.12a)$$

$$V_E = \frac{1}{16} (M_6^{00} - 3M_6^{10} - 3M_6^{01} + 9M_6^{11}) - 2M_N, \quad (5.12b)$$

where the Fermi momentum is related to the Wigner-Seitz cell by $k_F r_0 = (9\pi/8)^{1/3}$. W_{sr} can be expanded according to

$$W_{sr} = C \left[\frac{\langle r^2 \rangle^{1/2}}{r_0} \right]^3 [1 + \lambda_1 (k_F R_c)^2 + \lambda_2 (k_F R_c)^4 + \dots]. \quad (5.13)$$

The λ_i 's are small numbers depending on the precise values of the short range two-body potential in the various spin-isospin channels (for instance, $\lambda_1 = \frac{1}{25}$ and $\lambda_2 = -\frac{3}{1225}$ in the case of a pure central potential).

C is a constant proportional to μ^3 calculable in the model and depending on the exact value of the parameters. For instance, with $\mu = 0.7$ and $\alpha_S = 2.58$ one finds $C \simeq 25$ MeV. One may equally take V_{sr} as phenomenological and consider C a parameter. The numerical results show that the above model of repulsion leads to nuclear matter saturation. Although our description is only qualitative, one has a possible scenario of the saturation mechanism. At moderate density the binding comes from the leakage of the quarks and the gluon, which is more and more important as the density increases. Above saturation density, approximately, the rms radius of the bound nucleon decreases again and becomes smaller than the free rms radius at $r_0 \approx R_N$, where the size of the Wigner-Seitz cell equals the free nucleon radius. This effect is mostly due to the behavior of $W(\rho) = M_N(R) + B_q + B_\chi$, which tries to keep the nucleon radius close to r_0 . As can be seen in Fig. 3, the contribution of the gluons to the total energy gives two terms, W_{sr} and B_G , which tend to compensate for each other. An important saturation mechanism in this simple quark model is the quark kinetic energy, which increases beyond its free nucleon value for $r_0 < R_N$ (see Fig. 4). To calculate the total binding potential curve $W(\rho)$ in Fig. 4, we have used the parameters $m_{GB} = 2$ GeV, $\sigma_V = 0.24$ GeV, $C = 25$ MeV, and $m_q = 30$ MeV.

Choosing different values for m_{GB} , σ_V , or C , we would change the depth and the position of the minimum of the

FIG. 3. Gluonic and nongluonic contributions to the binding potential vs density ρ . Dashed curve: binding potential without gluon effect [Eq. (3.12)]. Dotted-Dashed curve: gluonic contributions [Eqs. (5.5) and (5.13)]. Solid curve: total binding potential.

curve $W(\rho)$, but not the general shape of the curve. In the same figure we show the change of the rms radius of the bound nucleon relative to the rms radius of the free nucleon, $\text{rms}=(r^2)^{1/2}=0.85$ fm. We see that below saturation density the nucleon is indeed larger in the nucleus, but at saturation density the effect is reversed. This theoretical result correlates well with the A dependence of the EMC effect which, in its simplest interpretation, can be explained by rescaling of the confinement size of the quarks in the bound nucleon. To make a comparison with the rescaling model more precise, we have marked in Fig. 4 also the rescaling factor of Ref. 26 for four typical nuclei. The average (not local) density of different nuclei is obtained by taking the hard sphere radius R_c equivalent to the rms radius. Besides the largest nucleus ^{157}Au , the agreement with $[\text{rms}(\text{bound})]/\text{rms}(\text{free})$ is very good. The discrepancy at high density shows the limitation of the model at high density.

FIG. 4. Binding potential (solid curve) and ratio of the bound nucleon rms to the free nucleon rms (dashed curve) vs average density ρ . Also shown as dots are the rescaling factors given in Ref. 26 for some typical nuclear average densities.

VI. AN APPLICATION: THE NOLEN-SCHIFFER ANOMALY

In a recent paper, Williams and Thomas²¹ have given a convincing explanation of the Nolen-Schiffer anomaly⁹ (NSA) using the color dielectric model without gluon effects. We will show in the following that the inclusion of the gluon fields gives a consistent explanation of the proton-neutron mass difference and the NSA.

The free neutron-proton mass difference contains a quark confinement term, a gluon magnetic contribution, and the Coulomb energy,

$$M_n - M_p = \Delta M_q + \Delta \Sigma_G + \Delta M_c. \quad (6.1)$$

The first term comes from the frequency difference between a u quark with mass m_u and a d quark with mass m_d . From Eq. (3.6), we have

$$\Delta M_q = \frac{\alpha_{c.m.}(m_d - m_u)}{2(\Omega_0 - 1)}. \quad (6.2)$$

The general structure of the gluon self-energy is

$$\Sigma_G = \frac{1}{2} \sum_{i>j} I_{ij}(m_i, m_j) \left\langle \sigma_i \cdot \sigma_j \sum_{a=1}^8 t_i^a t_j^a \right\rangle. \quad (6.3)$$

Here, $I_{ij}(m_i, m_j)$ is a radial interaction integral between a quark with mass m_i and a quark with mass m_j . The color matrix element is

$$\left\langle \sum_{a=1}^8 t_i^a t_j^a \right\rangle = -\frac{2}{3} \quad (i \neq j). \quad (6.4)$$

Thus, the gluon self-energy becomes

$$\Sigma_G = -\frac{1}{3} \sum_{i>j} I_{ij}(m_i, m_j) \langle \text{SU}(6) | \sigma_i \cdot \sigma_j | \text{SU}(6) \rangle. \quad (6.5)$$

Using the SU(6) wave functions of the neutron and the proton, we obtain

$$\Sigma_G(n) = -\frac{4}{3} I_{ud} + \frac{1}{3} I_{dd}, \quad (6.6a)$$

$$\Sigma_G(p) = -\frac{4}{3} I_{ud} + \frac{1}{3} I_{uu}, \quad (6.6b)$$

so that

$$\Delta \Sigma_G = \frac{1}{3} (I_{dd} - I_{uu}). \quad (6.7)$$

For small quark mass difference, $\Delta \Sigma_G$ is proportional to $m_d - m_u$ and has the form

$$\Delta \Sigma_G = A_G \alpha_S (m_d - m_u). \quad (6.8)$$

The coefficient $A_G = -0.022$ has been obtained from Ref. 22. We again consider that the chromoelectric quark self-energy is taken into account. Thus, the chromoelectric field does not contribute to neutron-proton mass difference. The Coulomb energy contribution is

$$\Delta M_c = \frac{A_c}{R}. \quad (6.9)$$

A_c is again determined from Ref. 22, $A_c = -0.58$ MeV fm. Taking, as before, $\alpha_S = 2.58$ and $\alpha_{c.m.} = 0.8$, we deduce from the experimental value of $M_n - M_p = 1.29$ MeV the quark mass difference:

$$\Delta m_q = m_d - m_u = 5.5 \text{ MeV}.$$

The different contributions to $M_n - M_p = 1.29 \text{ MeV}$ in order of decreasing importance are $\Delta M_q = 2.1 \text{ MeV}$, $\Delta \Sigma_G = -0.31 \text{ MeV}$, and $\Delta M_c = -0.5 \text{ MeV}$.

The Nolen-Schiffer anomaly⁹ is a systematic failure of Coulomb corrections to account for the observed mass difference of mirror nuclei:

$$M_{(Z>)} - M_{(Z<)} = (Z_{>} - Z_{<})(M_p - M_n) + E_c + \Delta, \quad (6.10)$$

where $M_p - M_n = -1.29 \text{ MeV}$ is the free neutron-proton mass difference. E_c represents the Coulomb correction and Δ is the unexplained part of the nuclear mass difference, the NSA. From the expression of the binding energy (5.3) and (5.5), it is apparent that the nucleon binding potential (5.8) depends on the quark mass m_q . Due to the nonvanishing quark mass difference Δm_q , we expect that a bound neutron and a bound proton feel a different potential. We consider two mirror nuclei as being constituted of the same even-even core and of one added valence proton or neutron. It has been shown by Williams and Thomas that the added nucleon does not modify the core. Thus, we interpret the binding potential difference

$$W_p - W_n = \Delta \quad (6.11)$$

of the valence proton and the valence neutron as the NSA Δ . If the valence proton and the valence neutron of the two mirror nuclei move in a medium with the same average density, we find Δ with the wrong sign. But—as remarked by Williams and Thomas—the valence proton has a larger rms radius than the valence neutron and therefore feels a smaller density. It is precisely the density difference which gives the correct sign of Δ . Introducing the core density $\rho(r)$ and the shell model wave functions $\varphi_p(r)$ and $\varphi_n(r)$ of the valence nucleon, we can calculate the average densities felt by the valence neutron and the valence proton,

$$\rho_{p,n} = \int d\mathbf{r} \rho(r) |\varphi_{p,n}(r)|^2. \quad (6.12)$$

The corresponding Wigner-cell radius $r_{0p,n}$ are defined according to

$$\frac{4}{3} \pi r_{0p,n}^3 = \rho_{p,n}^{-1}. \quad (6.13)$$

For the NSA, we deal with a valence nucleon moving in a region where the density is actually very low, so one can ignore all the density dependent corrections proportional to

$$\eta = \exp \left[-\frac{2m_q}{\chi_N} (r_0 - R) \right],$$

which is extremely small. The same reason makes the radius R of the valence nucleon cavity the same as the free bag radius R_N . From Eq. (5.7) one can determine the corresponding χ field felt by the valence nucleons. We are, in fact, interested in the difference $\chi_p - \chi_n$. For a small density difference one has

$$\begin{aligned} \frac{\Delta\chi}{\bar{\chi}} &= \frac{\chi_n - \chi_p}{\bar{\chi}} \\ &= \frac{r_{0p}^3 - r_{0n}^3}{\bar{r}_0^3 - R^3 + \frac{\alpha_S}{4\pi m_{GB}^2 \sigma_V^2 m_q^3 R^4} \left[\frac{\Omega_0}{2(\Omega_0 - 1)} \right]^2} > 0, \end{aligned} \quad (6.14)$$

where $\bar{r}_0^3 = (r_{0p}^3 + r_{0n}^3)/2$, and $\bar{\chi}$ is the average χ -field solution of (5.7) with the Wigner-Seitz radius $\bar{r}_0$. Because of the Coulomb interaction the valence proton sits more on the nuclear surface than the valence neutron. It therefore feels a lower nuclear density with a smaller χ than the neutron, i.e., $\chi_p < \chi_n$.

These χ fields are actually created by all the nucleons of the isospin symmetric core. Therefore the relevant quark mass appearing in (6.14) has to be the average quark mass $m_q = (m_u + m_d)/2$.

We now calculate the difference $W_p - W_n$ to first order in $\Delta m_q/m_q > 0$ and $\Delta\chi/\bar{\chi} > 0$. It receives four contributions: a quark, a χ , a gluon self-energy, and a gluon interaction energy contribution, corresponding, respectively, to Eqs. (5.3), (5.4), (5.5), and (5.13).

The quark contribution is

$$\Delta W_q = B_q(p) - B_q(n), \quad (6.15)$$

where $B_q(p,n)$ are the binding corrections of the quark energies due to tunneling, for the proton and the neutron,

$$B_q(p,n) = -\frac{\alpha_{c.m.} \Omega_0}{2R^2} \left[\frac{n_u(p,n)}{m_u} + \frac{n_d(p,n)}{m_d} \right] \chi_{p,n}. \quad (6.16)$$

$n_{u,d}(p,n)$ are the number of up and down quarks in both isospin states of the nucleon. It follows that

$$\Delta W_q = -\frac{3\alpha_{c.m.} \Omega_0}{2m_q R^2} \bar{\chi} \left[\frac{1}{3} \frac{\Delta m_q}{m_q} - \frac{\Delta\chi}{\bar{\chi}} \right]. \quad (6.17)$$

It is apparent that the $\Delta\chi$ term (proportional to the density difference [see Eq. (6.14)] gives a repulsive contribution contrary to the Δm term.

The χ term depends only on the density difference [see again (6.14)]

$$\Delta W_\chi = \frac{2\pi}{3} m_{GB}^2 \sigma_V^2 (\bar{r}_0^3 - R^3) \bar{\chi}^2 \left[\frac{r_{0p}^3 - r_{0n}^3}{\bar{r}_0^3 - R^3} - 2 \frac{\Delta\chi}{\bar{\chi}} \right]. \quad (6.18)$$

The gluon term comes from the difference of the gluon binding energy due to the chromomagnetic self-energy and short range interaction energy of the nucleon,

$$\begin{aligned} \Delta W_G &= B_G(p; \chi_p) - B_G(n; \chi_n) \\ &+ W_{sr}(p; \chi_p) - W_{sr}(n; \chi_n) \\ &\equiv \Delta B_G + \Delta W_{sr}. \end{aligned} \quad (6.19)$$

The general spin-isospin color structure of $B_G(p,n; \chi_p, \chi_n)$ has the form given by (6.3). We thus have, from (6.6),

$$B_G(p; \chi_p) = \frac{4}{3} B_G(m_u, m_d; \chi_p) - \frac{1}{3} B_G(m_u, m_u; \chi_p), \quad (6.20a)$$

$$B_G(n; \chi_n) = \frac{4}{3} B_G(m_u, m_d; \chi_n) - \frac{1}{3} B_G(m_d, m_d; \chi_n). \quad (6.20b)$$

When the quark masses are identical, $B_G(m_i, m_i; \chi)$ reduces to the gluon binding energy of an hypothetical nucleon made of three quarks with the same mass m_i . Formally, one has

$$B_G(m_i, m_i; \chi) = B_G(m_i; \chi), \quad (6.21)$$

and $B_G(m_i; \chi) < 0$ is the gluon binding energy given by (5.5) for a quark mass m_i and a color dielectric field χ .

It follows from (6.20) and (6.21) that

$$\Delta B_G = \frac{1}{3} [B_G(m_d; \chi_n) - B_G(m_u, \chi_p)] + \frac{4}{3} [B_G(m_u, m_d; \chi_p) - B_G(m_u, m_d; \chi_n)]. \quad (6.22)$$

To leading order in Δm_q and $\Delta \chi$, one has

$$B_G(m_u, m_d; \chi_p) - B_G(m_u, m_d; \chi_n) = \frac{\partial B_G(m_q; \bar{\chi})}{\partial \bar{\chi}} (\chi_p - \chi_n), \quad (6.23)$$

$$B_G(m_u; \chi_p) - B_G(m_d; \chi_n) = \frac{\partial B_G(m_q; \bar{\chi})}{\partial \bar{\chi}} (m_u - m_d) + \frac{\partial B_G(m_q; \bar{\chi})}{\partial \bar{\chi}} (\chi_p - \chi_n). \quad (6.24)$$

Therefore, ΔB_G is

$$\Delta B_G = \frac{1}{3} \frac{\partial B_G(m_q; \bar{\chi})}{\partial m_q} (m_d - m_u) - \frac{\partial B_G(m_q; \bar{\chi})}{\partial \bar{\chi}} (\chi_n - \chi_p). \quad (6.25)$$

It is easy to see that the two contributions are positive. We know that χ_p is smaller than χ_n . Thus, from Fig. 2 we see that there is less leakage of the gluon field for the proton. It follows that the valence proton feels less attraction due to the gluon self-energy than the valence neutron. It is also apparent that keeping χ fixed but increasing the quark mass from m_u to m_d makes the leakage of the gluon field less pronounced, and we get a positive contribution to $W_p - W_n$.

The short-range repulsive interaction term W_{sr} from gluon exchange in six quark clusters partially compensates for the attractive nucleon self-energy B_G in the medium. Taking into account ΔW_{sr} [Eq. (6.19)] for the proton-neutron mass difference has the same effect,

$$\Delta W_{sr} = -C \frac{\langle r^2 \rangle^{3/2}}{r_0^3} \frac{r_{0p}^3 - r_{0n}^3}{\bar{r}_0^3}. \quad (6.26)$$

This equation follows from Eq. (5.13) for small density. Therefore the positive terms of ΔB_G are partially cancelled by ΔW_{sr} .

Using the formula of ΔW_q , ΔW_χ , ΔB_G , and ΔW_{sr} from (6.17), (6.18), (6.25), and (6.26), together with the expression of $\Delta \chi$ given by (6.14), we can represent the NSA Δ as

$$\Delta = F(\bar{r}_0) \frac{r_{0p}^3 - r_{0n}^3}{\bar{r}_0^3} - G(\bar{r}_0) \frac{m_d - m_u}{m_q}, \quad (6.27)$$

where $F(\bar{r}_0)$ and $G(\bar{r}_0)$ are plotted in Fig. 5. The parameters (m_{GB} , σ_V , m_q , α_S , and $\alpha_{c.m.}$) are the same as used previously.

We will calculate the NSA for ^{15}O - ^{15}N , ^{17}F - ^{17}O , and ^{41}Sc - ^{41}Ca . For this purpose, we need the average densities seen by the valence particles in the above nuclei. We are guided by the estimates from the harmonic oscillator or Woods-Saxon model fitted to electron scattering data for the proton densities and corrected for Coulomb effects for the neutron densities.^{9,21} We see changes of rms radii by $\pm 1.5\%$ depending on the harmonic oscillator or Woods-Saxon description.²³ Consequently, we would attach an error of $\pm 5\%$ to the relative difference of Wigner-Seitz volumes $(r_{0p}^3 - r_{0n}^3)/\bar{r}_0^3$ appearing in Eq. (6.27). With the nuclear parameters listed in Table I, we reproduce a NSA (see Table I) which increases with $\bar{\rho} = 1/[(4\pi/3)\bar{r}_0^3]$. For the light nuclei $A = 15$ and 17 , Δ is 380 and 440 keV with errors of ± 50 keV, which make it compatible with the nuclear physics estimates⁹ of the anomaly. For the largest pair of mirror nuclei (^{41}Sc - ^{41}Ca), we find a larger Δ of $900 \text{ keV} \pm 100 \text{ MeV}$, which exceeds the estimate of 620 keV given by Nolen and Schiffer.⁹ Auerbach²⁴ has taken into account 3p-2h states and core polarization. He thereby reduced the nuclear physics estimate to 400 keV. It is probably fair to say that there is a considerable spread in the nuclear physics calculation of the anomaly. For larger nuclei there is a tendency that the NSA is about 7% of the Coulomb energy;⁹ this increases with average density.

FIG. 5. The functions $F(\bar{r}_0)$ (solid curve) and $G(\bar{r}_0)$ (dashed curve) vs the Wigner-Seitz cell size $\bar{r}_0$ [see Eq. (6.27)].

TABLE I. The Nolen-Schiffer anomaly (NSA) Δ calculated with and without gluons for three different mirror nuclei. The average valence nucleon Wigner-Seitz cell radius is $\bar{r}_0$. The relative difference of the Wigner-Seitz cells for the valence proton and the valence neutron is given in the third column.

Mirror nuclei	$\bar{r}_0$ (fm)	$\frac{r_{0p}^3 - r_{0n}^3}{\bar{r}_0^3}$	Δ with gluons (keV)	Δ without gluon (keV)	$\Delta(\text{expt})$ (keV)
$^{17}\text{F}-^{17}\text{O} (d_{5/2})$	1.78	0.183 ± 0.009	385 ± 47	730 ± 70	330^a
$^{15}\text{O}-^{15}\text{N} (p_{1/2})$	1.72	0.16 ± 0.008	444 ± 50	693 ± 75	340^a
$^{41}\text{Sc}-^{41}\text{Ca} (f_{7/2})$	1.56	0.115 ± 0.006	905 ± 90	743 ± 110	$300-400,^b 620^a$

^aReference 9.

^bReference 24.

This experimental behavior is correctly described by our calculation.

We also have calculated the NSA without gluon corrections, still keeping our parameters $m_u - m_d, m_q$ constant. We find an anomaly, $\Delta \approx 700$ keV (cf. Table I), which does not vary with the average density. The sensitivity to the difference $(r_{0p}^3 - r_{0n}^3)/\bar{r}_0^3$ is more critical in this case, which reflects itself in the larger errors (cf. Table I). In Ref. 21 an anomaly $\Delta \approx 400-500$ keV was obtained that decreases slightly with increasing average density. Their parameters, $(m_d - m_u)/m_q = (3 \text{ MeV})/(29 \text{ MeV})$ and $R_N = 0.83$ fm, are different from ours, $(m_d - m_u)/m_q = (5.5 \text{ MeV})/(30 \text{ MeV})$ and $R_N = 1.16$ fm. We remark that it is difficult to fit the free neutron-proton mass difference without gluons. With the values of Ref. 21, we would get [see Eqs. (6.2) and (6.9)] $\Delta \tilde{M}_q = 1.4 \alpha_{c.m.} \text{ MeV}$, $\Delta \tilde{M}_C = -0.6 \text{ MeV}$, and an overly small neutron-proton mass difference $M_n - M_p = \Delta \tilde{M}_q + \Delta \tilde{M}_C$. Depending on the treatment of c.m. corrections ($0.5 < \alpha_{c.m.} < 0.8$), one gets $M_n - M_p$ between 0.1 and 0.5 MeV.

There is another limit of validity which can be imposed on the model. Take a nucleon of the core; then the difference in binding energy between a neutron and a proton cannot be larger than the measured difference in removal energies for deep levels. The authors of Ref. 21 proposes a limit of 1 MeV, which seems reasonable. Using Eqs. (6.16) and (6.25) we find, for the mass differences of protons and neutrons in the core ($\chi_p = \chi_n = \chi_N$),

$$\begin{aligned}
 (M_p - M_n)_C &= \left[\frac{1}{3} \frac{\partial B_q}{\partial m_q} - \frac{1}{3} \frac{\partial B_G}{\partial m_q} \right] (m_u - m_d) \\
 &= \left[\frac{B_q}{3} - B_G^0 - \frac{B_G^i}{3} \right] \left[\frac{m_d - m_u}{m_q} \right]. \quad (6.28)
 \end{aligned}$$

Substituting B_q and B_G from Eqs. (5.3) and (5.5), we get $(M_p - M_n)_C = -1 \text{ MeV}$, consistent with the bound, at $\bar{r}_0 = 1.22$ fm, which corresponds to an average nuclear density of $\rho = 0.13 \text{ fm}^{-3}$. A calculation of the charge symmetry breaking (CSB) $(M_p - M_n)_C$ for deep levels without gluon corrections²⁵ was interpreted as an indicator that the color dielectric model is neither capable of satisfying the bound of 1 MeV nor of giving an increase of the nucleon radii compatible with the EMC effect. As demonstrated in Sec. V (see Fig. 4), a radius increase is

possible without violating the bound on $M_p - M_n$. The CSB remains small because the gluon contribution cancels the quark contribution to the asymmetry [$(B_q/3)(\Delta m_q/m_q) = -3.3 \text{ MeV}$; $-(B_G^0 + B_G^i/3)(\Delta m_q/m_q) = 2.3 \text{ MeV}$], and since B_q is proportional to χ_N we can have a mean field in nuclei three times larger than that from the estimate of Ref. 25. We have $\chi_N = 0.022$ and a barrier of 1.3 GeV separating quarks in different nucleons, whereas in Ref. 21 the upper limit for χ_N was 0.01. Thus, once gluon corrections are taken into account, the CSB allows a sufficiently large increase of the rms radius of the bound nucleon to successfully interpret the EMC data (cf. Sec. V).

VII. CONCLUSION

We have presented a schematic model for interacting quarks and gluons in the framework of color dielectrics. This idea is based on averages of high frequency gluon fluctuations which are taken into account through a scalar color dielectric field χ . The effective Lagrangian of this model is determined to a large extent by this averaging procedure. It has the peculiar feature that the quark mass over the color dielectric field (m_q/χ) regulates quark confinement. A different color dielectric field inside and outside of hadrons is associated with a small and large effective quark mass. In addition, the color dielectric field also controls the confinement of gluons by multiplying the gluon field tensors $F_{\mu\nu}F^{\mu\nu}$ with a color dielectric constant $\epsilon = \chi^\alpha$ ($\alpha > 1$) in the Lagrangian.

This paper analyzes both of these properties by looking at nuclear matter which has a nonvanishing color dielectric field. Of course, the treatment of the color dielectric model at finite baryon density is complicated. We approach the problem of baryon matter from the low density side. For vanishing nuclear density we have noninteracting nucleons which gradually change when they are immersed into a nuclear medium of increasing density. This method, however, is unable to describe the transition to quark matter.

In the low density approximation we have obtained four characteristic nuclear features which allow a comparison with experiment. The first consequence of a nonvanishing color dielectric field outside of the nucleon in nuclear matter is *nuclear binding*, which mainly comes from the

tunneling of quarks between nucleons. The gluon contribution to this mean potential is not very large and slightly repulsive. Its main importance is to favor baryon matter stability at normal nuclear density. At the saturation density the tendency of nucleons to swell beyond normal size is reversed because otherwise clustering generates a repulsive gluon interaction energy. This effect adds to the increasing kinetic energy of quarks, which is also repulsive. The change of the rms radius of the bound nucleon is large enough to allow a possible interpretation of the EMC effect in terms of the rescaling model; we note, however, that a full understanding of the EMC effect is only possible when the nuclear quark wave functions can be calculated in the infinite momentum frame. In this work we have defended our original proposal⁴ against objections raised by Williams and Thomas.^{21,25} These were based on the failure of the color dielectric model without gluons to account simultaneously for nuclear binding, EMC effect, and a *small charge symmetry breaking* in nuclei. But as shown in this work, here, the sensitivity to the up and down quark mass difference is less pronounced because gluon effects largely compensate for quark binding due to tunneling. Only the quark contribution to binding was taken into account in Refs. 21 and 25. The importance of gluon corrections is also visible in the calculation of the *Nolen-Schiffer anomaly (NSA)*. There, gluon effects allow a simultaneous description of the free neutron-proton mass difference and the NSA in light mirror nuclei.

A numerical solution of the complete color dielectric model, without gluon fields, for the free nucleon, has been achieved.^{13,14} It is therefore desirable in the future to solve the χ field equation in nuclear matter exactly and to take into account gluon interaction effects. In this way, a better understanding of baryon matter in terms of quarks and gluons may be reached over the full range of density including the transition to quark matter.

This work was performed in part at the Institut für Theoretische Physik, Heidelberg, and supported by the Bundesministerium für Forschung und Technologie.

APPENDIX

In this appendix we discuss the modification of the nonperturbed quark wave functions [Eq. (3.3)] of a given nucleon when the presence of the surrounding nucleons is taken into account. Using a solid state physics picture, one assumes that the nucleonic bag cavities are frozen on a cubic lattice with periodicity $2r_0$. Thus, the quarks move in a periodic "potential" $\beta\tilde{M}(\mathbf{r})$ and obey the Dirac equation

$$[-i\boldsymbol{\alpha}\cdot\nabla + \beta\tilde{M}(\mathbf{r})]\phi(\mathbf{r}) = \tilde{\omega}\phi(\mathbf{r}). \quad (\text{A1})$$

Inside the bag cavities of radius R , $\tilde{M}(\mathbf{r}) = m_q$; outside, $\tilde{M}(\mathbf{r}) = m_q/\chi_N$.

To solve (A1), one first introduces Bloch wave functions $\phi_{\mathbf{k},\alpha}(\mathbf{r})$:

$$\phi_{\mathbf{k},\alpha}(\mathbf{r}) = [C(\mathbf{k})]^{-1/2} \sum_{n=0}^{A-1} e^{i\mathbf{k}\cdot\mathbf{R}_n} \psi(\mathbf{r}-\mathbf{R}_n) \chi_\alpha. \quad (\text{A2})$$

The summation runs over all the A lattice sites and $\psi(\mathbf{r}-\mathbf{R}_n)$ is a nonperturbed quark wave function [Eq. (3.3)] centered on $\mathbf{R}_n$. α stands for a spin-isospin-color (SIC) index. The $\mathbf{k}$'s are inverse lattice vectors limited to the first Brillouin zone. Note that, for a given $\mathbf{k}$, there are only one-fourth of the 12 possible SIC states occupied.

By use of the property

$$\sum_{n=0}^{A-1} e^{i(\mathbf{k}-\mathbf{k}')\cdot\mathbf{R}_n} = \delta_{\mathbf{k},\mathbf{k}'}, \quad (\text{A3})$$

and choosing $C(\mathbf{k})$ according to

$$C(\mathbf{k}) = A \sum_{n=0}^{A-1} e^{i\mathbf{k}\cdot\mathbf{R}_n} \int d\mathbf{r} \psi^\dagger(\mathbf{r}) \psi(\mathbf{r}-\mathbf{R}_n), \quad (\text{A4})$$

one proves very easily that the Bloch wave functions are orthogonal, i.e.,

$$\int d\mathbf{r} \phi_{\mathbf{k},\alpha}^\dagger(\mathbf{r}) \phi_{\mathbf{k}',\alpha'} = \delta_{\mathbf{k},\mathbf{k}'} \delta_{\alpha,\alpha'}. \quad (\text{A5})$$

The Bloch wave functions are not exact solutions of the Dirac equation (A1) with the periodic mass distribution $\tilde{M}(\mathbf{r})$. However, their frequencies $\omega(\mathbf{k})$ can be calculated according to

$$\omega(\mathbf{k}) = \int d\mathbf{r} \phi_{\mathbf{k},\alpha}^\dagger(\mathbf{r}) [-i\boldsymbol{\alpha}\cdot\nabla + \beta\tilde{M}(\mathbf{r})] \phi_{\mathbf{k},\alpha}(\mathbf{r}). \quad (\text{A6})$$

$\omega(\mathbf{k})$ can be written as

$$\omega(\mathbf{k}) = \omega + \Delta\omega(\mathbf{k}), \quad (\text{A7})$$

where ω is the frequency solution of Eq. (3.5) and $\Delta\omega(\mathbf{k})$ is found to be

$$\Delta\omega(\mathbf{k}) = \frac{L + \sum_{n \neq 0} e^{i\mathbf{k}\cdot\mathbf{R}_n} J_n}{1 + \sum_{n \neq 0} e^{i\mathbf{k}\cdot\mathbf{R}_n} S_n}. \quad (\text{A8})$$

Using solid-state physics terminology, L is the crystal field integral,

$$L = \int d\mathbf{r} [\tilde{M}(\mathbf{r}) - M(\mathbf{r})] \bar{\psi}(\mathbf{r}) \psi(\mathbf{r}). \quad (\text{A9a})$$

[$M(\mathbf{r})$ is the mass distribution, centered in $\mathbf{R}_0=0$, given by Eq. (3.2).] J_n is the interaction integral

$$J_n = \int d\mathbf{r} [\tilde{M}(\mathbf{r}) - M(\mathbf{r})] \bar{\psi}(\mathbf{r}) \psi(\mathbf{r}-\mathbf{R}_n). \quad (\text{A9b})$$

S_n is the overlap integral.

$$S_n = \int d\mathbf{r} \psi^\dagger(\mathbf{r}) \psi(\mathbf{r}-\mathbf{R}_n). \quad (\text{A9c})$$

In order to construct a bound nucleon as a localized color singlet with center of mass located at the site $\mathbf{R}_n$, we introduce quark Wannier wave functions centered in $\mathbf{R}_n$,

$$\begin{aligned} W_{n,\alpha}(\mathbf{r}) &\equiv W_n(\mathbf{r}) \chi_\alpha \\ &= A^{-1/2} \sum_{\mathbf{k}} e^{-i\mathbf{k}\cdot\mathbf{R}_n} \phi_{\mathbf{k},\alpha}(\mathbf{r}). \end{aligned} \quad (\text{A10})$$

Using (A3) and

$$\sum_{\mathbf{k}} e^{i\mathbf{k}\cdot(\mathbf{R}_n - \mathbf{R}_p)} = A \delta_{n,p}, \quad (\text{A11})$$

it is straightforward to verify that these wave functions obey

$$\int d\mathbf{r} W_{n,\alpha}^\dagger(\mathbf{r}) W_{p,\alpha'}(\mathbf{r}) = \delta_{n,p} \delta_{\alpha,\alpha'} . \quad (\text{A12})$$

This means that the quark Wannier wave functions for quarks belonging to two different nucleons are orthogonal. These Wannier wave functions are not exact solutions of the Schrödinger equations, but their frequencies $\tilde{\omega}_n$ can be determined by

$$\tilde{\omega}_n = \int d\mathbf{r} W_n(\mathbf{r}) [-(\boldsymbol{\alpha} \cdot \nabla) + \beta \tilde{M}(\mathbf{r})] W_n(\mathbf{r}) . \quad (\text{A13})$$

$\tilde{\omega}_n$ is actually independent of n and turns out to be equal to the average of the Bloch frequencies $\omega(\mathbf{k})$,

$$\begin{aligned} \tilde{\omega}_n &\equiv \tilde{\omega} = \omega + \Delta\tilde{\omega} , \\ \Delta\tilde{\omega} &= \frac{1}{A} \sum_{\mathbf{k}} \Delta\omega(\mathbf{k}) . \end{aligned} \quad (\text{A14})$$

In this approach the wave function of a bound nucleon with center of mass located in $\mathbf{R}_n$ is

$$\psi_{\text{BN}}(\mathbf{r}_1, \mathbf{r}_2, \mathbf{r}_3) = W_n(\mathbf{r}_1) W_n(\mathbf{r}_2) W_n(\mathbf{r}_3) \chi_{\text{SIC}}(1, 2, 3) , \quad (\text{A15})$$

where $\chi_{\text{SIC}}(1, 2, 3)$ is the internal wave function of the three quarks coupled to spin $\frac{1}{2}$, isospin $\frac{1}{2}$, and color 0.

One can calculate the corrections $\Delta\tilde{\omega}$ restricting the summation over n to the (six) nearest neighbors. Thus, we find for the correction to the binding potential per nucleon, to leading order in $\chi_N/(m_q R)$,

$$\Delta\tilde{W} = 3\Delta\tilde{\omega}$$

$$\begin{aligned} &= -\frac{9}{4R} \left[\frac{\chi_N}{m_q R} \right]^2 e^{-4(m_q/\chi_N)(r_0-R)} \left[\frac{\Omega_0}{2(\Omega_0-1)} \right] \\ &\quad \times \left[\frac{R^2}{(2r_0-R)r_0} \right] \left[\Omega_0 - \frac{R}{2r_0-R} \right] . \end{aligned} \quad (\text{A16})$$

Taking $r_0 = 1.14$ fm ($\rho = 0.16$ fm $^{-3}$), which is at the limit of the validity of the model, we obtain $\Delta\tilde{W} = -2.7$ MeV. $\Delta\tilde{W}$ is actually a very rapidly decreasing function of density. For instance, we find $\Delta\tilde{W} = -0.5$ MeV at $\rho = 0.125$ fm $^{-3}$ and $\Delta\tilde{W} = -0.002$ MeV at $\rho = 0.1$ fm $^{-3}$.

In the same way, one can calculate the contribution of other nucleons to the source term of the χ field equation [Eq. (5.6)]. The additional source term on the right hand side of Eq. (5.7) is written to leading order in $\chi_N/m_q R$,

$$\frac{3}{4} \frac{3\alpha_{\text{c.m.}} \Omega_0}{2R} \left[\frac{\chi_N}{m_q R} \right]^2 e^{-(2m_q/\chi_N)(r_0-R)} .$$

This term has to be compared with the leading term of order $\chi_N/(m_q R)$ on the right hand side of Eq. (5.7). At $\rho = 0.16$ fm $^{-3}$ this gives a 5% correction which becomes 2.5% at $\rho = 0.125$ fm $^{-3}$ and 0.1% at $\rho = 0.1$ fm $^{-3}$.

The calculation of the gluon interaction energy between different nucleons, in the lattice model, also shows that these contributions are of higher order in $\chi_N/(m_q R)$ and $\eta = \exp[-(2m_q/\chi_N)(r_0-R)]$.

¹Proceedings of the Workshop on Nuclear Chromodynamics: Quarks and Gluons in Particles and Nuclei, Santa Barbara, 1985, edited by S. Brodsky and E. Moniz (World-Scientific, Singapore, 1985).

²J. J. Aubert *et al.*, Phys. Lett. **123B**, 275 (1985).

³A. Bodek *et al.*, Phys. Rev. Lett. **50**, 1431 (1983); R. G. Arnold *et al.*, *ibid.* **52**, 727 (1984).

⁴G. Chanfray, O. Nachtmann, and H. J. Pirner, Phys. Lett. **147B**, 249 (1984).

⁵G. Mack, Nucl. Phys. **B235**, 197 (1984).

⁶H. B. Nielsen and A. Patkos, Nucl. Phys. **B195**, 137 (1982).

⁷T. DeGrand, R. L. Jaffe, N. Johnson, and J. Kiskis, Phys. Rev. D **12**, 2060 (1975).

⁸B. Frois and J. M. Laget, La Recherche **14**, 1370 (1983); J. M. Cavedon *et al.*, Phys. Rev. Lett. **49**, 278 (1982); B. Frois *et al.*, Nucl. Phys. **A396**, 409c (1983).

⁹J. A. Nolen, Jr. and J. P. Schiffer, Annu. Rev. Nucl. Sci. **19**, 471 (1969); S. Schlomo, Rep. Prog. Phys. **41**, 957 (1978).

¹⁰Lattice Gauge Theories and Monte Carlo Simulations, edited by C. Rebbi (World-Scientific, Singapore, 1983).

¹¹M. Rosina, A. Schuh, and H. J. Pirner, Nucl. Phys. **A448**, 557 (1986).

¹²J. M. Drouffe, Nucl. Phys. **B218**, 89 (1983).

¹³W. Broniowski, M. K. Banerjee, and T. D. Cohen, submitted to Phys. Rev. Lett.

¹⁴A. Schuh and H. J. Pirner, Phys. Lett. **B173**, 19 (1986).

¹⁵N. R. Dagdeviren and A. K. Kerman (unpublished).

¹⁶M. Bickeböll, Diploma thesis, University of Bonn, 1984.

¹⁷M. Bickeböll, M. C. Birse, H. Marschall, and L. Wilets, Phys. Rev. D **31**, 2892 (1985).

¹⁸T. D. Lee, Phys. Rev. D **19**, 1802 (1979).

¹⁹M. Oka and K. Yazaki, in International Review of Nuclear Physics, 1984, edited by W. Weise, Vol. 1 p. 42, and references therein.

²⁰G. A. Miller, in International Review of Nuclear Physics, Ref. 19, Vol. 1, p. 253. G. A. Miller and L. S. Kisslinger, Phys. Rev. C **27**, 1669 (1983).

²¹A. G. Williams and A. W. Thomas, Phys. Rev. C **33**, 1070 (1986).

²²W.-Y.P. Hwang, Phys. Rev. D **31**, 2826 (1985).

²³H. R. Collard, L. R. B. Elton, and R. Hofstadter, in Landolt-Börnstein Gruppe J. Kernphysik und Kerntechnik, edited by H. Schopper (Springer, Heidelberg, 1967).

²⁴N. Auerbach, Phys. Rev. **98**, 273 (1983).

²⁵A. G. Williams and A. W. Thomas, Phys. Lett. **154B**, 320 (1983).

²⁶F. G. Close, R. G. Roberts, and G. G. Ross, Phys. Lett. **129B**, 346 (1983).

²⁷H. J. Pirner, Proceedings of the Continuous Electron Beam Accelerator Facility (CEBAF) Summer Workshop (1986), to be published as APS Report CERN-TH 4482/86.