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CURRENT ALGEBRA AND SUPERSYMMETRY^{*)}

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A B S T R A C T

The implications of supersymmetry and chiral Ward identities in supersymmetric gauge theories are explored using current algebra methods, and a critical assessment is made of the relative merits of the current algebra and effective Lagrangian approaches. Using the Ward identities directly, simple derivations are given of several important properties of the condensates in supersymmetric QCD, and of the generalized Dashen formulae. The corrections to these results in the presence of explicit, soft supersymmetry breaking are calculated. A concise formula is presented for the mass splittings within pseudo-Goldstone multiplets induced by soft supersymmetry breaking terms. It is shown that if this supersymmetry breaking is the $\theta = 0$ component of a chiral superfield, the supertrace of the pseudo-Goldstone masses vanishes. Using current algebra reduction formulae, the pseudo-Goldstone masses are calculated in supersymmetric Yang-Mills theory, and supersymmetric QCD for $N_F < N_C$ and $N_F = N_C$. Some differences are found between the current algebra and effective Lagrangian predictions, and their possible origins are discussed.

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1. INTRODUCTION

Current algebra and chiral effective Lagrangians enjoy enormous success in QCD in describing the spectrum and interactions of the pseudo Goldstone bosons of spontaneously broken chiral symmetry¹⁾. By current algebra, we mean here the method of using chiral Ward identities together with approximate techniques such as pole dominance, reduction of soft pions from external states, etc., directly on matrix elements to deduce relations of physical interest.

In theory, these approaches are completely equivalent, since both contain the same symmetry information and involve the same dynamical assumptions and approximations. In practice, however, they can be complementary. Effective Lagrangians provide a straightforward and, for subtle questions, more reliable means of calculation. However these calculations can often be long and uninformative, this being especially true of the supersymmetric examples considered here. The current algebra method, on the other hand, is quick and transparent, and frequently allows new relations to be seen which could escape detection in the effective Lagrangian approach.

In supersymmetric QCD (SQCD), effective Lagrangians²⁾ have played a major role in illuminating the remarkably rich and surprising low-energy structure. In this paper, we develop the current algebra approach. The supersymmetric and chiral Ward identities are exploited systematically to give simple and unified derivations of many key results in SQCD. Then soft supersymmetry (SUSY) breaking is introduced, and a number of new results are found, including a simple formula for the mass splittings induced between members of a pseudo Goldstone multiplet.

Our interest in soft SUSY breaking is partially motivated by applications of SQCD to composite models of quarks and leptons³⁾. The known light fermions are identified as the SUSY partners of the Goldstone bosons due to spontaneous chiral symmetry breaking in a confining gauge theory at a scale above 1 TeV, and/or as the 't Hooft fermions required by the anomaly matching conditions for an unbroken chiral symmetry. Explicit SUSY breaking terms of order much less than the confinement scale Λ are introduced by hand to obtain a realistic spectrum, their ultimate origin being at present unknown. Clearly in this situation it would be useful to have an easy way of determining the effect of soft SUSY breaking on the masses of the light particles, in particular whether a given breaking term raises the boson masses relative to the fermions. At the moment this involves extensive

calculations with effective Lagrangians⁴⁾, in which the systematic features of the results are hard to see. Using current algebra, we discover a concise mass formula which answers this need. In addition we find that the mass supertrace for a pseudo Goldstone multiplet vanishes to leading order when the SUSY breaking term is the lowest component of a chiral superfield.

At this point it is useful to recall some properties of SQCD. One surprising feature is that unlike conventional QCD, SQCD is very sensitive to the relative number of flavours N_F and colours N_C . For $N_F < N_C$, certain VEV's of squark bilinears tend to infinity in the limit of small (s)quark mass m . This is the now well-known phenomenon of the "runaway" vacuum in $N_F < N_C$ SQCD⁵⁾. For $N_F = N_C$, on the other hand, these VEV's are of $O(\Lambda^2)$, as would be expected for the condensates in a conventional confining theory.

It is important for our discussion that SQCD exhibits complementarity⁶⁾, that is, there is no distinction between the Higgs and confining phases. The theory may be analysed either as a weak-coupling Higgs model with spontaneously broken gauge symmetry, or equivalently as a confining colour gauge theory. We may choose to describe the theory in whichever picture is more convenient. Here we shall analyse SQCD for all N_F, N_C in the confining picture using gauge invariant composite operators as the fields describing the low-energy spectrum.

Another important feature of SQCD is the existence of an anomaly-free abelian chiral symmetry $U(1)_X$, a linear combination of the conventional anomalous axial $U(1)_A$ and the R-symmetry $U(1)_R$ of the superconformal algebra. This symmetry is special to SQCD and, as we shall see, is directly responsible for many of the unusual properties of the theory. It is especially challenging to current algebra since, when spontaneously broken, it gives rise to a light flavour singlet pseudo Goldstone multiplet which can couple non-trivially to explicit SUSY and chiral symmetry breaking operators.

The text is organised as follows. In section 2 the SUSY and chiral Ward identities are introduced and explained. Section 3 contains physical applications, and we derive generalised Dashen formulae⁷⁾⁻⁹⁾, the anomalous relation between the gluino and squark condensates found by Konishi¹⁰⁾, and the m dependence of condensates of chiral superfields¹¹⁾. Although these results are not new the derivations here are particularly simple and emphasise the role of the $U(1)_X$ symmetry. In section 4, the reduction formulae are presented which allow a pseudo Goldstone

particle to be removed from an external state in a matrix element, and the condition for this approximation to be valid is specified.

Soft SUSY breaking is introduced in section 5. In section 6 the corrections to the condensate relations due to these breaking terms are found and compared with the equivalent effective Lagrangian results. Our mass splitting formula for pseudo Goldstone multiplets is presented in section 7. Using a combination of reduction formulae, Dashen formulae and condensate relations it is shown how to express these mass differences in terms of physical masses and symmetry breaking parameters only. Then, in section 8, the formula is applied to SUSY Yang-Mills theory, $N_F < N_C$ SQCD and $N_F = N_C$ SQCD, and the results compared with those from the effective Lagrangians. In some cases discrepancies are found, and the possible origins of these are discussed. Section 9 contains our conclusions.

There are two appendices, the first describing the SUSY Ward identities in component form using a functional formalism, and the second developing the chiral Ward identities necessary to find the broken SUSY corrections to the Dashen formulae. In fact it is not possible to carry this programme to completion, and we explain the difficulties encountered in this approach and comment on earlier attempts appearing in the literature¹²⁾.

2. WARD IDENTITIES

2.1. Supersymmetry Ward identities

The Ward identity for unbroken SUSY is

$$i \partial^\mu Q_{\mu\alpha} + \omega_\alpha^Q \Gamma[\Phi] = 0 \quad (2.1)$$

where $Q_{\mu\alpha}$ is the supersymmetry current, and ω^Q is the variation operator $-i \delta_\alpha^Q \Phi \frac{\delta}{\delta \Phi}$ where $\delta_\alpha^Q \Phi = i [Q_\alpha, \Phi]$. The (anti-) commutators of components of real and chiral superfields with the SUSY charge Q_α are given in Appendix A. (For notations, conventions and further explanation of the formal methods used here, see refs. 13) and 8).

To use SUSY in current algebra we need to know not only how the component fields transform, but also how the states transform. These transformations are deduced from the observation that the state corresponding to a given field transforms as the functional derivative with respect to that field. For a chiral superfield $\phi(x, \theta) = \phi + \theta^\alpha \psi_\alpha + \theta^2 F$ we easily find

$$\frac{\delta}{\delta \phi(x, \theta)} = -\frac{1}{4} \left(\frac{\delta}{\delta F} - 2\theta^\alpha \frac{\delta}{\delta \psi^\alpha} + \theta^2 \frac{\delta}{\delta \phi} \right) \quad (2.2)$$

and the transformations of $\frac{\delta}{\delta F}$, $\frac{\delta}{\delta \psi^\alpha}$ and $\frac{\delta}{\delta \phi}$ are therefore known. Identifying $|\phi\rangle \sim \frac{\delta}{\delta \phi}$ etc. we deduce the transformation rules for the states:

$$\begin{aligned} Q_\alpha |F\rangle &= 2i |\psi_\alpha\rangle & Q_\alpha |F^*\rangle &= 0 \\ Q_\alpha |\psi_\beta\rangle &= -i \epsilon_{\alpha\beta} |\phi\rangle & Q_\alpha |\bar{\psi}_{\dot{\alpha}}\rangle &= -2i k_{\alpha\dot{\alpha}} |F\rangle \\ Q_\alpha |\phi\rangle &= 0 & Q_\alpha |\phi^*\rangle &= 4i k_{\alpha\dot{\alpha}} |\bar{\psi}^{\dot{\alpha}}\rangle \end{aligned} \quad (2.3)$$

The state $|F\rangle$ is not independent. For a chiral superfield of mass m the auxiliary field equation of motion $F = -m\phi^*$ allows us to deduce $|F\rangle$

$$|F\rangle = -m^{-1} |\phi^*\rangle \quad |F^*\rangle = -m^{-1} |\phi\rangle \quad (2.4)$$

Also, the on-shell fermion states are related:

$$k_{\alpha\dot{\alpha}} |\bar{\psi}^{\dot{\alpha}}\rangle = -\frac{1}{2} m |\psi_\alpha\rangle \quad k_{\alpha\dot{\alpha}} |\psi^\alpha\rangle = \frac{1}{2} m |\bar{\psi}_{\dot{\alpha}}\rangle \quad (2.5)$$

There are thus two independent bosonic and fermionic states corresponding to a chiral superfield.

It is convenient to define the even and odd parity scalar states $|\sigma\rangle = \frac{1}{\sqrt{2}} (|\phi\rangle + |\phi^*\rangle)$ and $|\pi\rangle = \frac{i}{\sqrt{2}} (|\phi\rangle - |\phi^*\rangle)$, for which

$$\begin{aligned} Q_\alpha |\sigma\rangle &= \sqrt{2} i m |\psi_\alpha\rangle & Q_\alpha |\pi\rangle &= \sqrt{2} m |\psi_\alpha\rangle \\ Q_\alpha |\psi_\beta\rangle &= -\frac{i}{\sqrt{2}} \epsilon_{\alpha\beta} (|\sigma\rangle - i |\pi\rangle) \end{aligned} \quad (2.6)$$

We now use these transformation rules to derive various SUSY relations amongst decay constants, i.e. matrix elements such as $\langle 0 | (T_0 - \bar{T}_0) | \pi \rangle$, and vertex functions, e.g. $\langle \psi^\alpha | T_\alpha | \pi \rangle$. For the decay constants of a chiral superfield $T(x, \theta) = T_0 + \theta^\alpha T_\alpha + \theta^2 T_F$ it is not difficult to show,

$$\begin{aligned} \langle 0 | T_0 | \phi^* \rangle &= 0 & \langle 0 | T_F | \phi \rangle &= 0 \\ \langle 0 | T_F | \phi^* \rangle &= -m \langle 0 | T_0 | \phi \rangle \\ \langle 0 | T_\alpha | \psi_\beta \rangle &= -\epsilon_{\alpha\beta} \langle 0 | T_0 | \phi \rangle \end{aligned} \quad (2.7)$$

In terms of σ, π states:

$$\begin{aligned} \langle 0 | T_F - \bar{T}_F | \pi \rangle &= m \langle 0 | T_0 - \bar{T}_0 | \pi \rangle \\ \langle 0 | T_F + \bar{T}_F | \sigma \rangle &= -m \langle 0 | T_0 + \bar{T}_0 | \sigma \rangle \\ \langle 0 | T_\alpha | \psi_\beta \rangle &= \frac{i}{\sqrt{2}} \epsilon_{\alpha\beta} \langle 0 | T_0 - \bar{T}_0 | \pi \rangle = -\frac{1}{\sqrt{2}} \epsilon_{\alpha\beta} \langle 0 | T_0 + \bar{T}_0 | \sigma \rangle \end{aligned} \quad (2.8)$$

Now consider the vertex functions for components of a real, even-parity superfield $C(x, \theta, \bar{\theta}) = C_0 + \theta^\alpha C_\alpha + \bar{\theta}_{\dot{\alpha}} \bar{C}^{\dot{\alpha}} + \dots$. We can derive many identities, the most important being:

$$\begin{aligned} \langle \psi_\beta | C_\alpha | \phi \rangle - \epsilon_{\alpha\beta} \langle \phi | C_0 | \phi \rangle &= 0 \\ \langle \psi_\beta | C_\alpha | \phi^* \rangle - \epsilon_{\alpha\beta} \langle \phi | C_0 | \phi^* \rangle - 2m \langle \psi_\beta | C_0 | \psi_\alpha \rangle &= 0 \\ \langle \phi | C_{\alpha\dot{\alpha}} | \phi^* \rangle + 2k_{\alpha\dot{\alpha}} \langle \phi | C_0 | \phi^* \rangle - 2m^2 \langle \bar{\psi}_{\dot{\alpha}} | C_0 | \psi_\alpha \rangle &= 0 \end{aligned} \quad (2.9)$$

Introducing the notations,

$$\begin{aligned} \langle \pi | C_0 | \pi \rangle &= C_{\pi\pi} & \langle \sigma | C_0 | \sigma \rangle &= C_{\sigma\sigma} \\ \langle \psi_\beta | C_0 | \psi_\alpha \rangle &= -\frac{1}{2} \epsilon_{\alpha\beta} C_{\psi\psi} \\ \langle \psi_\beta | C_\alpha | \pi \rangle &= \frac{1}{\sqrt{2}} \epsilon_{\alpha\beta} C_{\psi\pi} & \langle \psi_\beta | C_\alpha | \sigma \rangle &= \frac{1}{\sqrt{2}} \epsilon_{\alpha\beta} C_{\psi\sigma} \end{aligned} \quad (2.10)$$

we can rewrite the first two of eqs. (2.9) as

$$\begin{aligned} i C_{\psi\pi} &= C_{\pi\pi} - m C_{\psi\psi} \\ C_{\psi\sigma} &= C_{\sigma\sigma} - m C_{\psi\psi} \end{aligned} \quad (2.11)$$

Notice that $C_{\pi\sigma} = C_{\sigma\pi} = 0$ by parity.

In the same way, we find relations amongst the vertex functions of the chiral superfield $T(x, \theta)$, e.g.:

$$\begin{aligned}
 \langle \phi^* | T_0 | \phi^* \rangle &= 0 & \langle \phi | T_F | \phi \rangle &= 0 \\
 \langle \psi_\beta | T_\alpha | \phi \rangle - \epsilon_{\alpha\beta} \langle \phi | T_0 | \phi \rangle &= 0 \\
 \langle \psi_\beta | T_\alpha | \phi^* \rangle &= 0 \\
 \langle \psi_\beta | T_0 | \psi_\alpha \rangle + \frac{1}{2} m^{-1} \epsilon_{\alpha\beta} \langle \phi | T_0 | \phi^* \rangle &= 0 \quad (2.12)
 \end{aligned}$$

With the notation (2.10), we find here

$$\begin{aligned}
 i T_{\psi\pi} &= T_{\pi\pi} - T_{\sigma\sigma} = T_{\pi\pi} + i T_{\sigma\pi} - m T_{\psi\psi} \\
 T_{\psi\sigma} &= T_{\sigma\sigma} - T_{\pi\pi} = T_{\sigma\sigma} - i T_{\pi\sigma} - m T_{\psi\psi} \quad (2.13)
 \end{aligned}$$

In particular we have the very important result (from the third of eqs. (2.12))

$$T_{\psi\sigma} + i T_{\psi\pi} = 0 \quad (2.14)$$

This result is the basis of a vanishing supertrace mass formula for pseudo Goldstone multiplets to be derived in section 7. Notice that the analogous expression for the real superfield vertex functions does not vanish:

$$C_{\psi\sigma} + i C_{\psi\pi} = C_{\pi\pi} + C_{\sigma\sigma} - m C_{\psi\psi} \quad (2.15)$$

2.2 Chiral Ward identities

The Lagrangian for SQCD is

$$\begin{aligned}
 \mathcal{L}_{SQCD} &= -\frac{1}{24} \int d^4x d\theta \quad F^\alpha F_\alpha + \text{h.c.} \\
 &+ \frac{1}{24} \int d^4x d\theta d\bar{\theta} \quad (\bar{Q}_+ e^V Q_+ + Q_- e^{-V} \bar{Q}_-) \\
 &- \frac{1}{4} m \int d^4x d\theta \quad \text{Tr } Q_+ Q_- + \text{h.c.} \quad (2.16)
 \end{aligned}$$

where $Q_{\pm}^i = (s_{\pm} + \theta^{\alpha} q_{\pm\alpha} + \theta^2 F_{\pm})^i$ is a chiral superfield containing the quark and squark fields, i, j are flavour indices, and for simplicity we have taken all the quark masses equal, i.e. $m^{ij} = m \delta^{ij}$. V is the gauge vector superfield and F^{α} the field strength.

The gauge invariant composite operators which we expect to be relevant to the light mass spectrum of SQCD are the chiral superfields $S = g^2/32\pi^2 F^{\alpha} F_{\alpha}$ and $T^{ij} = Q_{+}^i Q_{-}^j$. (We will also use the notation $T^a = 2 \text{Tr} G^a T$ and $T^0 \equiv \text{Tr} T$, where G^a , $a = 1, \dots, N_F^2 - 1$, are flavour $SU(N_F)$ matrices with $\text{Tr} G^a G^b = \frac{1}{2} \delta^{ab}$).

Including anomalous symmetries, the global invariance of $\mathcal{L}_{\text{SQCD}}$ for $m = 0$ is $SU(N_F)_L \times SU(N_F)_R \times U(1)_V \times U(1)_A \times U(1)_R$. The chiral Ward identities of the theory are as follows. The non-abelian $SU(N_F)$ chiral symmetry with current A_{μ}^a is described by

$$i \partial^{\mu} A_{\mu}^a - m (T_F^a - \bar{T}_F^a) + \omega^a \Gamma = 0 \quad (2.17)$$

There are two anomalous axial symmetries, the usual $U(1)_A$ and an additional $U(1)_R$ symmetry which completes the superconformal algebra. $U(1)_R$ does not commute with SUSY. The $U(1)_A$ and $U(1)_R$ Ward identities are

$$i \partial^{\mu} A_{\mu} - n_A(T) m (T_F^0 - \bar{T}_F^0) + a (S_F - \bar{S}_F) + \omega^A \Gamma = 0 \quad (2.18)$$

$$i \partial^{\mu} R_{\mu} - (n_R(T) + 2) m (T_F^0 - \bar{T}_F^0) + r (S_F - \bar{S}_F) + \omega^R \Gamma = 0 \quad (2.19)$$

The weights and anomaly coefficients F^2 are $n_A(T) = 2$, $n_A(S) = 0$, $n_R(T) = -\frac{4}{3}$, $n_R(S) = -2$, $a = 2 N_F$, $r = -\frac{2}{3} (3N_C - N_F)$. The infinitesimal variations for components of a chiral superfield follow from $\delta^A T(x, \theta) = i n_A(T) T(x, \theta)$ and $\delta^R T(x, \theta) = i (n_R(T) + \theta \frac{\partial}{\partial \theta}) T(x, \theta)$. The corresponding finite transformations are $T(x, \theta) \xrightarrow{U(1)_A} e^{i n_A(T) \alpha} T(x, \theta)$ and $T(x, \theta) \xrightarrow{U(1)_R} e^{i n_R(T) \alpha} T(x, \theta e^{i \alpha})$.

In fact it is more convenient to work with different linear combinations of $U(1)_A$ and $U(1)_R$. We define $U(1)_X$ to be the combination which is anomaly-free, and $U(1)_Y$ to be the anomalous symmetry left unbroken by the mass term. That is,

$$i \partial^{\mu} X_{\mu} - (n_X(T) + 2) m (T_F^0 - \bar{T}_F^0) + \omega^X \Gamma = 0 \quad (2.20)$$

$$i \partial^\mu \gamma_\mu + \gamma (S_F - \bar{S}_F) + \omega \gamma \tau = 0 \quad (2.21)$$

where $n_X(T) = 2 N_F^{-1} (N_C - N_F)$, $n_X(S) = -2$, $n_Y(T) = -2$, $n_Y(S) = -2$ and $y = -2N_C$.

One of the radically new features of SQCD is the appearance of this anomaly-free flavour singlet axial symmetry $U(1)_X$. Many of the interesting and peculiar properties of SQCD have their origins in this symmetry and the Ward identity (2.20).

The Ward identities determine the effective Lagrangian describing the low momentum behaviour of SQCD. The construction is discussed in some detail from the present point of view in ref. 13). We have to solve the zero momentum form of the Ward identities (2.17), (2.20) and (2.21) for $\mathcal{L}_{\text{eff}}(S, T)$, which we split into a D term (the complementary functional) obeying the homogeneous identities and an F term (the particular integral) which satisfies the full inhomogeneous identities. Denoting the F term by $f(S, T)$ it must satisfy the $U(1)_X$ Ward identity,

$$-\frac{1}{4} \int d^4x d\theta \left(-i \delta^X_S \frac{\partial f}{\partial S} - i \delta^X_T \frac{\partial f}{\partial T} - (n_X(T) + 2) m \tau_\tau T \right) = 0 \quad (2.22)$$

This implies,

$$n_X(S) S \frac{\partial f}{\partial S} + n_X(T) T \frac{\partial f}{\partial T} + 2f - (n_X(T) + 2) m \tau_\tau T = 0 \quad (2.23)$$

with the solution,

$$f(S, T) = S \log (S^{N_C - N_F} \det T) + c S + m \tau_\tau T \quad (2.24)$$

with c arbitrary. Clearly $f(S, T)$ satisfies the non-abelian Ward identity (2.17). If we also impose the anomalous $U(1)_Y$ identity,

$$n_Y(S) S \frac{\partial f}{\partial S} + n_Y(T) T \frac{\partial f}{\partial T} + 2f + \gamma S = 0 \quad (2.25)$$

then c is fixed to be $-2N_C$. The effective Lagrangian for SQCD is therefore,

$$\begin{aligned} \mathcal{L}_{\text{eff}}(S, T) = & \frac{1}{24} \int d\alpha d\theta d\bar{\theta} \left(\tilde{d}(S, \bar{S}) + d(T, \bar{T}) \right) \\ & - \frac{1}{4} \int d\alpha d\theta \left\{ S \left(\log(S^{N_C - N_F} \det T) - 2N_C \right) + m \text{Tr} T \right\} \\ & + \text{h.c.} \end{aligned} \quad (2.26)$$

The D terms are not so constrained. They must of course be flavour singlets, and to satisfy the $U(1)_X$ Ward identity $d(S, \bar{S})$ and $d(T, \bar{T})$ must be $n_X = 0$ combinations of $S, \bar{S}$ or $T, \bar{T}$. So we may have $d(S, \bar{S}) = (S \bar{S})^P$ and $d(T, \bar{T}) = a(\text{Tr} T \bar{T})^Q + b(\text{Tr}(T \bar{T})^2)^R + \dots$ for example. Where it is necessary to specify the D terms for an explicit calculation, we make one of two simple choices - "canonical" kinetic terms, viz. $\tilde{d} \sim S \bar{S}$, $d \sim \text{Tr} T \bar{T}$ or "conformal" kinetic terms, viz. $\tilde{d} \sim (S \bar{S})^{1/3}$, $d \sim (\text{Tr} T \bar{T})^{1/2}$.

It is also sometimes convenient to consider the effective Lagrangian for T only. In that case we must solve eq. (2.23) for $f(T)$ omitting the S dependent term. This gives the effective Lagrangian

$$\mathcal{L}_{\text{eff}}(T) = \frac{1}{24} \int d\alpha d\theta d\bar{\theta} d(T, \bar{T}) - \frac{1}{4} \int d\alpha d\theta \left\{ \gamma (\det T)^{-\frac{1}{N_C - N_F}} + m \text{Tr} T \right\} + \text{h.c.} \quad (2.27)$$

provided $N_F < N_C$. The case $N_F = N_C$ is special. Since $n_X(T)$ is then zero, only the mass term survives in the F term in $\mathcal{L}_{\text{eff}}(T)$. This means that $\mathcal{L}_{\text{eff}}(T)$ contains no information about the axial anomaly. Since we believe the anomaly to be important for all N_F, N_C we always use $\mathcal{L}_{\text{eff}}(S, T)$ for $N_F = N_C$ SQCD, although for simplicity we often just consider $\mathcal{L}_{\text{eff}}(T)$ when $N_F < N_C$.

The combination of the SUSY and chiral Ward identities presented here provides a powerful tool for investigating the non-perturbative properties of SQCD. In the next section, we explore some of their immediate consequences.

3. APPLICATIONS OF THE SUPERSYMMETRY AND CHIRAL WARD IDENTITIES

3.1 Generalised Dashen formulae

As is now well-known, there are two Dashen formulae for the pions in SQCD. The first is the familiar formula¹⁴⁾ relating $F_{\pi\pi}^2 m_\pi^2$ to the squark condensate

$m \langle q_+ q_- + \bar{q}_+ \bar{q}_- \rangle$. The second, originally proposed in ref. 7) is bilinear in the pion mass and relates $F_\pi^2 m_\pi$ to a squark condensate $m \langle s_+ s_- + s_+^* s_-^* \rangle$. In this section we outline the derivation of these formulae, following Ref. 8).

We restrict ourselves to the flavour singlet sector, and construct from eq. (2.20) the $U(1)_X$ Ward identity,

$$\begin{aligned} \langle 0 | i \partial^\mu X_\mu (T_0 - \bar{T}_0) | 0 \rangle &= (n_X(T) + 2) m \langle 0 | (T_F - \bar{T}_F) (T_0 - \bar{T}_0) | 0 \rangle \\ &+ i n_X(T) \langle 0 | T_0 + \bar{T}_0 | 0 \rangle = 0 \end{aligned} \quad (3.1)$$

where the final term has been simplified using the fundamental property of the symmetry operator ω^X . (From now on, we simply write T_0, T_F for T_0^0, T_F^0 etc. since this should cause no confusion.) At zero momentum

$$m \langle 0 | (T_F - \bar{T}_F) (T_0 - \bar{T}_0) | 0 \rangle = i n_X(T) (n_X(T) + 2)^{-1} \langle T_0 + \bar{T}_0 \rangle \quad (3.2)$$

Similarly,

$$m \langle 0 | (T_F - \bar{T}_F) (T_F - \bar{T}_F) | 0 \rangle = i \langle T_F + \bar{T}_F \rangle \quad (3.3)$$

Provided $n_X(T) \neq 0$, i.e. $N_F \neq N_C$, eq. (3.2) indicates that if the condensate $\langle T_0 + \bar{T}_0 \rangle$ forms then $U(1)_X$ is spontaneously broken and there exists a light flavour singlet pseudo Goldstone boson (the η) coupling to $(T_0 - \bar{T}_0)$. Defining the decay constant as usual by $\langle 0 | X_\mu | \eta \rangle = i k_\mu F_\eta$ we have immediately $m \langle 0 | (T_F - \bar{T}_F) | \eta \rangle = i (n_X(T) + 2)^{-1} F_\eta m_\eta^2$. Then, using the SUSY identity (2.8), we find $m \langle 0 | (T_0 - \bar{T}_0) | \eta \rangle = i (n_X(T) + 2)^{-1} F_\eta m_\eta$. Saturating the Green's function in eq. (3.2) with the η ("pole dominance") leads to the linear Dashen formula,

$$F_\eta^2 m_\eta = n_X(T) (n_X(T) + 2) m \langle T_0 + \bar{T}_0 \rangle \quad (3.4)$$

$$= 4 N_C N_F^{-2} (N_C - N_F) m T_F \langle s_+ s_- + s_+^* s_-^* \rangle_{ij} \quad (3.5)$$

The conventional quadratic Dashen formula follows from the identity (3.3). However, the derivation is clearly subtle since for unbroken SUSY $\langle T_F + \bar{T}_F \rangle = 0$. The resolution of this puzzle requires a very careful treatment of auxiliary fields (in general, of the properties of insertions of gauge invariant composite operators which vanish using the equations of motion) and is discussed in ref. 8).

The result is that saturating eq. (3.3) with the η gives

$$F_\eta^2 m_\eta^2 = -2 N_C^2 N_F^{-2} m \text{Tr} \langle q_+ q_- + \bar{q}_+ \bar{q}_- \rangle_{ij} \quad (3.6)$$

For unbroken SUSY, $\langle T_F + \bar{T}_F \rangle = 0$ implies that the quark condensate is equal to a second squark condensate, and we find

$$F_\eta^2 m_\eta^2 = 8 N_C^2 N_F^{-2} m^2 \text{Tr} \langle s_+ s_+^* + s_- s_-^* \rangle_{ij} \quad (3.7)$$

$$= 2 (n_x(\tau) + 2)^2 m^2 \langle C_0 \rangle \quad (3.8)$$

in the notation of section 5 and Appendix B.

Notice that the conventional Dashen formula (3.6) is valid whether or not SUSY is broken. In contrast, the linear formula (3.5) holds only for unbroken SUSY. If SUSY is unbroken, these formulae apply also to the scalar (ρ) and fermion(χ) partners of the η , which of course lie in a chiral supermultiplet.

For $N_F = N_C$ the derivation of these formulae breaks down. In particular, from eq. (3.2) we see that there is no light particle coupling to $(T_0 - \bar{T}_0)$ or $(T_F - \bar{T}_F)$. In this case, the η has a mass of $O(\Lambda)$ and there is no flavour singlet pseudo Goldstone boson. This is confirmed by explicit calculations with $\mathcal{L}_{\text{eff}}(S, T)$ for $N_F = N_C$.

Similar Dashen formulae hold in the flavour singlet sector. Assuming for simplicity ^{F3} that the chiral symmetry $SU(N_F)_L \times SU(N_F)_R$ is spontaneously broken to flavour $SU(N_F)$, and defining the pion decay constant by $\langle 0 | A_\mu^a | \pi^b \rangle = i k_\mu F_\pi \delta^{ab}$,

$$F_\pi^2 (m_\pi)^{ab} = 2 N_F^{-1} \delta^{ab} m \text{Tr} \langle s_+ s_- + s_+^* s_-^* \rangle_{ij} \quad (3.9)$$

$$F_\pi^2 (m_\pi^2)^{ab} = - N_F^{-1} \delta^{ab} m \text{Tr} \langle q_+ q_- + \bar{q}_+ \bar{q}_- \rangle_{ij} \quad (3.10)$$

In conventional QCD the quark condensate is assumed to be $O(\Lambda^3)$ with F_π of $O(\Lambda)$. This leads to the well-known relation $m_\pi^2 = O(m)$. In SQCD the situation is less clear. Neither F_π nor the condensates can be assumed to be independent of m , but in fact have an m dependence which depends critically on the number of

of flavours N_F and colours N_C . The m dependence of the squark condensate in eqs. (3.5), (3.9) may be deduced from the $U(1)_X$ Ward identity (see section 3.3). However, the m dependence of the quark condensate is not determined by such symmetry considerations, and so it is not possible to separate unambiguously the m dependences of $m_{\pi(\gamma)}$ and $F_{\pi(\gamma)}$ from the Dashen formulae alone.

3.2. Anomalous condensate relation

One particularly striking result in SQCD is the relation discovered by Konishi¹⁰⁾ between the squark condensate $m \text{Tr} \langle s_+ s_- \rangle_{ij}$ and the gluino condensate $\langle \lambda^\alpha \lambda_\alpha \rangle$. It is remarkable because a naive use of SUSY, viz. commuting the SUSY charge through the individual fields comprising a composite operator, would suggest that both $m \text{Tr} \langle s_+ s_- \rangle_{ij} = 0$ and $\langle \lambda^\alpha \lambda_\alpha \rangle = 0$. The relation has since proved extremely important in instanton analyses of supersymmetric gauge theories¹⁸⁾⁻²⁰⁾, and is a crucial ingredient in proofs of the non-perturbative violation of the non-renormalisation theorem in SQCD and of the existence of dynamical SUSY breaking in certain chiral gauge theories²⁰⁾.

Konishi's derivation made use of a careful gauge-invariant point-split regularisation of the composite operators involved. There are then extra contributions to the naive SUSY identity coming from commuting the SUSY charge through the path ordered phase factor in this definition, and it is precisely these terms which destroy the naive result.

This "anomalous" condensate relation may also be obtained very simply by the combined use of the $U(1)_X$, $U(1)_Y$ and SUSY Ward identities^{F4}. From eqs. (2.20) and (2.21) we immediately find,

$$\langle 0 | i \partial^\mu X_\mu (S_0 - \bar{S}_0) | 0 \rangle - (n_X(\tau) + 2) m \langle 0 | (T_F - \bar{T}_F) (S_0 - \bar{S}_0) | 0 \rangle = -i \eta_X(s) \langle S_0 + \bar{S}_0 \rangle \quad (3.11)$$

$$\langle 0 | i \partial^\mu Y_\mu (T_0 - \bar{T}_0) | 0 \rangle + \gamma \langle 0 | (S_F - \bar{S}_F) (T_0 - \bar{T}_0) | 0 \rangle = -i \eta_Y(\tau) \langle T_0 + \bar{T}_0 \rangle \quad (3.12)$$

At zero momentum, the current terms vanish (for $m \neq 0$) since there are no massless pseudoscalar bosons coupling to X_μ or Y_μ . Now we use the following SUSY Ward identities, which hold for any chiral superfields,

$$\begin{aligned}\langle 0 | S_F T_0 | 0 \rangle &= \langle 0 | S_0 T_F | 0 \rangle \\ \langle 0 | S_F \bar{T}_0 | 0 \rangle &= 0\end{aligned}\quad (3.13)$$

to show

$$\langle 0 | (T_F - \bar{T}_F) (S_0 - \bar{S}_0) | 0 \rangle = \langle 0 | (T_0 - \bar{T}_0) (S_F - \bar{S}_F) | 0 \rangle \quad (3.14)$$

Combining eqs. (3.11), (3.12) and (3.14) gives,

$$\langle S_0 + \bar{S}_0 \rangle = -\gamma^{-1} n_Y(\tau) n_X(S)^{-1} (n_X(\tau) + 2) m \langle T_0 + \bar{T}_0 \rangle \quad (3.15)$$

$$= N_F^{-1} m \langle T_0 + \bar{T}_0 \rangle \quad (3.16)$$

That is, assuming parity is unbroken,

$$\frac{N_F g^2}{32 \pi^2} \langle \lambda^\alpha \lambda_\alpha \rangle = m T_F \langle S_+ S_- \rangle_{ij} \quad (3.17)$$

which is precisely Konishi's relation^{10),21)}.

An even simpler way to understand eq. (3.17) is given in Appendix B. Needless to say, this result is reproduced by $\mathcal{L}_{\text{eff}}(S, T)$ for all N_F, N_C .

3.3 m dependence of condensates

Another very nice application of the $U(1)_X$ and SUSY Ward identities allows us to deduce the m dependence of the VEV of the $\theta = 0$ component of any chiral superfield $R(x, \theta)$. This result was first obtained in ref. 11), the following derivation being a simplification of the arguments presented there [see also Ref. 22)].

Again from the $U(1)_X$ Ward identity at zero momentum we have

$$m \langle 0 | (T_F - \bar{T}_F) R_0 | 0 \rangle = i n_X(R) (n_X(\tau) + 2)^{-1} \langle R_0 \rangle \quad (3.18)$$

Now notice that a zero momentum insertion of $im(T_F + \bar{T}_F)$ into a Green's function is simply equivalent to the operation $m \frac{\partial}{\partial m}$, i.e.

$$m \frac{\partial}{\partial m} \langle R_0 \rangle = i m \langle 0 | (T_F + \bar{T}_F) R_0 | 0 \rangle \quad (3.19)$$

But then by the SUSY identity

$$\langle 0 | \bar{T}_F R_0 | 0 \rangle = 0 \quad (3.20)$$

we find

$$m \frac{\partial}{\partial m} \langle R_0 \rangle = - n_x(R) (n_x(T) + 2)^{-1} \langle R_0 \rangle \quad (3.21)$$

and so,

$$\langle R_0 \rangle \sim m^{-\frac{n_x(R)}{n_x(T)+2}} \quad (3.22)$$

In particular we have

$$\langle T_0 \rangle \sim m^{-\left(\frac{N_c - N_F}{N_c}\right)} \quad (3.23)$$

and consistently with eq. (3.16)

$$\langle S_0 \rangle \sim m^{N_F/N_c} \quad (3.24)$$

Again, these results are reproduced by $\mathcal{Z}_{\text{eff}}(S,T)$ or $\mathcal{Z}_{\text{eff}}(T)$.

We see immediately from eq. (3.23) that for $N_F < N_c$ the VEV $\langle T_0 \rangle \rightarrow \infty$ as $m \rightarrow 0$. This is the famous phenomenon of the vacuum "running to infinity" in massless SQCD⁵⁾, an effect now confirmed and explained by detailed instanton calculations¹⁸⁾⁻²⁰⁾.

4. REDUCTION FORMULAE

In current algebra it is possible to relate a matrix element containing a pseudo Goldstone boson in an external state to one with this particle removed. In this section we consider some "reduction formulae" of this type. They will be particularly useful in simplifying the mass formulae to be derived in section 7.

Consider first a generic Ward identity,

$$i \partial^\mu J_\mu - D + \omega \Gamma = 0 \quad (4.1)$$

We assume the symmetry with Noether current J_μ is spontaneously broken, and denote the corresponding Goldstone boson by π . Evaluating the Green's function $\langle 0 | (i \partial^\mu J_\mu - D)(-k) O(k) | 0 \rangle$ in two ways, viz.

$$\begin{aligned} & \langle 0 | (i \partial^\mu J_\mu - D)(-k) O(k) | 0 \rangle \\ &= \langle 0 | i \partial^\mu J_\mu - D | \pi \rangle \langle \pi | O | 0 \rangle \frac{1}{k^2 - m_\pi^2} + \langle 0 | (i \partial^\mu J_\mu - D) O | 0 \rangle_{\text{no pole}} \\ &= -F_\pi \langle \pi | O | 0 \rangle - \langle 0 | \omega \Gamma O | 0 \rangle_{\text{no pole}} \end{aligned} \quad (4.2)$$

and

$$\begin{aligned} \langle 0 | (i \partial^\mu J_\mu - D)(-k) O(k) | 0 \rangle &= - \langle 0 | \omega \Gamma O | 0 \rangle \\ &= - \langle 0 | \delta O | 0 \rangle \end{aligned} \quad (4.3)$$

we deduce,

$$\langle 0 | O(k) | \pi \rangle = \frac{1}{F_\pi} \langle \delta O \rangle - \frac{1}{F_\pi} \langle 0 | \omega \Gamma(-k) O(k) | 0 \rangle_{\text{no pole}} \quad (4.4)$$

This formula becomes useful provided we can discard the last term as being higher order in the symmetry breaking parameter implicit in D . For soft pions ($k \sim 0$) this is usually true. In fact we will normally assume it is also true for on-shell pions.

As an example in SQCD we can use the reduction formula to write, to leading order in m ,

$$\begin{aligned} \langle 0 | T_0 - \bar{T}_0 | 3 \rangle &= \frac{1}{F_3} \langle \delta^x (T_0 - \bar{T}_0) \rangle \\ &= \frac{1}{F_3} n_x(\tau) \langle T_0 + \bar{T}_0 \rangle \end{aligned} \quad (4.5)$$

which, if we substitute for the decay constant, is just the linear Dashen formula (3.4).

A very similar result holds if we replace the vacuum in eq. (4.4) by an arbitrary particle state $|b\rangle$. In this case we can prove,

$$\begin{aligned} & \langle b(-p+k) | O(p) | \pi(k) \rangle \\ &= \frac{1}{F_\pi} \langle b(-p+k) | \delta O(p-k) | 0 \rangle - \frac{1}{F_\pi} \langle b(-p+k) | \omega \Gamma(-k) O(p) | 0 \rangle \quad (4.6) \\ & \quad \text{no } \pi \text{ pole} \end{aligned}$$

In this expression, only the π pole coupling to $\omega \Gamma$ is removed from the final matrix element. This, and $\langle b | 0 | \pi \rangle$, both still contain any pole coming from a state coupling directly to 0. As in eq. (4.4), k is arbitrary. Notice that only the operator O is varied in the reduction, not the state $|b\rangle$. Again, this formula is useful if the final term can be argued to be higher order in the breaking.

In the mass formula of section 7, $O(p)$ becomes a SUSY breaking operator $\Delta(0)$ at zero momentum, and like $|\pi\rangle$, $|b\rangle$ is also a member of the pseudo Goldstone multiplet. An example is,

$$\langle \psi^{a\alpha}(k) | \Delta_\alpha(0) | \pi^b(k) \rangle = \frac{1}{F_\pi} \langle \psi^{a\alpha}(k) | \delta^b \Delta_\alpha(-k) | 0 \rangle \quad (4.7)$$

Provided SUSY is unbroken, we can write similar reduction formulae for the other particles σ and ψ in the flavour non-singlet pseudo Goldstone multiplet. The only change is that the variation δ is replaced by δ_σ or δ_λ as in eqs. (B.6) and (B.8). In the above example,

$$\langle \psi^{a\alpha}(k) | \delta^b \Delta_\alpha(-k) | 0 \rangle = \frac{1}{F_\pi} \langle \delta_\lambda^a \delta^b \Delta_\alpha \rangle \quad (4.8)$$

It is easy to check, at least when Δ is a chiral superfield, that this gives the same result as using the SUSY identity (2.8) to relate the matrix element with $|\psi\rangle$ to one with $|\pi\rangle$, then using the standard reduction formula for $|\pi\rangle$.

5. SOFT SUPERSYMMETRY BREAKING

We now introduce soft SUSY breaking. There are three types of soft breaking terms²³⁾ which can be introduced into the SQCD Lagrangian (2.16) viz. $m_\lambda (S_0 + \bar{S}_0)$, $\hat{\mu}^2 (T_0 + \bar{T}_0)$ and $\mu^2 C_0$, where $C_0 = \text{Tr}(s_+^* s_+ + s_-^* s_-)_{ij}$ is the $\theta = \bar{\theta} = 0$ component of the current superfield $C(x, \theta, \bar{\theta})$ (see Appendix B). (For simplicity, we again restrict ourselves to terms which do not explicitly break flavour.) Explicitly, these are

$$\mathcal{L}_{\text{SUSY break}} = - \int dx \left\{ \frac{g^2 m_\lambda}{32\pi^2} (\lambda^\alpha \lambda_\alpha + \bar{\lambda}_{\dot{\alpha}} \bar{\lambda}^{\dot{\alpha}}) + \hat{\mu}^2 \text{Tr}(s_+ s_- + s_+^* s_-^*)_{ij} + \mu^2 \text{Tr}(s_+ s_+^* + s_- s_-^*)_{ij} \right\} \quad (5.1)$$

We wish to study the effects of these terms on the condensates and pseudo Goldstone particle masses. For this we need the broken SUSY Ward identities generalising eq. (2.1), viz.

$$i \partial^\mu Q_{\mu\alpha} - \Delta_\alpha + \omega_\alpha^Q \Gamma = 0 \quad (5.2)$$

where $\Delta_\alpha = [Q_\alpha, \Delta_0]$, Δ_0 being the terms in eq. (5.1). That is,

$$\Delta_\alpha = i (m_\lambda S_\alpha + \hat{\mu}^2 T_\alpha + \mu^2 C_\alpha) \quad (5.3)$$

Now let $O_\beta(k_1, \dots, k_n)$ denote a product of operators. From eq. (5.2) we have the broken SUSY identity,

$$\begin{aligned} \langle 0 | i \partial^\mu Q_{\mu\alpha}(p) O_\beta(k_1, \dots, k_n) | 0 \rangle &= \langle 0 | \Delta_\alpha(p) O_\beta(k_1, \dots, k_n) | 0 \rangle \\ &+ \langle 0 | \delta_\alpha^Q(p) O_\beta(k_1, \dots, k_n) | 0 \rangle = 0 \end{aligned} \quad (5.4)$$

At zero momentum ($p = 0$), provided there is no massless fermion coupling non-derivatively to $Q_{\mu\alpha}$, i.e. provided SUSY is not spontaneously broken²⁴⁾, we have simply

$$\langle 0 | \delta_\alpha^Q O_\beta(k_1, \dots, k_n) | 0 \rangle = \langle 0 | \Delta_\alpha(0) O_\beta(k_1, \dots, k_n) | 0 \rangle \quad (5.5)$$

The broken SUSY corrections to the Ward identities we have been using so far are therefore given by zero momentum insertions of Δ_α .

In practice, we shall usually be forced to assume that $m_\lambda, \hat{m}, \mu \ll m \ll \Lambda$ to obtain relatively simple formulae. As explained in the introduction, this is also a sensible physical choice for these parameters if we have in mind applications to SUSY composite models.

If we are content to work to lowest order in the SUSY breaking parameters, we may utilise the following trick. Suppose that in the absence of SUSY breaking $\langle 0 | O(k_1 \dots k_n) | 0 \rangle$ is zero (O will generally be a sum of terms, each being a product of operators). Then, with the breaking m_λ (for example) we can expand $\langle 0 | O(k_1 \dots k_n) | 0 \rangle$ as $O_1 m_\lambda + O_2 m_\lambda^2 + \dots$. To lowest order we have trivially

$$\left(1 - m_\lambda \frac{\partial}{\partial m_\lambda}\right) \langle 0 | O(k_1, \dots, k_n) | 0 \rangle = 0 \quad (5.6)$$

Replacing the derivative $m_\lambda \frac{\partial}{\partial m_\lambda}$ by a zero momentum insertion of the breaking term, we find in general

$$\langle 0 | O(k_1, \dots, k_n) | 0 \rangle = i \langle 0 | \Delta_0(0) O(k_1, \dots, k_n) | 0 \rangle + O(m_\lambda, \hat{m}^2, \mu^2) \quad (5.7)$$

This lowest order result is often more useful than the complete formula (5.5).

6. CONDENSATES IN THE PRESENCE OF SOFT SUSY BREAKING

We are now ready to discuss the modifications induced by soft SUSY breaking to the condensate properties of sections 3.2 and 3.3, and to compare these current algebra predictions with those found in the effective Lagrangian $\mathcal{L}(S, T)$.

6.1 Anomalous condensate relation

The derivation of Konishi's anomalous condensate relation in section 3.2 makes use of the two SUSY identities (3.13). From the general formula (5.5) it is easy to check that the corresponding broken SUSY identities are

$$\langle 0 | S_F T_0 | 0 \rangle - \langle 0 | S_0 T_F | 0 \rangle = \frac{1}{4} \langle 0 | \Delta_\alpha(0) (S^\alpha T_0 - S_0 T^\alpha) | 0 \rangle$$

$$\langle 0 | S_F \bar{T}_0 | 0 \rangle = \frac{1}{4} \langle 0 | \Delta_\alpha(0) S^\alpha \bar{T}_0 | 0 \rangle \quad (6.1)$$

In this direct approach we are therefore faced with the problem of evaluating a 3-point Green's function. Even to lowest order in $\frac{m}{\Lambda}$, etc. this is very difficult, since there are many ways of inserting light intermediate states which can contribute.

Fortunately, we can circumvent this difficulty using the trick explained in section 5. To zeroth order, we have the relation

$$\langle S_0 + \bar{S}_0 \rangle - \frac{1}{N_F} m \langle T_0 + \bar{T}_0 \rangle = 0 \quad (6.2)$$

Consider first the breaking $\Lambda^2(T_0 + \bar{T}_0)$. To $O(\frac{\Lambda^2}{m^2})$ we have from eq. (5.7)

$$\langle S_0 + \bar{S}_0 \rangle - \frac{1}{N_F} m \langle T_0 + \bar{T}_0 \rangle \quad (6.3)$$

$$= -i\Lambda^2 \left(\langle 0 | (T_0 + \bar{T}_0) (S_0 + \bar{S}_0) | 0 \rangle - \frac{1}{N_F} m \langle 0 | (T_0 + \bar{T}_0) (T_0 + \bar{T}_0) | 0 \rangle \right) \\ = -\frac{\Lambda^2}{m^2} \langle 0 | T_0 + \bar{T}_0 | \rho \rangle \left(\langle \rho | S_0 + \bar{S}_0 | 0 \rangle - \frac{1}{N_F} m \langle \rho | T_0 + \bar{T}_0 | 0 \rangle \right)$$

saturating the Green's functions with the flavour singlet ρ . The advantage of the trick is clear - it reduces the problem from one of handling 3-point Green's function to one involving only 2-point functions. To this order, we can manipulate the terms on the RHS using exact SUSY identities. The decay constants can be simplified using the reduction formula. We find

$$\begin{aligned} \langle 0 | T_0 + \bar{T}_0 | \rho \rangle &= -i \langle 0 | T_0 - \bar{T}_0 | \gamma \rangle \\ &= -i \frac{1}{F_\gamma} \langle \delta^X (T_0 - \bar{T}_0) \rangle \\ &= \frac{1}{F_\gamma} n_X(\tau) \langle T_0 + \bar{T}_0 \rangle \end{aligned} \quad (6.4)$$

Similarly for $\langle 0 | S_0 + \bar{S}_0 | \rho \rangle$. Thus,

$$\langle S_0 + \bar{S}_0 \rangle = \frac{1}{N_F} m \langle T_0 + \bar{T}_0 \rangle \left\{ 1 - \frac{\Lambda^2}{m_\gamma^2} \frac{1}{F_\gamma^2} \langle T_0 + \bar{T}_0 \rangle n_X(\tau) (n_X(s) - n_X(\tau)) \right\} \quad (6.5)$$

and using the linear Dashen formula for $\langle T_0 + \bar{T}_0 \rangle$ we find

$$\langle S_0 + \bar{S}_0 \rangle = \frac{1}{N_F} m \langle T_0 + \bar{T}_0 \rangle \left\{ 1 + \frac{\Lambda^2}{m m_\gamma} \right\} \quad (6.6)$$

An almost identical calculation for the m_λ breaking gives,

$$\langle S_0 + \bar{S}_0 \rangle = \frac{1}{N_F} m \langle T_0 + \bar{T}_0 \rangle \left\{ 1 - \frac{1}{N_C - N_F} \frac{m_\lambda}{m_\gamma} \right\} \quad (6.7)$$

For the μ^2 breaking we have a further problem. We easily find

$$\langle S_0 + \bar{S}_0 \rangle = \frac{1}{N_F} m \langle T_0 + \bar{T}_0 \rangle \left\{ 1 - \frac{\mu^2}{m_\gamma^2} \frac{1}{F} \langle 0 | C_0 | \rho \rangle (n_x(s) - n_x(\tau)) \right\} \quad (6.8)$$

To express the correction entirely in terms of physical masses, we need to evaluate $\langle 0 | C_0 | \rho \rangle$ in the SUSY limit. Since C_0 is a real superfield, we cannot simply relate this to the corresponding γ decay constant, as we did for the chiral SUSY breakings above. Indeed, $\langle 0 | C_0 | \gamma \rangle = 0$.

We can however determine $\langle 0 | C_0 | \rho \rangle$ indirectly. From the $U(1)_A$ Ward identities (B.9) and (B.10), we can show

$$\begin{aligned} \frac{1}{2} \langle 0 | (M - N) (T_0 - \bar{T}_0) | 0 \rangle + n_A(\tau) m \langle 0 | (T_0 - \bar{T}_0) (T_0 - \bar{T}_0) | 0 \rangle \\ - a \langle 0 | (S_0 - \bar{S}_0) (T_0 - \bar{T}_0) | 0 \rangle = 0 \end{aligned} \quad (6.9)$$

Now, unbroken SUSY implies

$$\frac{1}{2} \langle 0 | (M - N) (T_0 - \bar{T}_0) | 0 \rangle = \langle 0 | C (T_F + \bar{T}_F) | 0 \rangle \quad (6.10)$$

We now saturate this term with the ρ , and the other two terms in eq. (6.9) with the γ . All the decay constant except $\langle 0 | C_0 | \rho \rangle$ are known from SUSY and the $U(1)_X$ Ward identities, and we determine

$$\langle 0 | C_0 | \rho \rangle = 2 n_x(\tau)^{-1} F_\gamma \quad (6.11)$$

(Since $\langle 0 | C_0 | \gamma \rangle = 0$ by parity, this also determines the corresponding fermion coupling, since SUSY then implies $\langle 0 | C_\alpha | \gamma_\beta \rangle = \frac{-1}{\sqrt{2}} \epsilon_{\alpha\beta} \langle 0 | C_0 | \rho \rangle$.)

Substituting back into eq. (6.8), we find

$$\langle S_0 + \bar{S}_0 \rangle = \frac{1}{N_F} m \langle T_0 + \bar{T}_0 \rangle \left\{ 1 + \frac{2 N_C}{N_C - N_F} \frac{\mu^2}{m_\gamma^2} \right\} \quad (6.12)$$

6.2 m dependence of condensates

The same methods also give the corrections to the m dependence formulae of section 3.3. Writing trivially for the m_λ breaking,

$$\langle R_0 \rangle = \langle R_0(m_\lambda=0) \rangle + m_\lambda \frac{\partial}{\partial m_\lambda} \langle R_0 \rangle + O(m_\lambda^2) \quad (6.13)$$

we have to lowest order,

$$\langle R_0 \rangle = \langle R_0(m_\lambda=0) \rangle - i m_\lambda \langle 0 | (S_0 + \bar{S}_0) | 0 \rangle \langle R_0 | 0 \rangle \quad (6.14)$$

$$= \langle R_0(m_\lambda=0) \rangle - \frac{m_\lambda}{m_\lambda^2} \langle 0 | S_0 + \bar{S}_0 | 0 \rangle \langle 0 | R_0 | 0 \rangle \quad (6.15)$$

Simplifying the decay constants using the reduction formula, e.g.

$$\begin{aligned} \langle 0 | R_0 | 0 \rangle &= -i \langle 0 | R_0 | 3 \rangle = -\frac{i}{F_3} \langle \delta^x R_0 \rangle \\ &= \frac{1}{F_3} n_x(R) \langle R_0 \rangle \end{aligned} \quad (6.16)$$

then relating $\langle S_0 + \bar{S}_0 \rangle$ to $\langle T_0 + \bar{T}_0 \rangle$ and using the linear Dashen formula, we find to lowest order in m_λ ,

$$\langle R_0 \rangle = \langle R_0(m_\lambda=0) \rangle \left\{ 1 - \frac{1}{N_F} \frac{n_x(R) n_x(S)}{n_x(T) (n_x(T)+2)} \frac{m_\lambda}{m_\eta} \right\} \quad (6.17)$$

In particular

$$\langle T_0 \rangle = \langle T_0(m_\lambda=0) \rangle \left\{ 1 + \frac{1}{N_c} \frac{m_\lambda}{m_\eta} \right\} \quad (6.18)$$

$$\langle S_0 \rangle = \langle S_0(m_\lambda=0) \rangle \left\{ 1 - \frac{N_F}{N_c (N_c - N_F)} \frac{m_\lambda}{m_\eta} \right\} \quad (6.19)$$

Completely analogous calculations for the other soft breakings give

$$\langle T_0 \rangle = \langle T_0 (\mu^2 = 0) \rangle \left\{ 1 - \frac{N_C - N_F}{N_C} \frac{\mu^2}{m m_\eta} \right\} \quad (6.20)$$

$$\langle S_0 \rangle = \langle S_0 (\mu^2 = 0) \rangle \left\{ 1 + \frac{N_F}{N_C} \frac{\mu^2}{m m_\eta} \right\} \quad (6.21)$$

and

$$\langle T_0 \rangle = \langle T_0 (\mu^2 = 0) \rangle \left\{ 1 - 2 \frac{\mu^2}{m_\eta^2} \right\} \quad (6.22)$$

$$\langle S_0 \rangle = \langle S_0 (\mu^2 = 0) \rangle \left\{ 1 + \frac{2 N_F}{N_C - N_F} \frac{\mu^2}{m_\eta^2} \right\} \quad (6.23)$$

The original condensate relations of sections 3.2 and 3.3 are valid for all N_F, N_C . However, in the derivation of the correction terms to both the Konishi formula and the m dependence, we have used pole dominance with the ρ . The corrections are therefore valid only when there is a light flavour singlet boson, i.e. for $N_F < N_C$. For $N_F = N_C$ the current algebra calculations above break down.

6.3 Relation between quark and squark condensates

With unbroken SUSY, $\langle T_F + \bar{T}_F \rangle = 0$. This is interesting since it provides a relation between the quark and squark condensates

$$\text{Tr} \langle q_+ q_- + \bar{q}_+ \bar{q}_- \rangle_{ij} = -4m \text{Tr} \langle s_+ s_+^* + s_- s_-^* \rangle_{ij} \quad (6.24)$$

If SUSY is broken $\langle T_F + \bar{T}_F \rangle$ is no longer zero and this relation is changed. We can determine the corrections using current algebra. Exactly as in section 6.2 above, we have

$$\langle T_F + \bar{T}_F \rangle = i \langle 0 | \Delta_0(0) (T_F + \bar{T}_F) | 0 \rangle \quad (6.25)$$

$$= (n_x(\tau) + 2)^{-1} F_\eta m^{-1} \langle 0 | \Delta_0 | \rho \rangle \quad (6.26)$$

where Δ_0 are the SUSY breaking terms. Using the reduction formula to express the decay constants as VEV's [or in the case of $\mu^2 C_0$ breaking, using eq. (6.11)

and the Dashen formula (3.8)] we find

$$\begin{aligned} \langle T_F + \bar{T}_F \rangle = & - (n_x(T) + 2)^{-1} \left\{ n_x(T) \frac{\hat{\mu}^2}{m} \langle T_0 + \bar{T}_0 \rangle + n_x(S) \frac{m_\lambda}{m} \langle S_0 + \bar{S}_0 \rangle \right. \\ & \left. + 4 n_x(T)^{-1} (n_x(T) + 2)^2 \frac{\mu^2 m}{m_\gamma^2} \langle C_0 \rangle \right\} \end{aligned} \quad (6.27)$$

That is

$$\begin{aligned} T_F \langle q_+ q_- + \bar{q}_+ \bar{q}_- \rangle_{ij} = & - 4m \langle C_0 \rangle - \left(\frac{N_c - N_F}{N_c} \right) \frac{\hat{\mu}^2}{m} \langle T_0 + \bar{T}_0 \rangle \\ & + \frac{N_F}{N_c} \frac{m_\lambda}{m} \langle S_0 + \bar{S}_0 \rangle - \frac{4 N_c}{N_c - N_F} \frac{\mu^2 m}{m_\gamma^2} \langle C_0 \rangle \end{aligned} \quad (6.28)$$

6.4 Condensates in the effective Lagrangian

The effective Lagrangian $\mathcal{L}_{\text{eff}}(S, T)$ for SQCD with arbitrary N_c , N_F and soft breaking terms is,

$$\begin{aligned} \mathcal{L}_{\text{eff}}(S, T) = & \frac{1}{24} \int d\alpha d\theta d\bar{\theta} \left(\hat{d}(S, \bar{S}) + d(T, \bar{T}) \right) \\ & - \frac{1}{4} \int d\alpha d\theta \left\{ S \left(\log(S^{N_c - N_F} \det T) - 2 N_c \right) + m T \right\} + \text{h.c.} \\ & - \int d\alpha \left\{ m_\lambda (S_0 + \bar{S}_0) + \hat{\mu}^2 (T_0 + \bar{T}_0) + \mu_L^2 (T^r T^{r*})^{1/2} \right\} \end{aligned} \quad (6.29)$$

where $T^r = \{T^0, T^a\}$. The term $\mu_L^2 (T^r T^{r*})^{1/2}$ in $\mathcal{L}_{\text{eff}}(S, T)$ represents the breaking term $\mu_{C_0}^2$ of the original theory, having the same transformation properties under the chiral symmetry group. Notice that μ_L^2 is not the same as μ^2 .

The effective potential is,

$$\begin{aligned} V = & K^{-1}(S, S^*) \left| \log(S^{N_c - N_F} \det T) - (N_c + N_F) \right|^2 \\ & + J_{rs}^{-1}(T, T^*) \left(S \frac{\partial}{\partial T^r} (\log \det T) - m \delta^{r0} \right) \left(S \frac{\partial}{\partial T^s} (\log \det T) - m \delta^{s0} \right)^* \end{aligned}$$

$$+ m_\lambda (S + S^*) + \hat{\mu}^2 (T + T^*) + \mu_L^2 (T^r T^{r*})^{1/2} \quad (6.30)$$

where $K = \frac{\partial^2 \tilde{d}}{\partial S \partial \bar{S}}$, $J_{rs} = \frac{\partial^2 d}{\partial T^r \partial \bar{T}^s}$, and we have omitted the subscripts on S_0, T_0 etc.

The condensates and masses are obtained by straightforward, but lengthy and uninformative, calculations from eq. (6.30). With a notation $\Xi = \{S^0, T^r\}$, the bosonic mass matrix is,

$$(m_{\Xi\Xi}^2)_{\alpha\beta} = J_{\alpha\gamma}^{-1} \frac{\partial^2 V}{\partial \Xi^\gamma \partial \Xi^\beta} \quad (m_{\Xi\Xi^*}^2)_{\alpha\beta} = J_{\alpha\gamma}^{-1} \frac{\partial^2 V}{\partial \Xi^\gamma \partial \Xi^{*\beta}} \quad (6.31)$$

etc., while the fermion mass matrix is,

$$(m_\Psi^2)_{\alpha\beta} = J_{\alpha\gamma}^{-1} \left(\frac{\partial^2 f}{\partial \Xi^\gamma \partial \Xi^\beta} - J_{\delta\epsilon}^{-1} \frac{\partial f}{\partial \Xi^\delta} \frac{\partial J_{\epsilon\alpha}}{\partial \Xi^\beta} \right) \quad (6.32)$$

where $f = S(\log(S^{N_C-N_F} \det T) - 2 N_C) - m T^0$.

We shall not display any details of these calculations, but simply comment on the results. For $N_F < N_C$, after evaluating the condensates and expressing the results in terms of the physical pseudo Goldstone masses, we reproduce precisely the formulae (6.6), (6.7) and (6.18) - (6.21) for the anomalous condensate relation and m dependence with soft breakings $\hat{\mu}^2$ and m_λ . For the $\mu_{C_0}^2$ breaking we find,

$$\langle S_0 + \bar{S}_0 \rangle = \frac{1}{N_F} m \langle T_0 + \bar{T}_0 \rangle \left\{ 1 + \frac{1}{2} \frac{\mu_L^2}{m m_\lambda} \right\} \quad (6.33)$$

$$\langle T_0 + \bar{T}_0 \rangle \sim m^{-\left(\frac{N_C-N_F}{N_C}\right)} \left\{ 1 - \frac{1}{2} \frac{N_C-N_F}{N_C} \frac{\mu_L^2}{m m_\lambda} \right\} \quad (6.34)$$

For the condensate $\langle T_F + \bar{T}_F \rangle$, the $\mathcal{L}_{\text{eff}}(S, T)$ results reproduce eq. (6.27) for m_λ and $\hat{\mu}^2$ breakings, while for μ^2 we find

$$\langle T_F + \bar{T}_F \rangle = -\frac{1}{2} \frac{N_C-N_F}{N_C} \frac{\mu_L^2}{m} \langle T_0 + \bar{T}_0 \rangle \quad (6.35)$$

To compare this with eq. (6.27), we can take the ratio of the linear and quadratic Dashen formulae (3.4), (3.8) to relate,

$$\langle C_0 \rangle = \frac{1}{2} \frac{N_C - N_F}{N_C} \frac{m_3}{m} \langle T_0 + \bar{T}_0 \rangle \quad (6.36)$$

It is now easy to check that all these effective Lagrangian results and the current algebra ones agree if we make the identification $\mu_L^2 = \frac{4N_C}{N_C - N_F} \frac{m}{m_3} \mu^2$. Notice however, that because only the combination $\mu^2 \langle 0|C_0|\rho \rangle$ occurs in the current algebra calculations, this freedom means that the effective Lagrangian is not providing a test of the explicit evaluation of $\langle 0|C_0|\rho \rangle$ in eq. (6.11).

To summarise, for $N_F < N_C$, all the current algebra formulae for condensates in the presence of arbitrary soft SUSY breaking are correctly reproduced, at the expense of much lengthier calculations, by the effective Lagrangian.

The current algebra formulae are no longer valid for $N_F = N_C$ due to the absence of a light flavour singlet multiplet. This is confirmed by $\mathcal{L}_{\text{eff}}(S, T)$ - for $N_F = N_C$ both pseudoscalars in the S, T^0 sector have masses of $\mathcal{O}(\Lambda)$. However, we can still calculate the condensates, and find the following expressions which now involve the mass difference Δm_3^2 of the two flavour singlet pseudoscalars:

$$\langle S_0 + \bar{S}_0 \rangle = \frac{1}{N_F} m \langle T_0 + \bar{T}_0 \rangle \left\{ 1 - \frac{1}{N_F} \frac{m_3}{\Delta m_3} \frac{m_\lambda}{m_3} + \mathcal{O}(m_\lambda^2, \mu^4, \mu^4) \right\} \quad (6.37)$$

$$\langle T_0 + \bar{T}_0 \rangle \sim 1 - \frac{\Delta m_3}{m_3} \left(\frac{1}{N_F} \frac{m_\lambda}{m_3} + \frac{\mu^2}{m m_3} + \frac{1}{2} \frac{\mu_L^2}{m m_3} \right) \quad (6.38)$$

$$\langle T_F + \bar{T}_F \rangle = \frac{m_\lambda}{m} \langle S_0 + \bar{S}_0 \rangle + \mathcal{O}(m_\lambda^2, \mu^4, \mu^4) \quad (6.39)$$

Finally, we emphasize that all the results of this section are independent of the choice of kinetic terms $\tilde{d}(S, \bar{S})$ and $d(T, \bar{T})$ in the effective Lagrangian.

7. PSEUDO GOLDSTONE MULTIPLY MASS FORMULAE

If SUSY is unbroken, the masses of the different members of a pseudo Goldstone chiral multiplet are equal. With soft SUSY breaking terms added to the Lagrangian this degeneracy is lifted. In this section we derive a simple formula for the mass splittings and show that to leading order the supertrace of the

masses vanishes for chiral SUSY breaking terms.

We show, to first order in the SUSY breaking,

$$(m_\psi^2 - m_\pi^2)^{ab} = -\sqrt{2} \langle \psi^{a\alpha} | \Delta_\alpha(0) | \pi^b \rangle \quad (7.1)$$

$$(m_\psi^2 - m_\sigma^2)^{ab} = \sqrt{2} i \langle \psi^{a\alpha} | \Delta_\alpha(0) | \sigma^b \rangle \quad (7.2)$$

where Δ_α is the soft SUSY breaking expression of eq. (5.3). Here, either the $\pi(\sigma)$ or ψ is on-shell. The matrix element includes any pole arising from a state coupling directly to $\Delta_\alpha(0)$. The same formulae hold for the flavour singlet multiplet.

We prove this for the flavour non-singlet multiplet, suppressing the flavour index for simplicity. The starting point is the broken SUSY identity ^{F5},

$$4 \langle 0 | T_F(k) T_F(-k) | 0 \rangle = \langle 0 | \Delta_\alpha(0) T^\alpha(k) T_F(-k) | 0 \rangle \quad (7.3)$$

These expressions have poles at $k^2 = m_\pi^2$, m_σ^2 or m_ψ^2 . Equating the residue of the pole at $k^2 = m_\pi^2$ gives

$$2 \langle 0 | (T_F - \bar{T}_F) | \pi \rangle = \langle 0 | \Delta_\alpha(0) T^\alpha(k) | \pi \rangle \Big|_{k^2 = m_\pi^2} \quad (7.4)$$

where the matrix element on the RHS is complete, i.e. it includes all intermediate states. Similarly,

$$2 \langle 0 | (T_F + \bar{T}_F) | \sigma \rangle = \langle 0 | \Delta_\alpha(0) T^\alpha(k) | \sigma \rangle \Big|_{k^2 = m_\sigma^2} \quad (7.5)$$

while from the residue of the pole at $k^2 = m_\psi^2$,

$$\langle 0 | \Delta_\alpha(0) T_F(k) | \psi^\alpha \rangle \Big|_{k^2 = m_\psi^2} = 0 \quad (7.6)$$

These formulae are exact, to all orders in the SUSY and chiral symmetry breaking.

Separating the contribution of the ψ intermediate state in eq. (7.5) we write,

$$2 \langle 0 | (T_F - \bar{T}_F) | \pi \rangle = \langle 0 | \Delta_\alpha(0) T^\alpha(k) | \pi \rangle \Big|_{k^2 = m_\pi^2} \Big|_{\text{no } \psi \text{ pole}} \\ - \frac{2i m_\psi}{m_\pi^2 - m_\psi^2} \langle 0 | T^\beta | \psi_\beta \rangle \langle \psi^\alpha(k) | \Delta_\alpha(0) | \pi(k) \rangle \Big|_{k^2 = m_\pi^2} \quad (7.7)$$

Similarly with σ . The LHS is $O(1)$ in the SUSY breaking. This can be balanced by the first term on the RHS, since the mass difference $(m_\pi^2 - m_\psi^2)$, which is $O(\Delta)$, occurs in the denominator. Since in the SUSY limit we have the identity (eq. (2.8)),

$$\langle 0 | T_F - \bar{T}_F | \pi \rangle = -\frac{i}{\sqrt{2}} m_\pi \langle 0 | T^\alpha | \psi_\alpha \rangle \quad (7.8)$$

we find the required formula,

$$m_\psi^2 - m_\pi^2 = -\sqrt{2} \langle \psi^\alpha(k) | \Delta_\alpha(0) | \pi(k) \rangle \Big|_{k^2 = m_\pi^2} + O(\Delta^2) \quad (7.9)$$

In the notation of eq. (2.10) we have to $O(\Delta)$,

$$m_\psi^2 - m_\pi^2 = 2 \Delta_{\psi\pi} \quad (7.10)$$

$$m_\psi^2 - m_\sigma^2 = -2i \Delta_{\psi\sigma} \quad (7.11)$$

The essential element in this proof is clear. In eq. (7.3) we have to pick out from the 3-point Green's function of naive $O(\Delta)$ a term of $O(1)$ in the SUSY breaking. This can only be provided by the contribution proportional to $(k^2 - m_\psi^2)^{-1} (k^2 - m_\pi^2)^{-1}$ from the ψ and π intermediate states, since this introduces a mass splitting term of $O(\Delta)$ in the denominator when either ψ or π is put on-shell.

From these mass splitting formulae, we can find the supertrace:

$$\text{Str } m^2 \equiv m_\pi^2 + m_\sigma^2 - 2 m_\psi^2 \\ = 2i (\Delta_{\psi\sigma} + i \Delta_{\psi\pi}) \quad (7.12)$$

If the SUSY breaking is either $m_\lambda(S_0 + \bar{S}_0)$ or $\hat{\mu}^2(T_0 + \bar{T}_0)$, i.e. involving a chiral superfield, then we see immediately from the SUSY Ward identity (2.14) that the supertrace vanishes to this order, i.e.

$$\text{Str } m^2 = O(m_\lambda^2) , O(\hat{\mu}^4) \quad (7.13)$$

However, this is not true of the other breaking term $\mu^2 C_0$. In this case the supertrace is already non-vanishing at $O(\mu^2)$.

The matrix elements in the mass formulae can be greatly simplified in any particular model. First, the reduction formulae of section 4 are applied to transform the matrix elements to VEV's. Then, using an appropriate Dashen formula, the result can be expressed entirely in terms of physical masses and symmetry breaking parameters.

In the following section we will show how this works in three models, SUSY Yang-Mills theory, $N_F < N_C$ SQCD and $N_F = N_C$ SQCD, and compare the current algebra results with those found using the effective Lagrangians.

8. PSEUDO GOLDSTONE MASSES IN SPECIFIC MODELS

8.1 Supersymmetric Yang-Mills theory

Experience with effective Lagrangians suggests that a reasonable description of the low energy dynamics is obtained by assuming the superfield $S(x, \theta)$, where $S_0 = g^2/32\pi^2 \lambda\lambda$, contains the relevant degrees of freedom, and considering the implications of the anomalous $U(1)_R$ symmetry²⁵⁾. Of course, since $U(1)_R$ is anomalous and there is no small chiral symmetry breaking parameter, the field S does not correspond to a true pseudo Goldstone multiplet, and current algebra techniques are not necessarily applicable. Nevertheless, we shall use this theory to test our ideas, in particular the mass formulae, and compare the results with those obtained from the effective Lagrangian, $\mathcal{L}_{\text{eff}}(S)$.

The $U(1)_R$ Ward identity, with soft SUSY breaking, is

$$i \partial^\mu R_\mu + (S_F - \bar{S}_F) + m_\lambda n_R(S) (S_0 - \bar{S}_0) + \omega^R \Gamma = 0 \quad (8.1)$$

where $n_R(S) = -2$, $r = -2 N_C$. Again, we use the Adler-Bardeen definition of the renormalised current R_μ .

With $m_\lambda = 0$, we readily find a linear Dashen formula,

$$\frac{F_\pi^2}{m_\pi} = - + n_R(S) \langle S_0 + \bar{S}_0 \rangle \quad (8.2)$$

(We denote the S multiplet by π, σ, ψ in this section.) There is also, of course, a quadratic formula.

The broken SUSY corrections to the condensates are found as in section 6.

We have

$$\langle S_0 + \bar{S}_0 \rangle = \langle (S_0 + \bar{S}_0) (m_\lambda=0) \rangle - i m_\lambda \langle 0 | (S_0 + \bar{S}_0) (S_0 + \bar{S}_0) | 0 \rangle \quad (8.3)$$

$$= \langle (S_0 + \bar{S}_0) (m_\lambda=0) \rangle + \frac{m_\lambda}{m_\pi^2} \frac{1}{F_\pi^2} n_R(S)^2 (\langle S_0 + \bar{S}_0 \rangle)^2 \quad (8.4)$$

Then using the Dashen formula (8.2), we find

$$\langle S_0 + \bar{S}_0 \rangle = \langle (S_0 + \bar{S}_0) (m_\lambda=0) \rangle \left(1 + \frac{1}{N_C} \frac{m_\lambda}{m_\pi} \right) \quad (8.5)$$

Just as in section 6.3, we can calculate the F component condensate:

$$\langle S_F + \bar{S}_F \rangle = \frac{1}{N_C} m_\lambda \langle S_0 + \bar{S}_0 \rangle \quad (8.6)$$

The mass formula reads in this case,

$$m_\psi^2 - m_\pi^2 = -\sqrt{2} i m_\lambda \langle \psi^\alpha | S_\alpha | \pi \rangle \quad (8.7)$$

and the supertrace is zero to this order. Using the reduction formula to remove the π , and a SUSY identity, we find

$$\langle \psi^\alpha | S_\alpha | \pi \rangle = \frac{1}{F_\pi} i \langle \psi^\alpha | S_\alpha | 0 \rangle (n_R(S) + 1) \quad (8.8)$$

$$= -\frac{1}{F_\pi} \sqrt{2} \langle \pi | S_0 - \bar{S}_0 | 0 \rangle (n_R(S) + 1) \quad (8.9)$$

$$\langle \psi^\alpha | S_\alpha | \pi \rangle = - \frac{\sqrt{2} i}{F_\pi^2} n_R(s) (n_R(s) + 1) \langle S_0 + \bar{S}_0 \rangle \quad (8.10)$$

Then, from the Dashen formula, we find

$$m_\psi^2 - m_\pi^2 = 2 \left(\frac{n_R(s) + 1}{r} \right) m_\lambda m_\pi = \frac{1}{N_c} m_\lambda m_\pi \quad (8.11)$$

These results are to be compared with the predictions from the effective Lagrangian

$$\mathcal{L}_{\text{eff}}(S) = \frac{1}{24} \int dx d\theta d\bar{\theta} d(s, \bar{s}) - \frac{1}{4} \int dx d\theta S (\log S^{N_c} - N_c) + \text{h.c.} - \int dx m_\lambda (S_0 + \bar{S}_0) \quad (8.12)$$

We consider two choices for $d(S, \bar{S})$ - canonical kinetic terms $S\bar{S}$ and conformal kinetic terms $(S\bar{S})^{1/2}$. The calculations are standard and the results are as follows. The condensate formulae (8.5) and (8.6) hold with either kinetic term. For the masses, we find with conformal terms⁴⁾,

$$\begin{aligned} m_\pi^2 &= m_0^2 \left(1 - \frac{5}{3} \frac{1}{N_c} \frac{m_\lambda}{m_\pi} \right) \\ m_\sigma^2 &= m_0^2 \left(1 - \frac{7}{3} \frac{1}{N_c} \frac{m_\lambda}{m_\pi} \right) \\ m_\psi^2 &= m_0^2 \left(1 - 2 \frac{1}{N_c} \frac{m_\lambda}{m_\pi} \right) \end{aligned} \quad (8.13)$$

and with canonical terms,

$$\begin{aligned} m_\pi^2 &= m_0^2 \left(1 + \frac{1}{N_c} \frac{m_\lambda}{m_\pi} \right) \\ m_\sigma^2 &= m_0^2 \left(1 + \frac{3}{N_c} \frac{m_\lambda}{m_\pi} \right) \\ m_\psi^2 &= m_0^2 \left(1 + \frac{2}{N_c} \frac{m_\lambda}{m_\pi} \right) \end{aligned} \quad (8.14)$$

In both cases the supertrace is zero, emphasizing the generality of this result. The mass splitting agrees with the current algebra calculation for the canonical kinetic terms only.

This agreement is encouraging. However, here we must conclude that the mass splittings of the "pseudo Goldstone" particles corresponding to the spontaneous breaking of the anomalous $U(1)_R$ are not quantities that are determined entirely by the symmetry. Clearly they are sensitive to finer details of the strong interaction dynamics as illustrated by their dependence on the choice of $\mathcal{L}_{\text{eff}}(S)$

8.2 Supersymmetric QCD, $N_F < N_C$

SQCD with $N_F < N_C$ is an interesting theory on which to test the mass formulae. For small values of the (s)quark mass m certain VEV's become large, as we found in section 3.3, and the theory has a "runaway" vacuum. Because of complementarity, the theory may be described equivalently as a weak-coupling Higgs model with spontaneously broken gauge symmetry, or as a confining gauge theory with chiral symmetry spontaneously broken by the condensates $\langle S \rangle$, $\langle T \rangle$ etc. It is this latter description we use here.

The condensates, with soft SUSY breaking, have already been studied in section 6, where we find complete agreement between the current algebra and effective Lagrangian results. Here we calculate the pseudo Goldstone multiplet mass splittings

Consider first the SUSY breaking $\hat{M}^2(T_0 + \bar{T}_0)$. The mass formula is,

$$(m_\psi^2 - m_\pi^2)^{ab} = -\sqrt{2} i \hat{M}^2 \langle \psi^{a\alpha} | T_\alpha | \pi^b \rangle \quad (8.15)$$

and the supertrace is zero. For the flavour non-singlets we need

$$\langle \psi^{a\alpha} | T_\alpha | \pi^b \rangle = \frac{1}{F_\psi F_\pi} \langle \delta^{a\alpha} \delta^b T_\alpha \rangle = -\frac{4\sqrt{2} i}{F_\psi F_\pi} \frac{1}{N_F} \delta^{ab} \langle T_0 \rangle \quad (8.16)$$

using the reduction formulae for π and ψ (see section 4 and Appendix B), or alternatively just for π and using a SUSY identity as in eq. (8.9). From the linear Dashen formula (B.18), we then obtain

$$m_\psi^2 - m_\pi^2 = -2 \frac{\Lambda^2}{m} m_\pi \quad (8.17)$$

In the flavour singlet sector we find, using π reduction and SUSY,

$$\langle \chi^\alpha | T_\alpha | \eta \rangle = - \frac{\sqrt{2} i}{F_\eta^2} n_\chi(\tau) (n_\chi(\tau) + 1) \langle T_0 + \bar{T}_0 \rangle \quad (8.18)$$

and with the linear Dashen formula (3.4),

$$m_\chi^2 - m_\eta^2 = -2 \frac{n_\chi(\tau) + 1}{n_\chi(\tau) + 2} \frac{\Lambda^2}{m} m_\eta = - \left(\frac{2N_c - N_F}{N_c} \right) \frac{\Lambda^2}{m} m_\eta \quad (8.19)$$

With the SUSY breaking $m_\lambda(S_0 + \bar{S}_0)$, the mass formula is

$$(m_\psi^2 - m_\pi^2)^{ab} = -\sqrt{2} i m_\lambda \langle \psi^{a\alpha} | S_\alpha | \pi^b \rangle \quad (8.20)$$

and $\text{Str } m^2 = 0$. Since S does not transform under non-abelian chiral transformations, we conclude that to lowest order in m_λ the mass splittings within the flavour non-singlet multiplet vanish, i.e.

$$m_\psi^2 - m_\pi^2 = O(m_\lambda^2) \quad (8.21)$$

For the flavour singlets,

$$\langle \chi^\alpha | S_\alpha | \eta \rangle = - \frac{\sqrt{2} i}{F_\eta^2} n_\chi(s) (n_\chi(s) + 1) \langle S_0 + \bar{S}_0 \rangle \quad (8.22)$$

Using the anomalous condensate formula (3.16), to relate $\langle S_0 + \bar{S}_0 \rangle$ to $\langle T_0 + \bar{T}_0 \rangle$, then the linear Dashen formula, gives

$$\begin{aligned} m_\chi^2 - m_\eta^2 &= - \frac{2}{N_F} \frac{n_\chi(s) (n_\chi(s) + 1)}{n_\chi(\tau) (n_\chi(\tau) + 2)} m_\lambda m_\eta \\ &= - \frac{N_F}{N_c (N_c - N_F)} m_\lambda m_\eta \end{aligned} \quad (8.23)$$

Finally for the $\mu^2 C_0$ breaking, we have

$$(m_\eta^2 - m_\pi^2)^{ab} = -\sqrt{2} i \mu^2 \langle \psi^{a\alpha} | C_\alpha | \pi^b \rangle \quad (8.24)$$

$$(m_\eta^2 - m_\sigma^2)^{ab} = -\sqrt{2} \mu^2 \langle \psi^{a\alpha} | C_\alpha | \sigma^b \rangle \quad (8.25)$$

As before, we conclude that $(m_\eta^2 - m_\pi^2)$ and $(m_\eta^2 - m_\sigma^2)$ are $O(\mu^4)$ for the flavour non-singlets, since $\delta^a C_0 = 0$. For the flavour singlets we can write,

$$\langle \chi^\alpha | C_\alpha | \eta \rangle = \frac{1}{F_3} \langle \chi^\alpha | \delta^\chi C_\alpha | 0 \rangle = \frac{i}{F_3} \langle \chi^\alpha | C_\alpha | 0 \rangle \quad (8.26)$$

and use the result following eq. (6.11) to obtain,

$$m_\chi^2 - m_\eta^2 = - \frac{2 N_F}{N_c - N_F} \mu^2 \quad (8.27)$$

Unfortunately, we are unable to evaluate $\langle \chi^\alpha | C_\alpha | \eta \rangle$, since in the absence of an anomaly-free Ward identity with fields coupling to the flavour singlet ρ or χ , we do not have an appropriate reduction formula. Furthermore, $\langle \chi^\alpha | C_\alpha | \rho \rangle$ cannot be transformed using SUSY identities into a sum of matrix elements each of which contains a pseudoscalar η .

These results are now to be compared with the effective Lagrangian predictions. For simplicity we use $\mathcal{L}_{\text{eff}}(T)$, eq. (2.27) with just the $\hat{\mu}^2$ breaking. The effective potential is

$$V = \frac{1}{2} f_r(\tau) f_s(\tau)^* + \mu^2 (\tau^0 + \tau^{0*}) \quad (8.28)$$

where

$$f_r(\tau) \equiv \frac{\partial f}{\partial \tau^r} = \frac{\partial}{\partial \tau^r} \left(\gamma (\det \tau)^{-\frac{1}{N_c - N_F}} + m \tau^0 \right) \quad (8.29)$$

The masses are calculated in the standard way, with the results:

$$\begin{aligned} m_\pi^2 &= m_\eta^2 - \gamma \kappa f_0'' f_0 \\ m_\sigma^2 &= m_\eta^2 + \gamma \kappa f_0'' f_0 \\ m_\eta^2 &= \gamma^2 (f'')^2 + 2 \gamma \gamma' f'' f_0 \end{aligned} \quad (8.30)$$

and for the flavour singlets

$$\begin{aligned}
 m_\gamma^2 &= m_\chi^2 - 2\kappa\kappa_0 f_{00} f_0 - \kappa^2 f_{000} f_0 \\
 m_\rho^2 &= m_\chi^2 + 2\kappa\kappa_0 f_{00} f_0 + \kappa^2 f_{000} f_0 \\
 m_\chi^2 &= \kappa^2 (f_{00})^2 + 2\kappa\kappa_0 f_{00} f_0
 \end{aligned} \tag{8.31}$$

where the condensed notation is as follows: $J_{ab} = J_{ab}^{-1}$, $\kappa = J_{00}^{-1}$,

$$J'_{ab} = \frac{\partial J_{ab}^{-1}}{\partial T^a}, \quad \kappa_0 = \frac{\partial J_{00}^{-1}}{\partial T^0}, \quad f_0 = \frac{\partial f}{\partial T^0}, \quad f''_{ab} = \frac{\partial^2 f}{\partial T^a \partial T^b}, \text{ etc.}$$

Here f_0 is $O(\mu^2)$, with all other terms of $O(1)$. Everything is evaluated at the VEV for T^0 , and is correct to $O(\mu^2)$.

We quote the result in this form to make the following important point. For the flavour non-singlets, notice that while the derivative J' appears in a term of $O(\mu^2)$ modifying the common mass, it does not appear in the mass differences. This means that the mass splittings $m_\gamma^2 - m_\pi^2$ and $m_\rho^2 - m_\sigma^2$ re-expressed in terms of physical masses and breaking parameters, are independent of the choice of kinetic terms. This is not true for the flavour singlets- there the mass splittings involve κ_0 , and we find different results for different kinetic terms.

We can also see directly from eqs. (8.30) and (8.31) that the supertrace vanishes, for both flavour singlets and non-singlets, independent of the choice of kinetic terms. This emphasizes the generality of this result. Recall that it is proved in the current algebra approach without having to use the reduction formula approximation.

The mass differences for the flavour non-singlets can be re-expressed as,

$$m_\gamma^2 - m_\pi^2 = m_\sigma^2 - m_\rho^2 = - \left(\frac{2N_C - N_F}{N_C} \right) \frac{\mu^2}{m} m_\pi \tag{8.32}$$

and for the flavour singlets, with canonical kinetic terms,

$$m_\chi^2 - m_\gamma^2 = m_\rho^2 - m_\chi^2 = - \left(\frac{2N_C - N_F}{N_C} \right) \frac{\mu^2}{m} m_\gamma \tag{8.33}$$

The non-uniqueness of the mass splittings for the flavour singlets in $\mathcal{L}_{\text{eff}}^{(T)}$ shows, as for SUSY Yang-Mills, that these are quantities that are sensitive to details of the underlying dynamics beyond spontaneously broken $U(1)_X$. Interestingly, the current algebra calculation again reproduces the result obtained with canonical kinetic terms in the effective Lagrangian.

For the flavour non-singlets, however, there is a clear disagreement between the two results. Since the effective Lagrangian result is unique, and presumably correct, we are forced to look again at the assumptions made in the current algebra derivation. The most likely source of error is in the use of the reduction formula for the two-state matrix elements. We therefore have to analyse the term omitted in eq. (4.6) and see whether it has a contribution of the same order as the term we have kept.

In this case the full reduction formula is

$$\langle \psi^{a\alpha}(k) | T_\alpha(0) | \pi^b(k) \rangle = \frac{1}{F_\pi} \langle \psi^{a\alpha}(k) | \delta^b T_\alpha(-k) | 0 \rangle - \frac{1}{F_\pi} \langle \psi^{a\alpha}(k) | \omega^b \Gamma(-k) T_\alpha(0) | 0 \rangle \quad (8.34)$$

no π pole

The first term on the RHS is naively $O(1)$. However, as we have seen it is actually $O(\frac{m_\pi}{m})$. We cannot estimate the second term reliably, but there is certainly one particularly dangerous contribution, from the flavour singlet X pole coupling to the SUSY breaking insertion T_α . We have (to zeroth order in $\hat{\mu}^2$),

$$\begin{aligned} & \frac{1}{F_\pi} \langle \psi^{a\alpha}(k) | \omega^b \Gamma(-k) T_\alpha(0) | 0 \rangle_{\text{no } \pi \text{ pole}} \\ &= \frac{2i}{n_X(T)+2} \frac{1}{m} \frac{F_3}{F_\pi} \langle \psi^{a\alpha}(k) | \omega^b \Gamma(-k) | X_\alpha(0) \rangle_{\text{no pole}} + \frac{1}{F_\pi} \langle \psi^{a\alpha}(k) | \omega^b \Gamma(-k) T_\alpha(0) | 0 \rangle_{\text{no pole}} \end{aligned} \quad (8.35)$$

The X pole gives an enhancement factor of m^{-1} , and it is quite plausible that this term (evaluated at $k^2 = m_\pi^2$) contributes at the same order as $\frac{1}{F_\pi} \langle \psi^{a\alpha} | \delta^b T_\alpha | 0 \rangle$, viz. $O(\frac{m_\pi}{m})$. Unfortunately we have no way to determine whether this is true.

We can get some more insight by looking back at the effective Lagrangian. For canonical kinetic terms we find that both m_π and $m_\gamma \sim m^{1 + (\frac{N_c - N_F}{N_c})}$, so that $O(\frac{m_\pi}{m}) \ll 1$. This means that here $\langle \psi | T_\alpha | \pi \rangle$ is actually suppressed from its naive value. With conformal terms, $\frac{m_\pi}{m}$ is $O(1)$. This is to be contrasted with conventional QCD where $m_\pi^2 \sim m$, and $\frac{m_\pi}{m}$ represents, as in the derivation of the

Dashen formula, an enhancement.

We are unable to come to any firm conclusion. This evidence suggests that keeping only the first term in the reduction formula (8.34) may be a bad approximation, and that we should at least also include the χ pole from the second term. Notice that this is a special feature of SQCD with $N_F < N_C$ - the existence of such a light flavour singlet particle which can couple directly to the SUSY breaking term is a consequence of spontaneously broken $U(1)_\chi$.

Unfortunately, we cannot check this hypothesis. Furthermore, it only makes it more mysterious why our results for the flavour singlets, and for SUSY Yang-Mills, should agree with the effective Lagrangian with canonical kinetic terms. Reluctantly we must admit that, in this example the approximations in the current algebra approach are out of control. While we can certainly trust the order of magnitude of the mass splittings derived from the mass formula with naive reduction, it seems that, in general, one cannot determine the precise numerical coefficients without extra dynamical assumptions.

8.3 Supersymmetric QCD, $N_F = N_C$

SQCD with $N_F = N_C$ is very different from the $N_F < N_C$ theory. Here since $n_\chi(T) = 2 N_F^{-1} (N_C - N_F)$, The $U(1)_\chi$ symmetry is not spontaneously broken by a VEV for $\langle T_0 + \bar{T}_0 \rangle$. For $m = 0$ therefore $U(1)_\chi$ remains unbroken, while for $m \neq 0$ it is broken both explicitly and spontaneously by the VEV $\langle S_0 + \bar{S}_0 \rangle$. The m dependence formulae show that $\langle T_0 + \bar{T}_0 \rangle$ is $O(\Lambda^2)$ while $\langle S_0 + \bar{S}_0 \rangle$ is $O(m\Lambda^2)$. For small m , no condensate becomes large ($\gg \Lambda$) as for $N_F < N_C$ SQCD. Furthermore, and as confirmed by $\mathcal{L}_{\text{eff}}(S, T)$, there is no light particle in the flavour singlet sector - both the pseudoscalars described by S, T^0 have masses of $O(\Lambda)$.

In this section, we use the current algebra mass formulae to compare the mass splittings in the flavour non-singlet pseudo Goldstone multiplet. Clearly this is an example where we expect current algebra to work well.

For the $\hat{\mu}^2(T_0 + \bar{T}_0)$ soft SUSY breaking, we have

$$(m_\gamma^2 - m_\pi^2)^{ab} = -\sqrt{2} i \hat{\mu}^2 \langle \psi^{a\alpha} | T_\alpha | \pi^b \rangle \quad (8.36)$$

$$= -\frac{4}{N_F} \frac{1}{F_\pi^2} \hat{\mu}^2 \delta^{ab} \langle \tau_0 + \bar{\tau}_0 \rangle \quad (8.37)$$

$$= -2 \frac{\hat{\mu}^2}{m} m_\pi \quad (8.38)$$

using the reduction and linear Dashen formulae as before. The supertrace is of course zero at this order. For the other breakings we again find mass splittings only at higher order, viz. $\mathcal{O}(m_\lambda^2)$ and $\mathcal{O}(\mu^4)$

The effective Lagrangian and potential are given in eqs. (6.29) and (6.30). A standard calculation gives, to $\mathcal{O}(\mu^2)$

$$\begin{aligned} m_\pi^2 &= m_\eta^2 - \frac{\hat{\mu}^2}{m} m_\eta \\ m_\sigma^2 &= m_\eta^2 + \frac{\hat{\mu}^2}{m} m_\eta \end{aligned} \quad (8.39)$$

with

$$m_\eta = S J_{ac}^{-1} \frac{\partial^2}{\partial \tau^c \partial \tau^a} (\log \det \tau) \quad (8.40)$$

We can also calculate the mass splittings in the case where we allow the SUSY breaking terms to be flavour dependent, viz. $\sum_r \hat{\mu}_r^2 (\tau^r + \bar{\tau}^r)$ where the index r denotes the diagonal generators of $U(N_F)$. For the special case of $N_F = N_C = 3$, the mass formula gives, for the off-diagonal multiplets (index m, n)

$$(m_\eta^2 - m_\pi^2)_{mn} = -2 \frac{m_\pi}{m} \left(\hat{\mu}_0^2 \delta_{mn} + \sum_{r \neq 0} \hat{\mu}_r^2 d_{r mn} \right) + \mathcal{O}(\hat{\mu}^4) \quad (8.41)$$

where $d^{abc} = \text{Tr} \{ G^a, \{ G^b, G^c \} \}$. The corresponding effective Lagrangian result is

$$(m_\eta^2 - m_\pi^2)_{mn} = -\frac{m_\pi}{m} \left(\hat{\mu}_0^2 \delta_{mn} + 2 \sum_{r \neq 0} \hat{\mu}_r^2 d_{r mn} \right) + \mathcal{O}(\hat{\mu}^4) \quad (8.42)$$

Surprisingly, we again find a discrepancy. Presumably, since in this example there are no light flavour singlet particles to couple directly to the SUSY breaking operator, we must conclude simply that because of the m dependence of m_π and F_π in SQCD the reduction formula for matrix elements with two external pseudo

Goldstone states is not valid. The contribution from the "no π pole" term in eq. (4.6) is not suppressed relative to the pole contribution. Needless to say, this is a discouraging result for the current algebra method.

The case $N_F = N_C$ is of course the most relevant for composite models, since here the SQCD vacuum does not run to infinity as $m \rightarrow 0$. Since in these models we want to identify the fermions in the pseudo Goldstone multiplets with quarks and leptons, and the bosons with as yet unseen SUSY partners, we are interested in soft SUSY breakings which can leave the fermion lighter than both the scalar and pseudoscalar in a multiplet. The supertrace formula immediately shows that the breakings with a chiral superfield, viz. m_λ and $\hat{\mu}^2$, can never satisfy this requirement. In the model with flavour symmetric quark and SUSY breaking mass terms, the remaining possibility, μ^2 type breaking, only affects the mass splittings at $O(\mu^4)$ which is beyond the scope of our analysis. To obtain results of phenomenological interest we should look at more complicated situations where these parameters are not identical, or where flavour symmetries are spontaneously broken. However, these developments do not involve any new points of principle, and we shall not pursue them here.

9. CONCLUSIONS

In this paper we have seen how exploiting the $U(1)_X$ and SUSY Ward identities using current algebra methods leads very directly to many important results in SQCD. In particular, the properties of the condensates which are responsible for many of the characteristic features of the theory, and the corrections to them due to soft SUSY breaking, are easily derived. For all these condensate relations, there is perfect agreement between the current algebra and effective Lagrangian results.

It would be interesting to extend these methods to include the other symmetries contained in the supercurrent, the real superfield whose first component is the $U(1)_R$ current and which contains both the supersymmetry current and energy-momentum tensor amongst its higher components^{26),13)}. However, it remains at present unclear how to use the wealth of information contained in the supercurrent Ward identity in SQCD. For example, before using the dilatation Ward identity, e.g. to provide further restrictions on the D terms in the effective Lagrangian, it is necessary to know what (combinations of) composite operators transform with

canonical dimensions, i.e. what are the appropriate renormalisation group invariants. Unfortunately, this requires the renormalisation theory of composite operators, particularly those associated with the axial anomaly, to be brought to the same level in SQCD as it is in QCD²⁷⁾, a task which despite a large literature²⁸⁾ remains unfinished. It should be straightforward to extend the results of this paper when such a complete renormalization theory becomes available.

The situation is less clear when we attempt to calculate the mass splittings within pseudo Goldstone multiplets induced by soft SUSY breaking. This is an interesting problem in view of applications of SQCD to composite models. We have presented a simple mass formula, and have proved that the mass supertrace vanishes to leading order if the soft SUSY breaking involves a chiral superfield. To express our predictions in terms of physical masses, we use a reduction formula together with a variety of our earlier results on condensates, generalised Dashen formulae, etc. In each model we have considered, SUSY Yang-Mills and SQCD with $N_F \leq N_C$, we have found discrepancies between these current algebra results and those found with the effective Lagrangian. The problem seems to lie in the reduction formula (4.6). It appears that in these SUSY models, the pseudo Goldstone pole contribution to the appropriate matrix elements is not enhanced, and so the pole dominance approximation inherent in the reduction formula is not valid.

Our initial remarks therefore seem to be justified. Effective Lagrangians appear to be the most reliable way to deduce the physical low-energy consequences of the broken symmetries of SQCD, although often at the expense of lengthy and unenlightening calculations. Results which do not depend on the particular choice of the effective Lagrangian's free parameters can be safely taken for good. In contrast, the direct application of Ward identities and current algebra allows easy and transparent derivations of physical relations in which the role of the symmetry is clearly seen. This allows new results to be found which could well go unnoticed in the effective Lagrangian. Nevertheless, current algebra predictions must be treated with caution, since it is very easy for important contributions to be omitted when Green's functions and matrix elements are approximated using techniques such as pole dominance. We may conclude that, although they are in theory equivalent, the current algebra and effective Lagrangian approaches are in practice complementary.

APPENDIX A. Functional SUSY Ward identities

The (anti-)commutation relations for the SUSY charge Q_α with the components of a chiral superfield $T(x, \theta) = T_0 + \theta^\alpha T_\alpha + \theta^2 T_F$ follow from

$$[Q_\alpha, T] = -i \frac{\partial}{\partial \theta^\alpha} T \quad [Q_\alpha, \bar{T}] = 4 \bar{\theta}^{\dot{\alpha}} \partial_{\alpha\dot{\alpha}} \bar{T} \quad (A.1)$$

Explicitly,

$$\begin{aligned} [Q_\alpha, T_0] &= -i T_\alpha & [Q_\alpha, \bar{T}_0] &= 0 \\ \{Q_\alpha, T_\beta\} &= 2i \epsilon_{\alpha\beta} T_F & \{Q_\alpha, \bar{T}_\beta\} &= 4 \partial_{\alpha\dot{\beta}} \bar{T}_0 \\ [Q_\alpha, T_F] &= 0 & [Q_\alpha, \bar{T}_F] &= -2 \partial_{\alpha\dot{\alpha}} \bar{T}^{\dot{\alpha}} \end{aligned} \quad (A.2)$$

For a real superfield $C(x, \theta, \bar{\theta}) = C_0 + \theta^\alpha C_{\alpha\lambda} + \bar{\theta}_{\dot{\alpha}} \bar{C}_{\dot{\alpha}\lambda} + \frac{1}{2} \theta^2 C_M + \frac{1}{2} \bar{\theta}^2 C_N + 2 \theta^\alpha \bar{\theta}^{\dot{\alpha}} C_{\alpha\dot{\alpha}} + \frac{1}{2} \bar{\theta}^2 \theta^\alpha C_{\lambda\alpha} + \frac{1}{2} \theta^2 \bar{\theta}_{\dot{\alpha}} \bar{C}_{\dot{\alpha}\lambda} + \frac{1}{4} \theta^2 \bar{\theta}^2 C_D$, we have

$$[Q_\alpha, C] = \left(-i \frac{\partial}{\partial \theta^\alpha} + 2 \bar{\theta}^{\dot{\alpha}} \partial_{\alpha\dot{\alpha}} \right) C \quad (A.3)$$

and so,

$$\begin{aligned} [Q_\alpha, C_0] &= -i C_{\alpha\lambda} \\ \{Q_\alpha, C_{\lambda\beta}\} &= i \epsilon_{\alpha\beta} C_M & \{Q_\alpha, \bar{C}_{\lambda\dot{\beta}}\} &= -2i C_{\alpha\dot{\beta}} + 2 \partial_{\alpha\dot{\beta}} C_0 \\ [Q_\alpha, C_M] &= 0 & [Q_\alpha, C_N] &= -i C_{\lambda\alpha} - 2 \partial_{\alpha\dot{\alpha}} \bar{C}_{\dot{\alpha}\lambda} \\ [Q_\alpha, C_{\beta\dot{\rho}}] &= \frac{i}{2} \epsilon_{\alpha\beta} \bar{C}_{\lambda\dot{\rho}} - \partial_{\alpha\dot{\rho}} C_{\lambda\beta} \\ \{Q_\alpha, C_{\lambda\beta}\} &= i \epsilon_{\alpha\beta} C_D + 4 \partial_{\alpha\dot{\beta}} C_{\rho\dot{\beta}} & \{Q_\alpha, \bar{C}_{\lambda\dot{\beta}}\} &= 2 \partial_{\alpha\dot{\beta}} C_M \\ [Q_\alpha, C_D] &= -2 \partial_{\alpha\dot{\alpha}} \bar{C}_{\dot{\alpha}\lambda} \end{aligned} \quad (A.4)$$

It is interesting and sometimes useful to have an explicit form for the functional SUSY Ward identity (2.1) in terms of superfield components. For an effective action $\Gamma[T, C]$ we have

$$i \partial^\mu Q_{\mu\alpha} = i T_\alpha \frac{\delta \Gamma}{\delta T_0} - 2i T_F \frac{\delta \Gamma}{\delta T_\alpha} + 4 \partial_{\alpha\dot{\beta}} \bar{T}_0 \frac{\delta \Gamma}{\delta \bar{T}_\beta} - 2 \partial_{\alpha\dot{\alpha}} \bar{T}^{\dot{\alpha}} \frac{\delta \Gamma}{\delta \bar{T}_F}$$

$$\begin{aligned}
& -i C_{\chi\alpha} \frac{\delta\Gamma}{\delta C_0} - i C_M \frac{\delta\Gamma}{\delta C_\chi^\alpha} + \left(-2i C_{\alpha\dot{\beta}} + 2\partial_{\alpha\dot{\beta}} C_0 \right) \frac{\delta\Gamma}{\delta \bar{C}_{\chi\dot{\beta}}} - \left(i C_{\lambda\alpha} + 2\partial_{\alpha\dot{\lambda}} \bar{C}_\chi^\alpha \right) \frac{\delta\Gamma}{\delta C_N} \\
& + \left(\frac{i}{2} \epsilon_{\alpha\dot{\beta}} \bar{C}_{\lambda\dot{\beta}} - \partial_{\alpha\dot{\beta}} C_{\chi\beta} \right) \frac{\delta\Gamma}{\delta C_{\rho\dot{\beta}}} + \left(i \epsilon_{\alpha\dot{\beta}} C_D + 4\partial_{\alpha\dot{\beta}} C_{\beta\dot{\beta}} \right) \frac{\delta\Gamma}{\delta C_{\lambda\dot{\beta}}} \\
& + 2\partial_{\alpha\dot{\beta}} C_M \frac{\delta\Gamma}{\delta \bar{C}_{\lambda\dot{\beta}}} - 2\partial_{\alpha\dot{\lambda}} \bar{C}_\lambda^\alpha \frac{\delta\Gamma}{\delta C_D} = 0 \quad (A.5)
\end{aligned}$$

Introducing sources K_0, K_α, K_F for the chiral superfield components T_0, T_α, T_F resp. $J_0, J_{\chi\alpha}, \dots$ for the real superfield components $C_0, C_{\chi\alpha}, \dots$, and $J_{\mu\alpha}^Q$ for the SUSY current $Q_{\mu\alpha}$, we can rewrite eq. (A.5) as a Ward identity for the generating functional $W[J_{\mu\alpha}^Q; K_0, \dots; \bar{K}_0, \dots; J_0, \dots]$

$$\begin{aligned}
& i\partial_\mu \frac{\delta W}{\delta J_\mu^Q} + iK_0 \frac{\delta W}{\delta K^\alpha} - 2iK_\alpha \frac{\delta W}{\delta K_F} + 4\bar{K}^{\dot{\beta}} \partial_{\alpha\dot{\beta}} \frac{\delta W}{\delta \bar{K}_0} + 2\bar{K}_F \partial_{\alpha\dot{\beta}} \frac{\delta W}{\delta \bar{K}_{\dot{\beta}}} \\
& + iJ_0 \frac{\delta W}{\delta J_\chi^\alpha} - iJ_{\chi\alpha} \frac{\delta W}{\delta J_M} - 2i\bar{J}_\chi^{\dot{\beta}} \left(\frac{\delta W}{\delta J_{\alpha\dot{\beta}}} + i\partial_{\alpha\dot{\beta}} \frac{\delta W}{\delta J_0} \right) + J_N \left(2\partial_{\alpha\dot{\beta}} \frac{\delta W}{\delta J_{\chi\dot{\beta}}} + i\frac{\delta W}{\delta J_\lambda^\alpha} \right) \\
& + J^{\beta\dot{\beta}} \left(\partial_{\alpha\dot{\beta}} \frac{\delta W}{\delta J_\chi^\alpha} + \frac{i}{2} \epsilon_{\alpha\dot{\beta}} \frac{\delta W}{\delta \bar{J}_\lambda^{\dot{\beta}}} \right) - iJ_\lambda^{\dot{\beta}} \left(\epsilon_{\alpha\dot{\beta}} \frac{\delta W}{\delta J_\beta^\alpha} - 4i\partial_{\alpha\dot{\beta}} \frac{\delta W}{\delta J_{\rho\dot{\beta}}} \right) \\
& + 2\bar{J}_\lambda^{\dot{\beta}} \partial_{\alpha\dot{\beta}} \frac{\delta W}{\delta J_M} + 2J_\beta \partial_{\alpha\dot{\beta}} \frac{\delta W}{\delta \bar{J}_{\lambda\dot{\beta}}} = 0 \quad (A.6)
\end{aligned}$$

(Note that the chiral superfield source is $K(x, \theta) = -\frac{1}{4} (K_F - 2\bar{\theta}^\alpha K_\alpha + \theta^2 K_0)$ in this notation. Similarly for $J(x, \theta, \bar{\theta})$.) The relations amongst Green's functions implied by unbroken SUSY can now be found by taking successive functional derivatives of eq. (A.6).

Here we are interested in decay constants and vertex functions. To recover the decay constant relations of section 2 we first construct the partial transform $\tilde{W}[K, \phi]$, where K is the source for T and ϕ is the chiral superfield describing the states. A decay constant $\langle 0|T_0|\phi \rangle$ is represented in this pure-

ly functional formalism by $\frac{\delta^2 \tilde{W}}{\delta K_0 \delta \phi}$. To be convinced of this, consider inserting intermediate states into the Green's function $\langle 0 | T T | 0 \rangle$. We have,

$$\langle 0 | T T | 0 \rangle = \sum_{\phi} \langle 0 | T | \phi \rangle \Delta_{\phi\phi} \langle \phi | T | 0 \rangle \quad (A.7)$$

where $\Delta_{\phi\phi}$ is the propagator. This is expressed in functional language as

$$\frac{\delta^2 W}{\delta K \delta K} = \sum_{\phi} \frac{\delta^2 \tilde{W}}{\delta K \delta \phi} \left(\frac{\delta^2 \tilde{W}}{\delta \phi \delta \phi} \right)^{-1} \frac{\delta^2 \tilde{W}}{\delta \phi \delta K} \quad (A.8)$$

The Ward identity for $\tilde{W}[K, \phi]$ is

$$\begin{aligned} i \partial_{\mu} \frac{\delta \tilde{W}}{\delta J_{\mu\alpha}} + i K_0 \frac{\delta \tilde{W}}{\delta K^{\alpha}} - 2i K_{\alpha} \frac{\delta \tilde{W}}{\delta K_F} + 4 \bar{K}^{\beta} \partial_{\alpha\beta} \frac{\delta \tilde{W}}{\delta \bar{K}_0} + 2 \bar{K}_F \partial_{\alpha\beta} \frac{\delta \tilde{W}}{\delta \bar{K}_{\beta}} \\ - i \gamma_{\alpha} \frac{\delta \tilde{W}}{\delta \phi} - 2i F \frac{\delta \tilde{W}}{\delta \psi^{\alpha}} + 4 \partial_{\alpha\beta} \psi^{\alpha} \frac{\delta \tilde{W}}{\delta \bar{\psi}_{\beta}} - 2 \partial_{\alpha\alpha} \bar{\psi}^{\alpha} \frac{\delta \tilde{W}}{\delta F^{\alpha}} = 0 \end{aligned} \quad (A.9)$$

Differentiating with respect to $\phi(x)$ and $K_{\alpha}(y)$, and evaluating at zero sources and fields, we find simply

$$\frac{\delta^2 \tilde{W}}{\delta K_F(y) \delta \phi(x)} = 0 \quad (A.10)$$

Differentiating w.r.t. $\phi(x)$ and $\bar{K}^{\alpha}(y)$, and Fourier transforming, gives

$$k_{\alpha\alpha} \frac{\delta^2 \tilde{W}}{\delta \bar{K}_0 \delta \phi} = 0 \quad (A.11)$$

Similarly we can show

$$\frac{\delta^2 \tilde{W}}{\delta K_F \delta \psi^{\alpha}} - k_{\alpha\alpha} \frac{\delta^2 \tilde{W}}{\delta K_{\alpha} \delta \bar{\psi}_{\alpha}} = 0 \quad (A.12)$$

$$\frac{\delta^2 \tilde{W}}{\delta K^{\alpha} \delta \psi^{\beta}} + \epsilon_{\alpha\beta} \frac{\delta^2 \tilde{W}}{\delta K_0 \delta \phi} = 0 \quad (A.13)$$

and many others. These are readily identified with the identities (2.7) in the text.

The vertex function identities can also be reproduced in this formalism. Since the method is now clear we give just one example. Differentiating eq. (A.9) three times, w.r.t. q , ψ^β and K_0 gives

$$\frac{\delta^3 \tilde{W}}{\delta \psi^\beta \delta K^\alpha \delta q} - \epsilon_{\alpha\beta} \frac{\delta^3 \tilde{W}}{\delta q \delta K_0 \delta q} = 0 \quad (A.14)$$

recovering the second of eqs. (2.12).

APPENDIX B. Superfield chiral Ward identities and Dashen formulae in the presence of soft SUSY breaking.

Here we consider the superfield extensions of the chiral Ward identities of section 2.2. We begin with the $U(1)_A$ symmetry. The Noether current A_μ is the $\theta\bar{\theta}$ component of the "current superfield" $C(x, \theta, \bar{\theta}) = \bar{Q}_+ e^V Q_+ + \bar{Q}_- e^{-V} Q_-$. The superfield Ward identity for C is

$$\frac{1}{24} \bar{D}^2 C + \frac{1}{4} n_A(T) m T - \frac{1}{4} a S + \omega_S^A \Gamma = 0 \quad (B.1)$$

$$\frac{1}{24} D^2 C + \frac{1}{4} n_A(T) m \bar{T} - \frac{1}{4} a \bar{S} + \bar{\omega}_S^A \Gamma = 0 \quad (B.2)$$

where the symmetry operator superfield $\omega_S^A = -i \delta^A_R \frac{\delta}{\delta R}$ for a generic chiral superfield $R(x, \theta)$ with $\delta^A_R = \text{in}_A(R) R$.

The component equations are easily derived. In the notation of Appendix A for components of $C(x, \theta, \bar{\theta})$ (but with $A_{\alpha\dot{\alpha}} = C_{\alpha\dot{\alpha}}$), we find, using eq. (2.2),

$$i \partial^\mu A_\mu - n_A(T) m (T_F - \bar{T}_F) + a (S_F - \bar{S}_F) + \omega^A \Gamma = 0 \quad (B.3)$$

where

$$\omega^A(R) = \left(\delta^A_{R_0} \frac{\delta}{\delta R_0} + \delta^A_{R^\alpha} \frac{\delta}{\delta R^\alpha} + \delta^A_{R_F} \frac{\delta}{\delta R_F} \right) + \left(\delta^A_{\bar{R}_0} \frac{\delta}{\delta \bar{R}_0} + \delta^A_{\bar{R}^\alpha} \frac{\delta}{\delta \bar{R}^\alpha} + \delta^A_{\bar{R}_F} \frac{\delta}{\delta \bar{R}_F} \right)$$

with $\delta^A_{R_0} = \text{in}_A(R) R_0$, etc. (B.4)

Similarly, defining $\Lambda_\alpha = C_{\lambda\alpha} - 2i \partial_{\alpha\dot{\alpha}} \bar{C}_\lambda^{\dot{\alpha}}$, we find

$$\frac{1}{2} \Lambda_\alpha - n_A(\tau) m T_\alpha + a S_\alpha + \omega_{\Lambda\alpha}^A \Gamma = 0 \quad (B.5)$$

where

$$\omega_{\Lambda\alpha}^A(R) = -2 \delta^A R_0 \frac{d}{dR^\alpha} + \delta^A R_\alpha \frac{d}{dR_F} \quad (B.6)$$

and the conjugate equation for $\bar{\Lambda}_{\dot{\alpha}}$, and defining $\Theta = C_D - \partial^2 C_0$,

$$\frac{1}{2} \Theta - n_A(\tau) m (T_F + \bar{T}_F) + a (S_F + \bar{S}_F) + \omega_\Theta^A \Gamma = 0 \quad (B.7)$$

where

$$\omega_\Theta^A(R) = \left(\delta^A R_0 \frac{d}{dR_0} + \delta^A R^\alpha \frac{d}{dR^\alpha} + \delta^A R_F \frac{d}{dR_F} \right) - \left(\delta^A \bar{R}_0 \frac{d}{d\bar{R}_0} + \delta^A \bar{R}_{\dot{\alpha}} \frac{d}{d\bar{R}_{\dot{\alpha}}} + \delta^A \bar{R}_F \frac{d}{d\bar{R}_F} \right) \quad (B.8)$$

There are two further equations of interest, for the Θ^2 and $\bar{\Theta}^2$ components of C :

$$\frac{1}{2} C_M - n_A(\tau) m \bar{T}_0 + a \bar{S}_0 + \omega_M^A \Gamma = 0 \quad (B.9)$$

$$\frac{1}{2} C_N - n_A(\tau) m T_0 + a S_0 + \omega_N^A \Gamma = 0 \quad (B.10)$$

where

$$\omega_M^A(R) = -\delta^A \bar{R}_0 \frac{d}{d\bar{R}_F} \quad \omega_N^A(R) = \delta^A R_0 \frac{d}{dR_F} \quad (B.11)$$

These are of course just the $\Theta = \bar{\Theta} = 0$ components of eqs. (B.1) and (B.2).

Taking the VEV of eq. (B.10) gives

$$\frac{1}{2} \langle C_N \rangle - n_A(\tau) m \langle T_0 \rangle + a \langle S_0 \rangle = 0 \quad (B.12)$$

With unbroken SUSY we must have $\langle C_N \rangle = 0$ since $C_N = -\frac{1}{2} i \{ \bar{Q}_{\dot{\alpha}}, \bar{C}_X^{\dot{\alpha}} \}$. We therefore recover the anomalous condensate relation (3.16) this time²¹⁾ from a simple combination of the SUSY and $U(1)_A$ superfield Ward identities.

From now on we shall just use eqs. (B.3), (B.5) and (B.7). Notice that only eq. (B.3) represents a true symmetry. The other two do not contain the derivative of a Noether current, and are not Ward identities for a symmetry of the theory. Nevertheless, as we see below, provided SUSY is unbroken they can play a similar role.

Clearly, analogous identities hold for the non-abelian chiral symmetry, except of course that here the anomaly term is absent. These identities can be used directly to obtain the individual Dashen formulae for the components $(\pi, \sigma, \psi)^a$ of the flavour non-singlet pseudo Goldstone chiral multiplet [see Ref. 29)]. Using the forms above for ω , ω_Λ and $\omega_\mathfrak{D}$, we readily show,

$$\langle 0 | i \partial^\mu A_\mu^a (T_0 - \bar{T}_0)^b | 0 \rangle - m \langle 0 | (T_F - \bar{T}_F)^a (T_0 - \bar{T}_0)^b | 0 \rangle = - \frac{2i}{N_F} \delta^{ab} \langle T_0 + \bar{T}_0 \rangle \quad (B.13)$$

$$\langle 0 | i \partial^\mu A_\mu^a (T_F - \bar{T}_F)^b | 0 \rangle - m \langle 0 | (T_F - \bar{T}_F)^a (T_F - \bar{T}_F)^b | 0 \rangle = - \frac{2i}{N_F} \delta^{ab} \langle T_F + \bar{T}_F \rangle \quad (B.14)$$

$$\frac{1}{2} \langle 0 | \Lambda_\alpha^a T_\beta | 0 \rangle - m \langle 0 | T_\alpha^a T_\beta^b | 0 \rangle = - \frac{2i}{N_F} \delta^{ab} \epsilon_{\alpha\beta} \langle T_0 \rangle \quad (B.15)$$

$$\frac{1}{2} \langle 0 | \mathfrak{D}^a (T_0 + \bar{T}_0)^b | 0 \rangle - m \langle 0 | (T_F + \bar{T}_F)^a (T_0 + \bar{T}_0)^b | 0 \rangle = - \frac{2i}{N_F} \delta^{ab} \langle T_0 + \bar{T}_0 \rangle \quad (B.16)$$

$$\frac{1}{2} \langle 0 | \mathfrak{D}^a (T_F + \bar{T}_F)^b | 0 \rangle - m \langle 0 | (T_F + \bar{T}_F)^a (T_F + \bar{T}_F)^b | 0 \rangle = - \frac{2i}{N_F} \delta^{ab} \langle T_F + \bar{T}_F \rangle \quad (B.17)$$

where again we assume a symmetric space chiral symmetry breaking. If for the moment we assume that at zero momentum we can discard the first term in each identity, and saturate the remaining Green's function with the relevant pseudo Goldstone component π , ψ or σ , we find the following Dashen formulae:

$$F_\pi^2 m_\pi = F_\psi^2 m_\psi = F_\sigma^2 m_\sigma = \frac{2}{N_F} m \langle T_0 + \bar{T}_0 \rangle \quad (B.18)$$

$$F_\pi^2 m_\pi^2 = F_\sigma^2 m_\sigma^2 = \frac{4}{N_F} m^2 \langle C_0 \rangle \quad (B.19)$$

where $C_0 = \text{Tr}(s_+ s_+^* + s_- s_-^*)_{ij}$. Notice that while we find both linear and quadratic Dashen formulae for π and σ , there is only a linear formula for ψ . Of course, with unbroken SUSY, $F_\pi = F_\sigma = F_\psi$ and $m_\pi = m_\sigma = m_\psi$.

It remains to check the validity of neglecting the terms $\langle 0 | \Lambda_\alpha T_\beta | 0 \rangle$, $\langle 0 | \mathfrak{D}(T_0 + \bar{T}_0) | 0 \rangle$, etc. at zero momentum. Since Λ_α and $\mathfrak{D}$ are not derivatives

of Noether currents this is not obviously correct. It is at this point, only, that we must use unbroken SUSY Ward identities. It is easy to check that,

$$\langle 0 | (\not{\partial} + 2i \not{\partial}^\mu A_\mu) T_0 | 0 \rangle = \frac{1}{2} \langle 0 | \Lambda^\alpha T_\alpha | 0 \rangle \quad (B.20)$$

$$\langle 0 | (\not{\partial} - 2i \not{\partial}^\mu A_\mu) T_0 | 0 \rangle = \frac{1}{2} \langle 0 | \bar{\Lambda}_\alpha [\bar{Q}^\alpha, T_0] | 0 \rangle \quad (B.21)$$

which implies

$$\langle 0 | \not{\partial} T_0 | 0 \rangle = \frac{1}{4} \langle 0 | \Lambda^\alpha T_\alpha | 0 \rangle = 2i \langle 0 | \not{\partial}^\mu A_\mu T_0 | 0 \rangle \quad (B.22)$$

Clearly, therefore, in the absence of a massless Goldstone multiplet, $\langle 0 | \not{\partial} T_0 | 0 \rangle$ and $\langle 0 | \Lambda^\alpha T_\alpha | 0 \rangle$ vanish at $k = 0$.

Now we consider the effects of soft supersymmetry breaking. As well as modifying the SUSY Ward identities, the introduction of the SUSY breaking terms (5.1) into $\mathcal{L}_{\text{SQCD}}$ also changes the chiral Ward identities. The corrections to the anomaly free identities are,

$$i \not{\partial}^\mu A_\mu^a - m (\tau_F - \bar{\tau}_F)^a + \hat{f}^2 (\tau_0 - \bar{\tau}_0)^a + \omega^a \Gamma = 0 \quad (B.23)$$

$$\frac{1}{2} \Lambda_\alpha^a - m T_\alpha^a + \omega_{\Lambda\alpha}^a \Gamma = 0 \quad (B.24)$$

$$\frac{1}{2} \not{\partial}^a - m (\tau_F + \bar{\tau}_F)^a + \hat{f}^2 (\tau_0 + \bar{\tau}_0)^a + \mu^2 C_0^a + \omega_{\not{\partial}}^a \Gamma = 0 \quad (B.25)$$

and for the flavour singlet $U(1)_X$,

$$i \not{\partial}^\mu X_\mu - (n_X(\tau) + 2) m (\tau_F - \bar{\tau}_F) + \hat{f}^2 n_X(\tau) (\tau_0 - \bar{\tau}_0) + n_X n_X(S) (S_0 - \bar{S}_0) + \omega^X \Gamma = 0 \quad (B.26)$$

It is natural to use these techniques to try to find the modifications of the Dashen formulae induced by soft SUSY breaking. The first step is to write the following identities, obtained from the above set of anomaly free Ward identities:

$$\begin{aligned} \langle 0 | i \not{\partial}^\mu A_\mu^a (\tau_0 - \bar{\tau}_0)^b | 0 \rangle &= m \langle 0 | (\tau_F - \bar{\tau}_F)^a (\tau_0 - \bar{\tau}_0)^b | 0 \rangle \\ &+ \hat{f}^2 \langle 0 | (\tau_0 - \bar{\tau}_0)^a (\tau_0 - \bar{\tau}_0)^b | 0 \rangle = -\frac{2i}{N_F} d^{ab} \langle \tau_0 + \bar{\tau}_0 \rangle \end{aligned} \quad (B.27)$$

$$\begin{aligned} \langle 0 | i \partial^\mu A_\mu^a (T_F - \bar{T}_F)^b | 0 \rangle &= m \langle 0 | (T_F - \bar{T}_F)^a (T_F - \bar{T}_F)^b | 0 \rangle \\ + \mu^2 \langle 0 | (T_0 - \bar{T}_0)^a (T_F - \bar{T}_F)^b | 0 \rangle &= - \frac{2i}{N_F} \delta^{ab} \langle T_F + \bar{T}_F \rangle \end{aligned} \quad (B.28)$$

$$\frac{1}{2} \langle 0 | A_\alpha^a T_\beta^b | 0 \rangle = m \langle 0 | T_\alpha^a T_\beta^b | 0 \rangle = - \frac{2i}{N_F} \delta^{ab} \epsilon_{\alpha\beta} \langle T_0 \rangle \quad (B.29)$$

$$\begin{aligned} \frac{1}{2} \langle 0 | \partial^a (T_0 + \bar{T}_0)^b | 0 \rangle &= m \langle 0 | (T_F + \bar{T}_F)^a (T_0 + \bar{T}_0)^b | 0 \rangle \\ + \mu^2 \langle 0 | (T_0 + \bar{T}_0)^a (T_0 + \bar{T}_0)^b | 0 \rangle &+ \mu^2 \langle 0 | C_0^a (T_0 + \bar{T}_0)^b | 0 \rangle = - \frac{2i}{N_F} \delta^{ab} \langle T_0 + \bar{T}_0 \rangle \end{aligned} \quad (B.30)$$

$$\begin{aligned} \frac{1}{2} \langle 0 | \partial^a (T_F + \bar{T}_F)^b | 0 \rangle &= m \langle 0 | (T_F + \bar{T}_F)^a (T_F + \bar{T}_F)^b | 0 \rangle \\ + \mu^2 \langle 0 | (T_0 + \bar{T}_0)^a (T_F + \bar{T}_F)^b | 0 \rangle &+ \mu^2 \langle 0 | C_0^a (T_F + \bar{T}_F)^b | 0 \rangle = - \frac{2i}{N_F} \delta^{ab} \langle T_F + \bar{T}_F \rangle \end{aligned} \quad (B.31)$$

and for the flavour singlet sector,

$$\begin{aligned} \langle 0 | i \partial^\mu X_\mu (T_0 - \bar{T}_0) | 0 \rangle &= (n_x(\tau) + 2) m \langle 0 | (T_F - \bar{T}_F) (T_0 - \bar{T}_0) | 0 \rangle \\ + \mu^2 n_x(\tau) \langle 0 | (T_0 - \bar{T}_0) (T_0 - \bar{T}_0) | 0 \rangle &+ m_\lambda n_x(s) \langle 0 | (S_0 - \bar{S}_0) (T_0 - \bar{T}_0) | 0 \rangle = -i n_x(\tau) \langle T_0 + \bar{T}_0 \rangle \end{aligned} \quad (B.32)$$

$$\begin{aligned} \langle 0 | i \partial^\mu X_\mu (T_F - \bar{T}_F) | 0 \rangle &= (n_x(\tau) + 2) m \langle 0 | (T_F - \bar{T}_F) (T_F - \bar{T}_F) | 0 \rangle \\ + \mu^2 n_x(\tau) \langle 0 | (T_0 - \bar{T}_0) (T_F - \bar{T}_F) | 0 \rangle &+ m_\lambda n_x(s) \langle 0 | (S_0 - \bar{S}_0) (T_F - \bar{T}_F) | 0 \rangle = -i (n_x(\tau) + 2) \langle T_F + \bar{T}_F \rangle \end{aligned} \quad (B.33)$$

To derive Dashen formulae from these equations we have to overcome several problems. First, if we continue to define F_π by $\langle 0 | A_\mu^a | \pi^b \rangle = i k_\mu F_\pi \delta^{ab}$ we need the corrections to $\langle 0 | (T_F - \bar{T}_F)^a | \pi^b \rangle$. These corrections arise because the soft SUSY breaking terms also break chiral symmetries. From eq. (B.23) we find

$$m \langle 0 | (T_F - \bar{T}_F)^a | \pi^b \rangle = i F_\pi \mu_\pi^2 \delta^{ab} \left(1 + \frac{\mu_\pi^2}{m m_\pi} \right) \quad (B.34)$$

Defining F_σ and F_ψ analogously, viz. $\frac{1}{2} \langle 0 | \theta^a | \pi^b \rangle = -F_\sigma m_\sigma^2 \delta^{ab}$ and $\frac{1}{2} \langle 0 | \Lambda_\alpha^a | \psi_\beta^b \rangle = -\frac{1}{\sqrt{2}} F_\pi m_\pi \epsilon_{\alpha\beta} \delta^{ab}$, we also have

$$m \langle 0 | (T_F + \bar{T}_F)^a | \sigma^b \rangle = -F_\sigma m_\sigma^2 \delta^{ab} \left(1 - \frac{\Lambda^2}{m m_\sigma} - \frac{\Lambda^2}{m^2} \right) \quad (B.35)$$

$$m \langle 0 | T_\alpha^a | \psi_\beta^b \rangle = -\frac{1}{\sqrt{2}} F_\pi m_\pi \epsilon_{\alpha\beta} \delta^{ab} \quad (B.36)$$

For the flavour singlet, with $\langle 0 | X_\mu | \gamma \rangle = i k_\mu F_\gamma$,

$$m \langle 0 | (T_F - \bar{T}_F) | \gamma \rangle = i F_\gamma m_\gamma^2 \left(1 + \frac{N_c - N_F}{N_c} \frac{\Lambda^2}{m m_\gamma} + \frac{N_F}{N_c(N_c - N_F)} \frac{m_\lambda}{m_\gamma} \right) \quad (B.37)$$

From eqs. (B.28) and (B.34) we can immediately write the quadratic Dashen formula for the π :

$$F_\pi^2 m_\pi^2 \left(1 + \frac{\Lambda^2}{m m_\pi} \right) = \frac{4}{N_F} m^2 \langle C_0 \rangle \quad (B.38)$$

and for the flavour singlet γ :

$$F_\gamma^2 m_\gamma^2 \left(1 + \frac{N_c - N_F}{N_c} \frac{\Lambda^2}{m m_\gamma} + \frac{N_F}{N_c(N_c - N_F)} \frac{m_\lambda}{m_\gamma} \right) = 8 \frac{N_c^2}{N_F^2} m^2 \langle C_0 \rangle \quad (B.39)$$

The second problem is more severe. We need to relate, for example, $\langle 0 | T_0 - \bar{T}_0 | \gamma \rangle$ to $\langle 0 | T_F - \bar{T}_F | \gamma \rangle$, i.e. we need the broken SUSY corrections to the decay constant identities in section 2. This is straightforward in principle, but requires care. The safest derivation involves comparing residues in Green's functions related by SUSY identities, as in ref. 8), rather than using the method of section 2 with transformations of states. We find, for example

$$\begin{aligned} m_\gamma \langle 0 | T_0 - \bar{T}_0 | \gamma \rangle &= \langle 0 | T_F - \bar{T}_F | \gamma \rangle \\ &= i m_\gamma \langle 0 | \Delta_0(0) (T_0 - \bar{T}_0) | \gamma \rangle - i \langle 0 | \Delta_0(0) (T_F - \bar{T}_F) | \gamma \rangle + \frac{i}{2} F_\gamma \langle \gamma | \Delta_0(0) | \gamma \rangle \end{aligned} \quad (B.40)$$

similar results holding for ρ , χ and in the flavour non-singlet sector.

The final problem is that the argument given above enabling us to discard terms such as $\langle 0 | \delta^a(T_0 + \bar{T}_0)^b | 0 \rangle$ at zero momentum relies on an unbroken SUSY Ward identity. With SUSY breaking, we have instead, to lowest order,

$$\langle 0 | \delta^a(0) (T_0 + \bar{T}_0)^b(0) | 0 \rangle = i \langle 0 | \Delta_0(0) \delta^a(0) (T_0 + \bar{T}_0)^b(0) | 0 \rangle \quad (\text{B.41})$$

etc. This zero-momentum 3-point Green's functions can contribute at the same order as the other 2-point Green's functions in eq. (B.30) and must be retained.

It is clear, therefore, that this approach to modifying the Dashen formulae is extremely complicated. We have to control both 3-point Green's functions and 2-point matrix elements with external states, and keep track of all possible intermediate states which can contribute at leading order. This may be possible but is a difficult task.

Fortunately, we were able to find an alternative, much simpler, way to derive the essential dynamical information. Our main interest is after all in the masses of the pseudo Goldstone particles rather than in the combinations with decay constants given by the Dashen formulae. The simple formula we derived in section 7 gives directly the mass splittings amongst the states in a pseudo Goldstone multiplet due to soft SUSY breaking.

A set of Ward identities essentially identical to eqs. (B.27)-(B.31) has been derived independently by Clark and Love¹²⁾ in their analysis of broken SUSY corrections to the Dashen formulae. However, while we agree with their derivation up to the point of obtaining the Ward identities, it appears that their subsequent discussion is not correct. In particular, they assume that the actual order of magnitude (in terms of the chiral and SUSY breaking parameters) of certain Green's functions is equal to their naive order. However, this neglects the possible enhancements due to pseudo Goldstone boson intermediate states. This is of course the crucial point in the derivation of the original Dashen formula in QCD, where a 2-point Green's functions of naive $O(m^2)$ is found to be $O(m_\pi^2)$ because of the π intermediate state, and is then equated to a VEV of $O(m)$. Consequently, in ref. 12), several terms are neglected which are of the same order as those kept and the final Dashen formulae quoted are therefore unreliable.

Footnotes

- F.1 This identification is not always innocuous. See the discussion in Appendix A of ref. 8).
- F.2 The current R_μ here is renormalised so that the Adler-Bardeen theorem holds. The anomaly coefficient r is determined by the one-loop β function of SQCD.
- F.3 The Vafa-Witten theorem¹⁵⁾, which states that the even-parity chiral symmetries are not spontaneously broken in vector gauge theories, is not applicable to SQCD¹⁶⁾. In contrast to conventional theories therefore, where the chiral symmetry breaking $G \rightarrow H$ is such that G/H is a symmetric space, supersymmetric gauge theories can have a richer structure of chiral symmetry breaking with flavour symmetries also being broken. In this case there may be many independent pion decay constants. This is analysed in ref. 17), where the further generalisations of Dashen's formula are quoted.
- F.4 The derivation presented here is that referred to in Konishi's original paper¹⁰⁾, footnote 4.
- F.5 This choice is not unique. Many similar Ward identities lead to the same conclusion.

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