ALTERNATIVE SUPERSTRING COMPACTIFICATIONS

WILL NOT GIVE NEW LOW-ENERGY MODELS

B.A. Campbell\textsuperscript{(*)}, J. Ellis

CERN - Geneva

and

D.V. Nanopoulos

Physics Department, University of Wisconsin
Madison, WI 53706, U.S.A.

\textbf{ABSTRACT}

Alternatives to Calabi-Yau and related compactifications, such as orbifolds with non-Abelian space groups, could in principle yield low-energy models based on a GUT group different from $E_6$, $SO(10)$ or $SU(5)$. We look for such models with chiral fermion generations, trilinear superpotential terms yielding quark and lepton masses as well as Higgs mixing, perturbative gauge couplings, and at least one extra $U(1)$ beyond the standard model. The only viable model based on an $SU(6)$ GUT group is equivalent to the minimal rank-5 $E_6$ model studied previously. There are no acceptable models based on the GUT groups $SU(N>7)$ or $SO(14)$. When combined with previous arguments disfavouring $SU(5)$ and $SO(10)$ GUT groups, as well as rank-6 $E_6$ models, our results point to a unique low-energy $SU(3)_C\times SU(2)_L\times U(1)_Y\times U(1)_X$ model inspired by the superstring.

\textsuperscript{*}) Permanent address: Department of Physics, University of Alberta, Edmonton, Alberta, Canada.

CERN-TH.4504/86
August 1986
One of the current ambiguities in attempts to extract low-energy four-dimensional physics from the superstring\(^1\) is the choice of compactification space for the six surplus dimensions. The first spaces considered\(^2\) were manifolds, but it has subsequently emerged that orbifolds may be equally satisfactory for compactifying strings\(^3\). Most of the compactification schemes discussed up to now have been chosen to preserve one supersymmetry in four dimensions\(^2\). This requirement is imposed in order to safeguard the gauge hierarchy, though some theorists have never banished their doubts whether four-dimensional supersymmetry is really necessary for this purpose. If one does indeed wish to compactify supersymmetrically, the holonomy group \(H\) of the curled-up space must be \(SU(3)\) in the case of a manifold\(^2\), or a point group \(P\) which is some discrete subgroup of \(SU(3)\) in the case of an orbifold\(^3\). This continuous or discrete holonomy group must then be embedded in the gauge group in a non-trivial manner. The first approach taken was to identify the spin connection with the gauge connection, arriving\(^2\) at some Calabi-Yau\(^4\) manifold or trivial perturbation\(^5\) thereof. The orbifold equivalents\(^3\) have \(P\) mapped into some standard \(SU(3)\) subgroup of \(E_6 \times E_6\). Generalizations of these simplest approaches have been proposed, in which a larger \([SU(4)\text{ or } SU(5)\text{?}]\) gauge bundle may be constructed on the manifold of compactification\(^6\), or \(P\) is mapped into a non-standard \(SU(3)\) subgroup of \(E_6 \times E_6\)\(^3\). There is a residual four-dimensional "grand unification" (GUT) gauge group \(G\) which is the subgroup of \(E_6 \times E_6\)\(^\dagger\) that commutes with the image \(H\) of the holonomy group postulated in one of the above scenarios: 
\[
[H,G] = 0.
\]
It is possible to break \(G\) to a smaller four-dimensional gauge group \(\tilde{G}\) by some generalization\(^7\) of the Hosotani\(^8\) mechanism. In the case of manifolds one can have non-zero vacuum expectation values for some Wilson line integrals representing a discrete symmetry group\(^8\). Analogously, in the case of orbifolds one may choose\(^3\) a non-trivial embedding of the translation part of the space group in the gauge group\(^\dagger\).

In this paper we classify the phenomenologically interesting four-dimensional GUT groups \(G\) and post-Hosotani groups \(\tilde{G}\) without appealing to model-dependent details of the compactification scheme. We do not commit ourselves to manifolds or orbifolds, standard or non-standard embeddings. We recall that, to be considered as a suitable candidate for a GUT group, \(G\) should admit chiral representations. This restricts one to the sequences \(SU(N)\): \(N \geq 5\) and \(SO(4M+2)\): \(M > 2\), or to \(E_6\). It is well known that the majority of these groups proved

*\) It has been suggested that additional gauge symmetries may arise from some orbifold compactifications. We believe that the arguments of this paper could be applied to such a case also, but we have not done so since no explicit example is known.
unsatisfactory in previous attempts at grand unification, and most models were based on SU(5)\(^9\), SO(10)\(^{10}\) or \(E_6\)\(^{11}\). In a superstring approach, the available groups are the \(E_6\), SU(5+6+9) and SO(10 or 14) subgroups of \(E_8\), and there are important restrictions on the matter representations. Among these groups, the SU(5) and SO(10) possibilities\(^6\) have already been disfavoured by phenomenological difficulties in the Higgs sector\(^12\), which will be recalled later\(^*\). Successful models\(^13\) have previously been based on \(E_6\) broken\(^7\) by the Hosotani mechanism\(^8\) to a rank 5 subgroup \(\tilde{G}\), but there are phenomenological obstacles to using subgroups \(\tilde{G}\) of rank 6\(^14\). The other candidate subgroups SU(N=6) and SO(14) of \(E_6\) have not been explored previously.

We require in this paper that the four-dimensional theory contains trilinear couplings which allow quark and lepton masses and mixing between the Weinberg-Salam supersymmetric Higgses \(H\) and \(\bar{H}\) (these are the two restrictions that give most of our results), that it be perturbative at energies below \(\mu_p\) without invoking an intermediate scale, and that it have at least one extra \(U(1)\) present down to low energies, which enables the \(H\bar{H}\) mixing coefficient to have an acceptable magnitude\(^12\). We find just one SU(6) model which has the same \(\tilde{G} = SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)_E\) as the previous rank-5 \(E_6\) model\(^13\), and the same gauge non-singlet particle content. We find no acceptable models based on SU(N>7) or on SO(14). We conclude that the low-energy gauge group \(\tilde{G} = SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)_E\) may be unique, though it may have an origin different from conventional Calabi-Yau compactification.

We now state more precisely our key assumptions, together with some motivations. The two most important assumptions are the following.

**A1** The matter particles occupy chiral representations of \(SU(3)_C \times SU(2)_L \times U(1)_Y\). This assumption does not need much motivation, save to note that the observed matter particles occupy a chiral representation of \(SU(3)_C \times SU(2)_L \times U(1)_Y\). The existence of mirror generations is not excluded by experiment, but in the absence of an intermediate scale they would cause the strong coupling \(\alpha_3\) to blow up before \(\mu_p\), thus conflicting with our later assumption A3. Note that the assumption A1 is somewhat stronger than simply requiring \(G\) to have chiral representations. It is for example possible to construct models which are chiral with respect to SU(7), but vector-like with respect to SU(5) and hence the standard model group \(SU(3)_C \times SU(2)_L \times U(1)_Y\).

\(^*\) Recall that only the SO(4M+2) and \(E_6\) groups are safe, i.e., automatically free of anomalies, which is an aesthetic advantage.
A2. The trilinear superpotential for the matter fields includes terms which could give masses to the known quark and lepton fields, and which contain a coupling $\bar{H}H$ of the Higgs doublets in the supersymmetric standard model to some neutral field $N$. Clearly quarks and leptons do have masses, but they could in principle have arisen from quartic or higher order superpotential terms, using products of several vacuum expectation values. Presumably one could construct models along these lines, but they would be very unattractive. Moreover, the $t$ quark mass is so large that it seems unlikely to arise from higher-order effects, and our results are valid even if one only requires the masses of one generalization to be obtainable from trilinear terms. Mixing between the supersymmetric standard model Higgses $H$ and $\bar{H}$ given by $\langle 0\vert H\vert 0 \rangle = 0(m_{H})$ is essential if one is to have both $\langle 0\vert H\vert 0 \rangle$, $\langle H\vert \bar{H} \rangle = 0(m_{H})$ and avoid an unacceptable axion $^{12}$. Here again we believe that a term so important to phenomenology should be among the trilinear superpotential terms which arise naturally from the superstring.

Two other assumptions which exclude some cases are the following.

A3. The model should be perturbative at energies below $m_{P}$ without invoking an intermediate scale. The distaste for coupling constants which blow up is presumably universal, but some readers may not be averse to intermediate scales. We exclude them here because of the phenomenological difficulties ($H\bar{H}$ mixings$^{13}$, no-scale dynamics$^{13}$, excess entropy generation$^{15}$) they have encountered in previous scenarios for model-building. These difficulties could perhaps be avoided in other dynamical schemes, be we do not explore such possibilities here as we do not wish to fish out the pond. The best prospects for avoiding our subsequent conclusions may be offered by relaxing this assumption of no intermediate scale, which we regard as our weakest.

A4. The unbroken gauge group $\mathcal{G}$ should contain at least one extra $U(1)$ beyond the standard model, which is present down to low energies. It was pointed out previously$^{12}$ that an extra $U(1)$ was desirable for the dynamical generation of small Higgs mixing $\langle 0\vert H\vert 0 \rangle = 0(m_{H})$ $H\bar{H}$, in models where the dominant source of supersymmetry breaking in the observable sector is a gaugino mass, as is generally believed$^{13},^{16},^{17}$. Models with no extra $U(1)$ run the risk that the scalar field $N$ whose vacuum expectation value mixes $H$ and $\bar{H}$ may either have $\langle 0\vert H\vert 0 \rangle = 0(m_{P})$, or form unacceptable cosmological domains$^{12}$. 
Having stated our assumptions, we can now analyze possible models starting with the SU(N=6) series. For this purpose it is important to recall the SU(9) decomposition of the adjoint $E_8$ representation of $E_8$, which is

$$E_8 = 80 + 84 + \overline{84} = \text{adjoint} + 9[3] + \overline{9}[\overline{3}]$$

where $N[m]$ denotes the $m$-fold antisymmetric representation of SU(N), with conjugate representations denoted by bars. The general decomposition rule

$$N[m] = N^{-1}[m] + \overline{N}^{-1}[m-1]$$

will be used repeatedly in the following. These observations mean that one only has $9[3]$ and $\overline{9}[\overline{3}]$ representations of SU(9) or $\overline{9}[\overline{2}]$, $\overline{9}[\overline{1}]$, $9[2]$ and $9[1]$ representations of SU(8) available for model-building. All antisymmetric SU(7) and SU(6) representations are available.

**SU(6)** In order to get trilinear quark and mass terms (A2), we need 10-10-5 and 10-3-3 couplings in SU(5), i.e., $(5[2])^2 \overline{5}[1]$ and 5$[1]$ of $5[2] \overline{5}[1]$ couplings. It is easy to check that these can only come from $(\overline{6}[2])^3$ and $\overline{6}[2] \overline{6}[\overline{1}]$ couplings respectively, with

$$[5] = \begin{pmatrix} 6[2] \\ (\overline{5}[2] \equiv T) + (\overline{5}[\overline{1}] \equiv H) \end{pmatrix}$$

$$\overline{[5]} = \begin{pmatrix} 6[\overline{1}] \\ (5[\overline{1}] \equiv \overline{H}) + 5[\overline{1}] \end{pmatrix}$$

where $T$ is the quark/lepton 10 of SU(5), $\overline{T}$ is the quark/lepton $\overline{5}$, $H$ is a $\overline{5}$ including the $Y = +\frac{1}{3}$ Higgs doublet, and $\overline{H}$ is a $\overline{5}$ including the $Y = -\frac{1}{3}$ Higgs doublet. The particle content (3) also admits $H\overline{H}$ mixing (A2) via a $6[2] \overline{6}[1]$ coupling with $N$ assigned to a $5[0]$ from a $\overline{6}[1]$. The set (3) is also free of SU(6) anomalies. Indeed, it has the same particle content as a $27$ representation of $E_6$ which can be decomposed as $27 = (15,1) + (\overline{6},2)$ under the maximal subgroup SU(6)$\times$SU(2)$_N$ of $E_6$. However, in the general SU(6) case considered here, SU(2)$_N$ need not be a symmetry, which means that some Yukawa couplings which would have forbidden by $E_6$ could now be allowed. Since the particle content is the same as the $E_6$ model (A3), we can take over the known result that three generations and no more are consistent with perturbation theory and the absence of an intermediate scale (A3). This also rules out additional light triplets from vector-like combinations of SU(6) representations in addition to the minimal set (3). Finally we note (A4) that SU(6) can be broken to SU(5)$\times$U(1), where the extra U(1) is
identical to the $U(1)^5$ of the minimal rank-5 low-energy group obtainable \(^7\) from $E_6$ by Hosotani breaking \(^8\), leading to $\mathcal{G} = SU(3)^c SU(2)_L \times U(1) \times U(1) \times U(1)^{13}$. 

In summary: $SU(6)$ gives us a unique low-energy model which has the same gauge and matter fields as the familiar minimal $E_6$ model. 

$SU(7)$: There are two possible ways to get $\mathbf{10-10-5}$ and $\mathbf{10-\overline{5}-\overline{5}}$ couplings (A2) from this gauge group, namely (i) $(\mathbf{7}[3])^2 \mathbf{7}[1]$ and $\mathbf{7}[3] \mathbf{7}[\overline{1}] \mathbf{7}[\overline{2}]$ respectively, or (ii) $(\mathbf{7}[2])^2 \mathbf{7}[3]$ and $\mathbf{7}[2] \mathbf{7}[\overline{1}] \mathbf{7}[\overline{2}]$ respectively. In case (i) the minimal representation content $\mathbf{7}[3] + \mathbf{7}[1] + \mathbf{7}[\overline{1}] + \mathbf{7}[\overline{2}]$ must be supplemented by an additional $\mathbf{7}[1]$ in order to be free of anomalies. However, the resulting model is vector-like with respect to $SU(5)$, and so fails assumption A1 \(^*)\). In case (ii), the minimal representation content $\mathbf{7}[2] + \mathbf{7}[3] + \mathbf{7}[\overline{1}]$ has four times the anomaly associated with a single $\mathbf{7}[1]$. To cancel the anomaly we can add either (a) $\mathbf{7}[2] + \mathbf{7}[\overline{1}]$, or (b) $\mathbf{7}[3]$, or (c) $\mathbf{7}[3] + \mathbf{7}[\overline{1}]$, or (d) $\mathbf{4} \mathbf{7}[\overline{1}]$. Of these, case (b) is vector-like and so fails assumption A1, but cases (b), (c) and (d) pass A1. In case (a), we can get Higgs mixing from a $\mathbf{7}[3] \mathbf{7}[\overline{1}] \mathbf{7}[\overline{2}]$ coupling and so pass A2, but there are no trilinear Higgs mixing terms in cases (c) or (d), which therefore fail A2. Finally, we must check the compatibility of assumptions A3 and A4 for the surviving model of case (a).

Its representation content is $\mathbf{7}[2] + \mathbf{7}[3] + \mathbf{7}[\overline{2}] + \mathbf{2} \mathbf{7}[\overline{1}]$, which has the following $SU(5)$ decomposition, corresponding to one conventional chiral generation ($\mathbf{10+\overline{5}}$) plus pieces real with respect to $SU(5)$:

\[
\begin{align*}
\left(5 \otimes 5 \right)_6 + & \left(5 \otimes \overline{5} \right) + \left(5 \otimes 5 \right) + \left(\overline{5} \otimes 5 \right) + \left(\overline{5} \otimes \overline{5} \right) + \\
& \left(\overline{5} \otimes 5 \right) + \left(\overline{5} \otimes \overline{5} \right) + \left(\overline{5} \otimes \overline{5} \right) + \left(\overline{5} \otimes 5 \right) + \\
+ & \left(5 \otimes 5 \right) + \left(\overline{5} \otimes \overline{5} \right) + \left(\overline{5} \otimes \overline{5} \right) + \left(\overline{5} \otimes 5 \right)
\end{align*}
\]

(4)

(The suffixes $n$ on the representations in (4) and subsequently indicate the $SU(N)$ indices which have been "neutralized" to obtain the indicated $SU(5)$ representation $5^2 m^n$.) Many of the $SU(3)$ 3 and $\overline{3}$ fields in the $\mathbf{5}[1]$, $\mathbf{5}[\overline{1}]$, $\mathbf{5}[2]$ and $\mathbf{5}[\overline{2}]$ $SU(5)$ representations in (4) must combine with each other and become massive in order to be consistent with assumption A3. One obstacle to achieving this would be the presence of an additional conserved $U(1)$ outside $SU(5)$, as required by assumption A4. There are two independent $U(1)$'s in $SU(7)/SU(5)$,\(^*)\)

\(^*)\) It would also give an antisymmetric mass matrix for the charge $+2/3$ quarks.
namely \( Q_6 \equiv \text{diag}(1,1,1,1,1,-5,0) \) and \( Q_7 \equiv (1,1,1,1,1,-6) \), and any additional conserved \( U(1) \) must be of the form \( aQ_6 + bQ_7 \). To be consistent with \( A_3 \), one of the \( 5^2 \) in Eq. (4) must combine with either the \( 5^2 \) or the \( 5^3 \). This is possible only if the extra \( U(1) \) is either \( a(Q_6 + 5Q_7) \) \( \text{diag}(1,1,1,1,1,0,-5) \) or is \( Q_6 \) itself. These two \( U(1)'s \) actually appear in equivalent \( SU(7) \to SU(6) \to SU(5) \times U(1) \) decompositions. However, even if one or the other of these \( U(1)'s \) is selected \( (A_4) \), the rest of the representation (4) still contains \( 6(3^3 + 3) \) of \( SU(3) \) which cannot be given large masses. Therefore this model conflicts with assumption \( A_3 \).

In summary: there is no SU(7) model satisfying our assumptions. In view of this failure, one might also expect failure with SU(8) and SU(9), which is indeed the case, though not necessarily for the same reasons. We will discuss these groups briefly, drawing attention to the new difficulties which arise.

**SU(8):** Although one can get the \( 10-10-5 \) quark and lepton mass terms from a \( (8[3])^{28}[2] \) coupling, there is no way to get a \( 10-5-5 \) coupling using only the available \( 8[3], 8[2], 8[3] \) and \( 8[2] \) representations. Thus one fails assumption \( A_2 \).

**SU(9):** The same in spades. The only non-zero trilinear Yukawa couplings are \( (9[3])^3 \) which do not contain \( SU(5) 10-5-5 \) couplings, thus failing \( A_2 \). Moreover, because only \( 9[3] \) and \( 9[3] \) representations are available, anomaly-free SU(9) models are automatically vector-like, thus failing even \( A_1 \).

In summary: the only SU(N>5) model is one based on \( SU(6) \times SU(5) \times U(1) \) which has the same gauge and particle content as the rank-5 subgroup of \( E_6 \) previously studied.

We now turn briefly to the possibility of SO(14): the \( 248 \) of \( E_8 \) contains the adjoint \( 91 \), spinorial \( 64+\bar{64} \), two \( 14 \)'s and a \( 1 \) representation of SO(14), so these are the representations available for model-building. Their \( SO(10) \times SU(2) \) decompositions are:

\[
\begin{align*}
91 &= (45,1,1) + (1,3,1) + (1,1,3) + (10,2,2) \\
64 &= (16,1,1) + (8,2,1) \\
14 &= (10,1,1) + (1,1,2) + (1,2,1)
\end{align*}
\]

(5)

There is a \( 64-64-14 \) coupling analogous to the \( 16-16-10 \) coupling of SO(10) which could in principle serve to provide quark and lepton masses, thus satisfying \( A_2 \). However, the \( HH \) mixing could only come from a \( 14-14-1 \) coupling, which is not
satisfactory since in the absence of low-energy gauge interactions for it, the vacuum expectation value of the $I$ is unlikely to have a small value $O(m_W)$, as was discussed previously in the case of SU(5) and SO(10)$^{12}$. A more serious difficulty is apparent from (5), namely that SO(14) models contain even numbers of conventional SO(10) generations and an equal number of mirror generations (a defect shared with other SO(N): $11 < N < 16$ groups). Therefore they do not obey assumption A1 and would also fail A3, preventing the construction of a satisfactory model.

We conclude that alternative compactifications which do not pass through the $E_6$ subgroup of $E_8$ will only lead to phenomenologically satisfactory models if they break $E_8$ to $SU(6) \times SU(5) \times U(1)$. The gauge boson and matter particle content of the resulting model would be identical to the model$^{13}$ based on the rank-5 subgroup $SU(3)_C \times SU(2)_L \times U(1) \times U(1)_Y$ of $E_6$, which we have previously argued to be favoured over rank-6 $E_6$ models$^{14}$ and over $SU(5)$ and SO(10) models. [Recall also that manifolds leading to these latter gauge groups, which have $(2,0)$ world-sheet supersymmetry, are known not to be solutions to the equations of motion$^{18}$.] Thus the uniquely preferred low-energy model which can be extracted from the superstring is that based on $SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)_E$ with three generations each containing 27 matter supermultiplets. It may in the future prove possible to arrive at this model by an alternative to Calabi-Yau compactification. However, existing orbifold compactifications$^{3}$ do not give this gauge group. To do so, one needs orbifolds with a non-Abelian space group, which may require starting from a six-dimensional manifold which is not Euclidean, and models of this type have not yet been studied.

ACKNOWLEDGEMENTS

We thank M. Duff, S. Ferrara, C. Gomez and K. Narain for useful discussions.
REFERENCES


    M. Dine, N. Seiberg, X.-G. Wen and E. Witten - Princeton University preprint (1986);