HIGGS PHYSICS AT FUTURE e^+e^- COLLIDERS

G.L. Kane^{*)} and J.J.G. Scanio⁺⁾

CERN - Geneva

A B S T R A C T

We examine the possibilities for searching for Higgs bosons, either directly or by studying the scattering of longitudinal W bosons, at e^+e^- colliders with $\sqrt{s} > 300$ GeV. Rates for various signals are presented and compared and the luminosity needed to find Higgs physics at a given $\sqrt{s}$ is estimated. Procedures to reduce some backgrounds by appropriate cuts are discussed.

^{*)} Physics Department, University of Michigan, Ann Arbor, MI 48109, U.S.A.

⁺⁾ Physics Department, University of Cincinnati, Cincinnati, OH 45221, U.S.A.

1. Introduction

The Higgs sector of the Standard Model is still not understood. No experiment has been published which could have found a Standard Model Higgs boson, whatever mass it had.

If M_H is less than about 40 GeV it can be found at SLC or LEP if these machines reach their design luminosities; for smaller luminosity the range in M_H decreases. If LEP II is eventually constructed that range will be extended to about 85 GeV, being limited there by background from $e^+e^- \rightarrow Z^0 Z^0$. The process considered in these energy regions for the production of the Higgs is $e^+e^- \rightarrow Z^0 H^0$, followed by $Z \rightarrow \ell^+ \ell^-$. For a review of all these topics see, for example, ref. 1.

If toponium should be found at a convenient mass, the Wilczek [2] process $(t\bar{t}) \rightarrow H\gamma$ will allow a search for M_H up to essentially the toponium mass.

No presently approved machine can find the Higgs particle if $M_H \geq 85$ GeV. In a minimal three generation Standard Model the Higgs mass should be greater than about 10 GeV [3]. If $m_t \geq 55$ GeV, or if a fourth generation of fermions exists, however, this lower limit disappears [4]. There is no upper limit to M_H . If $M_H \geq 1.5$ TeV, perturbation theory starts to break down, and observable effects begin to appear in the scattering of longitudinal W bosons and perhaps in the radiative corrections to some low energy quantities, as emphasized by Veltman and collaborators [5]. (W always includes $W^\pm$ and Z^0 for us unless the charge is specified).

For M_H up to 1.5 or 2 TeV it is probably justifiable to use perturbative calculations for estimates, since the one loop corrections to the Higgs width have been calculated [6], and the effects are numerically small (about 15% at $M_H = 1.5$ TeV). Production cross sections go as Γ_H/M_H so the one loop corrections to Γ_H are relevant.

At the SSC [7], a pp collider with luminosity of $10^{33} \text{ cm}^{-2} \text{ sec}^{-1}$, large numbers of Standard Model Higgs bosons would be produced, of order one/sec for $M_H \approx 100$ GeV, and about one longitudinal W pair per GeV per 10^7 sec in the 1.5 TeV region. If $M_H < 2m_t$, rare modes such as $H \rightarrow \gamma\gamma$ or $H \rightarrow \tau\tau$ may be detectable [8]. If $2m_t < M_H < 2M_W$, no way has been found at any proposed machine to look for a Higgs boson. For $2M_W < M_H \leq 600$ GeV the Higgs decays almost exclusively into WW, and, using leptonic modes such as $ZZ \rightarrow \ell^+ \ell^- \ell^+ \ell^-$ or using triggered modes, a Higgs resonance can probably be seen directly, over any Standard Model backgrounds. For $M_H \geq 600$ GeV it may be necessary to study longitudinal W bosons, and to use W pairs where one $W \rightarrow \ell \nu$ is the basis for a trigger, the other $W \rightarrow q\bar{q}$. The Standard Model backgrounds are serious. It may be possible to study the Higgs sector at the SSC for $M_{WW} \leq 1.5$ TeV, but definitive simulations have not been completed.

For large M_H the Higgs sector gets more difficult to study in the Standard Model because it is a clear prediction of the Standard Model that $\Gamma_H = 1/2 M_H^3$, where Γ_H and M_H are in TeV. For $M_H \geq 1$ TeV, $\Gamma_H \geq 500$ GeV. This prediction does not hold in a supersymmetric world [9], where more neutral Higgs particles exist but none of them have widths larger than 10 GeV. In a Standard Model world it is necessary to study the TeV

region and, because of the large Γ_H , it is necessary to study $W_L W_L$ scattering. If the superstring world view is correct, or if the world is supersymmetric on the electroweak scale, then it is more likely that at least one light Higgs exists ($M_H < M_W$), and the additional H^0 , $H^\pm$ are narrow resonances, possibly with masses in the region of several hundred GeV.

In this paper we wish to examine what can be studied in e^+e^- collisions [10]. Although it is not yet known whether TeV electron linear colliders can be built, there is great interest in determining their feasibility. One of the major reasons to go to higher energies is to study the Higgs sector in order to provide experimental input to the understanding of spontaneous symmetry breaking and the origin of mass. It is not simple to determine what machine (i.e., what $\sqrt{s}$ and what luminosity, $\mathcal{L}$) can do what Higgs physics. We will concentrate on the Standard Model Higgs here, rather than on the narrow Higgs spectrum suggested by supersymmetry.

An important point to note concerning the energy dependence of e^+e^- cross sections is that they fall into two categories. The first includes the familiar ones mediated by an s-channel γ or Z^0 , where σ falls like s^{-1} . Such cross sections are typically at most of order the $\mu^+\mu^-$ cross section, $\sigma_{\text{point}} \approx 10^{-37} \text{ cm}^2/\text{s}$, where $\sqrt{s}$ is in TeV units. The cross section for $e^+e^- \rightarrow W^+W^-$ at high energies is $[\pi\alpha_2^2 \ln(s/M_W^2)]/2s$ and also falls. $\alpha_2 = \alpha/\sin^2\theta_W$ where θ_W is the mixing angle. On the other hand, some cross sections are scaled not by s^{-1} but by M_W^{-2} , and do not decrease at all with energy! One example is $\sigma(e^+e^- \rightarrow \nu\bar{\nu}H) \approx [\alpha_2^3/16M_W^2] \ln(s/M_H^2) = 1.3 \ln(s/M_H^2) \times 10^{-37} \text{ cm}^2$ as $s \rightarrow \infty$ at fixed M_H ; technically this is only correct

for $M_H < 2M_W$ since additional contributions interfere for $M_H > 2M_W$. Another example is the production of W^+W^- by $\gamma\gamma \rightarrow W^+W^-$, where as $s \rightarrow \infty$, the cross section is $\sigma(e^+e^- \rightarrow e^+e^-W^+W^-) = (8\alpha^4/\pi M_W^2)(\ln^2 s/m_e^2) \int (dM^2/M^2) \ln(s/M^2)$, for integration over any fixed region in $M_{WW}^2 = M^2$. A similar result holds for our main process, the production of W pairs by $WW \rightarrow WW$ in the reaction $e^+e^- \rightarrow \nu\bar{\nu}W^+W^-$, but it is more complicated to express because of photon exchange diagrams, etc. The essential point is that because some processes scale as M_W^{-2} rather than as s^{-1} , they will be the dominant ones when $\sqrt{s} \geq 1$ TeV. Fig. 1 compares some cross sections. See also Fig. 22 for the cross section for the reaction $e^+e^- \rightarrow H^0Z^0$.

2. Calculation of WW Scattering and Heavy Higgs Production

It was first pointed out by Jones and Petkov [11] that heavy Higgs particles would be produced by WW fusion, as in fig. 2. This was rediscovered by Cahn and Dawson [12], and independently in ref. 13 in the context of a "real W " approximation. The structure functions needed to make calculations in the real W approximation were given in refs. 14,15; more recent discussions can be found in the report of the working group on the subject in ref. 7.

When $M_H > 2M_W$ the Higgs decays to WW as in fig. 3. This diagram is not gauge invariant, as emphasized in refs. 16 and 17 where the amplitudes for $WW \rightarrow WW$ with all the diagrams of fig. 4 included are given. Each of the Higgs exchange diagrams of fig. 4 gives an amplitude growing

as s , as does their sum. Each of the other three diagrams grows as s^2 , and their sum grows like s . The sum of all five diagrams gives a constant amplitude. On a narrow Higgs resonance the first contribution will dominate, but as Γ_H grows all diagrams become important.

Calculations in the real W approximation, where one calculates $WW \rightarrow WW$ for real W 's and then folds those amplitudes with the W structure functions to obtain the $e^+e^- \rightarrow \nu WW$ amplitudes, are important for two reasons. First, they have allowed approximate results to be obtained for the $e^+e^- \rightarrow \nu WW$ amplitudes some time before the availability of the complete perturbative amplitudes; the complete amplitudes are now being calculated by at least two groups [18]. Possibly more importantly, if WW scattering does really become strong (in this context see also ref. 17), $\sigma(WW \rightarrow WW)$ can be extracted from the data, and the polarization of the final W 's measured, using the real W formalism. Or, conversely, if some day there is a non-perturbative theory for $WW \rightarrow WW$ scattering, its predictions can be calculated and compared with experiment.

We have calculated fig. 5 with all $WW \rightarrow WW$ amplitudes included for the circle [19] in the real W approximation. In fig. 5 $\sqrt{s}$ is the total energy of the e^+e^- system and M_{WW} is the total energy of the WW system. Some of the results for angular distributions are shown in fig. 6. Note that the first diagram of fig. 4 gives an isotropic distribution so any angular dependence is due to the other contributions. In fig. 6, only longitudinal W 's are considered in the initial and final states. $z = \cos\theta$ where θ is the scattering angle of the W 's.

The cross sections $d\sigma/dM_{WW}$ for longitudinal W's in the initial and final states are shown in figs. 7-11 for various values of $\sqrt{s}$. The reader can then make estimates of what can be observed. Once $M_H > 2$ TeV in the Standard Model, $\Gamma_H \geq 4$ TeV so there will be no meaningful change in cross sections. On the other hand, as discussed in the introduction, calculations should be reliable to better than a factor of 2 for $M_H \leq 2$ TeV. The curves for $M_H = \infty$ are a way of estimating the kinds of effects that might be observed if a Higgs particle does not exist. We can conclude that if no light Higgs or resonant increase in the number of longitudinal W bosons is discovered, experiments could establish that $M_H \geq 2$ TeV and no high energy machine, whatever its energy or luminosity, could improve on that result. This question could be investigated at an accelerator with $\sqrt{s} \approx 4$ TeV and $\mathcal{L} > 1.5 \times 10^{33} \text{ cm}^{-2} \text{ sec}^{-1}$ or one with $\sqrt{s} \approx 10$ TeV and $\mathcal{L} > 2 \times 10^{33} \text{ cm}^{-2} \text{ sec}^{-1}$, or something in between, as can be seen from figs. 10 and 11. It will be difficult for the SSC to push M above about 1.5 TeV unless it can study either the W^+W^- mode where one $W \rightarrow q\bar{q}$ at luminosities above 10^{33} , or the Z^0Z^0 leptonic modes at even higher luminosities.

3. Backgrounds and Signatures

A number of remarks are relevant. Since e^+e^- colliders are cleaner than hadron ones, perhaps $H \rightarrow WW$ and WW scattering can be studied when both W 's decay to $q\bar{q}$. We have assumed that to be the case. Similarly, when $M_H < 2M_W$, we expect that the $H \rightarrow t\bar{t}$ mode can be exploited. There are backgrounds from direct WW and $t\bar{t}$ production, but presumably these can be removed by a cut on the total energy. There are backgrounds from $\gamma\gamma \rightarrow t\bar{t}$ and $\gamma\gamma \rightarrow WW$. We shall comment further on these below. The region $2m_t < M_H < 2M_W$ could be studied at any e^+e^- collider with $\sqrt{s} \geq 300$ GeV and $\mathcal{L} > 2 \times 10^{32}$. We have not considered effects of beam energy spread. Such energy resolution losses will not help, but we assume that they will not degrade the signal unduly.

Some backgrounds have been discussed previously. In ref. 20 it was argued that our main signal, from $e^+e^- \rightarrow \nu\bar{\nu}W^+W^-$ would be masked by initial photon radiation in $e^+e^- \rightarrow \gamma W^+W^-$. We have examined this problem and we believe that with appropriate cuts it can be eliminated. The essential physics point is that energy loss from an undetected γ is due to the escape of a single particle, while the undetected energy in $\nu\bar{\nu}$ is carried by two particles. Therefore, the missing mass peaks at zero for the γ , and at large masses for $\nu\bar{\nu}$. A quantitative example is shown in fig. 12. A cut on the missing mass can rather clearly separate these effects. The curve in fig. 12 which peaks at large missing mass is just $d\sigma(e^+e^- \rightarrow \nu\bar{\nu}W^+W^-)/dm$ at $\sqrt{s} = 2$ TeV and where W^+W^- is approximated by a Higgs particle of mass $M_H = 1.0$ TeV. The curve which peaks at zero missing mass attempts to indicate the effects of measuring resolution in the process $e^+e^- \rightarrow \gamma W^+W^-$.

The total cross section for this process is known [21] and at $\sqrt{s} = 2$ TeV is $\sigma_{\text{tot}}(e^+e^- \rightarrow \gamma W^+W^-) = 6.2 \times 10^{-2}$ pb. The missing photon should produce a delta function peak in m^2 at $m^2 = 0$, but assuming some measuring resolution Δm , an "effective" cross section $(d\sigma/dm)_{\text{eff}}$ can be defined such that $(d\sigma/dm)_{\text{eff}} \Delta m = \sigma_{\text{tot}}$. It is $(d\sigma/dm)_{\text{eff}}$ which is plotted. Clearly this curve is a hyperbola. If the missing mass resolution is 200 GeV, a vertical line is drawn at 200 GeV up to the dashed line and over to the vertical scale; the enclosed area is the photon contribution. A cut at 200 GeV separates the two contributions well.

The backgrounds from $\gamma\gamma \rightarrow WW$, as in the process depicted in fig. 13, have been calculated in ref. 22, with further calculations and discussion by Hikasa [23], who pointed out that the $\gamma\gamma \rightarrow WW$ background produces mainly transverse W's, while the signal of interest produces mainly longitudinal ones. If it is possible to determine the W polarization then the $\gamma\gamma \rightarrow WW$ background can be suppressed. In effect, what is being looked for is an increase in the fraction of longitudinal W's, f_L . To detect such an increase, f_L must not be too small at its maximum, or statistical and systematic uncertainties will mask the effect. For $\gamma\gamma \rightarrow WW$ the situation appears to be acceptable; the rate before any cuts are made is a few times above the expected signal for various $\sqrt{s}$, M_H . An angular cut such as $|\cos\theta| < 0.7$ reduces the $\gamma\gamma \rightarrow WW$ rate considerably, while reducing the signal essentially linearly. Thus the signal/background can be kept ≥ 1 before the polarization analysis. Examples at $\sqrt{s} = 2, 4$ TeV are shown in figs. 14, 15.

Another source of background that complicates the above analysis is W pairs from WW channels which do not couple to the Higgs. For example, $W_T W_T \rightarrow WW$ produces WW final states which do not show a significant Higgs signal, since the coupling of $W_T W_T$ to H is suppressed by $(M_W/M_H)^2$ relative to the $W_L W_L$ coupling. These final states must also not dilute the sample too much. There is need for concern because the luminosity of $W_T W_T$ is larger than that of $W_L W_L$ by a factor of $1/4 \ln^2[(p_T^{\max}/M_W)^2]$. We use $1/4 \ln^2[(M_{WW}^2/4M_W^2)]$ in our calculations. It turns out that this is not qualitatively a problem, though it must be included in quantitative studies.

Curves of $d\sigma(e^+e^- \rightarrow \nu\bar{\nu}W^+W^-)/dM_{WW}$ for various helicity combinations [19] are shown in fig. 16 for $\sqrt{s} = 2$ TeV and $M_H = 500$ GeV and in fig. 17 for $\sqrt{s} = 4$ TeV and $M_H = 1000$ GeV. The curves for the unpolarized cross section compared to that for longitudinal W 's in the initial and final states are compared in figs. 18,19. In figs. 20,21 we show the curves for a very large Higgs mass, $M_H \gg \sqrt{s}$. Notice that at large $\sqrt{s}$ and large M_{WW} the LLLL cross section becomes larger than the TTTT cross section. Here LLLL represents the process where all the gauge bosons are longitudinally polarized and TTTT represents the process where all the gauge bosons are transversely polarized.

One way to perform the polarization analysis [16,23] is to examine the W decay angular distribution for $W \rightarrow f_1 \bar{f}_2$, where $f_1 \bar{f}_2$ can be $l^\pm \nu$, $q\bar{q}$ or $l^+ l^-$. The longitudinal gauge bosons decay with a $\sin^2\theta$ distribution, and the transverse ones with a $(1 \mp \cos\theta)^2$ distribution for helicity ± 1 , where θ is the angle between the fermion direction and the W direction, evaluated in the W rest frame, and the distributions have been averaged

over the azimuthal angle. Since the fraction of W 's that are longitudinal has to be determined statistically, a detailed simulation including backgrounds and cuts and detection effects will be needed to determine how many events are needed to detect Higgs physics. In the next section we have normally used a requirement of 100 events/bin if $M_H \geq 600$ GeV.

The question of the charge states of the outgoing gauge bosons must also be considered. It may be difficult to distinguish between the decays of Z 's and W 's because they can both decay into two jets or there may be problems detecting both leptons in the Z decay. The process of $Z^0 Z^0$ fusion is suppressed relative to that of $W^+ W^-$ fusion by a factor of approximately 10 because of the coupling of the Z^0 to $e^+ e^-$ is about one third of that of the W^- to $e^- \nu$. However, the process $WW \rightarrow ZZ$ can certainly occur. In the LLLL channel where the amplitude can be approximated by the Higgs contribution one can estimate that $\sigma(W_L W_L \rightarrow W_L W_L) \approx 2\sigma(W_L W_L \rightarrow Z_L Z_L)$ because $\Gamma(H \rightarrow WW) \approx 2\Gamma(H \rightarrow ZZ)$. We expect $\sigma(W_T W_T \rightarrow W_T W_T)$ to be equal to or larger than $\sigma(W_T W_T \rightarrow Z_T Z_T)$ but a detailed calculation using the complete amplitudes for $WW \rightarrow ZZ$ should be made to examine this point, when these amplitudes are available. There is, of course, no two photon background in the production of $Z^0 Z^0$.

One should also estimate the contribution of WZ fusion. This contribution is down by about a factor of about 3 from the $W^+ W^-$ fusion process because of the Z^0 coupling to $e^+ e^-$. In addition there is neither a resonant Higgs process nor a two-photon background so we expect the dominant amplitude to be TTTT. The luminosity estimates of the next section will not be qualitatively affected by these additional processes and a refined esti-

mate will be made when the complete gauge boson scattering amplitudes become available.

4. What are the Machine Requirements to Study Higgs Physics?

We have tried to estimate what $\sqrt{s}$ and luminosity $\mathcal{L}$ are needed to study Higgs physics at a given mass scale. We have included $e^+e^- \rightarrow Z^0 H^0$ at lower energies where it is large enough to contribute. Representative graphs of $e^+e^- \rightarrow Z^0 H^0$ are shown in fig. 22. The form for the cross section is given, for example, in ref. 1. For high energies $\sigma(e^+e^- \rightarrow Z^0 H^0) \approx 1.2 \times 10^{-38} \text{ cm}^2/\text{s}$ where $\sqrt{s}$ is in TeV. In making our luminosity estimates we assume that we can use 20% of this cross section because of angular cuts and detection efficiencies.

Based on our calculations of cross sections for $e^+e^- \rightarrow H X$ including the correct gauge invariant W^+W^- amplitudes when $M_H > 2M_W$, and $e^+e^- \rightarrow Z^0 H^0$ as mentioned above, we have tried to guess how many events are needed to discover an effect. An angular cut keeping events for $|\cos\theta| < 0.7$ was used. With this cut the $\gamma\gamma$ background is effectively suppressed. Typically we required about 50 events to see an effect if $M_H \leq 600$ GeV, and 100 events/bin with bins of 100 to 200 GeV above that. The numbers are shown in fig. 23. Notice that $\mathcal{L}$ increases with increasing M_H , that it decreases with increasing $\sqrt{s}$ and that curves of constant $\mathcal{L}$ are essentially linear in the $\sqrt{s}$ vs M_H plot.

The numbers in fig. 23 clearly give the minimum $\mathcal{L}$ needed. If large detection inefficiencies are present they will go up. If significant cuts are needed to reduce backgrounds, again the required $\mathcal{L}$ will increase. Most importantly, we emphasize that with the luminosities shown, the effect could be detected, but not studied. To establish a Higgs boson at lower masses, it is important to see several decay modes and prove that they scale properly with mass. The $\gamma\gamma$ and gg modes are present at one loop but with observable rates if $M_H < 2m_t$, and are extremely interesting, involving radiative corrections and interference effects. At higher masses the dependence on W^+W^- vs Z^0Z^0 needs to be checked, and perhaps the $t\bar{t}$ mode can be found. If any other heavy particles exist they may couple strongly to a Standard Model Higgs. So probably a machine should be capable of eventually achieving about 50 times the luminosities in fig. 23, in order to be able to make definitive statements concerning the nature of heavy Higgs particles.

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Figure Captions

- Fig. 1 Various total cross sections in picobarns as a function of $\sqrt{s}$. The missing mass integral in $\sigma(e^+e^- \rightarrow e^+e^-W^+W^-)$ is performed from $M^2 = 4M_W^2$ to $M^2 = s$. The Higgs in $\sigma(e^+e^- \rightarrow \nu\bar{\nu}H)$ is taken to have a mass of 1.0 TeV.
- Fig. 2 Feynman diagram for $e^+e^- \rightarrow \nu\bar{\nu}H$ via W^+W^- fusion. The wavy lines are W's.
- Fig. 3 Feynman diagram for the production of a Higgs boson via W^+W^- fusion in e^+e^- collisions and the subsequent decay of the Higgs to W^+W^- . The wavy lines are W's.
- Fig. 4 Gauge invariant set of Feynman diagrams for W^+W^- scattering. The wavy lines denote gauge bosons and the dashed lines represent Higgs bosons.
- Fig. 5 Feynman amplitude for the process $e^+e^- \rightarrow \nu\bar{\nu}W^+W^-$ in the real W approximation.
- Fig. 6 $d\sigma(e^+e^- \rightarrow \nu\bar{\nu}W^+W^-)/dM_{WW}dz$ versus $z = \cos\theta$, for $\sqrt{s} = 2.0$ TeV and $M_H = 1.0$ TeV for various values of M_{WW} , in TeV. θ is the W scattering angle. Only longitudinal W's are considered in the initial and final states.

- Fig. 7 $d\sigma(e^+e^- \rightarrow \nu\bar{\nu}W^+W^-)/dM_{WW}$ versus M_{WW} for $\sqrt{s} = 0.5$ TeV for various values of M_H , in GeV. Only longitudinal W's are considered. There is an angular cutoff of $|z| < 0.9$.
- Fig. 8 As in Fig. 7 but with $\sqrt{s} = 1.0$ TeV.
- Fig. 9 As in Fig. 7 but with $\sqrt{s} = 2.0$ TeV.
- Fig. 10 As in Fig. 7 but with $\sqrt{s} = 4.0$ TeV.
- Fig. 11 As in Fig. 7 but with $\sqrt{s} = 10.0$ TeV.
- Fig. 12 Solid line: $d\sigma(e^+e^- \rightarrow \nu\bar{\nu}W^+W^-)/dm$ at $\sqrt{s} = 2.0$ TeV with W^+W^- approximated by a Higgs with $M_H = 1.0$ TeV. m is the missing mass of the $\nu\bar{\nu}$ system. Dashed line: $(d\sigma/dm)_{\text{eff}}$ where $(d\sigma/dm)_{\text{eff}}\Delta m = \sigma_{\text{tot}}(e^+e^- \rightarrow \gamma W^+W^-)$ at $\sqrt{s} = 2.0$ TeV. Δm represents a possible measuring resolution in the missing mass of the photon. See text for further comments.
- Fig. 13 Feynman amplitude for the process $e^+e^- \rightarrow e^+e^-W^+W^-$ in the real photon approximation.
- Fig. 14 The unpolarized $d\sigma(e^+e^- \rightarrow \nu\bar{\nu}W^+W^-)/dM_{WW}$ in the real W approximation for $\sqrt{s} = 2.0$ TeV, $M_H = 500$ GeV with curves for $d\sigma(e^+e^- \rightarrow e^+e^-W^+W^-)/dM_{WW}$ in the real photon approximation. The unpolarized cross section with no cuts, with a cut at $|z| < 0.7$ and the cross section for the production of longitudinal W's with no angular cuts are shown.

- Fig. 15 As in Fig. 14 but with $\sqrt{s} = 4$ TeV and $M_H = 1000$ GeV.
- Fig. 16 $d\sigma(e^+e^- \rightarrow \nu\bar{\nu}W^+W^-)/dM_{WW}$ where the subprocess $W_1W_2 \rightarrow W_3W_4$ has helicities $(H_1H_2H_3H_4)$. L=longitudinal, T=transverse. Here $\sqrt{s} = 2.0$ TeV and $M_H = 500$ GeV.
- Fig. 17 As in Fig. 16 but with $\sqrt{s} = 4$ TeV, $M_H = 1000$ GeV.
- Fig. 18 The unpolarized $d\sigma(e^+e^- \rightarrow \nu\bar{\nu}W^+W^-)/dM_{WW}$ compared with the cross section where all W's are longitudinally polarized.
- Fig. 19 As in Fig. 18 but with $\sqrt{s} = 4.0$ TeV, $M_H = 1000$ GeV.
- Fig. 20 Comparison of the unpolarized $d\sigma(e^+e^- \rightarrow \nu\bar{\nu}W^+W^-)/dM_{WW}$ with that where the W's have various polarizations. Here $\sqrt{s} = 2.0$ TeV, $M_H = \infty$.
- Fig. 21 As in Fig. 20 but with $\sqrt{s} = 4.0$ TeV.
- Fig. 22 $\sigma(e^+e^- \rightarrow H^0Z^0)$ for various values of M_H in GeV.
- Fig. 23 Minimum luminosity, $\mathcal{L}$, in units of $10^{33} \text{ cm}^{-2} \text{ sec}^{-1}$ to detect an effect in the Higgs sector, for a given $\sqrt{s}$ and M_H . $\mathcal{L}$ should probably be 50 times larger than these minimum values to study the associated physics.

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- Figure 2 -

- Figure 3 -

- Figure 4 -

- Figure 5 -

- Figure 6 -

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- Figure 9 -

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- Figure 18 -

- Figure 19 -

- Figure 20 -

- Figure 21 -

- Figure 22 -

- Figure 23 -