RADIATIVE GENERATION OF INTERMEDIATE MASS SCALES IN SUPERSTRING-INSPIRED

MODELS

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1. INTRODUCTION

In ordinary and supersymmetric GUTs much attention has been devoted to the implications of intermediate mass scales (IMS) between the ones of grand unification, M_X , and of electroweak breaking, m_w . On one hand, going beyond minimal $SU(5)$ to higher symmetries such as $SO(10)$ and E_6 , IMS appear naturally as intermediate steps in the symmetry breaking chain, through the usual Higgs mechanism of spontaneous symmetry breaking [1]. On the other hand, IMS generally imply new kinds of physical processes related to lepton and/or baryon number violation [2]. In particular, the right-handed neutrinos ν_R can acquire Majorana masses of the order of the intermediate mass scale M_I . This provides a simple explanation for the puzzle of (left-handed) neutrino masses m_ν , whose upper bounds are much below the masses m_χ of the charged lepton counterparts: in fact, the so-called "see-saw" mechanism gives $m_\nu \approx \frac{m_\chi^2}{M_I}$, which can be sufficiently small [3].

Nowadays, superstring theories [4] try to go one step beyond GUTs, aiming at the consistent unification of gravity with the other fundamental interactions. In particular, the heterotic superstring [5], formulated in 10 dimensions with a gauge group $E_8 \times E_8$, seems to be particularly promising for phenomenology, thanks to a class of compactifications [6,7] to four dimensions that can give chiral fermions, unbroken $N=1$ supersymmetry and a subgroup of E_6 as the effective low-energy gauge group.

The problem of implementing IMS in superstring-inspired models [8] has been considered by several authors [9-13]. The situation appears to be rather different from what occurs in GUTs: first of all, the usual Higgs mechanism for breaking the grand unification group has to be replaced by a different one, the so-called "flux-breaking mechanism" [14,9]; second, the gauge-non-singlet fields of the effective field theory below the compactification scale are assigned to definite representations of the symmetry group, and one cannot make use of additional representations. These

1.

ABSTRACT

The mechanism for generating radiativebreaking of the gauge symmetry at an intermediate mass scale is analyzed in E_6 superstring-inspired models. Such a breaking can be induced by large vacuum expectation values, which are developed along a flat direction of the scalar potential. Radiative one-loop corrections as well as possible contributions from non-renormalizable terms are taken into account. The minimization of the effective potential is carried out quantitatively in a specific model based on the gauge group $SU(3)_C \times SU(2)_L \times [U(1)]^3$. A numerical analysis is performed in order to establish the compatibility between the values obtained for M_I (in the range 10^7-10^{14} GeV) and those of the low-energy parameters ($\sin^2 \theta_w$, α_3 and α_{em}). The characteristic features of the example considered here are believed to remain valid in more general situations.

two facts put severe constraints on the realization of IMS and strongly limit the possible choices.

We are concerned here with those superstring-inspired models [6] which correspond to $d=4$, $N=1$ supergravity with gauge symmetry group $E_6 \times E_8'$. The first term E_6 is the gauge group of the observable sector, and it is broken down to a subgroup H of rank $r=5$ or 6 (via the flux-breaking mechanism) at the compactification scale $M_C \sim M_p$, where $M_p \sim 2.4 \times 10^{16}$ GeV is the reduced Planck mass. The second term E_8' refers to the hidden sector, which can provide the breaking of local $N=1$ supersymmetry through gaugino condensation [15] at a scale Λ_C : as a consequence, soft breaking terms appear in the observable sector, and one ends up with an effective low-energy theory with $N=1$ (softly broken) supersymmetry.

In general, two main phenomenological problems have to be faced in E_6 superstring-inspired models: the decay of the proton and the generation of approximately massless neutrinos. Due to the presence of extra charges $1/3$ quarks, say D and $\bar{D}$, in the matter supermultiplets, the proton would decay extremely fast unless $m_{D, \bar{D}}$ is very large or some of the possible Yukawa couplings are approximately zero. Concerning the neutrino mass, each generation contains a right-handed field $\nu_R \leftrightarrow (\nu^c)_L$, but it cannot acquire a large Majorana mass because neither appropriate Higgs fields are available, nor radiative corrections give significant effects. In addition, the conventional constraints on the grand unification mass M_X , the unified gauge coupling constant g_u and the electroweak angle θ_w , coming from the renormalization group analysis, must also be satisfied by every realistic model.

Among the class of models considered here, one which can be phenomenologically realistic in absence of any intermediate mass scale is the so-called "minimal" model [9, 10, 16], based on the gauge group $H_{\min} \equiv SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)_E$, which can be obtained from E_6 by non-abelian flux-breaking. Detailed calculations of radiative electroweak

symmetry breaking and of particle spectra have been performed in that model [16]. There the problem of too fast proton decay is eliminated by imposing the vanishing of appropriate Yukawa couplings, which can be attributed to the presence of discrete symmetries or to topological properties of the manifold of compactification [8, 9, 17]. A similar solution is adopted for the problem of neutrino masses: Dirac-type masses would be unacceptably large, but they can vanish if a specific Yukawa coupling is zero. The contribution of non-renormalizable terms in the superpotential, which could generate Majorana neutrino masses [11], would be too small in this model, due to the absence of an intermediate scale. Then one has at least three additional massless particles, the right-handed neutrinos ν_R , which might not coexist so comfortably with the present astrophysical limits [18], unless the neutral gauge boson to which they couple is sufficiently heavy.

Alternative, more traditional, solutions to these problems could be provided by superstring-inspired models which allow for IMS. For sufficiently high values of M_1 , the D-type quarks could become heavy enough to inhibit proton decay, and the ν_R neutrino could acquire a Majorana mass so that the see-saw mechanism (or some generalization of it) [19] would become operative.

These are, in our opinion, the main motivations for the interest in implementing superstring-inspired models with IMS. The aim of the present paper is to investigate the possibility of building models of this kind that are compatible with the low-energy phenomenology. As a first step, we shall be mainly concerned with the general issue: how to generate dynamically intermediate energy scales, extending to the present framework the knowledge of radiative symmetry breaking accumulated over the past years [20-23]. The various points will be illustrated by a simple model based on E_6 , broken at the scale $M_C \sim M_p$ to the rank-six subgroup $H_2 \equiv SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)_E$; in turn, H_2 is

2. CONDITIONS FOR IMS

To discuss the possibility of generating IMS in superstring-inspired models, we need to recall some of their basic features. We consider here the class of models which assume the compactification of the heterotic superstring on a six-dimensional manifold K of $SU(3)$ holonomy, with the spin connection ω identified with the gauge connection A [6].

On a simply connected manifold K_0 , the resulting four dimensional theory is a $N=1$ supergravity, with $E_6 \times E_8$ as a gauge group. Chiral matter superfields are singlets under the E_8 factor, and transform according to the fundamental representation of E_6 ; one has in general $n_g = 27 + \delta^\circ(27+27)$, with $n_g = \frac{|X(K_0)|}{2}$ and $\delta^\circ \equiv b_{1,1}(K_0) \geq 1$, where X and $b_{1,1}$ are topological invariants (the Euler characteristic and the Betti-Hodge number of the manifold K_0 , respectively). Simply connected manifolds have to face two very severe problems: 1) the number of chiral generations, n_g , is in general unacceptably high; 2) an acceptable Higgs breaking of E_6 down to the standard model group $H \equiv SU(3)_C \times SU(2)_L \times U(1)_Y$ is impossible, since the only H singlets contained in the 27 of E_6 are also singlets under $SU(5)$ and $SO(10)$.

Both difficulties can be avoided if K_0 admits a freely acting discrete symmetry group G , so that it can be replaced by the (non simply-connected) quotient manifold $K = K_0/G$ [6,9]. One has then $\chi(K) = \chi(K_0)/N(G)$, where $N(G)$ is the number of elements of G , so that $n_g = n_g^0/N(G)$ can become acceptably small. Moreover, a new mechanism for E_6 symmetry breaking becomes available, the so called "flux-breaking" [14,9]. Non-trivial gauge configurations on K can contribute to the vacuum expectation value (VEV) of Wilson-loop operators of the type

$$U_g = P \exp i \int_\Gamma A_g dx, \quad (2.1)$$

which can play the same role as Higgs bosons in the adjoint representation of E_6 . In (2.1) Γ is a non-contractible loop on K , which is the image of the path from x to gx ($g \in G$) on K_0 . The mapping $g \mapsto U_g$ is a homomorphism of G

broken to $H_1 \equiv SU(3)_C \times SU(2)_L \times U(1) \times U(1)$, at the scale M_I [24]. We do not attempt to make the model fully realistic and, in the spirit of the paper, we leave aside detailed solutions for the specific problems mentioned above.

The material of the paper will be organized as follows. We start in Section 2 discussing the general conditions that must be satisfied in superstring-inspired models to have an acceptable symmetry breaking at an intermediate mass scale: most of the content of this section is not new, but we try to be pedagogical and to clarify some obscure points. In Section 3 we introduce the specific model that will be used, in the rest of the paper, as a working example to illustrate our ideas. In Section 4 we develop our main point: going beyond the existing group-theoretical and algebraic discussions [9-13], we examine in detail the structure of the scalar potential, in connection with the possibility of generating a radiative breaking of the intermediate symmetry. The minimization of the potential is carried out taking into account the full one-loop corrections and the possible inclusion of additional non-renormalizable terms. In Section 5, we analyze the compatibility between the solutions previously found and the low-energy parameters ($\sin^2 \theta_w$, α_3 and α_{em}), requiring perturbative unification and acceptable initial conditions: a class of solutions survives, demonstrating the feasibility of such a scenario. Finally, we draw some conclusions and we comment on the perspectives for solving the remaining open problems.

onto a discrete subgroup $\bar{G}$ of E_6 . It was shown in ref. [9] that the fields $\psi(x)$ on K which remain light upon compactification are those fields $\psi(x)_0$ on K_0 which satisfy the condition:

$$\psi(g x)_0 = U_g \psi(x)_0, \quad (2.2)$$

expressing the invariance under the direct sum $G \oplus \bar{G}$.

Since massless gauge bosons of E_6 are invariant under the action of G , the subgroup of E_6 which remains unbroken after compactification is simply the maximal subgroup H which obeys the condition:

$$[H, \bar{G}] = 0. \quad (2.3)$$

It is a straightforward exercise [9, 10, 12] to find out all possible subgroups H of E_6 which contain H_0 : they can have rank 5 or 6, so that they contain at least an additional $U(1)$ factor with respect to H_0 . Of course, most of the possibilities do not represent good candidates for a realistic low-energy theory in the TeV region, because of phenomenological problems with proton decay, neutrino masses, the observed properties of low-energy interactions, etc.. For some of these models, the above problems would admit a solution if a further stage of symmetry breaking could take place at a sufficiently large IMS.

Any acceptable intermediate scale breaking must correspond to an approximately flat direction of the scalar potential, in order not to spoil the supersymmetric solution of the gauge hierarchy problem.

Therefore, the scalar gauge-non-singlet fields which acquire large VEVs must be self-conjugate multiplets contained in the $b_{1,1}$ pairs $(27 + \bar{27})$ and their VEVs must be approximately equal, to avoid a big D-term contribution to the scalar potential. Moreover, the superpotential couplings which could induce a big F-term contribution to the potential must also be absent.

It is therefore crucial, for the discussion of IMS, to understand which components of the $b_{1,1}$ $(27 + \bar{27})$ survive as massless fields on K after flux breaking. In particular, the generation of an IMS is impossible in the ab-

sence of these light 'survivors'.

The rule for the determination of the light survivors is set by eq. (2.2). Since the index theorem ensures that $n_g = n^0/N(G)$ chiral generations, corresponding to $n_g \cdot 27$ superfields, will survive on K , to determine the additional self-conjugate light pairs coming from the $b_{1,1}(\bar{27} + 27)$ it is convenient to concentrate on the $b_{1,1} \cdot \bar{27}$ on K_0 .

In general, they will transform as a sum of irreducible representations (IR) R_i of G . On the other hand, each $\bar{27}$ decomposes under $\bar{G}$ (which is a discrete subgroup of E_6) into a sum of IRS L_j .

The 'survivors' on K will correspond, according to (2.2), to the fields invariant under $G \oplus \bar{G}$. Given a multiplet $\bar{\psi}$ of H contained in a $\bar{27}$ of E_6 , it will belong to an IR L_j of $\bar{G}$, and therefore it will survive on K , if and only if it also belongs to an IR R_i of G such that the direct product representation $R_i \otimes L_j$ contains a singlet of $G \oplus \bar{G}$. If $\bar{\psi} \in \bar{27}$ is light, then the index theorem ensures that the self-conjugate $\psi \in 27$ will also remain light.

Historically, the first class of manifolds which was considered for model building were the so-called Y -manifolds, with $b_{1,1}=1$. In this particular case, the single $\bar{27}$ which is present is invariant under G , so that its surviving components on K are simply those invariant under $\bar{G}$. A complete classification of the survivors for the different subgroups H of E_6 can be found in ref. [12].

On the manifolds with $b_{1,1} > 1$, however, the situation is radically changed, since it is the general condition (2.2) that determines the survivors. If one is not working with a specific manifold with known discrete symmetries, one can contemplate the possibility that an arbitrary given set of H submultiplets, contained in the $b_{1,1}(\bar{27} + 27)$ of E_6 , can remain light on K , for an appropriate choice of the manifold K_0 and its discrete symmetry G .

Besides the existence of some light survivors with the appropriate quantum numbers, another necessary condition for the radiative generation of

3. AN ILLUSTRATIVE MODEL

In this section, we introduce a model representing a concrete example in which radiative generation of IMS breaking can occur. The model is based on the rank 6 group

$$H_2 = \text{SU}(3)_C \times \text{SU}(2)_L \times \text{U}(1)_Y \times \text{U}(1)_{Y'} \times \text{U}(1)_{Y''}, \quad (3.1)$$

where the three abelian factors are specified by the generators

$$Y = \sqrt{\frac{3}{5}} \left[\frac{1}{3} \tau_3 + \frac{1}{\sqrt{3}} \frac{Y_L}{L} + \frac{1}{\sqrt{3}} \frac{Y_R}{R} \right], \quad (3.2)$$

$$Y' = -\frac{1}{2\sqrt{10}} \left[\tau_3 + 2\sqrt{3} \frac{Y_L}{L} - 3\sqrt{3} \frac{Y_R}{R} \right],$$

$$Y'' = \sqrt{\frac{3}{8}} \left[\tau_3 - \frac{2}{\sqrt{3}} \frac{Y_L}{L} - \frac{1}{\sqrt{3}} \frac{Y_R}{R} \right],$$

which are expressed in terms of diagonal generators of the $\text{SU}(3)_C \times \text{SU}(3)_L \times \text{SU}(3)_R$ subgroup of E_6 .

For later use, we specify also the gauge couplings related to the different factors in the gauge group H_2 ; they will be denoted, with an obvious correspondence, by: $g_3, g_2, g_1, g'_1, g''_1$.

The quantum numbers of the submultiplets in the $\underline{27}$ of E_6 are shown in Table 1. Our notation corresponds to the following assignment in terms of the $\text{SO}(10)$ multiplets: $\underline{16}(Q, L, U, d^c, e^c, \nu^c) + \underline{10}(D, D^c, H, \bar{H}) + \underline{1}(N)$. The physical particles will be, in general, superpositions of these fields.

We assume that there are n_g generations plus additional components (survivors) of the $b_{1,1} \quad (27+2\bar{7})$ pairs. A minimal set of survivors is employed in our analysis it consists of L_g, H_g, ν_g^c and L_g^*, H_g^*, ν_g^{c*} . Their quantum numbers coincide with those of the corresponding components of the generations and with the conjugate ones. This set of light components

IMS is supersymmetry breaking: without susy breaking, the degeneracy along the flat directions of the scalar potential cannot be lifted by radiative corrections [25].

The favoured mechanism for supersymmetry breaking in superstring-inspired models is gaugino condensation in the hidden sector [15]. A plausible scenario assumes that a simple factor G' of the subgroup of E_8 unbroken after compactification becomes strongly interacting at a certain scale $\Lambda_c < M_p$, inducing VEVs for gaugino bilinears and triggering in such a way, under proper conditions, the breaking of local $N=1$ supersymmetry, with a gravitino mass $m_{3/2} \sim \frac{\Lambda_c^3}{M_p^2}$. One expects this breaking to be fed into the observable sector in the form of generic soft supersymmetry breaking terms: gaugino and scalar masses and trilinear scalar couplings.

In particular, there are indications [26] that the dominant source of supersymmetry breaking in the observable sector is likely to be a set of gaugino masses (proportional to the squares of the corresponding gauge couplings) $m_{1/2} \sim m_{3/2}^{5/3} / M_p^{2/3}$, to be imposed as boundary conditions at the scale Λ_c . Radiative corrections in the observable sector, taken into account by the renormalization group equations (RGEs), generate then other kinds of soft susy-breaking parameters at lower scales.

To have $m_{1/2} \sim 100 \text{ GeV} \div 1 \text{ TeV}$, one needs $m_{3/2} \sim 10^{10} - 10^{11} \text{ GeV}$, which corresponds in turn to $\Lambda_c \sim 3 \div 6 \times 10^{15} \text{ GeV}$.

This concludes our review of the general conditions for the radiative generation of IMS: a specific example will be worked out in detail in the following sections.

can be obtained only on a manifold K with $b_{1,1} > 1$.

We assume that the effective Lagrangian, at scales $M_1 \ll Q \ll M_x$, is a sum of supersymmetric terms, which are determined by the superpotential, and of soft-breaking terms. The superpotential can be split into three parts; the first one contains the Yukawa couplings among the generations and has the standard form:

$$W_g = h_L \bar{L} H e^c + h_d \bar{Q} H d^c + h_u \bar{Q} H u^c + h_v \bar{L} H \nu^c + h'_v \bar{D} d^c + \lambda_2 \bar{H} H N + \lambda_3 \bar{D} D N + \lambda_Q \bar{Q} D^c L + \lambda'_L \bar{D} u^c e + \lambda_B \bar{Q} Q D + \lambda'_D \bar{D}^c d^c u; \quad (3.3)$$

the second one contains the Yukawa couplings among the survivors only, and in the present case it reduces to

$$W_s = \lambda \bar{L} H \nu^c + \lambda_* \bar{L} H_* \nu_*^{c*}; \quad (3.4)$$

finally, the third part W_{NR} contains non-renormalizable terms which are expected, in general, to be originated in the effective low-energy theory from exchange of superheavy fields [10-13]. Here we write explicitly only those terms which will be relevant for the analysis of the scalar potential:

$$W_{NR} = \frac{f_1}{M_P} (\nu_s^c \nu_s^{c*})^2 + \frac{f_2}{M_P^3} (\nu_s^c \nu_s^{c*})^3 + \dots \quad (3.5)$$

For the sake of simplicity, we have excluded, from the superpotential, a part W_{gs} containing Yukawa couplings between generations and survivors, and analogous g - s terms in W_{NR} , e.g. of the type $\frac{1}{M_P} (\nu_s^c \nu_s^{c*})^2$. It might appear that some of these terms play an essential role for the solution of the phenomenological problems mentioned in Sect. 1. On the other hand, their introduction gives rise to other difficulties, and we neglect them altogether, to avoid unessential complications in our analysis.

Nevertheless, it is worthwhile to recall that the quoted problems are related to some of the extra terms which appear in (3.3), in addition to those of the standard model.

The two sets of couplings $(\lambda_Q \bar{Q} D^c L + \lambda'_L \bar{D} u^c e + h'_v \bar{D} d^c \nu^c)$ and $(\lambda_B \bar{Q} Q D + \lambda'_B \bar{D}^c d^c u)$ would generate too fast proton decay mediated by the D, D^c scalar components. With suitable assignment of the baryon number for D ($B(D) = +1/3$ or $-2/3$), either of the two sets conserves B , and one has to impose $\lambda_L = \lambda'_L = 0$ or $\lambda_B = \lambda'_B = 0$ to eliminate the other set which violates B . [10, 12, 16].

The coupling $\lambda_L H \nu^c$ would generate unacceptable Dirac neutrino masses, and the coupling $h'_v \bar{D} d^c \nu^c$ would also give a large mixing between the d and D quarks which would violate weak universality; to avoid these problems, one should put $\lambda_v = 0$ and $h'_v = 0$ (the latter condition can be relaxed if $\langle \nu^c \rangle = 0$) [16].

The vanishing of the dangerous couplings can be attributed to the presence of discrete symmetries [6, 9, 17]; an alternative possibility would be the generation of sufficiently high masses for D, D^c and ν^c [10, 12].

Constraints on the couplings are also imposed by the absence of FCNC at tree level. In particular, only one pair of the $(H, \bar{H})$ superfields with conventional Higgs quantum numbers should be significantly coupled with quarks and leptons, so that the other pairs should be either uncoupled or heavy enough [16, 24].

Going back to the effective Lagrangian, we have to specify the soft breaking part, L_{soft} , induced by the breaking of local supersymmetry occurring in the hidden sector. It contains, in general, the following terms

$$-L_{soft} = \sum_i m_i^2 |z_i|^2 + \frac{1}{2} \sum_a \mu_a (\lambda_a + h.c.) + \frac{1}{6} \sum_{i,j,k} (A_{ijk} W_{ijk} z_i z_j z_k + h.c.), \quad (3.6)$$

where z_i and λ_a stand for the scalar and gaugino fields, respectively; W_{ijk} are the third order derivatives of the superpotential with respect to the scalar fields, and A_{ijk} parameters with the dimension of a mass.

With the above pieces of information we are in a position of writing explicitly the scalar potential. In our analysis, only the terms containing the scalar components x and y of ν_s^c and ν_s^{c*} , respectively, will be relevant.

Another important ingredient of our analysis is the knowledge of the scale evolution of the coupling constants and of the soft-breaking parameters contained in the effective Lagrangian. The evolution is determined by a set of coupled RGEs, which are given in Appendix A.

A technical remark is in order concerning the use of these RGE's. The one-loop evolution of the gauge couplings (in the range between the unification scale M_X and m_W) is determined by uncoupled equations, provided a convenient basis is chosen for the corresponding generators. In fact, in an arbitrary basis, a mixing is expected to occur among the generators [27].

In our case, while the unbroken factors of the group H_2 (namely $SU(3)_C$ and $SU(2)_L$) will renormalize independently, the $U(1)$ factors, in general, will not, unless their generators are mutually orthogonal [28]. One can check immediately from Table 1 that each complete generation gives a vanishing contribution to the off-diagonal terms:

$$\text{Tr}(YY')_{27} = \text{Tr}(YY'')_{27} = \text{Tr}(Y'Y'')_{27} = 0.$$

On the other hand, the set of survivors ($L_s, H_s, \bar{V}_s^C$ and $L_s^*, H_s^*, \bar{V}_s^{C*}$) gives rise to the non-vanishing contribution

$$\text{Tr}(YY')_{\text{surv}} = \sqrt{24/5}$$

in the range between M_X and M_I (the survivors give no contribution below M_I , since they decouple at the IMS).

It follows that Y'' does not mix and g_1'' is renormalized independently. Instead, Y and Y' get mixed, so that it is convenient to move to a new basis $(\tilde{Y}, \tilde{Y}')$ defined by

$$\begin{aligned} \tilde{Y} &= \cos\theta Y + \sin\theta Y' \\ \tilde{Y}' &= -\sin\theta Y + \cos\theta Y' \end{aligned} \quad (3.7)$$

and

$$\text{Tr}(\tilde{Y}\tilde{Y}')_{\text{surv}} = 0. \quad (3.8)$$

Then, the corresponding couplings $\tilde{g}_1$ and $\tilde{g}_1'$ will obey uncoupled RGE's. As explained in [29], it is not restrictive to assume $-\pi/2 < \theta < \pi/2$. Therefore, from (3.8), one obtains two equivalent solutions:

$$\cos\theta = \sqrt{2/5}, \quad \sin\theta = -\sqrt{3/5} \quad \text{and} \quad \cos\theta = \sqrt{3/5}, \quad \sin\theta = \sqrt{2/5}.$$

In our analysis, we choose the first solution.

From now on we replace (3.1) by

$$\tilde{H}_2 = SU(3)_C \times SU(2)_L \times U(1)_{\tilde{Y}} \times U(1)_{\tilde{Y}'} \times U(1)_{\tilde{Y}''}, \quad (3.9)$$

and the corresponding gauge couplings are defined by: $\tilde{g}_3, \tilde{g}_2, \tilde{g}_1, \tilde{g}_1', \tilde{g}_1''$.

It can be shown that the electric charge is related to the couplings $\tilde{g}_2, \tilde{g}_1$ and $\tilde{g}_1'$ by [29]:

$$\frac{1}{e^2} = \frac{1}{\tilde{g}_2^2} + \frac{5}{3} \left(\frac{\cos^2\theta}{\tilde{g}_1^2} + \frac{\sin^2\theta}{\tilde{g}_1'^2} \right). \quad (3.10)$$

The RGE's given in Appendix A apply to the situation represented by (3.9).

A numerical study of these equations has been performed in the one-loop approximation. Suitable boundary conditions have to be imposed on the solutions.

For the gauge and Yukawa couplings, we have to fix such conditions at the scale M_X . Under our assumptions, only the couplings between survivors need to be considered:

$$\left. \begin{aligned} \tilde{g}_2 &= \tilde{g}_1 = \tilde{g}_1' = \tilde{g}_1'' = \tilde{g}_u \\ \lambda &= \lambda_* = \lambda_u \end{aligned} \right\} \quad (Q=M_X), \quad (3.11)$$

where $\lambda(M_X) = \lambda_*(M_X)$ is introduced just for the sake of simplicity.

The relevant soft-breaking parameters are: the masses $\mu_a = (u_2 \mu_1 \mu_1' \mu_1'')$ of the gaugino fields related to the $SU(2)$ and $U(1)$ factors in $\tilde{H}_2$, the common mass m_H of the scalar components in L_s, H_s, L_s^*, H_s^* , and the parameter A associated to the soft supersymmetry breaking interactions among the scalar survivors.

The initial conditions for these parameters are imposed at the scale

$$A_c: \left. \begin{aligned} \mu_a &= \frac{g_a^2}{g_u} \mu_u \\ m_x &= m_y = m_H = A = 0 \end{aligned} \right\} \quad (Q=A_c), \quad (3.12)$$

where $g_a = (g_2, g_1, g_1', g_1'')$.

In the numerical analysis, we have examined two different cases: $A_c = M_c$ and $A_c = 4 \times 10^{15}$ GeV [26].

As it will be discussed in the next section, the model naturally leads to IMS for reasonable values of M_c , A_c , g_u and λ_u . The appearance of IMS corresponds to the large VEVs developed by the x and y fields: $\langle x \rangle \approx \langle y \rangle \approx M_I$.

Due to such VEVs, the survivors L_s, H_s, L_s^* and H_s^* acquire masses of order M_I , from the term W_s (3.4), as it is evident from (B4, 5).

We should point out that such VEV's, being neutral under both $\tilde{Y}$ and $\tilde{Y}'$, cause the breaking at the scale M_I of the term $U(1)_{Y''}$ alone in $\tilde{H}_2$, so one is left with the $r=5$ symmetry group $H = SU(3)_C \times SU(2)_L \times U(1)_{Y'} \times U(1)_{Y''}$, for scales down to m_w . To go further down, one has to break H_1 at the electroweak scale m_w to $H = SU(3)_C \times U(1)_Q$. Also the breaking of H_1 can be generated radiatively, following the same lines discussed in analogous schemes [16, 24].

Finally, we should point out that the solutions found for M_I have to pass a further test to be acceptable: they have to be in agreement with the low-energy phenomenology, i.e. with the measured values of the parameters

$$\sin^2 \theta_w, \alpha_3 \text{ and } \alpha_{em}.$$

4. RADIATIVE GENERATION OF IMS

In this section we want to discuss the issue of a radiatively generated IMS for superstring inspired models. To be more specific, we refer to the model described in section 3; yet we expect that our conclusions are not restricted to this example but have a wider range of applications.

We are interested in the dependence of the tree level scalar potential V_0 on the fields x and y only, and we assume that the other fields coupled to x and y have zero or negligible VEV. Then, with the notation previously given, we obtain:

$$V_0 = m_x^2 |x|^2 + m_y^2 |y|^2 + \frac{g_u^2}{3} (|x|^2 - |y|^2)^2 + \dots + (n+1)^2 \frac{|f_n|^2}{M_p^{4n-2}} |x|^{2n} |y|^{2n} (|x|^2 + |y|^2) + \dots, \quad (4.1)$$

where the integer $n \geq 1$ in (4.1) is the smallest value compatible with the symmetries of the theory and the dots stand for possible higher dimensional terms. We want to stress the fact that, although V_0 in (4.1) refers to a particular model, nevertheless it is representative of a much more general situation. The only D-term which can accommodate a large IMS must be of the type given in (4.1) [10]. Soft breaking mass terms, as well as non-renormalizable terms, are a general feature of the models we are examining. No other terms in the potential are generally required by the se models.

Now we proceed to the minimization of V_0 neglecting, for the moment, the non-renormalizable terms.

In the case of exact supersymmetry we have: $m_{xy} = 0$. The potential displays the flat direction $|x| = |y|$. As stated by general theorems [25], the degeneracy among the minima of this valley cannot be removed by a loop computation, at any order of perturbation theory.

However, if scalars and gauginos have non-vanishing masses, supersymmetry is softly broken and, along the previous valley, we have:

$$V_o = (m_x^2 + m_y^2) |x|^2 \quad (4.2)$$

This direction is no longer flat: it has a curvature which, however, is small, since supersymmetry has to be broken by a small amount, to preserve the solution of the hierarchy problem.

If m_x^2 and m_y^2 are positive, V_o reaches the minimum at the origin $|x|=|y|=0$, but we must pay attention to the fact that loop corrections to V_o are no longer negligible. We have to take them into account since they can drastically shift the minimum [20].

The one-loop potential V_1 has the form [20,30] (see Appendix B):

$$V_1 = V_o + \Delta V_1 \quad (4.3)$$

$$\Delta V_1 = \frac{M}{64\pi^2} \text{Str } M^4 \ln \left[\frac{M^2}{Q^2} \right]$$

ΔV_1 depends on x and y through the tree-level mass matrix M . In the above expression for V_1 all parameters of the theory are functions of the renormalization point Q .

In the minimization of V_1 we proceed as follows. We momentarily ignore ΔV_1 . Hence the only effect of the radiative corrections is the dependence on Q of the parameters contained in V_o . Moreover, since we expect the D-term to be zero or small, we look at the $|x|=|y|$ direction where V_o is given by (4.2). If at a scale Q'

$$(m_x^2 + m_y^2)(Q') < 0,$$

then $\langle x \rangle > Q'$ since V_1 is decreasing for x close to Q' . On the other hand, if at a scale Q''

$$(m_x^2 + m_y^2)(Q'') > 0$$

then $\langle x \rangle < Q''$, since V_1 is increasing for x close to Q'' .

This suggests that V_1 reaches the minimum at the value $\langle x \rangle$ such that:

$$(m_x^2 + m_y^2) \langle x \rangle = 0 \quad (4.5)$$

(Throughout this section we assume a real, positive $\langle x \rangle$).

We have implemented condition (4.5) in our model with the renormalization group analysis contained in Appendix A. The model naturally leads to IMS for reasonable values of $M_x, \Lambda_c, g_u, \lambda_u$ ($\langle x \rangle$ does not depend on μ_u), as shown in Fig. 1. We observe that, even in absence of the non-renormalizable terms, large VEVs can develop in a range from 10^7 GeV (or less if λ_u is a very small) to about 10^{14} GeV. This upper limit is constrained by the actual location of Λ_c . The highest values are obtained by identifying Λ_c with M_x . However we have also considered the possibility that Λ_c is smaller than M_x . [26]

Next we study the effect of the term ΔV_1 . In fact, we expect that ΔV_1 does not essentially spoil the previous conclusions, since it is depressed by coupling constants and logarithms which must be small for the consistency of the perturbative calculation. However, in order to check this statement, we have explicitly worked out ΔV_1 for the model at hand (for its actual expression see the Appendix B). Along the direction $|x|=|y|$ and in the large $|x|$ limit, which is the one we are interested in, we obtain:

$$\Delta V_1 = B |x|^2 + C |x|^2 \log \frac{|x|^2}{Q^2} \quad (4.6)$$

where B and C are functions of the parameters of the theory, whose expressions are quoted in Appendix B.

The minimum $\langle x \rangle$ of V_1 must satisfy:

$$0 = V'(\langle x \rangle) = 2 \langle x \rangle \left[m_x^2 + m_y^2 + B + C + C \log \frac{\langle x \rangle^2}{Q^2} \right] \quad (4.7)$$

$$V''(\langle x \rangle) = 2 \left[m_x^2 + m_y^2 + B + C + C \log \frac{\langle x \rangle^2}{Q^2} \right] + 4 \frac{C}{Q^2} > 0 \quad (4.8)$$

We can consistently choose $Q=\langle x \rangle$ and drop the extremum $\langle x \rangle=0$ which has turned into a maximum [20], and the above conditions become:

$$(m_x^2 + m_y^2 + B + C) = 0 \quad (4.9)$$

$$C > 0, \quad (4.10)$$

where all the running parameters must be evaluated at $Q=\langle x \rangle$. In this case the minimum occurs at the value $\langle x \rangle$ such that the quantity $(m_x^2 + m_y^2 + B + C) \langle x \rangle$ vanishes. Since B and C are small compared to $(m_x^2 + m_y^2)$, the minimum will not be too different from the value found with the simplified condition (4.5). The deviations between the minima originating in our model from (4.5) and (4.9) are illustrated in Table 2. A pictorial representation of how the left-hand sides of equations (4.5) and (4.9) approach zero is contained in Fig. 2.

We can conclude that, in the absence of non-renormalizable terms for V_0 , an IMS is easily generated by radiative corrections and, with a good approximation, is given by the scale at which the combination $(m_x^2 + m_y^2)$ vanishes.

Finally, we investigate the effects of a non-renormalizable term for V_0 . Along the direction $|x|=|y|$ it is given by:

$$V_{NR} = 2(n+1)^2 \frac{|f_n|^2}{M_p^{4n-2}} |x|^{4n+2} \quad (4.11)$$

$$n = 1, 2, \dots$$

V_{NR} contains inverse powers of M_p growing with n and then, if n is sufficiently large, it will not affect the shape of the potential in the region $m_w \ll \langle x \rangle \ll M_p$ we are interested in. To be more precise, the conditions for a minimum of V_1 in $\langle x \rangle$ become:

$$(m_x^2 + m_y^2 + B + C + (4n+2)(n+1)^2 \frac{|f_n|^2}{M_p^{4n-2}} \langle x \rangle^{4n}) = 0 \quad (4.12)$$

$$4 \left[C + 4n(2n+1)(n+1)^2 \frac{|f_n|^2}{M_p^{4n-2}} \langle x \rangle^{4n} \right] > 0 \quad (4.13)$$

where all parameters are evaluated at $Q=\langle x \rangle$. In order to compare the minima $\langle x \rangle$ and $\langle x \rangle$ determined by conditions (4.5) and (4.12) respectively, we have looked for numerical solutions of (4.12) in our model, taking

$M_p = 2.4 \times 10^{18} \text{ GeV}$, $|f_n| = 1$, $\mu_u = 100 \text{ GeV}$, $n=1, 2$ and neglecting B and C . Results are shown in Fig. 1.

We note that, for each value of n , there exists a scale S_n below which the presence of V_{NR} is completely negligible. In other words the minima $\langle x \rangle$ below S_n are identical to the minima $\langle x \rangle$. In this range, the non-renormalizable term affects condition (4.5) by a correction which is totally ineffective. On the other hand, above S_n the effect of V_{NR} is dominant. It sets a scale which can hardly be much greater than S_n . For the model at hand we find S_1 around 10^9 GeV . In this case V_{NR} leads to an IMS which, in the most favorable case ($\lambda_u = 2.00$ and $\mu_u = 1000 \text{ GeV}$), reaches 10^{10} GeV . To obtain VEVs much greater than this, one has to forbid the $n=1$ term. Then, up to a scale $S_2 \sim 10^{13} \text{ GeV}$, V_{NR} can be neglected again. IMS still greater than $5 \times 10^{13} \text{ GeV}$ requires at least $n=3$ and λ_c close to M_x . Of course, our results have to be properly rescaled if one assumes values of $|f_n|$ significantly different from 1.

We conclude this section by noting that the non-renormalizable term of the smallest dimension does not strictly fix the IMS, but it rather provides an upper bound for it. VEVs much smaller of this bound can be accommodated, strongly depending on the boundary conditions imposed to the RGEs. Moreover, a little below such bound, the non renormalizable term becomes totally negligible in the minimum condition.

Also the second step of symmetry breaking, at the electroweak scale m_w , can be generated radiatively, according to the well established mechanism [16,24]. So far, the model appears to be satisfactory.

However, it must be considered as the first step towards the building of a realistic model with IMS. In fact, the main problems which motivate the request of IMS remain unsolved.

The model is too schematic to offer a clue to the solution of the problems of proton decay and neutrino masses, other than requiring the vanishing of the dangerous couplings in the superpotential.

A starting point might be to take into account the couplings between generations and survivors, that we have neglected in the superpotential.

In particular, for the neutrino case, it should help to include in (3.5) terms of the type $1/M_p (\bar{\nu}^c \nu^c \bar{\nu}^c \nu^c)$, or of higher dimension [11,12]. These terms would not affect the conclusions of our analysis of the scalar potential, provided ν^c has a vanishing (or $\ll |x|$) VEV. On the other hand, they would generate a ν_R -mass of the order of M_I^2/M , varying in the range of 10^6 to 10^8 GeV, for values of M_I taken from the allowed set given in Table 3. For the moment, these are only speculations, which should be tested in a more realistic model.

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5. RESULTS AND CONCLUSIONS

In the previous section we have analyzed a mechanism for generating radiatively gauge symmetry breaking at IMS, considering two sets of boundary conditions. Reasonable values of g_u , λ_u and M_X , Λ_c give rise to values for M_I in the range 10^7 to 10^{14} GeV.

We have checked that these solutions, at least in part, are compatible with the values of the low-energy parameters $\sin^2 \theta_w$, α_3 and α_{em} .

In order to make this check, we have made use of the solutions of the RGEs for the gauge couplings $\alpha = g^2/4\pi$ (in the one-loop approximation), given in Appendix A. Concerning α'_1 and α'_2 , one has simply to check that they remain small enough to justify the perturbative (one-loop) approximation. The other equations relate the couplings α_3 , α_{em} , $\alpha/\alpha_2 = \sin^2 \theta_w$ (at m_w) to the parameters α_u , M_I and M_X . Starting from the former set, we have obtained for α_u , M_I and M_X the values collected in Table 3. No acceptable values are obtained for α_u in the case $n=4$, so that we have restricted our analysis to the case $n=3$.

From the table, it appears that one gets agreement with the values obtained from the minimization of the scalar potential (in both cases $\Lambda = M_X$ and $\Lambda_c = 4 \times 10^{15}$ GeV), provided the Yukawa coupling λ_u takes definite values.

As we have remarked in section 4, values of IMS below 10^9 GeV are compatible with non-renormalizable terms of any dimension, while values in the range 10^9 to 10^{13} GeV require the absence of the $n=1$ term. Scales beyond $5 \cdot 10^{13}$ GeV are realized only if also the $n=2$ term is absent and Λ_c is close to M_X .

In conclusion, the model possesses the characteristic feature of radiative breaking of gauge symmetry at IMS, and it is in agreement with the phenomenological requirements at the electroweak scale. We believe that the illustrated mechanism for the generation of IMS is representative of a more general situation, and it keeps its validity beyond the specific scheme considered in this paper.

APPENDIX A

In this Appendix we collect the supersymmetric one-loop renormalization group equations [31] for the soft breaking scalar masses and trilinear scalar couplings, as well as for Yukawa couplings of the light $b_{1,1}$ (27+27) components considered in the text, and for the gauge coupling constants and the relative gaugino masses of the model previously described.

To solve the RGEs, we must specify the boundary conditions, imposed at the unification scale M_x or at the condensation scale Λ_c .

Following the lines of ref. [29], we shall assume for the scalar and gaugino masses and the trilinear scalar couplings:

$$\left. \begin{aligned} A &= A_* = 0 \\ m_x^2 &= m_y^2 = m_z^2 = m_{Ls}^2 = m_{Hs}^2 = m_{Ls*}^2 = 0 \\ \mu_a &= \frac{g_a^2}{g_u^2} \mu_u \end{aligned} \right\} \quad (\text{at } Q=\Lambda_c) \quad (A1)$$

For the Yukawa and gauge couplings, we fix:

$$\left. \begin{aligned} \lambda &= \lambda_* = \lambda_u \\ g_3 &= g_2 = g_1 = g_1' = g_1'' = g_u \end{aligned} \right\} \quad (\text{at } Q=M_x) \quad (A2)$$

With our boundary conditions, with the charge assignments given in Table 1, and working in the basis defined in (3.7), one may realize that λ and λ_* , A and A_* , m_x^2 and m_y^2 , m_z^2 , m_{Hs}^2 , m_{Ls}^2 and m_{Ls*}^2 evolve in exactly the same way, according to the RGEs:

$$\frac{d\lambda}{dt} = \frac{\lambda}{8\pi^2} \left(-\frac{3}{2} g_2^2 - \frac{1}{2} g_1'^2 - g_1''^2 + 2\lambda^2 \right) \quad (A3)$$

$$\frac{dA}{dt} = \frac{1}{8\pi^2} \left(3\mu_2 g_2^2 + \mu_1 g_1'^2 + 2\mu_1' g_1''^2 + 4\lambda^2 A \right) \quad (A4)$$

$$\begin{aligned} \frac{dm_{Hs}^2}{dt} &= \frac{1}{8\pi^2} \left[-3\mu_2^2 g_2^2 - \mu_1'^2 g_1'^2 - \frac{2}{3} \mu_1''^2 g_1''^2 + \right. \\ &\quad \left. + (2m_{Hs}^2 + m_x^2 + A^2)\lambda^2 \right] \end{aligned} \quad (A5)$$

$$\frac{dm_x^2}{dt} = \frac{1}{8\pi^2} \left[-\frac{8}{3} \mu_1'^2 g_1'^2 + 2(2m_{Hs}^2 + m_x^2 + A^2)\lambda^2 \right] \quad (A6)$$

The RGEs for g_a and μ_a are:

$$\frac{dg_a^2}{dt} = \frac{b_a + \Delta b_a}{8\pi^2} g_a^4, \quad M_I \leq Q \leq M_x \quad (A7)$$

$$\frac{dg_a^2}{dt} = \frac{b_a}{8\pi^2} g_a^4, \quad m_w \leq Q \leq M_I \quad (A8)$$

$$\frac{d\mu_a}{dt} = \frac{g_a^2}{8\pi^2} (b_a + \Delta b_a) \mu_a, \quad M_I \leq Q \leq \Lambda_c \quad (A9)$$

$$\frac{d\mu_a}{dt} = \frac{g_a^2}{8\pi^2} \mu_a, \quad m_w \leq Q \leq M_I \quad (A10)$$

in all the above formulas, $t = \ln Q$; b_a and Δb_a are given in Table 4.

While the RGEs for λ , A , m_{Hs}^2 , m_x^2 are solvable only numerically, eq. (A7,8) may be solved analytically, giving the following result for the generic gauge constant $\alpha_a = \frac{g_a^2}{4\pi}$ ($\alpha = \alpha_3, \alpha_2, \alpha_1, \alpha_1'$)

$$\alpha_a(M_w)^{-1} = \alpha_u^{-1} + \frac{b_a}{2\pi} \ln \frac{M_x}{m_w} + \frac{\Delta b_a}{2\pi} \ln \frac{M_x}{M_I} \quad (A11)$$

with $\alpha_u = \frac{g_u^2}{4\pi}$;

The evolution of g_I^n stops at $Q = M_I$, yielding:

$$\alpha_I^n (M_I)^{-1} = \alpha_u^{-1} + \frac{b_u^n + \Delta b_1^n}{2\pi} \ln \frac{M_X}{M_I}. \quad (A12)$$

The generic gaugino mass evolves like

$$\mu_a(Q) = \frac{g_a^2(Q)}{g_u^2} \mu_u \quad (A13)$$

We can work out from eq. (A11) expressions for $\ln \frac{M_X}{M_I}$ and $\ln \frac{M_X}{m_w}$, once given $\alpha_{em} = \alpha_w(m)$, $\alpha_3 = \alpha_3(m)$ and $\sin^2 \theta_w = \sin^2 \theta_w(m)$:

$$\ln \frac{M_X}{M_I} = \frac{2\pi}{\alpha_{em}} \cdot \frac{18}{5} \left(\sin^2 \theta_w - \frac{3}{8} \right) + 2 \frac{(\alpha_{em} - \frac{3}{8})}{\alpha_3}, \quad \Delta b_1 - 3\Delta b_2 \quad (A14)$$

$$\ln \frac{M_X}{m_w} = \frac{2\pi}{\alpha_{em}} \cdot \left[\frac{\Delta b_2 + \frac{\Delta b_1}{3}}{5} \left[\sin^2 \theta_w - \frac{3}{8} \right] + \frac{1}{3} (\Delta b_2 - \Delta b_1) \left[\frac{\alpha_{em} - \frac{3}{8}}{\alpha_3} \right] \right], \quad \Delta b_1 - 3\Delta b_2 \quad (A15)$$

where Δb_1 is defined as follows:

$$\Delta b_1 = \frac{2}{5} \Delta b_1 + \frac{3}{5} \Delta b_1. \quad (A16)$$

We get, from Table 4, $\Delta b_2 = 2$ and $\Delta b_1 = \frac{6}{5}$, so we can deduce that

$$\ln \frac{M_X}{M_I} = -\frac{5}{24} \cdot \frac{2\pi}{\alpha_{em}} \left[-\frac{18}{5} \left(\sin^2 \theta_w - \frac{3}{8} \right) + 2 \left[\frac{\alpha_{em} - \frac{3}{8}}{\alpha_3} \right] \right], \quad (A17)$$

$$\ln \frac{M_X}{m_w} = -\frac{5}{24} \cdot \frac{2\pi}{\alpha_{em}} \left[\frac{4}{5} \left(\sin^2 \theta_w - \frac{3}{8} \right) + \frac{4}{15} \left[\frac{\alpha_{em} - \frac{3}{8}}{\alpha_3} \right] \right]. \quad (A18)$$

APPENDIX B

In this Appendix we sketch the derivation of the one-loop potential V_1 . We are interested in the dependence of the potential on the fields x and y alone and we assume vanishing or negligible VEVs for all other fields in the theory. Under these conditions the one-loop potential is given by [20, 30]

$$V_1(x, y) = V_0(x, y) + \Delta V_1(x, y) \quad (B1)$$

$$\Delta V_1(x, y) = \frac{M}{64\pi^2} \sum_J (-1)^{2J} (2J+1) M_J^4 \ln \left(\frac{M_J^2}{Q^2} \right) \quad (B2)$$

The dependence on x , y of the term ΔV_1 is contained in M_J , the mass matrix for spin J particles, which must be computed by giving arbitrary VEVs to the fields x and y . We have included in M_J only those particles whose masses depend on x and/or y , namely: the gaugino and the gauge vector boson associated to $U(1)_{Y'}$; the scalar and spinor components of the supermultiplets $L_s, H_s, L_s^*, H_s^*, \nu_s^c, \nu_s^{c*}$.

We list below the explicit expressions of the mass matrices after diagonalization:

$$M_1^2 = \frac{4}{3} g_1'^2 \sum^2 \quad (B3)$$

$$M_{\frac{1}{2}}^2 = \text{diag} \left\{ \lambda^2 |x|^2 \quad (4 \text{ times}), \right. \\ \left. \lambda_s^2 |y|^2 \quad (4 \text{ times}), \right. \\ \left. 0, \frac{1}{2} \left[\mu_1'^2 + \frac{8}{3} g_1'^2 \sum^2 + \sqrt{\mu_1'^4 + \frac{16}{3} g_1'^2 \mu_1'^2} \right]^2, \right. \\ \left. \frac{1}{2} \left[\mu_1'^2 + \frac{8}{3} g_1'^2 \sum^2 - \sqrt{\mu_1'^4 + \frac{16}{3} g_1'^2 \mu_1'^2} \right]^2 \right\} \quad (B4)$$

27.

$$M_0^2 = \text{diag} \left\{ \begin{aligned} &m_H^2 + \lambda^2 |x|^2 + \lambda A |x|, \text{ (4 times)}, \\ &\frac{m_H^2 + \lambda^2}{2} |x|^2 - \lambda A |x|, \text{ (4 times)}, \\ &\frac{m_H^2 + \lambda^2}{2} |y|^2 + \lambda A_x |y|, \text{ (4 times)}, \\ &\frac{m_H^2 + \lambda^2}{2} |y|^2 - \lambda A_x |y|, \text{ (4 times)}, \\ &m_x^2 + \frac{2}{3} g_1^2 \Delta^2, \text{ (4 times)}, \\ &m_x^2 - \frac{2}{3} g_1^2 \Delta^2, \end{aligned} \right.$$

$$\frac{1}{2} \left(\frac{m_x^2 + m_y^2}{x} + \frac{4}{3} g_1^2 \Delta^2 \right) \left[\frac{m_x^2 - m_y^2}{x} + \frac{8}{3} g_1^2 \Delta^2 \right] + \frac{16}{9} g_1^4 (\Sigma^4 - \Delta^4)$$

$$\frac{1}{2} \left(\frac{m_x^2 + m_y^2}{x} + \frac{4}{3} g_1^2 \Delta^2 \right) \left[\frac{m_x^2 - m_y^2}{x} + \frac{8}{3} g_1^2 \Delta^2 \right] + \frac{16}{9} g_1^4 (\Sigma^4 - \Delta^4)$$

where

$$\Sigma^2 = |x|^2 + |y|^2 \quad \Delta^2 = |x|^2 - |y|^2$$

As a useful check we have computed the supertrace of M^2 :

$$\begin{aligned} \text{Str} M^2 &= \sum_J (-1)^{2J} (2J+1) M_J^2 = \\ &= -2\mu_1^2 + 8m_H^2 + 8m_x^2 + 2m_y^2 + 2m_x^2 \end{aligned} \quad (B6)$$

which corresponds to the result quoted in reference [3].

If we further assume suitable RGE boundary conditions for the various parameters present in (B3,4,5), we can consistently choose:

$$m_x^2 \approx m_y^2 \quad m_H^2 = m_x^2 \quad A = A_x \quad \lambda = \lambda_x \quad (B7)$$

Finally, taking the large $|x|$ limit in the $|x|=|y|$ direction, we find:

$$\begin{aligned} \Delta V_1 &= B(g_1^2, \lambda, \mu_1^2, m_x, m_H, A) |x|^2 + \\ &+ C(g_1^2, \lambda, \mu_1^2, m_x, m_H) |x|^2 \ln \left(\frac{|x|^2}{Q^2} \right) \end{aligned} \quad (B8)$$

28.

and

$$B = \frac{1}{64\pi^2} \left[16\lambda^2 (m_H^2 + 2A^2) + 32\lambda^2 m_H^2 \ln \lambda^2 + \right. \\ \left. + \frac{8}{3} g_1^2 (m_x^2 - 10\mu_1^2) + \frac{16}{3} g_1^2 (m_x^2 - 2\mu_1^2) \ln \frac{8}{3} g_1^2 \right] \quad (B9)$$

$$C = \frac{1}{64\pi^2} \left[32\lambda^2 m_H^2 + \frac{16}{3} g_1^2 (m_x^2 - 2\mu_1^2) \right] \quad (B10)$$

TABLE 1

Transformation properties under $SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)_{Y'}$ of the chiral superfields contained in the 27 of E_6 . The appropriate normalization factor for the $U(1)$ generators is indicated in brackets.

CHIRAL SUPERFIELDS	$SU(3)_C$	$SU(2)_L$	$U(1)_Y$ $\left(\frac{1}{\sqrt{3}}\right)$	$U(1)_{Y'}$ $\left(\frac{1}{\sqrt{40}}\right)$	$U(1)_{Y''}$ $\left(\frac{1}{\sqrt{40}}\right)$
Q	3	2	+1/6	-1	-2
D	3	1	-1/3	+2	+4
u^c	$\bar{3}$	1	-2/3	-1	-2
d^c	$\bar{3}$	1	+1/3	-2	+4
D^c	$\bar{3}$	1	+1/3	+3	-2
L	1	2	-1/2	-2	+4
H	1	2	+1/2	+2	+4
$\bar{H}$	1	2	-1/2	+3	-2
ν^c	1	1	0	0	-8
e^c	1	1	1	-1	-2
N	1	1	0	-5	-2

TABLE 2

Numerical results for $\langle x \rangle$ satisfying conditions (4.5) and (4.9) for various values of $\alpha_\lambda \lambda^2/4\pi$, in the two cases (a) $\Lambda_c = M = 5 \times 10^{16}$ GeV and (b) $\Lambda_c = 4 \times 10^{15}$ GeV, $M_x = 5 \times 10^{16}$.

$\alpha_\lambda = \frac{\lambda^2}{4\pi}$	(a)		(b)	
	$\langle x \rangle$ $m_x^2 + m_y^2 = 0$	$\langle x \rangle$ $m_x^2 + m_y^2 + B+C=0$	$\langle x \rangle$ $m_x^2 + m_y^2 = 0$	$\langle x \rangle$ $m_x^2 + m_y^2 + B+C=0$
0.318	7.0×10^{14}	1.7×10^{14}	9.4×10^{13}	2.3×10^{13}
0.244	3.6×10^{14}	1.1×10^{14}	5.9×10^{13}	1.7×10^{13}
0.179	1.6×10^{14}	5.2×10^{13}	3.2×10^{13}	1.0×10^{13}
0.124	4.8×10^{13}	1.8×10^{13}	1.4×10^{13}	5.0×10^{12}
0.079	8.5×10^{12}	3.6×10^{12}	3.6×10^{12}	1.5×10^{12}
0.045	4.5×10^{12}	2.3×10^{11}	3.5×10^{11}	1.8×10^{11}
0.020	1.3×10^9	7.4×10^8	2.8×10^9	1.8×10^9

Values of $\lambda^2/4\pi$ at M_X required to generate an IMS compatible with given values of α_{em} , α_3 and $\sin^2\theta_w$ at m . Both cases $A_C = M_X$ and $A_C = 4 \times 10^{15}$ GeV are considered. $\alpha_{em} = 1/128$, $m = 80$ GeV and $n = 3$.

TABLE 3

$\alpha_3(m) = \alpha_u$	$\sin^2\theta_w$	M_X	M_I	$\lambda^2/4\pi$ $A_C = M_X$	$\lambda^2/4\pi$ $A_C = 4 \cdot 10^{15}$ GeV
0.111	0.215	1.32×10^{17}	5.05×10^{14}	0.191	> 0.300
0.120	0.215	1.67×10^{17}	1.09×10^{14}	0.103	0.269
0.111	0.220	6.75×10^{16}	1.27×10^{13}	0.081	0.117
0.120	0.220	8.55×10^{16}	2.74×10^{12}	0.052	0.062
0.111	0.225	3.46×10^{16}	3.19×10^{11}	0.045	0.046
0.120	0.225	4.38×10^{16}	6.88×10^{10}	0.032	0.030
0.111	0.230	1.77×10^{16}	8.00×10^9	0.029	0.027
0.120	0.230	2.24×10^{16}	1.73×10^9	0.021	0.018
0.111	0.235	9.05×10^{15}	2.01×10^8	0.020	0.018
0.120	0.235	1.15×10^{16}	4.34×10^7	0.015	0.013

TABLE 4

Coefficients for the RGEs of the gauge couplings in one-loop approximation. g_1'' evolves with $b_1'' + \Delta b_1''$ from M_X to M_I , where $U(1)_{Y''}$ breaks down. With the analogous notation used for g_a , we define $\Delta b_a = (\Delta b_3, \Delta b_2, \Delta b_1, \Delta b_1'')$.

g_a	b_a	Δb_a	$b_a + \Delta b_a$
g_3	$3n - 9$ g	0	$3n - 9$ g
g_2	$3n - 6$ g	+2	$3n - 4$ g
g_1	$3n$ g	0	$3n$ g
g_1'	$3n$ g	+2	$3n + 2$ g
g_1''	---	---	$3n + 8/3$ g

FIGURE CAPTIONS

Fig. 1 - The intermediate mass scale M_I as a function of the coupling λ_u in the two cases:

(a) $\Lambda_c = M_x = 5 \times 10^{15} \text{ GeV}$

(b) $\Lambda_c = 4 \times 10^{15} \text{ GeV}, M_x = 5 \times 10^{16} \text{ GeV}$

Solid lines correspond to the case in which the effective superpotential does not contain non-renormalizable terms.

Dashed and dotted lines correspond to the presence of the $n=2$ and $n=1$ non-renormalizable terms, respectively. The representative value $\mu = 100 \text{ GeV}$ has been assumed.

Fig. 2 - Evolution of the quantities $(m_x^2 + m_y^2)/\mu^2$ (dashed lines) and $(m_x^2 + m_y^2 + B + C)/\mu^2$ (solid lines), as functions of the energy scale Q , for the representative values $\lambda_u = 0.75$, $\alpha_u = 0.111$ and $M_x = 5 \times 10^{16} \text{ GeV}$. Case (a) corresponds to $\Lambda_c = M_x$, case (b) to $\Lambda_c = 4 \times 10^{15} \text{ GeV}$.

Fig. 1

Fig. 2

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