The superstring theory with an $E_8 \times E_8$ gauge group in 10 dimensions may lead, after compactification, to a four dimensional $E_6$ unified theory [1]. Two phenomenologically interesting features of $E_6$ superstring-inspired models are the possible existence of an extra neutral gauge boson ($Z'$) below the TeV scale [2], and of many new fermions [3]. With the high precision measurements that can be achieved in $e^+e^-$ colliders (SLC/LEP) at the $Z^0$ pole, the effects of the $Z'$ may be observed. If there is mixing between the two neutral bosons, the mass of the standard $Z^0$ will be lowered [4] and its couplings to fermions will be changed. Many authors [5] have shown that, although the standard model fits all low energy neutral current data, as well as the $W$ and $Z^0$ mass measurements, other models with one extra $Z$ are consistent with the data, for $M_{Z'}$ well below 1 TeV and small mixing between the two neutral gauge bosons. It has also been shown that the measurements of forward-backward and left-right asymmetries at the $Z^0$ pole could provide evidence of deviations from the standard model as long as the mixing angle, $\phi$, was greater than $\sim 0.01$ radians [6]. Here we will see that measurable changes in the cross section to $\mu$-pairs, in the $Z^0$ width, and in the branching ratios to fermions could also be observed.

Under $E_6$, the fermions belong to a 27 representation. In the 27, there are two new neutral singlets, denoted $N$ and $n$. The $N$ is a member of the 16 representation of $SO(10)$, often denoted $\nu^c$ in the literature. The $n$ is the member of the 27 of $E_6$ which is an $SO(10)$ singlet. These neutral singlets are possibly the only new fermions in the 27 representation of $E_6$ which are light enough to be produced at energies below 100 GeV [7]. We study the effects of these two light neutral singlets on the $Z^0$ width and cross section to $\mu$-pairs.

After briefly reviewing the notation, we will derive the constraints on the parameters of the model from neutral current data, the $W$ and $Z$ mass measurements, and the cross section for $p\bar{p} \rightarrow Z \rightarrow e^+e^-$. The results for the largest possible deviations from the standard model for the $Z^0$ width and the cross section to $\mu$-pairs will then be presented. Finally, we will discuss the possibility of directly observing a light $Z'$.

Consider the following breakdown of $E_6$:

$$E_6 = SO(10) \times U(1)_Y \rightarrow SU(5) \times U(1)_X \times U(1)_Y \ .$$

(1)
The leading order result from the summation by parts procedure. Thus, for each \( q \),

\[
\int_D \psi \phi = \int_D \phi \psi \quad \text{for } D \text{ open},
\]

where \( \psi \phi \) is the term with total width of

\[
\int_0^1 \phi \psi = \frac{1}{2} \int_D \int_0^1 \phi \psi.
\]

In the following, we will assume that this corresponds to the

\[
\frac{1}{d} \int D \phi \psi \quad \text{for } D \text{ bounded},
\]

where

\[
\int D \phi \psi = \left( \frac{1}{d} \int D \phi \right) \left( \frac{1}{d} \int D \psi \right).
\]

The effective Lagrangian for neutral current processes can thus be written in terms of

\[
\mathcal{L}^{\text{eff}} = \mathcal{L} - \lambda \phi \phi N - \lambda \phi \phi N - \lambda \phi \phi N.
\]
have masses below $M_Z/2$. We present the results in Table 2. The deviations from the standard model are defined as $\Delta \Gamma = \Gamma_{SM} - \Gamma$ and $\Delta \sigma = \sigma_{SM} - \sigma$. To obtain the standard model values ($\sigma_{SM}$ and $\Gamma_{SM}$) for each case, we use the same value of $M_Z$ as for the mixed case, but assume standard coupling to the fermions with $z = 0.225$. For $\sigma (e^+ e^- \rightarrow \mu^+ \mu^-)$, we find that, for several cases, shifts of several hundred pb from the standard result are possible. Thus, the cross section to $\mu$-pairs, which will be measured to 50 pb [12], is clearly a good test of $Z^0$-$Z'$ mixing. For example, at $\theta = 0^\circ$, very large deviations in the cross section to $\mu$-pairs can occur. This is fortunate, since the left-right asymmetry at this angle is somewhat less sensitive to the mixing [6]. The asymmetry depends on $g_L^2 - g_R^2$, while the cross section depends on $g_L^2 + g_R^2$, and at $0^\circ$, shifts in the left-handed coupling are the same as shifts in the right-handed coupling. Only if $\theta \approx 40^\circ$-$50^\circ$ will $\Delta \sigma$ be too small to be observable. The reason for this is that, at the pole, $\sigma \propto (g_L^2 + g_R^2)$, and around 45$^\circ$, the changes in $g_L$ for the muon are almost exactly compensated for by changes in $g_R$.

The effect of mixing on the measurement of the total width, while not as dramatic, could nevertheless be measurable. For some cases, we find deviations of about 100 MeV, which can be seen, assuming that an accuracy of 50 MeV is achievable [12].

We also consider the cases where there are three generations of light singlets $n$ or $N$. The presence of one of these singlets allows an additional decay channel to the $Z'$ and hence reduces the $Z' \rightarrow e^+ e^-$ branching ratio, which softens the $p \bar{p}$ constraint. Thus, a lighter $Z'$, and hence a larger mixing angle, is allowed. The results for $n$ are shown in Table 3, where we list only the few angles for which there is a difference in the amount of mixing allowed. Comparing Tables 2 and 3, we see that, for angles below $50^\circ$, there may be measurable differences between the case where no $n$'s are included and the case where we have included 3 generations of $n$. The reason for this is that, for an unmixed $Z'$, $n$ couples most strongly at $\theta = 0^\circ$. Thus, at small $\theta$, where large mixing angles $\phi$ are allowed, the $n$ coupling to the $Z^0$ is relatively large. For the other singlet, $N$, the results remain extremely close to the case where there are no extra neutrals, since $N$ couples strongly only around $90^\circ$, where the mixing $\phi$ is too small.

As noted above, the measurement of the total width is not expected to give as dramatic changes from the standard model as the cross section to $\mu$-pairs. On the other hand, the partial widths exhibit large deviations (the effects partially cancel when we sum over all fermions). Since the partial width is proportional to $M_Z$ for massless fermions, the quantity $\Gamma (Z^0 \rightarrow f \bar{f})/M_Z$ is a function only of the $Z^0$ couplings to fermions. To compare the maximal deviations from the standard model in the partial widths to fermions, we plot $\Gamma (Z^0 \rightarrow \mu^+ \mu^-)/M_Z$ as a function of $\theta$ in Fig. 1a. With an expected precision of $2\%$ [12] on the muon partial width, the cases where the maximum mixing occurs can easily be tested for all $\theta$, with the exception of $\sim 50^\circ$, where, as noted above, the muon coupling is almost standard. For the cases of maximal mixing, large deviations are also expected in the branching ratio to quarks (Fig. 1b). Although the measurement will be less precise, the region where $\theta$ is small could be tested (we have neglected the correction factor for the mass of the $b$-quark, since it changes the result by less than 1%). In Fig. 1c, we plot the partial width to three generations of neutrinos, both with and without three generations of light singlets $n$. The addition of a decay mode into neutral particles increases the partial width of the $Z^0$, but the effect of the extra $n$'s may be measurable only for small $\theta$, where large mixings are allowed.

We should emphasize that $\phi = 0$ is always consistent with the data, and in that limit, the standard model results at the $Z^0$ pole will be reproduced. For this reason, the $E_6$ models cannot be ruled out even if all data at the $Z^0$ pole is consistent with the standard model (although some theoretical reason for the absence of mixing must be given). Certain values of $\theta$ can be ruled out only if deviations are observed which are larger than the ones listed in the tables and in the figures.

Finally, it is interesting to note that a $Z'$ lighter than the $Z^0$ is consistent with the data for certain values of $\theta$. For each angle $\theta$, there is a range of values for $\phi$ where the cross section for $p \bar{p} \rightarrow Z' \rightarrow e^+ e^-$ becomes very small, even for $M_{Z'}$ below the $Z^0$ mass. When $\theta$ is around $10^\circ$, this range of values for $\phi$ is also allowed by the neutral current data. At $\theta = 10^\circ$, for $\phi = 0.3$, the smallest allowed mass for $Z'$ is 85.2 GeV. Such a light $Z'$ could be directly observed. However, the coupling of the electron to $Z'$ is very weak, so that the best signal can be seen in $e^+ e^- \rightarrow$ hadrons. In Fig. 2, we see that for $M_{Z'} = 85.2$ GeV and $\phi = 0.3$, the $Z'$ cross section is about one third of the $Z^0$ cross section, and is unmistakable.
with m^* = 40 GeV.

Table 1: Z (φ) charge for the standard particles and extra Z^8 neutrals.

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References


Acknowledgments
Table 3: Same as Table 2, but with 3 generations of $n$ included.

Figure Captions

Figure 1: Ratio of $\Gamma(Z^0 \rightarrow f\bar{f})$ to $M_Z$ as a function of $\theta$:

a) $Z^0 \rightarrow \mu^+\mu^-$ for $\phi^+$ (maximum positive mixing) (dot), and $\phi^-$ (maximum negative mixing) (dash).

b) $Z^0 \rightarrow c\bar{c}$ for $\phi^+$ (dash-dot) and $\phi^-$ (dot) and $Z^0 \rightarrow b\bar{b}$ for $\phi^+$ (dash) and $\phi^-$ (dash-dot).

c) $Z^0 \rightarrow 3$ generations of $\nu$'s for $\phi^+$ (dash) and $\phi^-$ (dash-dot) and $Z^0 \rightarrow 3$ generations of $\nu$'s and $n$'s for $\phi^+$ (dash-dot) and $\phi^-$ (dot).

The solid lines correspond to the standard model.

Figure 2: Cross section for $e^+e^- \rightarrow$ hadrons vs. centre-of-mass energy, $\sqrt{s}$, for $\theta = 10^\circ$, $\phi = 0.3$ and $M_{Z'} = 85.2$ GeV.