EXPERIMENTAL TESTS OF THE ELECTROWEAK THEORY

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PROLOGUE

The success of the Electroweak Theory of Glashow, Salam, and Weinberg is well known. The theory agrees with the results of a large number of experiments of great variety. Perhaps no less impressive is the fact that all experiments which reported in the past a conflict with the theory turned out to be wrong.

These lectures aim at reviewing the status of experimental tests of the Electroweak Theory, for an audience which largely consists of young theorists. Hence the review avoids experimental details as much as possible but rather focuses on the principle of the experimental techniques, on the significance of the results, and on the likely future development.

The written version does not follow exactly the course of the lectures, but largely so. Some topics, such as the electroweak physics aspects of the W and Z bosons, have been added. Also, new results which were not available at the time the lectures were presented (July 1985), have been included wherever possible.
A WORD ON NOMENCLATURE

The definitions and symbols of the neutral-current coupling constants in the literature are confusing. We find for the fermion $f$: $g_1^f, g_1^A, g_1^h, g_1^k, c_1^f, c_1^h, c_1^k, \nu_f, \alpha_f, \epsilon_f, \epsilon(f); \epsilon(f_1^L), \epsilon(f_1^R); \not \text{not to speak of the isospin-oriented notation, } \alpha, \beta, \gamma, \delta \text{ and } \alpha, \tilde{\alpha}, \gamma, \tilde{\delta}; \text{ and of } h_{VV}, h_{VA}, h_{AA}. \text{ Moreover, the coupling constants used in } e^+e^- \text{ physics are numerically twice as large as the ones used in neutrino physics.} \]

The neutrino community has not succeeded in imposing its definition. The time has come to admit this defeat. We propose to adopt the definition of the vector and axial-vector coupling constants of $e^+e^-$ physics, and to retain the definition of the chiral coupling constants of neutrino physics.

We require a symbol for the coupling constants which lends itself to its interpretation as such. The symbol should have two variables, one for the type of coupling ($V, A, L, R$), the other to specify the fermion ($e, u, d, \ldots$). It should not involve brackets. Writing the coupling constant squared should not necessitate brackets either.

We propose the symbol $g$ for the neutral-current coupling constants, with a first lower suffix denoting the type of coupling, and a second lower suffix denoting the type of fermion.

The coupling constants of the fermion $f$ then read as follows:

\[
g_{Vf} = 2T_{3L} - 4Q_f \sin^2 \theta_w
\]

\[
g_{Af} = 2T_{3L}
\]

\[
g_{Lf} = (g_{Vf} + g_{Af})/4 = T_{3L} - Q_f \sin^2 \theta_w
\]

\[
g_{Rf} = (g_{Vf} - g_{Af})/4 = -Q_f \sin^2 \theta_w.
\]
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1. INTRODUCTION

1.1 Phenomenology of the Electroweak Theory

The Glashow–Salam–Weinberg unified gauge theory of electroweak interactions [1], hereafter called Electroweak Theory, has been described extensively in the literature [2]. Here, we restrict ourselves to recalling those features which are relevant to the discussion of our subject.

The fundamental fermionic constituents of matter, leptons and quarks, are arranged in three generations: \((\nu_e, e, u, d); (\nu_\mu, \mu, c, s); \text{ and } (\nu_\tau, \tau, t, b)\). They carry quantum numbers of weak isospin \((T)\) and of weak hypercharge \((Y)\). These quantum numbers are coupled via an analogue of the Gell-Mann–Nishijima relation, \(Q = T_3 + Y/2\), where \(Q\) denotes the fermion's electric charge. Left-handed fermions are grouped into isodoublets whereas right-handed fermions are isosinglets. The quantum number assignments are summarized in Table 1.

The gauge group of the Electroweak Theory is \(SU(2)_L \otimes U(1)\). The gauge symmetry is spontaneously broken at \(\sim 100\) GeV energy, without spoiling the renormalizability of the theory. The finite masses of the \(W\) and \(Z\), and of the fundamental fermions, are acquired through the interaction with a particle of spin \(0\) and weak isospin \(1/2\), the Higgs boson. Its mass is not specified within the Electroweak Theory.

The fundamental fermions are placed into left-handed isodoublets, in order to reproduce the known structure of the weak charged-current (CC) interaction. At the same time, the strength and the structure of the weak neutral-current (NC) interaction is specified in terms of a single parameter, the electroweak mixing parameter \(\sin^2 \theta_w\). Its physical meaning is the amount of admixture of electromagnetic current to the third, neutral, component of the weak CC:

\[
J^{\text{NC}}_\mu = J^2_\mu - \sin^2 \theta_w J^\text{em}_\mu.
\]

Table 1

Assignment of weak isospin \(T\) and weak hypercharge \(Y\)
to the fundamental fermions\(^a\)

<table>
<thead>
<tr>
<th></th>
<th>Q</th>
<th>T</th>
<th>T_3</th>
<th>Y</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\nu_e)</td>
<td>0</td>
<td>1/2</td>
<td>1/2</td>
<td>-1</td>
</tr>
<tr>
<td>(\bar{\nu}_e)</td>
<td>-1</td>
<td>1/2</td>
<td>-1/2</td>
<td>-1</td>
</tr>
<tr>
<td>(u_L)</td>
<td>2/3</td>
<td>1/2</td>
<td>1/2</td>
<td>1/3</td>
</tr>
<tr>
<td>(d_L)</td>
<td>-1/3</td>
<td>1/2</td>
<td>-1/2</td>
<td>1/3</td>
</tr>
<tr>
<td>(c_R)</td>
<td>-1</td>
<td>0</td>
<td>0</td>
<td>-2</td>
</tr>
<tr>
<td>(s_R)</td>
<td>2/3</td>
<td>0</td>
<td>0</td>
<td>4/3</td>
</tr>
<tr>
<td>(b_R)</td>
<td>-1/3</td>
<td>0</td>
<td>0</td>
<td>-2/3</td>
</tr>
</tbody>
</table>

\(^a\) The primed quarks denote the Kobayashi-Maskawa rotated weak interaction eigenstates.
Table 2
NC coupling constants of the fundamental fermions
(numerical values for $\sin^2 \theta_w = 0.225$)

<table>
<thead>
<tr>
<th></th>
<th>$\nu_e, \nu_\mu, \nu_\tau$</th>
<th>$e, \mu, \tau$</th>
<th>$u, c, t$</th>
<th>$d, s, b$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Vector $g_V$</td>
<td>1</td>
<td>$-1 + 4 \sin^2 \theta_w$ (−0.10)</td>
<td>$1 - (8/3) \sin^2 \theta_w$ (0.40)</td>
<td>$-1 + (4/3) \sin^2 \theta_w$ (−0.70)</td>
</tr>
<tr>
<td>Axial-vector $g_A$</td>
<td>1</td>
<td>$-1$</td>
<td>1</td>
<td>$-1$</td>
</tr>
<tr>
<td>Left-handed $g_L$</td>
<td>1/2</td>
<td>$-1/2 + \sin^2 \theta_w$ (−0.28)</td>
<td>$1/2 - (2/3) \sin^2 \theta_w$ (0.35)</td>
<td>$-1/2 + (1/3) \sin^2 \theta_w$ (−0.43)</td>
</tr>
<tr>
<td>Right-handed $g_R$</td>
<td>0</td>
<td>$\sin^2 \theta_w$ (0.23)</td>
<td>$- (2/3) \sin^2 \theta_w$ (−0.15)</td>
<td>$(1/3) \sin^2 \theta_w$ (0.08)</td>
</tr>
</tbody>
</table>

This current gives rise to an effective weak NC Lagrangian, for instance for neutrino-quark scattering:

$$\mathcal{L}_{\text{eff}}^{NC}(\nu q) = -(G_F/2\sqrt{2})[\bar{\nu}_\mu(1-\gamma_5)\nu][\bar{q}_\mu\gamma_\mu(g_{Vq} - g_{Aq}\gamma_5)q] .$$

The vector and axial-vector coupling constants of the fundamental fermion $f$ are given by

$$g_{Vf} = 2T_{3L} - 4Q_f \sin^2 \theta_w$$

$$g_{Af} = 2T_{3L} .$$

The Lagrangian is often used in terms of left- and right-handed ('chiral') coupling constants:

$$\mathcal{L}_{\text{eff}}^{NC}(\nu q) = -(G_F/\sqrt{2})[\bar{\nu}_\mu(1-\gamma_5)\nu][\bar{q}_\mu\gamma_\mu\{g_{Lq}(1-\gamma_5) + g_{Rq}(1+\gamma_5)\}]q] ,$$

with

$$g_{Lq} = (g_{Vq} + g_{Aq})/4$$

$$g_{Rq} = (g_{Vq} - g_{Aq})/4 .$$

The resulting NC coupling constants for the fundamental fermions are given in Table 2.

We note that only the neutrino NC has a $V-A$ structure, whereas all other currents have different $V$ and $A$ strengths. Equivalently, they consist of (left-handed) $V-A$ and (right-handed) $V+A$ terms.

The Electroweak Theory comprises electromagnetic interactions, mediated by the massless photon; weak CC interactions, mediated by the massive $W$ bosons; and the weak NC interaction, mediated by the $Z$ boson. The masses of the $W$ and $Z$ bosons are given by

$$m_W^2 = \frac{\alpha}{(G_F \sin^2 \theta_w \sqrt{2})}$$

and

$$m_Z = m_W \cos \theta_w .$$
The Electroweak Theory, although not tested in all details, has already survived eight major experimental disasters, and is today unambiguously supported by a large variety of experimental results.

At present, there is no experimental fact in electroweak physics that requires a wider theory than the Electroweak Theory. Yet too many aspects of it appear arbitrary. The strong desire of our theoretical colleagues for elegance, simplicity, and the absence of arbitrary parameters, drives the search for the wider theory encompassing the Electroweak Theory.

The theory as formulated by Glashow, Salam, and Weinberg is the most 'economic', or 'minimal' version. In the following, the term 'Electroweak Theory' refers to this minimal version.

1.2 Extensions of the minimal theory

Many possible extensions of the minimal Electroweak Theory have been suggested in the literature, mostly triggered by wrong experimental results. These extensions may be grouped into 'minor' extensions which can be accommodated within the Electroweak Theory, and 'major' extensions which go beyond the Electroweak Theory, such as Grand Unified Theories, Supersymmetry, and Compositeness. Here, we are concerned with minor extensions only:

i) Finite neutrino masses: In Table 1 right-handed neutrinos do not appear because neutrinos are supposed to be massless, and hence exist as left-handed particles only. Right-handed neutrinos could be accommodated, at the expense of introducing further free parameters into the theory.

ii) Further generations of leptons and quarks: The Electroweak Theory has no explanation of why there are three generations of leptons and quarks; an explanation of this puzzle can only come from beyond the Electroweak Theory. Meanwhile, more generations can be accommodated at the expense of additional free parameters.

iii) More complicated Higgs sector: The introduction of one neutral, scalar Higgs particle is the most economic solution, but there exist other possibilities which involve more neutral and also charged Higgs particles. Additional parameters will then emerge, and the relation $m_Z = m_W / \cos \theta_W$ will break down. The ratio $Q = m_W^2 / (m_Z^2 \cos^2 \theta_W)$, expected to be 1 in the minimal theory, can be tested experimentally.

1.3 Electroweak radiative corrections

A fundamental property of the Electroweak Theory is its renormalizability which was proved by 't Hooft [3]. As a consequence, higher order corrections are finite and calculable. They must be taken into account properly for precise experimental tests of the theory. Higher order corrections differ from experiment to experiment.

For many years, experiments were not accurate enough to be much worried by higher order corrections. Only in recent years has the experimental precision evolved such that shifts due to higher order corrections are comparable to, or even larger than, the experimental errors. This makes it possible to detect and measure deviations from theoretical predictions performed in lowest non-trivial order (at 'tree level' or in 'Born approximation'). In principle, although not comparable in the degree of precision, this programme is analogous to the measurement of, for example, the Lamb shift in the hydrogen atom, as a test of QED.

When calculating higher order corrections to, for instance, the electroweak mixing parameter $\sin^2 \theta_W$, a certain arbitrariness comes in, which is not present at the tree level: the choice of the renormalization scheme. Other than in QED there is no unique way to define the renormalized parameters of the theory. Rather, a precise definition of these parameters, together with a prescription of how to carry out the higher order corrections, has to be made.

In the literature, two renormalization schemes have been widely used. They can be most easily visualized with the two following definitions of $\sin^2 \theta_W$ (which coincide at the tree level):
i) \( \sin^2 \theta_w = e^2/g^2 \)

ii) \( \sin^2 \theta_w = 1 - (m_W/m_Z)^2 \).

In definition (i), \( \sin^2 \theta_w \) is the ratio of the electromagnetic to the weak coupling constants. In higher order, both become running coupling constants as a function of \( Q^2 \), the momentum transfer of the process under consideration. By convention, \( Q^2 = m_W^2 \) has been chosen as a point of reference, yielding the relation

\[
\sin^2 \theta_w(m_W) = e^2(Q^2 - m_W^2)/g^2(Q^2 = m_W^2),
\]

where the running coupling constants are evaluated in the ‘modified minimal subtraction’ (MS) renormalization scheme, at the momentum transfer \( Q^2 = m_W^2 \) [4].

In definition (ii), \( \sin^2 \theta_w \) is specified by physical particle masses, at any order of perturbation. This scheme is called the ‘on-shell (OS)’ scheme [5].

In 1983, a Workshop on Electroweak Radiative Corrections was held at Trieste [6]. It recommended that all calculable electroweak observables should be expressed by the following set of three independent quantities:

i) \( \alpha = (137.03604 \pm 0.00011)^{-1} \), the fine structure constant measured at \( Q^2 = 0 \);

ii) \( G_F = (1.16637 \pm 0.00002) \times 10^{-5} \text{ GeV}^{-2} \), the Fermi constant measured from the muon lifetime, after QED radiative corrections;

iii) \( m_Z = 91.2 \pm 1.7 \text{ GeV} \), the Z mass measured in pp collisions.

Clearly, \( \alpha \) and \( G_F \) are at present known with much better precision than \( m_Z \). However, after the turn-on of SLC and LEP we expect \( m_Z \) to be measured to \( \pm 0.02 \text{ GeV} \), thus completing the set of precisely measured independent quantities.

It seems by now a widely accepted procedure to calculate electroweak observables with \( \alpha \), \( G_F \), and \( m_Z \) as input quantities, employing the OS renormalization scheme.

To \( O(\alpha) \), electroweak radiative corrections may \textit{a priori} contain terms

\[
O\left((\alpha/\pi) \ln \left(m_W^2/\mu^2\right)\right) = 0.034 \quad \text{for } \mu^2 = 0.003 \text{ GeV}^2
\]

\[
O\left((\alpha/\pi) \ln \left(m_W/Q^2\right)\right) = 0.014 \quad \text{for } Q^2 = 20 \text{ GeV}^2
\]

\[
O\left((\alpha/\pi) \ln \left(m_I/Q^2\right)\right) = -0.013 \quad \text{for } m_I = 0.3 \text{ GeV}, Q^2 = 20 \text{ GeV}^2
\]

\[
O\left((\alpha/\pi)(1/\sin^2 \theta_w)\right) = 0.010
\]

\[
O\left(\alpha/\pi\right) = 0.002
\]

where \( \mu^2 \) is the momentum transfer of the processes from which \( \alpha \) and \( G_F \) are determined, \( Q^2 \) is the typical momentum transfer of the process under consideration, and \( m_I \) is a typical fermion mass. The most important contributions arise from ‘leading logarithms’ of the type \((\alpha/\pi) \ln \left(m_W^2/\mu^2\right)\), which arise from the renormalization of the electric charge due to vacuum polarization, from the scale \( \mu^2 \) to the scale \( m_W^2 \). Retaining only the leading logarithms (‘leading log approximation’), they can easily be summed to all orders by renormalization group techniques.

Rather than working in the leading log approximation it has become customary to calculate complete corrections of \( O(\alpha) \), although numerically the difference may not be large. The expected size of electroweak radiative corrections is of the order of \( 1\% \), and is to be taken into account in experiments with few per cent precision or better. In future high-precision experiments at the \( Z \) pole it will even be necessary to include \( O(\alpha^2) \) radiative corrections.

Electroweak radiative corrections have been calculated for a variety of electroweak observables. Much work has been done already, but a great deal remains to be done.

2. TESTS OF QED

Quantum electrodynamics is the archetype of a successful gauge theory. It has been tested by the most precise experiments which have ever been performed. Yet no serious discrepancy exists between
theory and experiment down to one part per million! Its formulation due to Feynman, Schwinger and Tomonaga is nearly 40 years old, yet important progress is still being made in both theory and experiment.

Experimental tests of QED can be grouped into three categories:

i) energy-level measurements in simple bound systems such as the hydrogen atom,

ii) measurements of the anomalous magnetic moments of charged leptons,

iii) colliding-beam reactions at large momentum transfer.

In our review of experimental tests of QED, no attempt at being complete is made. We rather focus on a few highlights.

2.1 Energy-level measurements in simple bound systems

Simple bound systems which have been studied are positronium, muonium, and the hydrogen atom. Here, the hyperfine splitting of the hydrogen ground state and the Lamb shift of the hydrogen 2S state are recalled.

Figure 1 shows the relevant energy levels of the hydrogen atom. The level splitting of the 2P_{3/2} and 2P_{1/2} states, the fine structure, is caused by the spin-orbit interaction of the electron. The interaction of the magnetic moment of the orbital electron with the magnetic moment of the proton leads to the hyperfine splitting of the ground state.

The measurement of the hydrogen hyperfine splitting frequency yielded the most precise number known in experimental physics: the frequency is measured to 1 part per 10^{12}! QED calculations of the hyperfine splitting, although being in agreement with experiment, fall short by 7 orders of magnitude in accuracy:

\[ \text{Exp: } \Delta \nu = 1420405.7517864 \pm 0.0000017 \text{ kHz} \quad [7] \]

\[ \text{QED: } \Delta \nu = 1420404 \pm 6 \text{ kHz} \quad [8]. \]

The higher order corrections to \( \Delta \nu \) have been calculated up to \( \mathcal{O}(\alpha^2) \) [8]. A more precise theoretical prediction seems unlikely since detailed information on the spin-dependent structure functions of the proton would be needed.

The Lamb shift of the hydrogen 2S state (Fig. 1) is caused by the interaction of the electron with the quantized electromagnetic field. It acts primarily on S states since the overlap of the electron wave function with the proton must be finite. It splits the otherwise degenerate 2S_{1/2} and 2P_{1/2} levels. The Lamb shift consists of two parts:

i) The electron emits and reabsorbs virtual photons. This diminishes the Coulomb potential hence the S levels are pushed higher (by about 1080 MHz).

ii) The Coulomb potential seen by the electron is increased because of virtual electron-positron pairs; hence the S levels are pulled lower (by about 30 MHz).

![Diagram of energy levels in the hydrogen atom](image)

**Fig. 1** The structure of the lower energy levels in the hydrogen atom.
Theory and experiment are of comparable precision, and in good agreement with each other:

\[ \text{Exp: } \Delta \nu = 1057.878 \pm 0.014 \text{ MHz} \quad [9] \]
\[ \text{QED: } \Delta \nu = 1057.898 \pm 0.026 \text{ MHz} \quad [10]. \]

The higher order corrections to \( \Delta \nu \) are calculated up to \( O(\alpha^2) \), with some minor discrepancies between different calculations [10].

### 2.2 Anomalous magnetic moment of charged leptons

The anomalous magnetic moment of the electron or muon is defined via

\[ \mu = g(e/2mc)(\hbar/2) = (eh/2mc)(1 + a), \]

where \( \mu \) is the magnetic moment, and \( g \) the gyromagnetic ratio. Hence we have the relation

\[ a = \frac{1}{2} (g - 2), \]

which explains why the pertinent experiments are referred to as '\( g - 2 \)' experiments.

The precision of electron \( g - 2 \) experiments was increased by an order of magnitude when Van Dyck et al. [11] managed to study single electrons held in an electromagnetic trap at liquid-helium temperature. Their result compares well with a QED calculation which comprises higher order corrections up to \( O(\alpha^2) \):

\[ \text{Exp: } a_e = (1159652.4 \pm 0.2) \times 10^{-9} \quad [11] \]
\[ \text{QED: } a_e = (1159652.38 \pm 0.26) \times 10^{-9} \quad [10]. \]

The anomalous magnetic moment of the muon has been measured in a series of \( g - 2 \) experiments at CERN. Since the last experiment of this series is the most precise measurement so far carried out at accelerators, and at the same time constitutes a beautiful example of experimental ingenuity, we shall briefly recall its main features.

Consider a longitudinally polarized muon moving slowly in a homogeneous magnetic field \( B \). Then its momentum vector will turn (in non-relativistic approximation) with the orbit frequency

\[ \omega_c = eB/mc. \]

The spin precession frequency is

\[ \omega_s = 2\mu B/\hbar = (1 + a_e)(eB/mc) = (1 + a_e)\omega_c. \]

Hence, the evolution with time of the muon polarization \((\vec{s} \cdot \vec{p})\), with spin \( \vec{s} \) and momentum \( \vec{p} \) measures directly the anomalous magnetic moment \( a_e \):

\[ \omega_s - \omega_c = d\theta/dt = a_e(eB/mc), \]

where \( \theta \) is the angle between \( \vec{s} \) and \( \vec{p} \).

In the third and so far last muon \( g - 2 \) experiment carried out at CERN [12], a momentum-selected pion beam was injected into a magnetic muon storage ring, 14 m in diameter (Fig. 2). Only muons which originated from forward pion decay were stored in the ring. They were highly polarized. The direction of the electron emitted in the decay in flight of the muon served as an analyser for the muon-spin direction. The decay electrons were measured by a series of electromagnetic shower counters. The requirement of large electron energies ensured that only those electrons were recorded which were emitted close to the direction of motion in the muon rest frame. The anomalous magnetic moment then shows up as a modulation of the counting rate with time, on top of the exponential decrease due to the (strongly time-dilated) muon decay. Figure 3 shows the...
observed modulation of the counting rate. From this, an experimental value of $a_u$ was obtained, which was in very good agreement with the QED prediction:

Exp: $a_u = (1165924 \pm 8.5) \times 10^{-9}$ \hspace{1cm} [12]

QED: $a_u = (1165921 \pm 8.3) \times 10^{-9}$ \hspace{1cm} [13].

The QED prediction comprises complete higher order corrections up to $O(\alpha^3)$, and estimates of $O(\alpha^4)$ and $O(\alpha^5)$ contributions. Its accuracy is at present limited by the hadronic contribution to $a_u$, $(66.7 \pm 8.1) \times 10^{-9}$. It could be improved by a better measurement of the $e^+e^-$ annihilation cross-section into hadrons, which provides an interesting example of a link between a high-precision
low-energy experiment, and high-energy experiments. The weak contribution to \( a_{\mu} \approx 2 \times 10^{-9} \) as calculated in the Electroweak Theory [13], is too small to be detectable as yet.

2.3 Colliding-beam reactions at large momentum transfer

In \( e^+e^- \) colliding beams, tests of QED at large \( Q^2 = 2 \times 10^3 \text{ GeV}^2 \) or, alternatively, at small interaction distances \( 1/\sqrt{Q^2} \approx 4 \times 10^{-3} \text{ fm} \), have been performed. For processes such as \( e^+e^- \to e^+e^- \to e^+e^- \to \mu^+\mu^- \), and \( e^+e^- \to \tau^+\tau^- \), the QED photon propagator is customarily multiplied by a form factor containing a parameter \( \Lambda \) which is a measure of the mass of the exchanged system, or of the radius of the leptons [14]:

\[
1/k^2 \to (1/k^2) F(q^2) \quad \text{with} \quad F(q^2) = 1 \pm [q^2/(q^2 - \Lambda^2)]
\]

where \( q^2 \) for time-like photons and \( q^2 \) for space-like photons.

In \( e^+e^- \to \mu^+\mu^- \) and \( e^+e^- \to \tau^+\tau^- \), only the s-channel contributes. In Bhabha scattering, \( e^+e^- \to e^+e^- \), both s- and t-channel diagrams contribute to the cross-section. In the case of exact validity of QED, \( \Lambda = \infty \) and hence \( F(q^2) = 1 \).

At PETRA, the experimental teams have chosen to publish their data after subtraction of \( O(\alpha) \) electroweak corrections to \( d\sigma/\Omega \), the differential cross-section obtained in lowest non-trivial order:

\[
d\sigma/\Omega = (d\sigma_{\text{exp}}/\Omega) (1 - \delta_{\text{em}} - \delta_{\text{w}}).
\]

The QED \( O(\alpha) \) correction \( \delta_{\text{em}} \) comprises hard photon bremsstrahlung, soft photon bremsstrahlung, and virtual corrections. The weak correction \( \delta_{\text{w}} \) from \( Z \) exchange is small at PETRA energies (see Section 6.2). The thus corrected cross-sections are then compared to \( (d\sigma/\Omega) F^2(q^2) \), with good agreement between experiment and QED. Figures 4 and 5 show the ratio of the measured to calculated QED cross-sections, as a function of \( s \), for \( e^+e^- \to \mu^+\mu^- \) and \( e^+e^- \to \tau^+\tau^- \). The effect of a finite \( \Lambda_+ \) (\( \Lambda_- \)) is to increase (decrease) the cross-section. The data require \( \Lambda_+ \approx 300 \text{ GeV} [15] \), which corresponds to an upper limit of the lepton radius of

\[
r_{e,\mu,\tau} < \hbar c/\Lambda = 0.7 \times 10^{-3} \text{ fm}.
\]

This can be seen as a demonstration of the point-like nature of the charged leptons, including the \( \tau \) lepton, or alternatively of the validity of QED up to \( Q^2 = 2 \times 10^3 \text{ GeV}^2 \).

![Graph](image)

**Fig. 4** Ratio of the measured to calculated cross-section of \( e^+e^- \to \mu^+\mu^- \), in lowest non-trivial order, as a function of \( s \).
2.4 Photon mass

QED requires a massless photon as carrier of the electromagnetic interaction. The best upper limit, obtained in the laboratory from a test of Coulomb's law, is $m_\gamma < 1.5 \times 10^{-14}$ eV [16]. A significantly better limit was obtained from a measurement of Jupiter's magnetic field, at 134 locations between 2.8 and 13.1 Jupiter radii, in the course of the Pioneer-10 mission [17].

The advantage of measuring the field variation in the vicinity of Jupiter lies in the large size of the object. The fractional changes of the field are proportional to $(m_\gamma D)^2$, where $D$ is the size of the magnetic dipole source [17]. Maxwell's equations with a non-zero photon mass become the 'Proca equations'. They have been used to predict the field values for a magnetic dipole field (with optional contributions from higher multipoles). In the fit to the data the photon mass was left as a free parameter, yielding an upper limit

$$m_\gamma < 6 \times 10^{-16} \text{ eV}.$$ 

A better but controversial limit has been obtained from the study of the magnetic field of even larger objects: galaxies.

2.5 Conservation of electric charge

Another fundamental aspect of QED which is subject to experimental test is the conservation of the electric charge. A Soviet team performed a search for the nuclear reaction

$$^{71}\text{Ga}_{31} \rightarrow ^{71}\text{Ga}_{32} + \gamma,$$

which implies a violation of conservation of electric charge. A lower limit for the lifetime

$$\tau > 2.3 \times 10^{23} \text{ yr}$$

was observed [18].

2.6 Summary concerning QED

Experimentally, there is no evidence for a non-zero photon mass, for a non-conservation of electric charge, or for a modification of the photon propagator up to $Q^2 = 2 \times 10^3$ GeV$^2$. Complete higher order calculations up to $O(\alpha^3)$ have been calculated. They are needed to obtain agreement between theory and experiment. The accuracy achieved is such that strong interaction effects are observable; for the time being, they limit further progress in some cases.

QED is the best theory we have.
3. NEUTRINOS AND THEIR MASSES

The existence of neutrinos was postulated in 1930 by Pauli in his famous letter addressed to 'Meine lieben radioaktiven Damen und Herren' who were assembled at a conference in Tübingen [19]. The role of the neutrinos was to restore conservation of momentum and angular momentum in nuclear β-decay. Pauli felt that they would be unobservable for ever. Shortly afterwards, in 1934, Fermi used neutrinos as indispensable building blocks in his theory of nuclear β-decay [20].

Today, half a century later, the properties of neutrinos are well understood. Neutrino beams of high energy and high intensity are a heavily used tool for experimentation. Large statistics are routinely available. Yet there are basic mysteries about neutrinos: Are they Dirac or Majorana particles? Are they truly massless?

3.1 Dirac or Majorana neutrinos?

Our standard way of describing neutrinos is in terms of two distinct particles per generation: a left-handed neutrino, and its antiparticle, the right-handed antineutrino. In the Electroweak Theory, neutrinos are massless. However, reality may be different. We follow here the thoughtful argumentation of Kayser [21].

In the Dirac theory the neutrino and the antineutrino are distinct particles: \( \nu \neq \bar{\nu} \). Equally consistent with the known experimental facts is the notion that the neutrino is its own antiparticle, \( \nu = \bar{\nu} \), then referred to as a Majorana particle.

Let the neutrino have a non-zero, albeit very small, mass. The physical difference between a Dirac and a Majorana neutrino can then be exhibited by applying a CPT transformation, and/or a Lorentz boost, to a left-handed neutrino \( \nu_L \) (see Fig. 6a). Under CPT, \( \nu_L \) transforms into \( \bar{\nu}_R \). Since \( \nu_L \) is massive, it can be overtaken with a sufficiently large Lorentz boost. Seen from the frame of the fast observer, the neutrino moves backward but is spinning the same way as in the original frame: \( \nu_L \) has turned into \( \nu_R \). Under CPT, this \( \nu_R \) transforms into \( \bar{\nu}_L \). The latter would equally emerge from a Lorentz boost along the direction of motion of \( \bar{\nu}_R \). The four states thus obtained are distinct particles if the neutrino belongs to the Dirac species.

On the other hand, the particle obtained from \( \nu_L \) after a Lorentz boost, a right-handed particle, may be identical with \( \bar{\nu}_R \). The results of the CPT transformation and of the Lorentz boost then coincide: there are only two distinct particles if the neutrino belongs to the Majorana species. Notice that in the latter case, because of the identity of neutrino and antineutrino, there is no lepton-number conservation. It is this feature which makes Majorana neutrinos attractive for Grand Unified Theories, in which the lepton number is in general violated.

---

![Fig. 6](image-url)  

*Fig. 6* a) The four distinct states of a Dirac neutrino, and b) the two distinct states of a Majorana neutrino.
The distinction between Dirac and Majorana neutrinos makes sense only for massive neutrinos. In the limit of massless neutrinos the distinction disappears because, in the Dirac neutrino case, the right-handed neutrino and the left-handed antineutrino states, even if existing, would never participate in any interaction (unless right-handed weak currents should exist). The only visible particles would be a left-handed state called by convention a neutrino, and a right-handed state called by convention an antineutrino. It would then be a matter of semantics to call massless neutrinos Dirac or Majorana particles, i.e. to think either of a particle and its distinct antiparticle with opposite helicity or of the two helicity states of one and the same particle.

As of today, there is no compelling theoretical prediction of finite neutrino masses. There is no compelling reason either why neutrinos should be massless.

3.2 Mass limits from weak decay kinematics

Experimental upper mass limits for all three known neutrino flavours have been reported [22–29]. They are summarized in Table 3. The mass limits vary by orders of magnitude between the different neutrino flavours. In the case of $\nu_e$, where the highest precision has been obtained, a claim was made of a non-zero mass [22]. This claim came at the time when cosmologists and Grand Unified Theory builders strongly favoured a non-zero neutrino mass, and caused a tremendous experimental activity.

**Table 3**

Summary of neutrino mass measurements from weak decay kinematics

<table>
<thead>
<tr>
<th>Neutrino</th>
<th>Decay mode</th>
<th>Mass</th>
<th>Ref.</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\bar{\nu}_e$</td>
<td>$^3\text{H} \rightarrow ^3\text{He} + e^- + \bar{\nu}_e$</td>
<td>$&lt; 55$ eV (90% c.l.)&lt;br&gt;$14 &lt; m_{\bar{\nu}_e} &lt; 46$ eV (99% c.l.)&lt;br&gt;$&lt; 18$ eV (95% c.l.)</td>
<td>23, 22, 24</td>
</tr>
<tr>
<td>$\nu_\mu$</td>
<td>$\pi^- \rightarrow \mu^+ + \nu_\mu$</td>
<td>$&lt; 500$ keV&lt;br&gt;$&lt; 250$ keV 90% c.l.</td>
<td>25, 26</td>
</tr>
<tr>
<td>$\nu_\tau$</td>
<td>$\tau \rightarrow 5\pi^- (\pi^0)\nu_\tau$</td>
<td>$&lt; 125$ MeV</td>
<td>27</td>
</tr>
<tr>
<td></td>
<td>$\tau \rightarrow 5\pi^0 (\pi^0)\nu_\tau$</td>
<td>$&lt; 84$ MeV 95% c.l.</td>
<td>28</td>
</tr>
<tr>
<td></td>
<td>$\tau \rightarrow 3\pi^0 \nu_\tau$</td>
<td>$&lt; 70$ MeV</td>
<td>29</td>
</tr>
</tbody>
</table>

The most precise measurement of the $\nu_e$ mass comes from the tritium $\beta$-decay: $^3\text{H} \rightarrow ^3\text{He} + e^- + \bar{\nu}_e$. This decay is convenient because of the low end-point energy (18.6 keV) of the $\beta$-spectrum, and of the long half-life ($T_{1/2} = 12.3$ yr). A finite $\bar{\nu}_e$ mass would show up as a distortion of the $\beta$-spectrum near the end point, shifting the end point by $mc^2$ towards lower values of energy. This is most easily visualized in the ‘Kurie plot’ of the $\beta$-spectrum: allowed $\beta$-decay spectra such as from $^3\text{H}$ decay appear as a straight line against a linear scale of $\beta$-ray energy (see Fig. 7).

After the claim of a finite $\bar{\nu}_e$ mass was made by the Moscow ITEP group [22], a hot discussion on potential systematic errors of this experiment started among experts. The criticism concerns uncertainties in the effective Q-value of the $^3\text{H}$ decay since the $^3\text{H}$ atom is bound in a valine molecule
(C₂H₆NO₂), and in the understanding of the energy resolution of the magnetic spectrometer. Since this concerns delicate details in a very sophisticated experimental set-up, trimmed over many years to achieve the smallest possible errors, we will have to wait for several groups to agree on a result before a definitive conclusion on a non-zero neutrino mass can be drawn. It is fair to say, however, that a \( \nu_e \) mass of \( \sim 10 \) eV is compatible with all presently known data.

Throughout the world there are about 10 experimental teams preparing or carrying out experiments on tritium decay. In one or two years time we might expect consensus about the validity of the result of the Moscow ITEP group.

In 1985, a claim was made by Simpson [30] of a \( \bar{\nu}_e \) mass of \( 17.1 \pm 0.2 \) keV which would be equally emitted in tritium decay. Meanwhile, several groups [31] have reported data which are in variance with his result: no neutrino of 17 keV mass has been found in the nuclear \( \beta \)-decay of \( ^{35}S \).

The best limit on the \( \nu_\mu \) mass comes from a study of the \( \pi \rightarrow \mu \nu_\mu \) two-body decay. An earlier experiment [25], carried out at SIN, measured the momenta of \( \pi^+ \) and of \( \mu^+ \) from decays in the forward direction, in the same magnetic spectrometer. The momentum difference determines the mass of the \( \nu_\mu \). Another experiment, also carried out at SIN [26], measured precisely the muon momentum from pion decay at rest. Together with the precisely known pion mass, this gave the so far most stringent limit, \( m_{\nu_\mu} < 250 \) keV (90% c.l.).

Limits on the mass of the \( \nu_\tau \) come from studies of \( \tau \) decays. Because of the large mass of the \( \tau \), the precision is considerably worse than for the \( \nu_\tau \). The MARK II and the HRS Collaborations at PEP studied the decay channel \( \tau \rightarrow 5\pi^+(\pi^n)\nu_\tau \), and the ARGUS Collaboration at DESY studied the channel \( \tau \rightarrow 3\pi^+\nu_\tau \). High statistics and good momentum resolution have allowed the latter team to push the limit as low as \( m_{\nu_\tau} < 70 \) MeV (95% c.l.). The limit comes from a fit to the neutrino energy spectrum which is obtained from the difference between the beam energy and the three-pion energy. The low-energy end of this spectrum is sensitive to the \( \nu_\tau \) mass. The data of the ARGUS Collaboration are shown in Fig. 8. There seems to be room for an improvement of the mass limit with increased statistics.

3.3 Double \( \beta \)-decay

The process of double \( \beta \)-decay addresses at the same time two fundamental problems: Do neutrinos have a non-zero mass? And if so, are they Dirac or Majorana particles? The transition

\[
(A,Z) \rightarrow (A,Z+2) + 2e^- (+2\nu)
\]

is referred to as 'double \( \beta \)-decay'. Neutrinoless double \( \beta \)-decay can be studied in the decays of even-even nuclei, where the competing normal \( \beta \)-decay, \( (A,Z) \rightarrow (A,Z+1) + e^- + \bar{\nu}_e \), is
energetically forbidden owing to the pairing energy in nuclei. It can proceed through second-order weak interaction with emission of two neutrinos, which is a process of no fundamental importance. The interest focuses on double $\beta$-decay without the emission of two neutrinos. This latter decay mode is strongly phase-space favoured over the former. The experimental signature would be a fixed energy transfer to the final-state electrons. Figure 9 shows the diagram of this process. It manifestly violates lepton-number conservation, and hence is forbidden for Dirac neutrinos.

Neutrinoless double $\beta$-decay is allowed for Majorana neutrinos. However, it cannot proceed if the neutrinos are massless. The neutrino emitted at one vertex is left-handed, but only a right-handed neutrino can be absorbed at the other vertex (always assuming that there exist no right-handed weak currents). The only way that a fermion has both helicity states is by having a finite mass. Hence the existence of neutrinoless double $\beta$-decay would demonstrate that neutrinos have a finite mass, and are Majorana particles.

Searches for neutrinoless double $\beta$-decay have a long history. They have been very much refined with the years and have achieved an impressive level of sensitivity. For a recent review, see Ref. [32]. No conclusive signal has been observed as yet. Table 4 gives a summary of the most stringent experimental limits on the half-lives of neutrinoless double $\beta$-decay. Most of the experiments [33–38] look for the appearance of a signal at 2.04 MeV, summing the energies of the two electrons, in the transition $^{76}\text{Ge} \rightarrow ^{76}\text{Se}$, employing large Ge semiconductor detectors carefully shielded against cosmic-ray background and natural radioactivity. Since $^{76}\text{Ge}$ constitutes 7.8% of natural Ge, source

<table>
<thead>
<tr>
<th>Group</th>
<th>Reaction</th>
<th>Half-life results</th>
</tr>
</thead>
<tbody>
<tr>
<td>Battelle–South Carolina [33]</td>
<td></td>
<td>$T_{1/2} &gt; 1.3 \times 10^{22}$ yr</td>
</tr>
<tr>
<td>CalTech [34]</td>
<td></td>
<td>$T_{1/2} &gt; 2.0 \times 10^{22}$ yr</td>
</tr>
<tr>
<td>Osaka [35]</td>
<td></td>
<td>$T_{1/2} &gt; 2.2 \times 10^{22}$ yr</td>
</tr>
<tr>
<td>Guelph–Downsview–Kingston [36]</td>
<td></td>
<td>$T_{1/2} &gt; 3.2 \times 10^{22}$ yr</td>
</tr>
<tr>
<td>UCSB–LBL [37]</td>
<td></td>
<td>$T_{1/2} &gt; 5.0 \times 10^{22}$ yr</td>
</tr>
<tr>
<td>Milan [38]</td>
<td></td>
<td>$T_{1/2} &gt; 1.2 \times 10^{23}$ yr</td>
</tr>
<tr>
<td>Heidelberg [39]</td>
<td>$^{128}\text{Te}, ^{130}\text{Te}$ abundance</td>
<td>$T_{1/2}(^{130}\text{Te})$ = $1.03^{+1.13}_{-1.03} \times 10^{-4}$</td>
</tr>
</tbody>
</table>

**Table 4**

Summary of half-life results on neutrinoless double $\beta$-decay
and detecting material are the same. As an example, Fig. 10 shows part of a high-statistics spectrum of the UCSB-I.B.L group [37]. There is no evidence for a peak at 2.04 MeV.

Other than searching directly for the emission of two electrons, an abnormal abundance of \((A, Z + 2)\) isotopes was looked for, in ore containing the \((A, Z)\) nucleus. These experiments have the advantage of integrating over a geological period of observation; however, they are unable to discriminate between double \(\beta\)-decay with and without neutrino emission. The best result comes from a geochemical study of the relative abundance of the xenon isotopes \(^{129}\)Xe and \(^{130}\)Xe originating from \(^{128}\)Te and \(^{130}\)Te decay [39].

Unfortunately, in order to extract neutrino mass limits from half-lives, a good knowledge of nuclear transition matrix elements is needed which poses non-trivial problems [32, 40]. The half-life is inversely proportional to \(m_{\nu_e}^2\). Hence the \(m_{\nu_e}\) mass limit improves slowly with the half-life limit. Using the calculations of Haxton et al. [40], the best half-life limit of the Milan group is equivalent to

\[
m_{\nu_e} < 3.7 \text{ eV} \quad [38].
\]

The geochemical result of the Heidelberg group is equivalent to

\[
m_{\nu_e} < 5.6 \text{ eV} \quad [39].
\]

These limits may be uncertain by factors of up to three. Hence one cannot conclude that these results are at variance with the claim of the Moscow ITEP group [22]. Further improvement of the limits on neutrinoless double \(\beta\)-decay would be very interesting. Fortunately, significant experimental progress can be expected in the near future.
3.4 Neutrino flavour oscillation

We are familiar with the fact that the weak interaction eigenstates of quarks are linear superpositions of quark-mass eigenstates. If neutrinos are massive, it is natural to assume that an analogous mixing phenomenon occurs.

Consider N neutrino mass eigenstates with mass \( m_i \), \( i = 1, ..., N \). In weak interaction emission or absorption processes, neutrinos with definite flavour are involved. The latter N 'flavour eigenstates' are presumably related to the mass eigenstates by an orthonormal mixing matrix \( U \) [analogous to the Kobayashi–Maskawa (KM) mixing matrix in the quark sector, see Section 4.1]:

\[
|\nu_i\rangle = \sum_j U_{ij} |\nu_j\rangle; \quad \ell = e, \mu, \tau, ..., \quad \text{and} \quad i = 1, 2, 3, ... .
\]

At creation, each flavour is a definite mixture of mass eigenstates. Since their mass is different, they move with different speed. At the point of absorption, the mass composition will differ from that at creation; hence there will be an admixture of other flavours. Note that the phenomenon of 'neutrino flavour oscillation' requires a mass difference of the involved mass eigenstates.

Neutrino oscillation experiments are usually analysed in terms of two neutrino flavours only, for example \( \nu_e \) and \( \nu_\mu \). Since there is no convincing experimental evidence of the occurrence of neutrino oscillations, this procedure is justified although a more general analysis would involve all three known neutrino flavours. The orthonormal matrix reduces to a \( 2 \times 2 \) matrix with one neutrino mixing angle \( \theta \) as a free parameter:

\[
\begin{bmatrix}
|\nu_e\rangle \\
|\nu_\mu\rangle
\end{bmatrix} = \begin{bmatrix}
\cos \theta & \sin \theta \\
-\sin \theta & \cos \theta
\end{bmatrix} \begin{bmatrix}
|\nu_1\rangle \\
|\nu_2\rangle
\end{bmatrix} .
\]

Then we obtain for the intensity of the initial neutrino flavour at a distance \( L \) from the point of creation:

\[
\frac{I(L)}{I(0)} = 1 - \sin^2 2\theta \sin^2 \left(1.267 \Delta m^2 [eV^2] L [m]/E [MeV] \right),
\]

where \( \Delta m^2 \) is the difference of the squares of the neutrino masses, \( |m_{\nu_1}^2 - m_{\nu_2}^2| \), and \( E \) the neutrino energy. There is an oscillatory behaviour with an 'oscillation length'

\[
L_{osc}[m] = \frac{\pi E [MeV]}{1.267 \Delta m^2 [MeV]^2} .
\]

There are two ways of observing the oscillation: either at a fixed neutrino energy as a function of the distance (Fig. 11a), or at a fixed distance as a function of the energy (Fig. 11b). In the latter case, the oscillatory character becomes the more rapid the lower the energy. With an apparatus with finite energy resolution one would average over the rapid oscillations. The average change of counting rate is then \( \frac{1}{2} \sin^2 2\theta \).

Neutrino-oscillation experiments are conveniently classified as 'disappearance' or 'appearance' experiments. In the latter case, one looks for the appearance of a new neutrino flavour amidst a beam which at its creation consists of another flavour only. The advantage is good sensitivity; the

![Fig. 11 Neutrino oscillation: a) with fixed energy \( E \) as a function of distance \( L \), and b) at fixed \( L \) as a function of \( E \).](15)
disadvantage is that one looks for specific neutrino flavours only. The disappearance experiment, on the other hand, is equally sensitive to all possible neutrino flavours, but faces larger statistical and systematic errors since any depletion is to be seen in the total event sample.

Experiments aim at exploring the smallest possible $\Delta m^2$. The sensitivity of $\Delta m^2$ is given by the ratio $L/E$: for a given oscillatory effect, $\Delta m^2$ is the smaller the larger the ratio $L/E$. Large distances and small neutrino energies are called for in a good oscillation experiment.

Figure 12 gives an overview of the sensitivity of present-day experiments, for the neutrino mass (more precisely: for $\sqrt{\Delta m^2}$).

Experiments which look at fixed $L$ for an oscillation with energy are plagued by uncertainties in the expected neutrino energy spectrum in the absence of oscillation. Experiments which look at fixed $E$ for a deviation from the expected rate have the same difficulty of depending on a comparison with a predicted event rate. Systematic errors from absolute normalization can be overcome by experiments employing two detector stations at different distances $L$.

For an interesting discussion of the quantum mechanics of neutrino oscillation, the reader is referred to an article of Kayser [41].

In the following subsections, we shall assume CPT and CP conservation, which implies equal oscillation amplitudes for $\nu_i \rightarrow \nu_j$, $\nu_j \rightarrow \nu_i$, $\bar{\nu}_i \rightarrow \bar{\nu}_j$, and $\bar{\nu}_j \rightarrow \bar{\nu}_i$.

### 3.4.1 Nuclear reactor experiments

Nuclear reactor experiments look for a departure from the expected energy spectrum of $\bar{\nu}_e$ originating from nuclear $\beta$-decay of fission products. Typical energies are $E = 3$ MeV, typical distances $L = 30$ m. The flux of $\bar{\nu}_e$ is detected via inverse $\beta$-decay: $\bar{\nu}_e + p \rightarrow e^+ + n$. Since the energies involved are too low to create $\mu$ or $\tau$ leptons in CC interactions (NC reactions are ‘blind’ with respect to neutrino oscillations), nuclear reactor experiments can only look for the disappearance of $\bar{\nu}_e$: $\bar{\nu}_e \rightarrow \bar{\nu}_x$.

Table 5 lists the parameters of experiments carried out at nuclear-reactor power plants [42–46]. Of these, only the Bugey reactor experiment reports evidence for neutrino oscillation: the group observes less $\nu_e$ interactions at 18.3 m distance than expected from the measurement at 13.6 m. They claim evidence for neutrino oscillation $\bar{\nu}_e \rightarrow \bar{\nu}_x$, with $\sin^2 2\theta = 0.25$ and $\Delta m^2 = 0.20$ eV$^2$, with a significance at the 3$\sigma$ level. The data reported by the other groups, in particular the Gösgen reactor experiment, invalidate the Bugey result, as can be seen from Figs. 13a,b which show the excluded regions in the $\Delta m^2 - \sin^2 2\theta$ plane of $\nu_e \rightarrow \nu_x$ and $\nu_e \rightarrow \nu_\tau$ transitions. Notice that a result on the absence of $\bar{\nu}_e \rightarrow \bar{\nu}_x$ transitions implies, in particular, the absence of $\nu_e \rightarrow \nu_\mu$ and $\nu_e \rightarrow \nu_\tau$ transitions. However, a claim on the existence of $\bar{\nu}_e \rightarrow \bar{\nu}_x$ oscillations such as Bugey’s could involve $\nu_e \rightarrow \nu_\mu$, $\nu_e \rightarrow \nu_\tau$, or even $\nu_e \rightarrow \nu_{\text{new}}$ transitions, or a combination of those.

The Bugey experiment is currently being repeated with an improved apparatus.

Figure 14 shows a typical positron energy spectrum from inverse $\beta$-decay, as observed in nuclear-reactor neutrino experiments.
Table 5

Neutrino oscillation experiments at nuclear reactors

<table>
<thead>
<tr>
<th>Group</th>
<th>Reactor</th>
<th>Distance L (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>CalTech–ISN–TU Munich [42]</td>
<td>I.L. Grenoble (F)</td>
<td>8.8</td>
</tr>
<tr>
<td>CalTech–SIN–TU Munich [43]</td>
<td>Gösgen (CH)</td>
<td>37.9, 45.9, 64.7</td>
</tr>
<tr>
<td>Grenoble–LAPP Annecy [44]</td>
<td>Bugey (F)</td>
<td>13.6, 18.3</td>
</tr>
<tr>
<td>UC Irvine [45]</td>
<td>Savannah River (USA)</td>
<td>18.2, 23.7</td>
</tr>
<tr>
<td>Kurchatov [46]</td>
<td>Rovno (USSR)</td>
<td>18, 25</td>
</tr>
</tbody>
</table>

Fig. 13  Excluded regions in the $\Delta m^2 - \sin^2 2\theta$ plane, for oscillations: a) $\nu_e \leftrightarrow \nu_x$, b) $\nu_x \leftrightarrow \nu_x$, and c) $\nu_x \leftrightarrow \nu_x$.

Fig. 14  Positron energy spectrum from $\bar{\nu}_e + p \rightarrow e^+ + n$ (from Ref. [43]).
3.4.2 Accelerator experiments

Accelerator experiments look either for the appearance of a new neutrino flavour, or for the disappearance of neutrinos in a beam of definite flavour at creation. Typical energies of the most sensitive experiments are \( E = 1 \) GeV, typical distances \( L = 1 \) km.

For small \( \Delta m^2 \), i.e. near the sensitivity limit, the oscillatory term

\[
\sin^2 2\theta \sin^2 (1.267 \Delta m^2 L/E) \Rightarrow \sin^2 2\theta (1.267 \Delta m^2 L/E)^2.
\]

Hence, a reduction of the allowed amplitude for oscillation, due to better experimental precision, will primarily result in an improvement of the limit on the mixing parameter, \( \sin^2 2\theta \), and hardly in an improvement of the limit on \( \Delta m^2 \).

A number of dedicated oscillation experiments have been carried out at accelerators, together with re-analyses of data taken in earlier experiments, in the light of possible neutrino oscillations. The most significant limits come from the experiments [47–54] which are listed in Table 6 together with their experimental parameters.

### Table 6

Neutrino oscillation experiments at accelerators

<table>
<thead>
<tr>
<th>Group</th>
<th>Reaction</th>
<th>Accelerator</th>
<th>Typical L/E</th>
</tr>
</thead>
<tbody>
<tr>
<td>FNAL 15 ft [47]</td>
<td>( \nu_\mu \to \nu_e, \nu_\mu \to \nu_\tau )</td>
<td>FNAL</td>
<td>1200 m/30 GeV</td>
</tr>
<tr>
<td>FNAL hybrid [48]</td>
<td>( \nu_\mu \to \nu_\tau )</td>
<td>FNAL</td>
<td>750 m/30 GeV</td>
</tr>
<tr>
<td>CDHS [49]</td>
<td>( \nu_\mu \to \nu_\kappa )</td>
<td>CERN-PS</td>
<td>850 m/1 GeV</td>
</tr>
<tr>
<td>CHARM [50]</td>
<td>( \nu_\mu \to \nu_e, \nu_\mu \to \nu_\kappa )</td>
<td>CERN-PS</td>
<td>850 m/1 GeV</td>
</tr>
<tr>
<td>CCFR [51]</td>
<td>( \nu_\mu \to \nu_\kappa )</td>
<td>FNAL</td>
<td>700 m/100 GeV</td>
</tr>
<tr>
<td>BNL [52]</td>
<td>( \nu_\mu \to \nu_e )</td>
<td>BNL-AGS</td>
<td>100 m/1 GeV</td>
</tr>
<tr>
<td>BEBC–APPW [53]</td>
<td>( \nu_\mu \to \nu_e, \nu_\mu \to \nu_\tau )</td>
<td>CERN-PS</td>
<td>850 m/1 GeV</td>
</tr>
<tr>
<td>Serpukhov [54]</td>
<td>( \nu_\mu \to \nu_\kappa )</td>
<td>Serpukhov</td>
<td>200 m/5 GeV</td>
</tr>
</tbody>
</table>

None of the listed experiments has reported evidence for neutrino oscillation. The limits obtained on \( \sin^2 2\theta \) and \( \Delta m^2 \) can be seen in Figs. 13a,b,c. The same comment as made above applies: limits on \( \nu_\mu \to \nu_e \), although of more general validity, are interpreted as particular limits on \( \nu_\mu \to \nu_e \) and \( \nu_\mu \to \nu_\tau \).

Several new dedicated experiments at accelerators are in preparation. Experiments with \( L/E \) ratios similar to those found in earlier experiments, have the same sensitivity on \( \Delta m^2 \) but will chiefly improve the limits on \( \sin^2 2\theta \) by virtue of better accuracy. What is wanted is a significantly larger \( L/E \) ratio, to provide sensitivity to smaller \( \Delta m^2 \). Unfortunately the two newly explored neutrino beams at LAMPF and at the Neutron Spallation Source at RAL originate from slow or stopped pion decay (and subsequent muon decay), with typical energy \( E = 30 \) MeV, and typical distance \( L = 30 \) m. Hence the \( L/E \) ratio is the same as at high-energy accelerators.

3.4.3 Other searches for neutrino flavour oscillation

The Sun is an abundant source of \( \nu_e \)'s which are created in the fusion of light elements (starting from hydrogen) into heavier ones. The flux at the Earth's surface is \( \sim 10^{11} \) cm\(^{-2}\) s\(^{-1}\). The typical
energy is ~ 1 MeV, the distance between Sun and Earth is $1.5 \times 10^{11}$ m. Hence the observation of oscillation phenomena is possible for $\Delta m^2$ as low as $10^{-11}$ eV$^2$.

There is one, and so far only one, experiment to measure the solar neutrino flux at the Earth's surface, to compare with the prediction resulting from our understanding of the thermonuclear reaction in the Sun. Since 1970, Davis et al. [55] have been conducting an underground experiment in the Homestake Mine (South Dakota, USA), designed to detect the reaction

$$\nu_e + ^{37}\text{Cl} \rightarrow e^- + ^{37}\text{Ar}.$$ 

The $^{37}\text{Ar}$ nuclei thus produced can be counted by means of their electron capture decay ($T_{1/2} = 35$ d).

The surprising, and hitherto unexplained, result of this experiment is that the observed flux of $\nu_e$ from the Sun is $(1.9 \pm 0.3)$ 'Solar Neutrino Units', SNU (1 SNU $= 1 \times 10^{-36}$ captures per target atom and second). This experimental rate [55] falls short by a factor of 3 compared to the theoretical expectation [56] of $(7.6 \pm 1.1)$ SNU, where the error has the meaning of one standard deviation. This discrepancy is referred to as the 'solar neutrino puzzle' which we have had to live with for the last 15 years.

The experimental technique is highly non-trivial; yet an experimental error seems unlikely after so many years of criticism and refinement. Figure 15 demonstrates that the rate observed over the last 15 years is pretty constant. Hence today's thinking is rather concerned with the processes of neutrino creation and emission from the Sun. A review of various suggestions has been given by Haxton [57]. Suffice it to say that there might be alterations of the standard model of the Sun, which are consistent with the findings of Davis et al. In a few years, another neutrino detector based on capture in $^{71}\text{Ga}$ will come into operation. This detector will be sensitive also to the medium part of the solar neutrino spectrum, unlike the capture in $^{37}\text{Cl}$ which selects neutrinos from the high-energy end of the spectrum only.

Another explanation of the solar neutrino puzzle is in terms of $\nu_e$ oscillating into another neutrino flavour. In view of the large error of the depletion in the measured $\nu_e$ flux it has little meaning to speculate how many neutrino flavours would have to be involved in the mixing. Our conservative view is that the solar neutrino puzzle needs clarification but cannot be taken at this time as convincing evidence of neutrino oscillations.

Neutrinos born in the Earth's atmosphere are another subject for oscillation studies. The neutrinos come from $\pi$ and $K$ decay in cosmic-ray showers. They can travel across the whole Earth and reach a detector located at the opposite surface. Thus the typical length is the Earth's diameter, $L = 1.3 \times 10^4$ km; the typical energy of cosmic-ray neutrinos is $E = 1$ GeV [58].

A search for oscillation phenomena of atmospheric neutrinos can be carried out with some existing detectors, designed to look for nucleon decay. These detectors are mounted deep underground, in order to have maximum shielding against background from cosmic-rays. The signature of neutrino oscillation of the type $\nu_x \rightarrow \nu_y$ would be an up/down asymmetry in the flux of atmospheric neutrinos. In practice, this measurement faces great difficulties since the flux of

![Fig. 15](image-url)
atmospheric neutrinos depends, via their parent particles, on the Earth's magnetic field, as does the up/down flux ratio. Furthermore, the correlation between the reconstructed event direction and the neutrino direction is poor, and the event statistics are scarce. Finally, there is a systematic uncertainty as to whether neutrinos travelling in vacuum behave the same way as neutrinos traversing high-density matter: since the forward scattering cross-section on electrons is different for $\nu_e$ and $\nu_\mu$, flavour impurity is generated when neutrinos traverse matter. This phenomenon of 'matter oscillation' [59] may obscure the observation of oscillation with atmospheric neutrinos.

Results on a search for oscillations of atmospheric neutrinos have been reported by the IMB Collaboration [60]. They operate a 3300 t (fiducial mass) water detector, at a depth of 1570 m water equivalent, in the Morton Salt Mine (Cleveland, Ohio). Their principal aim is a search for Cherenkov light emitted in nucleon decay reactions. The chief background is caused by events which originate from interactions of atmospheric neutrinos. Using only neutrino interactions that are entirely contained within the fiducial volume, 135 events with a single-prong topology were retained. A sample of 25 events pointed in the upward-going 1/5 of the solid angle, and equally 25 events pointed in the downward-going 1/5 of the solid angle. For maximal mixing, $\sin^2 2\theta = 1$, the data exclude the interval $2.2 \times 10^{-5} \text{ eV}^2 < \Delta m^2 < 11.2 \times 10^{-3} \text{ eV}^2$, neglecting however matter oscillation effects in the analysis.

3.5 Search for new neutrinos

In the classical neutrino experiment carried out by Danby et al. [61] at the Brookhaven AGS, it has been shown that the $\nu_\mu$ is distinct from the $\nu_e$. There is a lot of indirect evidence for the existence of the $\nu_\tau$ as a sequential neutrino [62]; however the $\nu_\tau$ remains today's best established but experimentally unobserved particle, and is considered here a 'new' neutrino.

3.5.1 Proton beam-dump experiments

In order to detect the interactions of the $\nu_e$, taken here as an example of a new neutrino $\nu_\tau$, one needs besides an adequate neutrino detector a flux of new neutrinos which is as large as possible. On the other hand, the overwhelming background from $\nu_e$ and $\nu_\mu$ must be suppressed. This is best achieved in a proton beam-dump experiment, the recipe of which is simple: take as many protons as one can afford, with the highest available energy; dump them into a block of matter with the shortest possible hadronic absorption length $\lambda_{abs}$ (to minimize pion and kaon decays into known neutrinos); and locate the detector just as close to the dump as not to be swamped completely by the flux of muons. The flux of new neutrinos, which is thought to originate from the semi-leptonic decay of heavy quarks produced in the proton-nucleus collisions, remains unaffected because the decay path of the heavy quarks is short compared to $\lambda_{abs} \sim 15$ cm. Standard neutrinos, however, are suppressed by 3 orders of magnitude as compared to normal neutrino beams, where pions and kaons have a decay path of $\sim 300$ m. By convention, the neutrino flux which scales with the inverse dump density, is referred to as 'non-prompt', in contrast to the 'prompt' flux which is independent of the dump density.

Various proton beam-dump experiments have been carried out over the past decade with the aim of looking for new penetrating (neutrino-like) particles. The most significant results come from experiments carried out at CERN and at FNAL. The situation has been reviewed recently by Huth [63].

First, it is worth recalling that in the first round of beam-dump experiments at CERN, a 'new' source of 'old' neutrinos was found: a flux of $\nu_e$ and $\bar{\nu}_e$ from semi-leptonic charm quark decay, providing the first evidence of appreciable charm production in hadronic collisions, in disagreement with earlier (wrong) upper limits from emulsion-scan experiments. This claim was first made by the BEBC Collaboration [64], and quickly confirmed by two other groups [65, 66].
A fair amount of interest was aroused by an indication of the prompt \( \nu_e + \bar{\nu}_e \) flux being lower than the prompt \( \nu_e + \bar{\nu}_e \) flux, in a second round of beam-dump experiments at CERN [67–69].

Because of the conflict of this result with the notion of \( e-\mu \) universality, a third round of beam-dump experiments was carried out at CERN, confirming within errors equal \( \nu_e + \bar{\nu}_e \) and \( \nu_\mu + \bar{\nu}_\mu \) fluxes of prompt origin. This result is supported by the findings of a FNAL beam-dump experiment [70]. Hence we feel entitled to dismiss any claim of new physics derived from an inequality of prompt \( \nu_e + \bar{\nu}_e \) and \( \nu_\mu + \bar{\nu}_\mu \) fluxes in proton beam-dump experiments.

Still, the neutrino flux from proton beam dumps must contain an appreciable flux of \( \nu_\tau \), primarily from \( F \to \tau \nu_\tau \) decay. Searches for \( \nu_\tau \) interactions concentrate on the observation of a finite decay path of the \( \tau^- \) created in \( \nu_\tau + \text{N} \to \tau^- + \text{X} \) interactions. This is a domain of bubble chambers equipped with high-resolution optics. A dedicated proton beam-dump experiment is under preparation at FNAL, utilizing protons near to 1 TeV energy. This experiment, which is planned for 1989 or so, is the only one in the foreseeable future which has a chance of pinning down experimentally the elusive \( \nu_\tau \).

### 3.5.2 Couplings to new massive neutrinos

In this subsection, we make the same physics assumption as made before when discussing neutrino flavour oscillations (Section 3.4): a neutrino flavour eigenstate \( |\nu_i\rangle \) is a linear superposition of mass eigenstates \( |\nu\rangle \), not degenerate in mass:

\[
|\nu_i\rangle = \sum U_{i\ell} |\nu\rangle, \quad \text{with } \ell = e, \mu, \tau, \ldots, \quad i = 1, 2, 3, \ldots
\]

The coefficients \( U_{i\ell} \) are elements of a non-diagonal orthonormal mixing matrix. This matrix is constructed such that the dominant couplings are between \( |\nu_e\rangle \) and \( |\nu_i\rangle \), \( |\nu_\mu\rangle \) and \( |\nu_\mu\rangle \), etc. Besides the phenomenon of neutrino flavour oscillation, we expect then two further manifestations of a potentially existing new mass eigenstate in the interactions of \( \nu_e \) or \( \nu_\mu \), by virtue of its admixture with strength \( |U_{i\ell}|^2 \), where \( \ell = e \) or \( \mu \) [71]:

i) Kinks or satellite lines in the charged-lepton momentum spectrum observed in suitable nuclear \( \beta \)-decays, or in the two-body decay of pseudoscalar mesons (\( \pi, K, D, F, \ldots \)). The strength of admixtures will be proportional to \( g|U_{i\ell}|^2 \), where \( g \) denotes a kinematical enhancement factor which takes into account that a massive neutrino is less helicity-suppressed.

ii) In-flight neutrino decays. The rate of decays will be proportional to \( g|U_{i\ell}|^2|U_{j\ell'}|^2 \), where \( |U_{i\ell}|^2 \) comes from the production, and \( |U_{j\ell'}|^2 \) from the decay.

No experiment has as yet reported evidence for a massive neutrino from such studies, apart from Simpson's claim for a 17.1 keV neutrino, which is contradicted by other experiments and which we disregard (see Section 3.2). Experimental limits are given in much the same way as for neutrino oscillations, in a two-dimensional array of \( |U_{i\ell}|^2 \) versus the mass \( m_\ell \) of the hypothetical new mass eigenstate. Notice that this mass eigenstate may also be the one closely coupled to \( \nu_e \). Reviews of the experimental situation have recently been given by Sarkar–Cooper [72] and Deutsch [73].

From the absence of kinks in the \( ^3\text{H} \) and \( ^{64}\text{Cu} \) \( \beta \)-spectra, upper limits have been obtained for \( |U_{i\ell}|^2 \) which are shown in Fig. 16 (taken from Ref. [74]). Other limits come from searches for secondary peaks in the momentum spectrum of charged leptons from pion and kaon two-body decay. Because of its helicity suppression, the decay \( \pi, K \to e\nu_e \) is particularly sensitive for a measurement of \( |U_{i\ell}|^2 \).

Conventional neutrino beams of \( \nu_e \) and \( \nu_\mu \) may contain an admixture of a massive neutrino flavour \( \nu_\tau \) (\( |\nu_\tau\rangle \) is the massive flavour eigenstate associated with the mass eigenstate \( |\nu_\tau\rangle \)). By virtue of neutrino mixing, \( \nu_\tau \) may decay via a CC reaction as shown in Fig. 17. The non-observation of, for example, an \( e^-e^- \) pair from decays such as \( \nu_e \to e^-e^-\nu_e \), for \( m_{\nu_\tau} > 2m_e \), in a \( \nu_e \) beam, puts an upper limit on \( |U_{i\ell}|^4 \). The lack of sensitivity due to the 4th power of \( |U_{i\ell}| \) is compensated by the high
neutrino flux which makes searches for neutrino decays competitive. Upper limits from experimental searches for neutrino decays are included in Fig. 16.

In an analogous way, limits can be obtained on $|U_{e2}|^2$ and $|U_{e3}|U_{e1}|$ [72, 73].

Limits on the mixing of massive neutrinos may prove very important in the future, once we have a theory capable of predicting masses and mixing angles of leptons and quarks.

3.6 How many neutrino generations?

3.6.1 Cosmological arguments

Cosmological constraints on the number of neutrino generations are provided by the observed abundances of the light elements D, $^3$He, $^4$He, and $^7$Li, produced in the early phase of the Universe. The amount of primordial nucleosynthesis depends on the nucleon to photon ratio, and on the number of neutrino generations $N_\nu$, where each generation is meant to comprise a left-handed neutrino and a right-handed antineutrino.

From the observed abundance of $^4$He, a limit on $\Delta N_\nu \approx 4$ was deduced [75], where $\Delta N_\nu$ denotes the number of additional neutrino generations beyond the third. A later analysis, based on a lower abundance of $^4$He, gave a more restrictive limit: $\Delta N_\nu \approx 1$ [76]. Recently, Ellis et al. [77] argued in favour of a more conservative limit

$$\Delta N_\nu \approx 2.5,$$

in view of larger systematic errors than had been taken into account before, in the experimental input data and in the cosmological model.

3.6.2 Limits from $W$ and $Z$ production

The total width of the W and Z bosons is expected to be given by all possible decays into lepton and quark doublets. Hence if more fermions exist than known at present, they may increase the widths $\Gamma(W \rightarrow \text{all})$ and $\Gamma(Z \rightarrow \text{all})$. If the new leptons and quarks are too massive the respective decay channels are kinematically forbidden. By analogy with the situation in the three known generations, the associated neutrinos will be light compared to $m_W$ and $m_Z$. Hence we expect the width $\Gamma(Z \rightarrow \text{all})$ to be the most sensitive quantity.

In the Electroweak Theory, $\Gamma(Z \rightarrow \text{all}) \approx 2.8 \text{ GeV}$ for $\sin^2 \theta_w = 0.23$, where the bulk of the $O(\alpha)$ electroweak radiative corrections is taken into account by utilizing $\alpha(m_Z^2)$ rather than $\alpha(0)$ in the tree level expressions. Each further neutrino generation adds $\sim 180 \text{ MeV}$ to the $Z$ width.
The width $\Gamma(Z \rightarrow \text{all})$ can be directly measured; however it can be more precisely inferred from the ratio of the cross-sections of W and Z production with subsequent electronic decay:

$$R = \frac{\sigma(pp \rightarrow ZX, Z \rightarrow e^+e^-)}{\sigma(pp \rightarrow WX, W \rightarrow e\nu)} = \left[\frac{\sigma(pp \rightarrow ZX)}{\sigma(pp \rightarrow WX)}\right] \cdot \left[\frac{\Gamma(Z \rightarrow e^+e^-)}{\Gamma(Z \rightarrow \text{all})}\right] \cdot \left[\frac{\Gamma(W \rightarrow e\nu)}{\Gamma(W \rightarrow \text{all})}\right].$$

The ratio $R$ is experimentally known. The ratio $\sigma(pp \rightarrow ZX)/\sigma(pp \rightarrow WX)$ has been calculated from QCD by Altarelli et al. [78]. Fortunately, most of the theoretical uncertainties cancel in the ratio. Inserting the predictions of the Electroweak Theory for $\Gamma(Z \rightarrow e^+e^-)$, $\Gamma(W \rightarrow e\nu)$, and $\Gamma(W \rightarrow \text{all})$, with the assumption $m_t = 40$ GeV, and no additional fermion beyond the known three generations contributing to the W width, $\Gamma(Z \rightarrow \text{all})$ can be determined. The measurements of the UA1 and UA2 groups gave the results [79]:

$$R > 0.077 \ (90\% \ c.l.) \quad \Delta N_e < 7 \quad \text{(UA1)}$$
$$R > 0.094 \ (90\% \ c.l.) \quad \Delta N_e < 2.6 \pm 1.7 \quad \text{(UA2)}.$$

The error in the UA2 upper limit reflects theoretical uncertainties. The laboratory limit on the number of new neutrino generations is consistent with the cosmological bound, and leaves room for a few more fermion generations to be discovered, at most.

3.7 Summary concerning neutrinos

If neutrinos are massless, there remains the $\nu_e$ and, maybe, a few more neutrinos to be seen experimentally, and that's it. If neutrinos have a non-zero mass, a rich pattern of physics questions emerges, susceptible to experimental test. A tremendous amount of experimental activity is going on to answer these questions. Yet, apart from unconfirmed and disputed claims, the experimental situation today is consistent with the notion of three massless neutrino flavours: $\nu_e$, $\nu_\mu$, and $\nu_\tau$.

4. QUARK MIXING AND CP VIOLATION

4.1 The Kobayashi–Maskawa mixing matrix

In the quark sector, it has been an established fact for a long time that the eigenstates participating in the weak interaction do not coincide with the mass eigenstates. The weak interaction eigenstates are connected to the mass eigenstates by an orthonormal mixing matrix $U_{KM}$ known as the Kobayashi–Maskawa (KM) matrix [80]. Its effects show up in weak quark decays, and in neutrino–quark scattering. By convention, the weak interaction eigenstates of the up-type quarks (charge $+2/3$) are identical with the mass eigenstates, whereas the down-type quarks (charge $−1/3$) undergo a rotation,

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} U_{ud} & U_{us} & U_{ub} \\ U_{cd} & U_{cs} & U_{cb} \\ U_{td} & U_{ts} & U_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix},$$

resulting in the following weak hadronic CC:

$$J_\mu = \bar{u} \gamma_\mu d \gamma_5 (1 - \gamma_3) U_{KM} \begin{pmatrix} d \\ s \\ b \end{pmatrix}.$$
\[
U = \begin{pmatrix}
\begin{array}{ccc}
c_1 & -s_1 c_3 & -s_1 s_3 \\
 s_1 c_2 & c_1 c_2 s_3 - s_2 s_3 e^{i\delta} & c_1 c_2 s_3 + s_2 s_3 e^{i\delta} \\
 s_1 s_2 & c_1 s_2 s_3 + c_2 s_3 e^{i\delta} & c_1 s_2 s_3 - c_2 s_3 e^{i\delta}
\end{array}
\end{pmatrix},
\]

where \( s_i = \sin \theta_i \) and \( c_i = \cos \theta_i \) (\( i = 1, 2, 3 \)).

A non-trivial phase angle \( \delta \) implies CP violation. One of today's topical questions is whether the observed CP violation in the \( K^0 - \bar{K}^0 \) system has its origin in quark mixing.

The Kobayashi–Maskawa parametrization is not the only one possible. Alternative parametrizations have been proposed in the literature, for instance that of Maiani [81].

In the absence of theoretical guidance, the elements of the KM matrix must be determined from experiment. Two levels of precision are then possible:

i) Each element is determined from the experimental constraints, and the constraint of orthonormality. The existence of further generations is allowed. This procedure maintains generality, at the expense of precision.

ii) On top of the other constraints, the number of generations is limited to three. This is less general but leads to better precision.

### 4.2 Measurement of the quark mixing parameters

In this discussion, we use the numerical values as quoted by the Particle Data Group [82].

#### 4.2.1 The matrix element \( U_{ud} \)

This matrix element comes from a comparison of nuclear \( \beta \)-decay rates (\( \propto |U_{ud}|^2 \)) with the rate of muon decay. A recent refined evaluation [83] gives the result

\[ |U_{ud}| = 0.9729 \pm 0.0012, \]

which is substantially more precise than earlier values.

#### 4.2.2 The matrix element \( U_{us} \)

This matrix element is determined from the rates of a number of semileptonic hyperon decays, and \( K_{es} \) decays. A recent analysis [84] gives the result

\[ |U_{us}| = 0.221 \pm 0.002. \]

#### 4.2.3 The matrix element \( U_{cd} \)

This matrix element is determined from the difference of the cross-sections of single charm-quark excitation, in CC \( \pi N \) and \( \pi N \) scattering, where \( N \) denotes the nucleon. The experimental signature consists of 'opposite-sign dimuon' events, where the second muon comes from semileptonic decay of the excited charm quark:

\[
\begin{align*}
v_\mu + d &\to \mu^- + c (c \to s_\mu^+ \nu_\mu) : \propto (U + D) |U_{cd}|^2 \\
v_\mu + s &\to \mu^- + c (c \to s_\mu^+ \nu_\mu) : \propto 2S |U_{cs}|^2 \\
\bar{v}_\mu + \bar{d} &\to \mu^+ + \bar{c} (\bar{c} \to s_\mu^- \bar{\nu}_\mu) : \propto (\bar{U} + \bar{D}) |U_{cd}|^2 \text{ (negligible)} \\
\bar{v}_\mu + \bar{s} &\to \mu^+ + \bar{c} (\bar{c} \to s_\mu^- \bar{\nu}_\mu) : \propto 2\bar{S} |U_{cs}|^2.
\end{align*}
\]

The quantities \( U, D, \ldots \) denote the structure functions of \( u, d, \ldots \) quarks in the proton, integrated over the full range of the fractional momentum \( x \) (the 'Bjorken variable') carried by the respective quark.
The result obtained by the CDHS Collaboration [85] is

\[ |U_{cd}| = 0.24 \pm 0.03 \].

4.2.4 The matrix element \( U_{cs} \)

As can be seen in the preceding subsection, the cross-section of opposite-sign dimuons is sensitive to \( |U_{cs}|^2 \). From a comparison of the differential cross-sections \( d\sigma(p\bar{p} \rightarrow \mu^-\mu^+X)/dx \) and \( d\sigma(\bar{p}N \rightarrow \mu^+\mu^-X)/dx \), the term \( 2S|U_{cs}|^2 \) can be isolated. Unfortunately, the wanted \( |U_{cs}|^2 \) is multiplied by the \textit{a priori} unknown structure function integral of the strange quark, \( S \), which is part of the proton sea. The strange sea is expected to be suppressed with respect to the (independently measured) non-strange sea, \( \bar{U} + \bar{D} \). Assuming conservatively \( 2S \lesssim \bar{U} + \bar{D} \), a lower bound on \( |U_{cs}| \) is obtained [85]:

\[ |U_{cs}| > 0.59 \ (90\% \ c.l.) \].

Another bound on \( |U_{cs}| \) can be obtained from the comparison of the decay rate of \( D \rightarrow \bar{K}e^+\nu_e \) with the one expected from the theory of semileptonic decays of pseudoscalar mesons. With a conservative estimate of the \( D \) meson form factor, the lower bound

\[ |U_{cs}| > 0.66 \]

is obtained, consistent with the bound from neutrino scattering.

4.2.5 The ratio of \( |U_{ub}| \) and \( |U_{cb}| \)

The reaction \( e^+e^- \rightarrow \tau(4S) \rightarrow \bar{b}b \) is a rich source of \( B \) mesons. In the semileptonic decay of \( b \)-quarks, the high-energy end of the charged-lepton spectrum is sensitive to whether the decay proceeds via \( b \rightarrow u \) or \( b \rightarrow c \). Hence a fit to the lepton energy spectrum as a sum of contributions from \( b \rightarrow u \) and \( b \rightarrow c \) determines the ratio of \( |U_{ub}| \) and \( |U_{cb}| \). Using only the high-momentum end of the spectrum, where no model assumptions about the \( b \rightarrow c \) transitions enter, Thorndike [86] obtains the limit

\[ |U_{ub}|/|U_{cb}| < 0.19 \].

A less conservative limit, which utilizes also the end-point energies of the \( b \rightarrow c\ell^-\bar{\nu}_\ell \) spectrum (and is dependent on the mass of the final-state hadron system), is [86]

\[ |U_{ub}|/|U_{cb}| < 0.14 \].

These limits are, in principle, susceptible to substantial improvement if the \( b \rightarrow u \) and \( b \rightarrow c \) transitions could be separated by another method: the tagging of secondary decay vertices from \( c \)-decay [87].

4.2.6 The matrix element \( U_{cb} \)

The semileptonic decay rate of the \( b \)-quark will be proportional to \( \alpha|U_{cb}|^2 + \beta|U_{ub}|^2 \), where \( \alpha \) and \( \beta \) are phase-space factors. The semileptonic decay rate is obtained from the \( b \)-lifetime and the semileptonic branching ratio. The measured semileptonic branching ratio of \( b \)-quarks is 12.1 \( \pm \) 0.8\% [86].

The measurement of the lifetime of the \( b \)-quark represents a formidable experimental challenge. The technique adopted at \( e^+e^- \) colliders is to measure the impact parameter of tracks originating from \( b \)-decay. An event sample enriched with \( e^+e^- \rightarrow \bar{b}b \) events is selected either by leptons from semileptonic decays, or by event shape. A finite lifetime is then inferred from a broader impact parameter distribution than obtained from a reference sample depleted of \( e^+e^- \rightarrow \bar{b}b \) events. Thus
the lifetime is measured on a statistical basis. To illustrate the difficulties, Fig. 18 shows the impact parameter distributions from an enriched and depleted sample, as measured by the MARK II Collaboration [88].

The world average of the lifetime of the b-flavoured hadrons is

$$\tau_B = (1.26 \pm 0.16) \times 10^{-12} \text{ s} [86].$$

A nice demonstration of such a finite b-quark lifetime can be seen in Fig. 19, which shows an event found in an emulsion scan by the WA75 Collaboration [89] at CERN. It is interpreted as a $B^- \bar{B}^0$ pair produced in the interaction of a 350 GeV $\pi^-$ with a target nucleus, with both $B^-$ and $\bar{B}^0$ decaying into charmed mesons.

The b-quark lifetime translates into the constraint for $U_{cb}$ [82]:

$$0.027 < |U_{cb}| < 0.053.$$

4.2.7 Numerical results for $U_{ik}$

From the experimental constraints listed above, together with the constraint of orthonormality, the following allowed ranges of the matrix elements $U_{ik}$ are obtained, when allowing more than three quark generations [82, 90]:

$$\begin{pmatrix}
0.9710 & -0.9748 & 0.218 - 0.224 & 0 & -0.01 \\
0.192 & -0.288 & 0.66 - 0.98 & 0.037 - 0.053 & 0 \\
0 & -0.14 & 0 & -0.72 & 0 & -0.999
\end{pmatrix} \quad (90\% \text{ c.l.}).$$
Restricting, in addition, the number of generations to three, leads to the following smaller ranges [82, 90]:

\[
\begin{pmatrix}
0.9742 - 0.9756 & 0.219 - 0.225 & 0 & -0.008 \\
0.219 - 0.225 & 0.973 - 0.975 & 0.037 - 0.053 \\
0.002 - 0.018 & 0.036 - 0.052 & 0.9986 - 0.9993 \\
\end{pmatrix}
\)  
(90% c.l.)

The numerical values of the matrix elements \(U_{ik}\) support the notion that
i) transitions within one generation are favoured,
ii) transitions between 1st and 2nd generation are allowed,
iii) transitions between 2nd and 3rd generation are forbidden,
iv) transitions between 1st and 3rd generation are doubly forbidden.

4.3 Measurement of \(\epsilon' / \epsilon\)

In the frame of the Electroweak Theory, the only possible source of CP violation is the phase angle \(\delta\) in the KM quark mixing matrix. The important question then is: Does this source of CP violation accommodate all experimental facts known about CP violation in the \(K^0 - \bar{K}^0\) system?

In our discussion here, we follow largely the review of Winstein [91].

CP violation was discovered in the Nobel-prize winning experiment of Christenson, Cronin, Fitch, and Turlay [92], carried out at the Brookhaven AGS. When looking at the angular distribution of the \(\pi^+ \pi^-\) system from \(K_L^0\) decays, they discovered a small but significant excess of \(\pi^+ \pi^-\) at zero forward angle, on top of a large background covering a larger angular range. Figure 20 reveals how tempting it must have been to simply disregard the observation as a ‘small systematic error’. Yet the authors not only convinced themselves about the validity of the effect, they also changed the world of particle physics by introducing the CP-violating decay \(K_L^0 \to \pi^+ \pi^-\).

The CP violation in the \(K^0 - \bar{K}^0\) system, the only system where CP violation has been seen so far, is in the usual parametrization caused by the admixture of the other CP eigenstate in the physical states of definite masses and lifetimes, \(K_S^0\) and \(K_L^0\):

\[
\begin{align*}
|K_S^0\rangle &= \frac{1}{\sqrt{2}} \left( |K_1^0\rangle + \epsilon |K_2^0\rangle \right) \\
|K_L^0\rangle &= \frac{1}{\sqrt{2}} \left( |K_1^0\rangle - \epsilon |K_2^0\rangle \right)
\end{align*}
\]

where \(|K_1^0\rangle\) and \(|K_2^0\rangle\) are the CP eigenstates

\[
\begin{align*}
|K_1^0\rangle &= (1/\sqrt{2}) \left( |K_1^0\rangle + |\bar{K}_1^0\rangle \right) \quad (\text{CP} = +1) \\
|K_2^0\rangle &= (1/\sqrt{2}) \left( |K_1^0\rangle - |\bar{K}_1^0\rangle \right) \quad (\text{CP} = -1).
\end{align*}
\]

![Angular distribution of the \(\pi^+ \pi^-\) system originating from \(K_L^0\) decays (from Ref. [92]).](image-url)

Fig. 20
Two CP-violation phenomena have been observed in the $K^0 - \bar{K}^0$ system:

i) The $K^0_L$ prefers decays into $\pi^- e^+ \nu_e$ to decays into $\pi^+ e^- \bar{\nu}_e$. Analogously, the $K^0_L$ prefers decays into $\pi^- \mu^+ \nu_\mu$ to decays into $\pi^+ \mu^- \bar{\nu}_\mu$.

ii) The $K^0_L$ decays also into $\pi^+ \pi^-$ and $\pi^0 \pi^0$, besides the dominant decay into $3\pi$.

The rare $2\pi$ decays (CP = +1) of the $K^0_L$ can proceed

i) via the $\epsilon$-admixture of $|K^0_L\rangle$ which decays into $2\pi$ under normal weak interaction, i.e. respecting CP conservation (this is assumed by the superweak model of CP violation in the $K^0 - \bar{K}^0$ system, suggested by Wolfenstein [93]: in this model, the CP-violating decay $K^0_L \rightarrow 2\pi$ is forbidden, and the source of CP violation in $K^0_L$ is the $\Delta S = 2$ transition $K^0_L \rightarrow \bar{K}^0_L$, caused by a 'superweak' second-order weak interaction);

ii) via a CP violating $\Delta S = 1$ amplitude of $K^0_L$, referred to as 'direct' CP violation.

The underlying processes of (i) and (ii) are depicted in Figs. 21a and b. The additional decay amplitude into the $2\pi$ final state of Fig. 21b leads to an observable effect in the comparison of $K^0_L \rightarrow \pi^0 \pi^0$ and $K^0_L \rightarrow \pi^+ \pi^-$ decays, because of a different mixture of (strong) isospin $I = 0$ and $I = 2$ $2\pi$ final states:

$$\mathcal{A}(K^0_L \rightarrow \pi^0 \pi^-)/\mathcal{A}(K^0_L \rightarrow \pi^+ \pi^-) = \eta_{\pi^-} = \epsilon + \epsilon'$$

$$\mathcal{A}(K^0_L \rightarrow \pi^0 \pi^0)/\mathcal{A}(K^0_L \rightarrow \pi^0 \pi^-) = \eta_{\pi^0} = \epsilon - 2\epsilon'$$

where $\epsilon'$ is a measure of 'direct' CP violation. In the absence of direct CP violation, $\epsilon'$ vanishes thus rendering $\eta_{\pi^-}$ equal to $\eta_{\pi^0}$.

The parameter $\epsilon$ can, in principle, be calculated in terms of the KM matrix elements, on the basis of the 'box diagrams' involving $\Delta S = 2$ transitions, shown in Fig. 21a. A summary of the theoretical situation has been given by Buras [94]. In short, the calculations of $\epsilon$ are not inconsistent with the experimental value

$$|\epsilon| = (2.27 \pm 0.08) \times 10^{-3} [91].$$

However, the theoretical uncertainties are still large, and with time a dilemma in accommodating the experimental value might arise.

Dominantly through the 'penguin' diagram shown in Fig. 21b, $\epsilon'$ is also expected to be non-zero in the Electroweak Theory. The calculation of $\epsilon'$ meets difficulties too [94]. The predictions for the experimentally accessible ratio $\epsilon'/\epsilon$ depend on the top-quark mass, on the KM matrix elements, and on the parameter $B$ which relates the calculable box diagrams of Fig. 20a to experimentally measured

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Fig. 21 a) $\Delta S = 2K^0 - \bar{K}^0$ transitions, and b) $\Delta S = 1$ decay of $K^0 \rightarrow 2\pi$, named for apparent reasons a 'penguin diagram'.

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transition amplitudes. The calculations indicate a positive value of $\epsilon'/\epsilon$ of a few parts in 1000, strongly dependent on the top-quark mass.

Experimentally, a large effort is under way to measure $\epsilon'/\epsilon$ with a precision of the order of $10^{-3}$. The problems associated with obtaining such a level of precision have been described by Wahl [95]. The experiments measure the ratio

$$|\eta_{00}|^2/|\eta_{+ -}|^2 \approx 1 - 6 \text{ Re } (\epsilon'/\epsilon),$$

profiting from the increase in sensitivity by a factor of 6. The two measurements are

$$\text{Re } (\epsilon'/\epsilon) = 0.0017 \pm 0.0082 \quad [96]$$

$$\text{Re } (\epsilon'/\epsilon) = -0.0046 \pm 0.0058 \quad [97].$$

Both the experimental and theoretical precisions are too poor, as yet, to allow significant conclusions on the existence of direct CP violation. More precise experiments are under way: experiments NA31 and PS195 at CERN, and experiment E731 at FNAL. They aim at an overall experimental error of $(1-2) \times 10^{-3}$ in $\text{Re } (\epsilon'/\epsilon)$. Experiments at proposed 'kaon factories' such as LAMPF II, TRIUMF II, aim at decreasing the error by another factor of 2.

4.4 Quark–antiquark transitions of heavy quarks

In this section, we disregard entirely the small effects of potential CP violation, but ask the question whether effects such as $D^0-\bar{D}^0$ and $B^0-\bar{B}^0$ mixing might be observable, just like $K^0-\bar{K}^0$ mixings.

Taking the $K^0-\bar{K}^0$ system as reference, and forgetting the small difference between $|K_0^0|$ and $|K_0^0|$, and between $|K_0^0|$ and $|K_0^0|$, we have a mass difference $\Delta m$ between $K_1^0$ and $K_2^0$,

$$\Delta m = m(K_1^0) - m(K_0^0) = 3.52 \times 10^{-15} \text{ GeV} = 0.535 \times 10^{10} \text{ s}^{-1},$$

to be compared to the decay width difference, which we approximate by the decay width of the $K_1^0$,

$$\Gamma(K_1^0) = 1.2 \times 10^{10} \text{ s}^{-1}.$$ 

The mass difference $\Delta m$ is caused by the second-order $\Delta S = 2$ weak interaction shown in Fig. 21a, while the decay is caused by a normal $\Delta S = 1$ weak interaction. For transitions $K^0 \leftrightarrow \bar{K}^0$ to occur the lifetime has to be large enough compared to the typical time needed for $K^0 \leftrightarrow \bar{K}^0$ transitions, which is characterized by $\Delta m$:

$$\Delta m/\Gamma \approx 1.$$

In the $K^0-\bar{K}^0$ system we notice a good match: $\Delta m/\Gamma = 0.48$.

We now proceed, following Ali and Jarlskog [98] to estimate $\Delta m/\Gamma$ for massive quark flavours.

To this end, the approximate parametrization of the KM matrix in terms of the Cabibbo angle $\theta_C$, $\lambda = \sin \theta_C \sim 0.22$, inspired by Wolfenstein [99], proves useful:

$$U_{KM} = \begin{pmatrix}
1 & \lambda & \lambda^3 \\
\lambda & 1 & \lambda^2 \\
\lambda^2 & \lambda^3 & 1
\end{pmatrix}$$

Fig. 22 Box diagrams for: a) $K^0-\bar{K}^0$, b) $D^0-\bar{D}^0$, c) $B^0-\bar{B}^0$, and d) $B^0_s-\bar{B}^0_s$ transitions.
From the box diagrams in Fig. 22 we can then read off the following estimates for \( \Delta m \) and \( \Gamma \):

**Case of \( K^0 \):**

\[
\Delta m \propto |U_{ud} U_{us} + U_{cd} U_{cs} + U_{td} U_{ts}|^2 \sim \lambda^2 \\
\Gamma(s \to u) \propto |U_{us}|^2 \sim \lambda^2 \\
\Rightarrow \frac{\Delta m}{\Gamma} = O(1) \quad \text{Good!}
\]

**Case of \( D^0 \):**

\[
\Delta m \propto |U_{ud} U_{ub} + U_{ct} U_{us} + U_{cb} U_{ub}|^2 \sim \lambda^2 \\
\Gamma(c \to s) \propto |U_{us}|^2 \sim 1 \\
\Rightarrow \frac{\Delta m}{\Gamma} = O(\lambda^2) \quad \text{Bad!}
\]

**Case of \( B^0_d \):**

\[
\Delta m \propto |U_{ub} U_{ud} + U_{cb} U_{cd} + U_{tb} U_{td}|^2 \sim \lambda^6 \\
\Gamma(b \to c) \propto |U_{ub}|^2 \sim \lambda^4 \\
\Rightarrow \frac{\Delta m}{\Gamma} = O(\lambda^2) \quad \text{Bad!}
\]

**Case of \( B^0_s \):**

\[
\Delta m \propto |U_{ub} U_{us} + U_{cb} U_{cs} + U_{tb} U_{ts}|^2 \sim \lambda^4 \\
\Gamma(b \to c) \propto |U_{ub}|^2 \sim \lambda^4 \\
\Rightarrow \frac{\Delta m}{\Gamma} = O(1) \quad \text{Good!}
\]

Hence the prospects for significant mixing in heavy-quark systems up to the b-quark are only good in the case of \( B^0_d-\bar{B}^0_s \). Clearly, the short lifetime of the b-quarks makes the observation of, for example, regeneration effects such as in the \( K^0-\bar{K}^0 \) system impossible. The standard experimental signature of mixing effects is "wrong-sign" leptons, for example in the following sequence after the production of a \( B^0_d-\bar{B}^0_s \) pair:

\[
\begin{align*}
B^0_d &\to X\mu^- \bar{\nu}_\mu \\
\bar{B}^0_s &\to B^0_d \to X\mu^- \bar{\nu}_\mu \text{ (rather than } \bar{B}^0_s \to X\mu^+ \nu_\mu) 
\end{align*}
\]

Quark-antiquark mixing leads to 'like-sign' lepton pairs. The first experimental indication of a ratio of like-sign muon pairs to opposite-sign muon pairs larger than expected (interpreted as \( B^0_d-\bar{B}^0_s \) mixing in \( pp \) collisions) has recently been reported by the UA1 Collaboration [100].

The chances of seeing mixing effects of \( B^0_d-\bar{B}^0_s \) in \( e^+e^- \) annihilations, even with large statistics collected at the Z pole, have been estimated to be small [98].

5. **THE STRUCTURE OF THE WEAK CHARGED CURRENT**

In the framework of the Electroweak Theory, the weak CC interaction has \( V-A \) structure. Originally this was inferred from the study of nuclear \( \beta \)-decay. It took about 25 years, from the first formulation of the weak CC interaction by Fermi, to establish its Lorentz structure. This period is not the most brilliant chapter of experimental particle physics: the experimenters had a hard time to make their choice between the options S, V, T, A, and P, or linear combinations of these. Since then, the \( V-A \) structure has been corroborated in many low-energy tests involving nuclear \( \beta \)-decay, muon decay, and pion decay.

Although the left-handed character of the weak CC is well established the search for right-handed currents continues: in high-precision measurements of muon decay, or in tests involving large momentum transfer.
5.1 Precision measurement of $\xi$ in muon decay

A precise measurement of the high-energy end of the spectrum of $e^+$ emitted in polarized $\mu^+$ decay has been carried out by an LBL group at TRIUMF [101]. The rate of $e^+$ emission opposite to the muon spin was measured as a function of the $e^+$ momentum. As shown in Fig. 23 this configuration is particularly sensitive to a contribution of $V+A$ in muon decay since the population from $V-A$ tends to zero. The direction of the spin of stopped muons at $\theta = \pi$ with respect to the $e^+$ momentum was assured by its origin from $\pi^+ \rightarrow \mu^+ \nu$ decay (producing left-handed $\mu^+$), and a strong magnetic field preventing depolarization. The same magnetic field could also be used to measure the $(V-A$ allowed) $e^+$ spectrum at $\theta = 0$, by a precession of the muon spin by an angle $\pi$. Thus the spectrum which one would measure with $V+A$ muon decay was measured with the same apparatus, thereby reducing the systematic errors of the measurement. Figure 24 shows schematically the $e^+$ spectra obtained under $\theta = 0$ and under $\theta = \pi$.

The measurement gave an improvement of the error on the asymmetry parameter $\xi$ of muon decay by an order of magnitude. At $x = p_e/p_{\mu}(\text{max}) = 1$ and $\theta = \pi$, the differential rate of $e^+$ becomes

$$d^2\Gamma/dx d(cos \theta) \propto 1 - \xi P_\mu,$$

where $P_\mu$ is the $\mu^+$ polarization. For $P_\mu = 1$ and $\xi = 1$, as predicted by $V-A$, the rate vanishes, providing a sensitive measure of $\xi$.

Expressed in terms of a limit on the $V+A$ amplitude in muon decay, the result is

$$G(V+A)/G(V-A) < 0.029 \ (90\% \ c.l.).$$

---

**Fig. 23** Orientation of the secondary particles in polarized $\mu^+$ decay, near the end point of the positron spectrum; $\theta$ denotes the angle between the muon spin and the $e^+$ momentum; a) $e^+$ emission opposite to muon spin, b) $e^+$ emission along the muon spin.

---

**Fig. 24** $e^+$ momentum spectra at $\theta = 0 \ (V+A)$ and at $\theta = \pi \ (V-A)$, where $\theta$ denotes the angle between $\mu^+$ spin and $e^+$ momentum.
The result can also be interpreted as a limit on the mass $m_R$ of a 'right-handed' $W_R$, that is the intermediate boson mediating a right-handed NC interaction. The existence of such bosons, presumably much heavier than the standard ‘left-handed’ $W_L$, is required in left-right symmetric models with a gauge group $SU(2)_L \otimes SU(2)_R \otimes U(1)$ [102]. More generally, the physical bosons $W_L$ and $W_R$, mediating $V-A$ and $V+A$ interactions, might be rotated by an angle $\theta$ with respect to the mass eigenstates $W_1$ and $W_2$:

$$W_L = \cos \theta \, W_1 - \sin \theta \, W_2$$
$$W_R = \sin \theta \, W_1 + \cos \theta \, W_2.$$ 

$V+A$ muon decay would then proceed with an amplitude

$$\mathcal{G}(V+A) \propto \left( \sin^2 \theta / m_1^2 \right) + \left( \cos^2 \theta / m_2^2 \right),$$

as compared to the $V-A$ amplitude

$$\mathcal{G}(V-A) \propto \left( \cos^2 \theta / m_1^2 \right) + \left( \sin^2 \theta / m_2^2 \right).$$

The above limit on $\mathcal{G}(V+A)/\mathcal{G}(V-A)$ then translates into an allowed domain of $m_1^2/m_2^2$ against the mixing angle. The domain resulting from this experiment is shown in Fig. 25. Independently of the mixing angle $\theta$, a mass $< 400$ GeV of the heavy intermediate boson mass eigenstate $W_2$ is excluded when taking $80$ GeV for the mass of the lighter mass eigenstate $W_1$.

This mass limit $m_2 > 400$ GeV is the more impressive, since the inherently bad sensitivity (the decay rate varies with the 4th power of the intermediate boson mass!) has been overcome by the precision of a brilliantly designed and conducted experiment. Notice, however, that the quoted limit would be meaningless if the possibly heavy right-handed neutrino associated with the right-handed leptonic current had a mass $m_{\nu_R} \approx 10$ MeV!
Another test of the $V-A$ structure of the currents involved in muon decay, albeit less precise, has been carried out by the CHARM Collaboration in their measurement of the cross-section of inverse muon decay, $\nu_\mu + e^- \rightarrow \mu^- + \nu_e$, in a high-energy neutrino beam [103]. Agreement with the $V-A$ prediction was found.

5.2 Search for $V+A$ currents in neutrino–quark scattering

Today, it is an established fact that in scattering reactions with large enough momentum transfer, the nucleon can be regarded as consisting of free point-like quarks, three valence quarks, and a ‘sea’ of quark–antiquark pairs. This Quark-Parton Model (QPM) of the nucleon works amazingly well, to better than 10\% accuracy, provided one does not study the $Q^2$-dependence of the structure functions where the deviation from the QPM is most apparent. In this and the subsequent section, we average out the (small) $Q^2$-dependence by considering data integrated over $Q^2$, and argue in the frame of the QPM. Neutrino–nucleon ($\nu N$; even more correctly: neutrino–nucleus) scattering data are interpreted as the scattering of neutrinos on free quarks: $\nu_\mu + q \rightarrow \mu^- + q'$. The interest in studying this reaction for the possible admixture of $V+A$ currents lies in its large $Q^2$: $(Q^2) \sim 30\text{ GeV}^2$, much higher than studied in particle decays.

Because of the $V-A$ helicity configurations in neutrino–quark scattering in the centre-of-mass system (see Fig. 26), the $y$-distribution for $q \bar{q}$ or $\bar{q}q$ scattering in the laboratory system is flat, and is $(1-y)^2$ in the case of $\nu \bar{q}$ or $\bar{\nu}q$ scattering. The variable $y = E_{\text{lab}}^{\nu}/E_{\text{lab}}^{\nu}$ denotes the fractional energy transfer of the incoming neutrino to the quark at rest ($0 \leq y \leq 1$).

Figure 27 shows the $y$-distribution of $\nu N$ and $\bar{\nu} N$ scattering as measured with a high-energy neutrino beam [104]. The data are consistent with the notion of $\nu$ and $\bar{\nu}$ scattering on quarks, with a small admixture of antiquarks from the sea of the nucleon. This proves, however, nothing about the $V-A$ nature of neutrino–quark scattering: after all, the validity of $V-A$ has been taken for granted when exploring the quark–parton structure of the nucleon, in particular for disentangling its quark and antiquark content, which is not possible with electron or muon scattering.

![Fig. 26](image-url) **Fig. 26** Fermion helicity configurations for $V-A$ neutrino–fermion scattering.

![Fig. 27](image-url) **Fig. 27** $y$-distribution of $\nu N$ and $\bar{\nu} N$ scattering.
How does one look then for a $V + A$ admixture in the quark CC without interpreting $V + A \equiv q$ scattering as $V - A \equiv \bar{q}$ scattering, i.e. as an increase of the antiquark sea? With the help of the second independent variable present in deep inelastic $\pi N$ scattering, $x$, the fractional nucleon momentum carried by the quark struck by the neutrino ($0 \leq x \leq 1$). The antiquark sea is confined to small values of $x$ ($x \approx 0.4$), whereas the (valence) quark distribution extends to larger $x$. Hence, confining the search to $x > 0.5$ where only quarks are present, a $(1 - y)^2$ contribution on top of the flat $y$-distribution of $q \bar{q}$ scattering would signify a $V + A$ contribution in either the quark or lepton CC. Actually, the sensitivity of the search is much increased when utilizing antineutrinos: the $V - A$ cross-section of $\bar{q}$ scattering vanishes, at large $x$, for $y \to 1$.

A search for a non-zero cross-section of $\bar{q}N$ scattering in the region $y > 0.66$, $x > 0.5$, has been made by the CDHS Collaboration [105]. Unlike the case of muon decay discussed in the preceding section, this search is sensitive only to a CC product of the type $J_{LH}^\pi \times J_{RH}^\nu$; the product $J_{RH}^\pi \times J_{LH}^\nu$ does not apply since the incoming neutrino beam is left-handed. The interaction is mediated by a boson which is born as $W_L$ but absorbed as $W_R$. With the same formalism as in the preceding section, the amplitude for such a $V + A$ admixture is

$$\alpha(V + A) \propto (\cos \theta \sin \theta / m_1^2) - (\cos \theta \sin \theta / m_2^2) .$$

Hence, the experiment is chiefly sensitive to the mixing angle $\theta$, since the amplitude vanishes for $\theta \to 0$.

The amplitude ratio $q = \alpha(V + A) / \alpha(V - A)$ has been measured:

$$|q^2| < 0.009 \ (90\% \ c.l.) .$$

This limit translates into the allowed domain of $m_1^2/m_2^2$ versus the mixing angle $\theta$, shown in Fig. 25. The limit is considerably less restrictive than that obtained from muon decay. However, it applies to large $Q^2$, $(Q^2) = 33 \text{ GeV}^2$, and is not limited to a small mass of the right-handed neutrino associated with the right-handed quark current.

5.3 Muon polarization in neutrino–quark scattering

The $V - A$ formulation of neutrino–quark scattering predicts that all involved fermions are left-handed, and all antifermions right-handed. Hence the outgoing high-energy $\mu^+$ in the deep inelastic $\bar{v}_e N$ CC scattering process

$$\bar{v}_e + N \to \mu^+ + X$$

must have $P = +1$.

Actually, any linear combination of $V$ and $A$ preserves the fermion helicity in the scattering process, whereas interactions of the $S$, $P$, or $T$ type involve a spin flip. Since the incoming $\bar{v}_e$, coming from $\pi^-$ and $K^-$ decay, are right-handed, a measurement of $P = +1$ of the outgoing $\mu^+$ proves only that a linear combination of $V$ and $A$ is at work, and not $S$, $P$, or $T$. Together with the result discussed in the preceding section, $V - A$ emerges as the only viable Lorentz structure of the CC. The $y$-distribution alone would not allow this conclusion since the $V - A$ $y$-distribution could be mimicked by a suitable combination of $S$, $P$, and $T$ terms [106].

A beautiful experiment to measure the polarization of outgoing $\mu^+$ in high-energy $\bar{v}_e$–nucleon reactions was carried out by the CHARM Collaboration [107]. Events were selected where the vertex was located in the CDHS detector, serving as instrumented target for $\bar{v}_e$ interactions, and the outgoing $\mu^+$ was stopped in the CHARM detector, serving as polarimeter. The spin of the stopped $\mu^+$ precessed in the horizontal plane under the influence of a weak vertical magnetic field. In the decay of a $\mu^+$, the $e^+$ is preferentially emitted along the direction of the muon spin. Hence the forward–backward asymmetry in the $e^+$ emission as a function of time measures (i) the degree of
muon polarization via the amplitude of the modulation, and (ii) the sign of the polarization via the phase of the modulation. The data are shown in Fig. 28.

The result was $P = +0.82 \pm 0.14$, in agreement with the expectation of $+1$, for $(Q^2) = 4 \text{ GeV}^2$. This corresponds to an upper limit $\sigma(S,P,T)/\sigma(V,A) < 0.20$ (95% c.l.).

6. WEAK NEUTRAL CURRENT PROCESSES

6.1 Overview

The Electroweak Theory predicts a wealth of weak NC processes, shown schematically in Fig. 29. The structure of all these processes is unambiguously predicted. Experimental work which has been done, or is within reach, is marked with a large star (precise results), a small cross (low-precision results), or a black circle (accessible in the near future).

The entries in Fig. 29 refer to the following reactions:

i) Precise results: $\bar{u}D$ scattering ($[eu]$ and $[ed]$), and $\nu_eN$ scattering ($N =$ nucleon; $[\nu_eu]$ and $[\nu_ed]$).

ii) Low-precision results: $\nu_ee$ scattering $[\nu_ee]$, $\nu_eN$ scattering ($[\nu_eu]$ and $[\nu_ed]$), $\nu_eC$ scattering $[\nu_eC]$, $e^+e^- \rightarrow \mu^+\mu^-$ $[\nu_e\mu]$, $e^+e^- \rightarrow c\bar{c} [ec]$, $e^+e^- \rightarrow \tau^+\tau^-$ $[\nu_e\tau]$, $e^+e^- \rightarrow b\bar{b} [eb]$, $\mu C$ scattering ($C =$ carbon; $[\nu_\mu u]$ and $[\nu_\mu d]$), trident production in $\nu_eN$ scattering $[\nu_e\mu_\mu]$, $J/\psi$ production in $\nu_eN$ scattering $[\nu_eC]$, and $\nu_eN$ scattering $[\nu_eN]$.

iii) Possible results: Bhabha scattering $[ee]$, $e^+e^- \rightarrow s\bar{s}$ $[es]$, $e^+e^- \rightarrow t\bar{t}$ $[et]$, and parity violation in nuclear transitions $([uu]$, $[ud]$, and $[dd])$.

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![Fig. 29](image-url)

**Fig. 29** Overview of NC reactions.
It is not the scope of these lectures to review in detail all the listed reactions. In the following sections, we rather focus on reactions with significant results, or on reactions which have the potential of high precision.

The NC consists, by construction, of V and A pieces only:

$$\Gamma^{NC}_\mu = \Gamma_{\gamma}(g_{\nu\mu} - g_{A\mu}) \gamma^\mu,$$

where the vector and axial-vector coupling constants are listed in Table 2 (Section 1.1). Unlike the CC case, where the Lorentz structure is always $V - A$, the NC Lorentz structure is a mixture of $V - A$ and $V + A$, the relative amount of which depends on the fermion involved. The absence of S, P, or T terms in the NC Lorentz structure is subject to experimental test.

The NC is constructed as a mixture of the neutral component of the weak CC, and of the electromagnetic current. Hence it consists dominantly of an isovector part, plus an admixture of an isoscalar piece from the electromagnetic current. The (weak) isospin structure of the NC is also subject to experimental test.

Furthermore, the NC is constructed as a flavour-conserving current. Flavour-changing NCs are absent in the Electroweak Theory. This absence is to be verified experimentally.

The programme of testing the predictions of the Electroweak Theory in NC reactions looks as follows:

i) Measure the NC coupling constants of the fundamental fermions in as many reactions as possible. Check their independence of $Q^2$. Check their generation universality. Check their consistency with V and A terms only, and with $T = 1$ and $T = 0$ pieces only. Verify the absence of flavour-changing NC reactions.

ii) Verify that all NC processes are correctly described, after electroweak radiative corrections, by one single value of $\sin^2 \theta_w$.

6.2 Lepton-lepton scattering

6.2.1 Neutrino-electron scattering

The scattering of neutrinos off electrons is of particular interest, since the process involves an elastic collision of point-like leptons. No uncertainties due to the internal structure of extended hadrons arise. However, the price to pay for this simplicity is high: the cross-section is lower than the $\nu N$ cross-section by a factor of the order of $m_N/m_e = 2000$, rendering both the accumulation of large statistics and the separation from background a formidable experimental challenge.

Neutrino-electron scattering comprises four experimentally accessible processes:

i) $\nu_e + e^- \rightarrow \nu_e + e^-$,

ii) $\bar{\nu}_e + e^- \rightarrow \bar{\nu}_e + e^-,$

iii) $\nu_\mu + e^- \rightarrow \nu_\mu + e^-,$

iv) $\bar{\nu}_\mu + e^- \rightarrow \bar{\nu}_\mu + e^-.$

Of these, (i) and (ii) proceed via both NC and CC reactions, whereas (iii) and (iv) proceed via a NC reaction only. The cross-sections of these four processes are shown in Fig. 30. Notice the strong deviation of the $\nu_e e^-$ and $\bar{\nu}_e e^-$ cross-sections from the $V - A$ ("CC only") prediction, arising from the interference of the CC and NC amplitudes. Notice further the equality of the $\nu_\mu e^-$ and $\bar{\nu}_\mu e^-$ cross-sections for $\sin^2 \theta_w = 0.25$, which is close to the value realized by Nature.

The process of $\nu_e e^-$ scattering has been observed recently at LAMPF [108]. The $\nu_e$ flux originates from the decay of $\mu^+$, which themselves are decay products of stopped $\pi^+$. A sample of $51 \pm 17$ events has been found, corresponding to a cross-section of

$$\sigma(\nu_e e^-) = (8.9 \pm 3.5) \times 10^{-42} E_\nu [\text{GeV}] \text{ cm}^2,$$

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consistent with the expectation for $\sin^2 \theta_w = 0.225$. This cross-section requires a destructive interference between CC and NC amplitudes (see Fig. 30).

The process $\bar{\nu}_e + e^- \rightarrow \bar{\nu}_e + e^-$ has been seen at the Savannah River Nuclear Reactor Plant [109]. The experimenters found agreement of their observations with the V–A cross-section. Taking into account the theoretically requested interference between CC and NC amplitudes, this limits, within a large error, the electroweak mixing parameter to values around $\sin^2 \theta_w \sim 0.30$ (see Fig. 30).

Scattering of $\nu_e$ and $\bar{\nu}_e$ off electrons has been studied extensively at high-energy accelerators. From the kinematical constraint for the energy and angle for the final-state electron,

$$E_e \theta_e^2 < 2m_e,$$

the experimental signature is a high-energy electron emitted under very small angle in the forward direction, together with the absence of hadronic activity. Although bubble chambers played an important role at the beginning, the field is now dominated by electronic detectors. The most significant results have so far been obtained by the CHARM Collaboration [110], and an experiment carried out at BNL [111]. The angular distributions of $\nu_e e$ and $\bar{\nu}_e e$ candidate events in both experiments are shown in Fig. 31. It is evident from the figure that the separation of signal and background, in particular in the case of the CHARM experiment, is a somewhat difficult task.

The present world averages of the $\nu_e e$ and $\bar{\nu}_e e$ scattering cross-sections are [112]:

$$\sigma(\nu_e e) = (1.55 \pm 0.20) \times 10^{-42} E_e [\text{GeV}] \text{ cm}^2,$$

$$\sigma(\bar{\nu}_e e) = (1.26 \pm 0.21) \times 10^{-42} E_e [\text{GeV}] \text{ cm}^2.$$
Values of $\sin^2 \theta_w$ may be determined from these cross-sections. However, as was first realized by the CHARM Collaboration, a better quantity to use is the cross-section ratio $\sigma(\nu_e e)/\sigma(\bar{\nu}_e e)$ which is strongly dependent on $\sin^2 \theta_w$, in the region of interest (see Fig. 32). Based on the expectation that many systematic errors would cancel in the cross-section ratio, a systematically more precise measurement of $\sin^2 \theta_w$ may be achieved. Using this method, the present world average is [112]:

$$\sin^2 \theta_w = 0.212 \pm 0.021 \text{ (stat)} \pm 0.009 \text{ (syst)}.$$

No higher order electroweak radiative correction has been applied to the above cross-sections. However, we expect these corrections to cancel largely in the ratio of cross-sections so that $\sin^2 \theta_w$ is virtually unaffected. This value of $\sin^2 \theta_w$, albeit less precise, is well compatible with the measurements in $\nu N$ scattering.

A new detector, built by the CHARM II Collaboration, is currently starting operation at CERN. The design goal of the experiment is to reduce the overall error on $\sin^2 \theta_w$ from $\nu_e e$ and $\bar{\nu}_e e$ scattering to 0.005.
6.2.2 $e^+e^-$ annihilation into charged leptons

With the advent of the $e^+e^-$ storage rings PETRA and PEP, it became possible to study the interference between single-photon and $Z$ exchange in $e^+e^-$ annihilation. In the Born approximation, taking the QED cross-section (the 'point-like' cross-section)

$$\sigma_{\mu\mu} = \sigma(e^+e^- \rightarrow \mu^+\mu^-) = 4\pi\alpha^2/(3s) = 86 \text{ nb}/(s [\text{GeV}^2])$$

as a reference, the ratio of the cross-section for $e^+e^-$ annihilation into $f\bar{f}$ (where $f$ denotes a fermion other than the electron) to $\sigma_{\mu\mu}$ is

$$R_{\mu} = \sigma(e^+e^- \rightarrow f\bar{f})/\sigma_{\mu\mu} = Q_f^2 - 2X_1 Q_f g_{\gamma\nu\bar{e}e\nu'} + \chi_3(g_{e\nu}^2 + g_{e\bar{e}}^2)(g_{\nu\nu'}^2 + g_{\nu'\nu}^2)$$

with

$$\chi_1 = g/s/[m_f^2 - 1 + \Gamma_f^2/(s - m_f^2)]$$
$$\chi_2 = g^2/s^2/[l(s/m_f^2 - 1)^2 + (\Gamma_f/m_f^2)^2]$$
$$g = G_F/(8\pi\alpha\sqrt{2}) = 4.5 \times 10^{-5} \text{ GeV}^{-2}.$$ 

At $s \ll m_f^2$, the deviation from the QED cross-section ratio is dominated by the interference term $-2X_1 Q_f g_{\gamma\nu\bar{e}e\nu'}$.

An inspection of the NC coupling constants (Table 2 in Section 1.1) shows that the interference term always has negative sign. In the case of annihilation into $\mu^+\mu^-$ or $\tau^+\tau^-$, the numerical value will be very small since the vector coupling constant of the charged leptons is close to zero.

Another quantity which is easy to measure is the forward–backward asymmetry of the final-state fermions. Denoting by $\theta$ the angle between the incoming $e^-$ direction and the direction of the outgoing fermion, the differential cross-section is

$$d\sigma(e^+e^- \rightarrow f\bar{f})/d\cos\theta = (\pi\alpha^2/2s)[R_{\mu}(1 + \cos^2\theta) + F \cos\theta]$$

$$F = -4X_1 Q_f g_{\gamma\nu\bar{e}e\nu'} + 8X_2 g_{\nu\nu'} g_{\nu\nu''} g_{\alpha\beta} g_{\alpha\beta'}. $$

The forward–backward asymmetry $A_{FB}$ is then

$$A_{FB} = 3F/(8R_{\mu}).$$

At $s \ll m_f^2$, $A_{FB}$ reduces to

$$A_{FB} = -3X_1 g_{\alpha\beta} g_{\alpha\beta'}/(2Q_f),$$

which always has a negative sign. Since $|g_{\alpha\beta} g_{\alpha\beta'}|$ is expected to be 1, $|A_{FB}|$ is much larger than the interference term in $R_{\mu}$, and hence more easily accessible to experimental test.

Figure 33 shows $R_{\mu}$ and $A_{FB}$ as a function of $\sqrt{s}$ for $e^+e^- \rightarrow \mu^+\mu^-$. For comparison, the effect of O($\alpha$) electroweak radiative corrections [113] is also shown. Their dominant effect arises from a reduction of the centre-of-mass energy because of initial-state bremsstrahlung, and an increase of $m_Z$ by about 3% compared to the Born approximation prediction with the same value of $\sin^2\theta_W$.

The forward–backward asymmetry of the processes $e^+e^- \rightarrow \mu^+\mu^-$ and $e^+e^- \rightarrow \tau^+\tau^-$ has been measured by many groups at PETRA and PEP. The most significant asymmetry is expected from PETRA since its maximum centre-of-mass energy ($\sqrt{s} \approx 46$ GeV) was substantially higher than at PEP ($\sqrt{s} = 29$ GeV), and the asymmetry increases with the square of the centre-of-mass energy. A compilation [114] of angular distributions of $e^+e^- \rightarrow \mu^+\mu^-$ events and the results for the asymmetry are shown in Figs. 34 and 35. In first approximation, the measured points pretty much follow the prediction of the Electroweak Theory. However, the data points have a tendency to fall below the
expectation for $m_Z = 93$ GeV, i.e. they favour a smaller $Z$ mass. The average value of the axial-vector coupling constant is, assuming $|g_{Ae}| = 1$,

$$|g_{Ae}| = 1.13 \pm 0.07.$$  

This is the most precise determination of a single NC coupling constant. It differs by 2σ from the assignment of the Electroweak Theory.

The measurements of the asymmetry for $e^+e^- \rightarrow \tau^+\tau^-$ are also shown in Fig. 35. In contrast to the muon case, here the data points have rather a tendency to lie above the expectation for $m_Z = 93$ GeV. Altogether, involving $\mu^-\tau$ universality and combining $\mu^-\mu^-$ and $\tau^+\tau^-$ data, the agreement with the Electroweak Theory is good.

The asymmetry at PETRA/PEP energies is essentially given by the product of the axial-vector coupling constants of the participating fermions, which are independent of $\sin^2 \theta_W$. How do the PETRA groups obtain a measurement of $\sin^2 \theta_W$ from the asymmetry? The dependence comes from the factor $\chi_1$ in the expression for the asymmetry. Expressing $g$ in terms of $m_Z$ and $\sin^2 \theta_W$, we obtain
\[ \chi_1 = \left[\frac{1}{16 \sin^2 \theta_w \cos^2 \theta_w}\right] \left[\frac{s}{s-m^2} \right], \]

which is dependent on \( \sin^2 \theta_w \). Determined this way, the strength of the weak NC interaction is measured by \( \sin^2 \theta_w \), whereas all other determinations of \( \sin^2 \theta_w \) assume the same strength for NC and CC, and rather determine \( \sin^2 \theta_w \) from the fermion coupling constants. Of course, both determinations of \( \sin^2 \theta_w \) coincide in the framework of the Electroweak Theory.

The average value of \( \sin^2 \theta_w \) from \( e^+e^- \) annihilation into leptons is [114]

\[ \sin^2 \theta_w = 0.210 \pm 0.019 \text{ (exp)} \pm 0.013 \text{ (from } m_Z), \]

entirely consistent with other determinations. The overall experimental error and the error due to the uncertainty on \( m_Z \) are given in turn. The specific interest in this determination of \( \sin^2 \theta_w \) lies in the large \( Q^2 \), and the purely leptonic character of the reactions.

Two comments should be added. Firstly, all given formulae refer to unpolarized colliding beams. Secondly, the data shown are all corrected for \( O(\alpha) \) QED effects [but not for \( O(\alpha) \) weak effects], that is hard photon bremsstrahlung, soft photon bremsstrahlung, and one-loop virtual contributions. The \( O(\alpha) \) QED correction of the measured asymmetry amounts typically to \( \Delta A = 0.01 \).

The data from the forward-backward asymmetry in \( e^+e^- \) annihilation into leptons, together with the cross-sections of \( \nu e \) scattering, can be analysed together in order to obtain best-fit values of the vector and axial-vector coupling constants of the electron. The results are shown in Fig. 36 as

Fig. 36 Allowed domains in the \( g_{Ae}-g_{Ve} \) plane, from \( \nu e \) scattering, from \( e^+e^- \rightarrow \mu^+\mu^- \) and \( e^+e^- \rightarrow \tau^+\tau^- \), from \( \bar{e}D \) scattering, and from \( \mu C \) scattering; \( e-\mu-\tau \) universality is assumed.
allowed domains in the $g_{Ae} - g_{Ve}$ plane. The four regions allowed from $\nu_e e$ and $\bar{\nu}_e e$ scattering are reduced to two domains when taken in conjunction with data from $e^+e^-$ annihilation into leptons. The remaining ambiguity is removed by the $\nu_e e$ scattering result from LAMPF, in favour of the solution predicted by the Electroweak Theory: $g_{Ae} = -1$, $g_{Ve} = 0$.

6.3 Neutrino–quark scattering

6.3.1 Scattering on isoscalar nuclei

Measuring the ratio of NC to CC scattering of neutrinos off heavy nuclei (nearly isoscalar targets) is still the most precise way of determining $\sin^2 \theta_w$. The present precision will not be surpassed before the first data on the Z pole at SLC and/or LEP become available.

In order to extract $\sin^2 \theta_w$ recourse was made in the past to the QPM description of the nucleon structure. Restricting ourselves to a world with u and d quarks only, we obtain by inserting the NC coupling constants the ratios of NC to CC cross-sections:

$$R_v = (NC/CC)v = 1/2 - \sin^2 \theta_w + (5/9) \sin^4 \theta_w (1 + r)$$

$$R_p = (NC/CC)p = 1/2 - \sin^2 \theta_w + (5/9) \sin^4 \theta_w (1 + 1/r)$$

$$r = CC_p/CC_v$$

These formulae, which are readily derived within the QPM, have recently also been derived by Llewellyn Smith [115] in a more general way, relying on isospin invariance only, thus paving the way to a determination of $\sin^2 \theta_w$ in a largely model-independent fashion. The QPM is then needed only for minor corrections such as the scattering off the strange sea, and the onset of charm production.

As a matter of fact, the numerical results for $\sin^2 \theta_w$ are nearly identical when extracted from $R_v$ and $R_p$ in either the QPM approximation, or else with the Llewellyn Smith formula. We take this numerical agreement as further evidence that the QPM provides a very good description of the inner structure of the nucleon. A particular advantage of the Llewellyn Smith formula is that the effect of the non-strange nucleon sea, and of a violation of the Callan–Gross relation, is nearly automatically taken into account.

The sensitivity of $R_v$ on $\sin^2 \theta_w$ is five times larger than that of $R_p$, for $\sin^2 \theta_w = 0.225$. Beam time is thus better invested in neutrino running.

The measurement of the NC to CC ratio has been a key issue for neutrino experiments ever since the discovery of NCs. The more precise results have been obtained by electronic detectors, because they can accumulate much higher statistics than bubble chambers. Electronic detectors use dense materials as the target for the neutrino beam, and hence profit from the systematically more reliable determination of $\sin^2 \theta_w$ from neutrino scattering on isoscalar nuclei (bubble chambers have specialized in studies on proton and neutron targets, which give valuable additional information but rely fully on the QPM).

The early series of measurements of the NC to CC ratio was terminated by a CDHS measurement in 1977, which reported $\sin^2 \theta_w = 0.24 \pm 0.02$ [116], a much lower value than the then-accepted world average $\sin^2 \theta_w = 0.31 \pm 0.03$ [117]. Electroweak radiative corrections were not yet an issue at that time. In the following years, a number of experiments were carried out at CERN and FNAL. The four most precise results on $\sin^2 \theta_w$ [118–121] are listed in Table 7.

On top of the experimental errors on $\sin^2 \theta_w$ quoted in Table 7, an additional ‘theoretical’ error of about $\pm 0.006$ arises from the incomplete knowledge of parameters in the QPM description of the nucleon which has been used in all cases to obtain $\sin^2 \theta_w$. A detailed analysis of those errors has been given by Geweniger [122].
Table 7

Results for $\sin^2 \theta_w$ from semileptonic neutrino scattering\textsuperscript{a)}

<table>
<thead>
<tr>
<th>Group</th>
<th>Target material</th>
<th>$\sin^2 \theta_w = 1 - m_W^2/m_Z^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>CCCFR [118]</td>
<td>Iron</td>
<td>0.242 ± 0.012</td>
</tr>
<tr>
<td>CDHS [119]</td>
<td>Iron</td>
<td>0.227 ± 0.012</td>
</tr>
<tr>
<td>CHARM [120]</td>
<td>Marble</td>
<td>0.209 ± 0.016</td>
</tr>
<tr>
<td>FMM [121]</td>
<td>Sand</td>
<td>0.247 ± 0.018</td>
</tr>
</tbody>
</table>

\textsuperscript{a)} O(\alpha) electroweak radiative corrections in the OS scheme applied.

The large statistics of $\nu$N and $\bar{\nu}$N events allowed a study of the NC y-distribution. For the particular case of neutrino scattering off an isoscalar target (which, for simplicity, consists of u and d quarks only), we expect y-distributions of the form

\[
\text{CC: } \frac{d\sigma}{dy} \propto U + D + (\bar{U} + \bar{D})(1 - y)^2 \\
\text{NC: } \frac{d\sigma}{dy} \propto [U + D + (\bar{U} + \bar{D})(1 - y)^2] (g_{1u}^2 + g_{1d}^2) + [\bar{U} + \bar{D} + (U + D)(1 - y)^2] (g_{2u}^2 + g_{2d}^2). 
\]

Numerically, $g_{1u}^2 + g_{1d}^2 = 0.30$, and $g_{2u}^2 + g_{2d}^2 = 0.03$. Hence, the CC and NC y-distributions are very similar, with a 10% admixture of $V + A$ in NC scattering. The neutrino NC y-distribution is slightly steeper than the CC y-distribution, and slightly flatter for antineutrinos. This expectation is borne out by experiment: Fig. 37 shows the y-distributions of NC and CC reactions, for $\nu$N and $\bar{\nu}$N scattering, as measured by the CHARM Collaboration [123].

The chief result is that in $\nu$N and $\bar{\nu}$N scattering the NC Lorentz structure is very close to $V - A$. However, neither the CDHS [124] nor the CHARM [123] Collaboration, who studied in detail the NC y-distributions, were able to demonstrate, from the shape of the y-distribution alone, the existence of a $V + A$ admixture with a significance larger than 2\sigma. Including the information from the

![Fig. 37](image-url) 

\textit{Fig. 37 } y-distributions of: CC and NC scattering of neutrinos and antineutrinos off isoscalar nuclei (from Ref. [126]).
**Fig. 38** Comparison of measurements of $R_v$ and $R_z$ with the prediction of the Electroweak Theory. All results are scaled to the experimental conditions of the CDHS experiment (from Ref. [122]).

**difference of $R_v$ and $R_z$,** the evidence for a $V+A$ admixture increased to the 4$\sigma$ level in either experiment (if NC had pure $V-A$, we would expect $R_v = R_z$). A summary of the most significant measurements of $R_v$ and $R_z$ as of 1984 is shown in Fig. 38. All experiments are consistent with $R_v > R_z$; however a more precise measurement of $R_z$ would be desirable.

Hypothetical S and P contributions in the NC Lorentz structure would show up as a term $\propto \gamma^2$ in the $y$-distribution of $\nu N$ and $\bar{\nu} N$ scattering [106]. Such a term has not been seen. The best upper limit has been given by the CHARM Collaboration [123]:

$$\sigma(S, P) / \sigma(V, A) \leq 0.03 \text{ (95\% c.l.)}.$$

In 1984, a third round of experiments was carried out at CERN, with much larger statistics than was available before, and with reduced systematic uncertainties of the neutrino flux. The final results [125, 126] of the two groups which launched into this programme of high-precision measurements of $\sin^2 \theta_w$ are given in Table 8. The experimental results are compatible. Both groups claim an

**Table 8**

Results on high-precision measurements of $\sin^2 \theta_w$
from semileptonic neutrino scattering$^a$

<table>
<thead>
<tr>
<th>Group</th>
<th>Target material</th>
<th>$\sin^2 \theta_w \equiv 1 - m_\nu / m_Z$</th>
</tr>
</thead>
<tbody>
<tr>
<td>CDHS [125]</td>
<td>Iron</td>
<td>$0.225 \pm 0.005 \text{ (exp)} \pm 0.003 \text{ (theor)}$ $+ 0.013 \text{ (m_e \text{ - 1.5 GeV})}$</td>
</tr>
<tr>
<td>CHARM [126]</td>
<td>Marble</td>
<td>$0.236 \pm 0.005 \text{ (exp)} \pm 0.003 \text{ (theor)}$ $+ 0.012 \text{ (m_e \text{ - 1.5 GeV})}$</td>
</tr>
</tbody>
</table>

$^a$ $O(\alpha)$ electroweak radiative corrections in the OS scheme applied.
experimental error on $\sin^2 \theta_w$ as low as \( \pm 0.005 \), which is a significant improvement over previous experiments. The theoretical error is reduced by virtue of the Llewellyn Smith formula [115]. The theoretical error excludes the uncertainties on the effective charm-quark mass, $m_c$. The dependence on $m_c$ is explicitly given, with $m_c = 1.5$ GeV as the central value. With $m_c = (1.5 \pm 0.3)$ GeV the total theoretical error increases to 0.005.

The systematic errors of the CDHS and CHARM experiments are quite different; hence a combined result from both experiments may be justified. The result is

$$\sin^2 \theta_w = 0.230 \pm 0.004 \text{(exp)} \pm 0.005 \text{(theor)}.$$ 

No other effort in semileptonic neutrino scattering is in sight which could further improve the accuracy on $\sin^2 \theta_w$.

The complete O($\alpha$) electroweak radiative corrections in the OS renormalization scheme have been applied. These corrections, which are dominated by O($\alpha$) QED corrections, have been calculated by several authors, with reasonable numerical agreement of their results [127]. The net effect of the radiative corrections is to lower $\sin^2 \theta_w$ by \( \sim 0.01 \). Figure 39 shows the NC and CC scattering diagrams at the Born approximation level, the diagrams for O($\alpha$) bremsstrahlung of real photons, and the diagrams for virtual O($\alpha$) electroweak corrections (vertex, self-energy, and box diagram corrections).

By comparing the measured $R_e$ with the one that would have been measured with the average value of $\sin^2 \theta_w$ obtained from the $\bar{p}p$ collision experiments, the CDHS group [125] have determined the so far most precise value of $q = m_\nu/(m_\nu \cos^2 \theta_w)$,

$$q = 0.998 \pm 0.011,$$

in agreement with the expectation $q = 1$ (see Section 1.1).

---

**Fig. 39** Diagrams for NC and CC neutrino–quark scattering at the Born approximation level, and at the level of O($\alpha$) electroweak radiative corrections.
The value of $\sin^2 \theta_W$ is well compatible with recent determinations from the W and Z masses (see Section 7.3). This good agreement holds only if $O(\alpha)$ electroweak radiative corrections are applied. Without these corrections, the values of $\sin^2 \theta_W$ from semileptonic $\nu N$ scattering and from the W mass would differ by more than 2\sigma. This complies with the expectation; however, a more precise measurement of the W mass is needed to render the effect of the radiative corrections significant.

6.3.2 Scattering on protons and neutrons

The NC/CC ratio from neutrino scattering on isoscalar targets provides information on $g_{L\nu}^d + g_{L\bar{\nu}}^u$ and $g_{R\nu}^u + g_{R\bar{\nu}}^d$. The analogous measurement with a proton target provides the combinations $2g_{L\nu}^d + g_{L\bar{\nu}}^u$ and $2g_{R\nu}^u + g_{R\bar{\nu}}^d$, whereas a measurement with a neutron target provides $g_{L\nu}^d + 2g_{L\bar{\nu}}^u$ and $g_{R\nu}^u + 2g_{R\bar{\nu}}^d$. These different combinations are to be measured in order to disentangle the NC coupling constants of the u- and d-quarks. The experiments have been carried out with bubble chambers filled with liquid hydrogen or liquid deuterium. In the case of deuterium, the even (odd) number of tracks originating from the interaction vertex classifies, in principle, the event as neutrino–neutron (neutrino–proton) scattering.

The most precise measurements of the NC to CC ratio for proton and neutron targets are listed in Table 9 [128–133]. However, the quoted published ratios cannot be compared directly with each other since they are not corrected for the losses from the different selection criteria. After correction for such losses [131], the experiments agree within errors. They also agree with the expectation from the values of the NC coupling constants, for $\sin^2 \theta_W = 0.225$. The most precise experiment [131] quotes, after $O(\alpha)$ electroweak radiative corrections, $\sin^2 \theta_W = 0.225 \pm 0.030$.

Table 9

<table>
<thead>
<tr>
<th>Group</th>
<th>$R_P^p$</th>
<th>$R_p^p$</th>
<th>$R_p^n$</th>
<th>$R_P^p$</th>
</tr>
</thead>
<tbody>
<tr>
<td>BEBC–ABCMO [128]</td>
<td>0.51 ± 0.04</td>
<td>0.33 ± 0.04</td>
<td>0.25 ± 0.02</td>
<td>0.57 ± 0.09</td>
</tr>
<tr>
<td>BEBC–TST [129]</td>
<td>0.47 ± 0.04</td>
<td>0.26 ± 0.04</td>
<td>0.34 ± 0.02</td>
<td></td>
</tr>
<tr>
<td>BEBC–D2 [130]</td>
<td>0.49 ± 0.05</td>
<td>0.34 ± 0.02</td>
<td></td>
<td></td>
</tr>
<tr>
<td>BEBC–BBCMO [131]</td>
<td>0.38 ± 0.03</td>
<td>0.22 ± 0.03</td>
<td></td>
<td></td>
</tr>
<tr>
<td>SIMMT (FNAL 15 ft) [132]</td>
<td>0.49 ± 0.06</td>
<td>0.36 ± 0.06</td>
<td></td>
<td></td>
</tr>
<tr>
<td>ACP (FNAL 15 ft) [133]</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The data from proton and neutron targets, together with those from isoscalar targets, allow the determination of $g_{L\nu}^d$, $g_{L\bar{\nu}}^u$, $g_{R\nu}^u$, and $g_{R\bar{\nu}}^d$. The remaining sign ambiguities can be resolved with the help of exclusive and semi-inclusive neutrino scattering processes. A first complete determination of the four ‘chiral’ coupling constants of the u- and d-quarks has been given by Sehgal [134] in 1977. A more refined and updated analysis has been given in 1981 by Kim et al. [135].

6.4 Charged lepton–quark scattering

6.4.1 $e D$ and $\mu C$ scattering

In 1978, many years of hard work at SLAC came to fruition: in the scattering of longitudinally polarized high-energy electrons off unpolarized deuterium nuclei, a parity-violating cross-section
asymmetry was found by a SLAC-Yale Collaboration [136]. Since the electromagnetic interaction conserves parity, the parity-non-conserving asymmetry was attributed to the weak NC interaction. The experiment demonstrated the interference of the neutral weak current and the electromagnetic current.

Because of the importance of this experiment, it may be appropriate to recall briefly its ingenious set-up. Longitudinally polarized electrons were emitted from a GaAs crystal, which was optically pumped by circularly polarized laser light. The electrons were then accelerated in the Stanford Linear Accelerator, with negligible loss of polarization. The polarization was reversed on a pulse-to-pulse basis. The electrons which were scattered under an angle of 4° from an unpolarized liquid-deuterium target, were momentum analysed in a magnetic spectrometer, and detected in a lead-glass shower counter.

The SLAC-Yale experiment gave its result at the right moment. Two atomic physics experiments (see Subsection 6.4.3), which searched for parity-violating effects in optical transitions of heavy nuclei, failed to see such effects at the level predicted by the Electroweak Theory. This was disturbing, and caused a fair amount of concern. The atomic physics experiments measure the weak NC interaction between electrons of the atomic shell, and quarks inside the nucleus: \( e^- + q \rightarrow e^- + q \). This is the same reaction as measured by the SLAC e'D experiment, albeit at much smaller \( Q^2 \). The asymmetry measured at SLAC was a great relief for the electroweak physicists community. Years later the same type of experiment was carried out by the BCDMS Collaboration at CERN, who observed a cross-section asymmetry in the scattering of left-handed \( \mu^+ \) and right-handed \( \mu^- \) off unpolarized carbon nuclei [137].

Consider a charged lepton with two polarization states \( P_1, P_2 \) (for instance, \( P_1 = +1 \) and \( P_2 = -1 \)). We define two types of asymmetries: \( A^\pm \) [138] where \( \pm \) refers to the sign of the lepton electric charge, and \( B \) [139]:

i) \[ A^\pm (P_1, P_2) = \frac{[\sigma^+ (P_1) - \sigma^+ (P_2)]/[\sigma^+ (P_1) + \sigma^+ (P_2)]}{g} = -gQ^2(P_1 - P_2)[g_{V\ell}A_q \pm g_{A\ell}V_q] \]

\[ g = G_F/(8\pi\alpha\sqrt{2}) = 4.5 \times 10^{-5} \text{ GeV}^{-2} \]

\[ A_q = (3/5)(g_{A\ell} - 2g_{V\ell})f(y) \]

\[ f(y) = [1 - (1 - y)^2]/[1 + (1 - y)^2] \]

\[ V_q = (3/5)(2g_{V\ell} - g_{V\ell}) \]

where the inelasticity \( y = (E - E')/E \) denotes the fraction of the incident lepton's energy which is transferred to the struck quark, and \( Q^2 \) the momentum transfer.

ii) \[ B(P_1, P_2) = \frac{[\sigma^+ (P_1) - \sigma^- (P_2)]/[\sigma^+ (P_1) + \sigma^- (P_2)]}{-gQ^2[2g_{A\ell}A_q + (P_1 - P_2)g_{V\ell}A_q - (P_1 + P_2)g_{A\ell}V_q]} \]

The SLAC-Yale e'D experiment measured, with \( e^- \) with longitudinal polarization \( P_1 = +P \) and \( P_2 = -P \) the asymmetry

\[ A^- (P, -P) = -2gQ^2P(g_{V\ell}A_q - g_{A\ell}V_q). \]

The BCDMS \( \mu^- \)C experiment measured, with \( \mu^- \) and \( \mu^- \) obtained from forward \( \pi \) and K decay \( (P_1 = -P, P_2 = +P) \),

\[ B(-P, P) = -2gQ^2(g_{A\ell} - Pg_{V\ell})A_q. \]

In both formulae the nucleon structure has been simplified such as to comprise valence u- and d-quarks only. The asymmetries arise from the interference of single-photon and Z exchange, and are proportional to \( Q^2 \). The SLAC-Yale experiment, at low \( Q^2 \), observed an asymmetry \( A \) of the order of \( 10^{-4} \); the BCDMS experiment, at much larger \( Q^2 \), an asymmetry \( B \) of the order of \( 10^{-2} \).
The SLAC-Yale results for the asymmetry A are shown in Fig. 40, as a function of the inelasticity \( y \). The measurements are in agreement with the Electroweak Theory and give, after \( O(\alpha) \) electroweak radiative corrections in the MS scheme \([140]\), a value of

\[
\sin^2 \theta_w = 0.215 \pm 0.020 \text{(exp)} \pm 0.020 \text{(thor)},
\]

where the theoretical error arises from uncertainties in the QPM description of the nucleon. The BCDMS results for the asymmetry B are shown in Fig. 41. They are also consistent with the Electroweak Theory, with a value of \( \sin^2 \theta_w \), after \( O(\alpha) \) electroweak radiative corrections in the OS scheme \([141]\), of

\[
\sin^2 \theta_w = 0.23 \pm 0.08.
\]

Either of the two experiments is sensitive to a particular linear combination of the vector and axial-vector coupling constants of the charged leptons (using, however, the theoretical values—confirmed by eN scattering experiments—of the NC coupling constants of the u- and d-quarks). The allowed domains in the \( g_{AC}-g_{W} \) plane are shown in Fig. 36 (Subsection 6.2.2). Whereas the SLAC-Yale experiment does not discriminate between the two solutions left from \( \nu_e e \) and \( \bar{\nu}_e e \) scattering, and \( e^+e^- \rightarrow \mu^+\mu^- \) and \( e^+e^- \rightarrow \tau^+\tau^- \), the BCDMS experiment decides in favour of the solution predicted by the theory, confirming the choice derived from the \( \nu_e e \) and \( \bar{\nu}_e e \) scattering experiments. Thus all sign ambiguities of the NC coupling constants of the charged leptons are resolved.

6.4.2 \( e^+e^- \) annihilation into quarks

Charged lepton–quark scattering is also studied, at today’s highest values of \( Q^2 \) (albeit time-like rather than space-like as in the preceding subsection), at PETRA and PEP in the \( e^+e^- \) annihilation of quarks. The ratio of the hadronic cross-section, \( \sigma(e^+e^- \rightarrow \bar{q}q) \), to the point-like cross-section is, in lowest order,

\[
R = 3 \sum_{q} Q_q^2 - 6 \chi_{1g} g_{Ve} \sum_{q} Q_q g_{Vq} + 3 \chi_{2}(g_{Vq}^2 + g_{Aq}^2) \sum_{q} (g_{Vq}^2 + g_{Aq}^2),
\]
where the sums run over all excited quark flavours. The factors $\chi_1$ and $\chi_2$ have been given in Subsection 6.2.2. At $s \ll m_\pi^2$, the electroweak interference term is expected to be small since $g_{ve}$ is close to zero, but not as small as in $e^+e^- \rightarrow \mu^+\mu^- : Q_\alpha g_{ve} \ll \Sigma Q g_{gq}$. From a compilation of recent data [114] on $R$, see Fig. 42, one might infer the onset of the $Z$ resonance, at the highest energies accessible at PETRA. These data are, however, not precise enough to make quantitative statements about the electroweak contribution, since the uncertainties of the QCD corrections in the quark sector are dominant.

In much the same way as in the case of charged leptons (Subsection 6.2.2) the forward-backward asymmetry offers a more sensitive measurement. The asymmetry is expected to be larger for quarks than for charged leptons. However, measuring the asymmetry of a specific quark flavour is experimentally a difficult task. So far, $e^+e^- \rightarrow \bar{c}c$ has been isolated via the decay sequence $D^* \rightarrow D$, and relatively pure samples of $e^+e^- \rightarrow \bar{b}b$ events have been selected by requesting electrons or muons with large transverse momentum with respect to the axis of the event. As an example, Fig. 43 displays the forward-backward asymmetry of $e^+e^- \rightarrow \bar{b}b$ measured by the JADE Collaboration [142].

From data from the JADE, MARK J, PLUTO, and TASSO Collaborations at PETRA, and from the HRS, MAC, MARK II, and TPC Collaborations at PEP, the following average axial-vector coupling constants have been determined [114]:

$$g_{Ac} = 1.0 \pm 0.3$$

$$g_{Ab} = -0.9 \pm 0.2$$

consistent with the assignments of +1 and −1, respectively.

6.4.3 Parity violation in atomic transitions

The interference between single-photon and $Z$ exchange in the NC amplitudes of electron–quark scattering can also be studied in optical transitions between atomic levels. The effect is due to the weak NC interaction between electrons of the shell and quarks in the nucleus. The atomic levels are no longer pure eigenstates of parity but receive a small admixture of opposite parity, causing a mixture of electric and magnetic dipole transitions. Their interference implies a rotation of the polarization plane of a linearly polarized laser beam, or a different absorption cross-section of right- and left-circularly polarized laser light.

The chief interest in measuring parity-violation effects in atoms lies in the small $Q^2 \sim R_{\text{atom}}^2 \sim 10^{-8}$ GeV$^2$ of the process.
Table 10

Experimental results and theoretical predictions of parity-violation effects in heavy atoms (from Ref. [144])

<table>
<thead>
<tr>
<th>Experiment</th>
<th>Atom</th>
<th>(\lambda) (nm)&lt;sup&gt;a&lt;/sup&gt;</th>
<th>Result&lt;sup&gt;b&lt;/sup&gt;</th>
<th>Prediction&lt;sup&gt;b&lt;/sup&gt;</th>
</tr>
</thead>
<tbody>
<tr>
<td>Novosibirsk 1978 [145]</td>
<td>Bi</td>
<td>648</td>
<td>(-20.2 \pm 2.7)</td>
<td>(-12.5^{+1.6}_{-1.3})</td>
</tr>
<tr>
<td>Oxford 1984 [146]</td>
<td>Bi</td>
<td>648</td>
<td>(-9.3 \pm 1.5)</td>
<td>(-10.0 \pm 2.5)</td>
</tr>
<tr>
<td>Moscow 1984 [146]</td>
<td>Bi</td>
<td>648</td>
<td>(-7.8 \pm 1.8)</td>
<td>(-13 \pm 2)</td>
</tr>
<tr>
<td>Seattle 1984 [147]</td>
<td>Bi</td>
<td>876</td>
<td>(-10.4 \pm 1.7)</td>
<td>(-10.0 \pm 2.5)</td>
</tr>
<tr>
<td>Seattle 1984 [148]</td>
<td>Pb</td>
<td>1280</td>
<td>(-9.9 \pm 2.5)</td>
<td>(-13 \pm 2)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Experiment</th>
<th>Atom</th>
<th>(\lambda) (nm)&lt;sup&gt;a&lt;/sup&gt;</th>
<th>Result&lt;sup&gt;c&lt;/sup&gt;</th>
<th>Prediction&lt;sup&gt;c&lt;/sup&gt;</th>
</tr>
</thead>
<tbody>
<tr>
<td>Berkeley 1981 [149]</td>
<td>Tl</td>
<td>293</td>
<td>(-1.80 \pm 0.65)</td>
<td>(-1.15 \pm 0.35)</td>
</tr>
<tr>
<td>Berkeley 1984 [150]</td>
<td>Tl</td>
<td>293</td>
<td>(-1.73 \pm 0.33)</td>
<td>(-1.6 \pm 0.2)</td>
</tr>
<tr>
<td>Paris 1982 [151]</td>
<td>Cs</td>
<td>539</td>
<td>(-1.34 \pm 0.33)</td>
<td>(-1.6 \pm 0.2)</td>
</tr>
<tr>
<td>Paris 1984 [152]</td>
<td>Cs</td>
<td>539</td>
<td>(-1.78 \pm 0.38)</td>
<td>(-1.6 \pm 0.2)</td>
</tr>
</tbody>
</table>

<sup>a</sup> Wavelength of laser light.
<sup>b</sup> Im \(E_p^p/M_1\), in units of \(10^{-8}\), where \(E_p^p\) is the (imaginary) amplitude of the parity-violating E1 transition, and \(M_1\) the amplitude of the parity-conserving M1 transition.
<sup>c</sup> Im \(E_p^p/\beta\) in mV/cm, where \(E_p^p\) is the same as in (b), and \(\beta\) is the vector polarizability of the atom in an external electric field.

Parity violation in atoms occurs mainly through the product of the axial-vector coupling of the electron, and the vector coupling of the u- and d-quarks: firstly, the vector coupling of the electron is small; secondly, the axial-vector quark coupling involves a spin flip of the nucleus; since the quark spins cancel each other in pairs of nucleons, only a small fraction of all quarks contained in unpaired nucleons remain to participate in the reaction.

Because of the coherent action of the quarks, parity-violation effects are enhanced in heavy nuclei. Still, to render the predicted effects observable, strong enhancement factors are needed, such as suppression of the normal parity-conserving transition.

The first two experiments looking at optical rotation of the plane of linearly polarized laser light in Bi vapour reported the absence of parity violation at the level predicted by the Electroweak Theory [143]. Both experiments were wrong. Later, new measurements and new experiments confirmed, within errors, the validity of the Electroweak Theory in predicting parity-violating transitions in heavy atoms.

A recent review of the theoretical and experimental situation has been given by Piketty [144]. The experimental results [145–152] are listed in Table 10, together with an average of theoretical predictions.

As a measure of the current sensitivity of the atomic physics experiments on \(\sin^2 \theta_W\), we give its value, after \(O(a)\) electroweak radiative corrections in the MS scheme [153], as determined in the Paris experiments [151, 152]:

50
\[ \sin^2 \theta_w = 0.205 \pm 0.043 \text{ (exp)} \pm 0.045 \text{ (theor)}, \]

where the (sizeable!) theoretical error comes from uncertainties in the atomic-physics calculations.

This value of \( \sin^2 \theta_w \) is compatible with other determinations at a Q^2 larger by 11 orders of magnitude!

### 6.5 Limits on flavour-changing neutral currents

In the frame of the Electroweak Theory flavour-changing NCs are absent by construction. The most stringent experimental test of the absence of weak s \( \rightarrow \) d transitions is the branching ratio [154]

\[ \Gamma(K_L^0 \rightarrow \mu^+\mu^-)/\Gamma(K_L^0 \rightarrow \text{all}) = 8.1^{+7.8}_{-7.0} \times 10^{-9}. \]

The small but finite branching ratio of the decay \( K_L^0 \rightarrow \mu^+\mu^- \) is quantitatively explained by a combination of a weak CC and an electromagnetic interaction.

The second best test for the absence of weak s \( \rightarrow \) d transitions comes from the upper limit [155]

\[ \Gamma(K^+ \rightarrow \pi^+\nu\bar{\nu})/\Gamma(K^+ \rightarrow \text{all}) < 1.4 \times 10^{-7} \text{ (90\% c.l.)}. \]

This decay is of particular interest since it proceeds in the frame of the Electroweak Theory only as a second-order weak CC reaction, at a level \( \ll 10^{-10} \). The decay rate would also serve for the ‘counting’ of the number of light neutrino generations. At kaon factories such as LAMPF II or TRIUMF II, branching ratios as low as \( 10^{-11} \) would be experimentally accessible.

The most stringent limits on flavour-changing NC reactions involving heavy quarks come from the absence of \( D^0 \)-meson (EMC Collaboration [156]) and \( B^0 \)-meson (CLEO Collaboration [157]) decays into an opposite-sign muon pair:

\[ \Gamma(D^0 \rightarrow \mu^+\mu^-)/\Gamma(D^0 \rightarrow \text{all}) < 3.4 \times 10^{-4} \text{ (90\% c.l.)} \]
\[ \Gamma(B^0 \rightarrow \mu^+\mu^-)/\Gamma(B^0 \rightarrow \text{all}) < 2 \times 10^{-4} \text{ (90\% c.l.)}. \]

### 6.6 Summary concerning NC processes

The absence of flavour-changing NC processes is tested to an impressive level of accuracy for light quarks. The limits for heavy quarks are much less stringent as yet.

The individual NC coupling constants have been determined, including their relative sign, for the fermions of the first generation. Some NC coupling constants of the higher fermion generations have been determined, too. No significant deviation from generation universality of the NC coupling constants has been found. The best-determined coupling constant is \( g_{\Lambda u} = -1.13 \pm 0.07 \), measured to 6\% precision (and deviating at the 2\( \sigma \) level from the theoretical value \( g_{\Lambda u} = -1 \)).

All measured coupling constants are, within a typical 10\% error, consistent with the theoretical assignments with one single value of \( \sin^2 \theta_w \). Henceforth, electroweak radiative corrections should be included also at the level of the NC coupling constants.

Values of \( \sin^2 \theta_w \) have been measured in many processes, ranging in \( Q^2 \) from \( 10^{-8} \text{ GeV}^2 \) to \( 10^3 \text{ GeV}^2 \). No deviation, within errors, from the notion of one single parameter has been found. There is no stringent proof of the necessity of electroweak radiative corrections as yet. However, the agreement between different experiments improves at the 2\( \sigma \) level when they are taken into account.

The more precise values of \( \sin^2 \theta_w \), after radiative corrections in the OS scheme, from ‘low-energy’ NC reactions are:

i) \( \sin^2 \theta_w = 0.212 \pm 0.029 \) from \( \nu e \) scattering,
ii) \( \sin^2 \theta_w = 0.210 \pm 0.023 \) from \( e^+e^- \rightarrow e^+e^- \) (\( l = \mu, \tau \)),
iii) \( \sin^2 \theta_w = 0.230 \pm 0.006 \) from \( \nu N \) scattering,
iv) \( \sin^2 \theta_w = 0.215 \pm 0.028 \) from \( \bar{e}D \) scattering,
where we have taken the optimistic view that all errors can be added in quadrature. The weighted average

\[ \sin^2 \theta_w = 0.228 \pm 0.006 \]

is compatible with all results from low-energy NC reactions. This value of \( \sin^2 \theta_w \) can be directly compared with the one obtained from the W and Z masses (Section 7.3).

7. W AND Z

For many years, the W and Z were the best established but unobserved particles. It needed the transformation of the CERN SPS into a \( \bar{p}p \) collider, the provision of a high-luminosity \( \bar{p}p \) source, and a huge experimental effort to record and analyse the data from high-energy \( \bar{p}p \) collisions in order to establish experimentally the existence of W and Z.

The idea dates back to 1976, when Rubbia, together with Cline and McIntyre [158], proposed the conversion of the CERN SPS into a \( \bar{p}p \) collider. This modification was proposed even before it started operation as a 400 GeV proton accelerator, at the very end of 1976.

The first \( \bar{p}p \) collisions were observed in July 1981. In the years 1982 and 1983, enough statistics were accumulated to allow the discovery of the W [159, 160] and the Z [161, 162]. They were found at their predicted mass values, based on the value of \( \sin^2 \theta_w \) obtained from neutrino- and electron-nucleon scattering experiments. This constituted another triumph of the Electroweak Theory. Much higher statistics have been accumulated since, at \( \sqrt{s} = 546 \) GeV and \( \sqrt{s} = 630 \) GeV.

The W and Z bosons have unambiguously been identified in their leptonic decay modes only. Here, we restrict ourselves to the electronic decay modes \( W \rightarrow e^+\nu \) and \( Z \rightarrow e^+e^- \) because large and comparable event samples exist from both the UA1 [163] and UA2 [164] Collaborations. Unlike the UA1 detector, the UA2 detector has no muon detection capability. The (less precise) results from the smaller sample of events with muonic decay modes \( W \rightarrow \mu^+\nu \) and \( Z \rightarrow \mu^+\mu^- \), observed by the UA1 Collaboration, agree in all aspects with the results from the electronic decay modes [165]. A recent summary of the experimental situation of the W and Z measurements has been given by DiLella [166]. Here, we follow closely his discussion of the subject.

7.1 Electroweak aspects of the W

The experimental signature of the W boson is an isolated (i.e. free of nearby hadrons) electron with large \( p_T \) (> 15 GeV), associated with large missing transverse momentum \( p_T^{\text{miss}} \), identified with the neutrino transverse momentum \( p_T \). Figure 44 shows the distributions of \( p_T \) as observed by UA1

![Fig. 44](image)

The \( p_T \) distribution of \( W \rightarrow e^+\nu \) decay candidates as observed: a) by UA1 and b) by UA2. The broken lines give the estimated hadronic background.

52
Table 11

$W \rightarrow e\nu$ event samples and background

<table>
<thead>
<tr>
<th>pT threshold (GeV)</th>
<th>UA1</th>
<th>UA2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of events</td>
<td>172</td>
<td>119</td>
</tr>
<tr>
<td>Hadronic background</td>
<td>5.3 ± 1.9</td>
<td>5.6 ± 1.7</td>
</tr>
<tr>
<td>$Z \rightarrow e$ (detected) e (undetected)</td>
<td>-</td>
<td>4.4 ± 0.8</td>
</tr>
<tr>
<td>$W \rightarrow \tau\nu_{\tau}$ ($\tau \rightarrow e\nu_{e}\nu_{\tau}$)</td>
<td>9.1 ± 0.5</td>
<td>1.7 ± 0.2</td>
</tr>
<tr>
<td>$W \rightarrow \tau\nu_{\tau}$ ($\tau \rightarrow \nu\pi^0$ + hadrons)</td>
<td>2.7 ± 0.4</td>
<td>-</td>
</tr>
<tr>
<td>$W \rightarrow e\nu$ signal</td>
<td>155 ± 13</td>
<td>107 ± 11</td>
</tr>
</tbody>
</table>

and UA2 after all cuts. The UA1 distribution is nearly free of background, thanks to the hermetic closure of their detector, whereas the UA2 distribution has a fair amount of hadronic background below $p_T = 25$ GeV. The status of affairs may best be represented by the event numbers and estimated backgrounds of the final event samples, given in Table 11.

The W mass is determined from the distribution of the ‘transverse mass’ $m_T$ defined as

$$m_T = \sqrt{2p_T^2(1 - \cos \Delta \phi)},$$

where $\Delta \phi$ is the difference in azimuthal angle of the momentum vectors $\vec{p}_T$ and $\vec{p}_T'$. The distribution in $m_T$ is less sensitive to the W transverse motion, contrary to the distribution of $p_T$. Figure 45 shows the

![Graphs showing the transverse mass distribution of $W \rightarrow e\nu$ event candidates, as observed by UA1 and UA2.](image)

**Fig. 45** The transverse mass distributions of $W \rightarrow e\nu$ event candidates, as observed a) by UA1 and b) by UA2.
$m_T$ distributions of the UA1 (after further cuts of $p_T > 30$ GeV and $p_T > 30$ GeV) and UA2 event samples. The best-fit values of the W mass are

\[ UA1: \quad m_W = 83.5^{+1.1}_{-1.0} \text{(stat)} \pm 2.7 \text{(syst)} \text{ GeV} \quad [163] \]

\[ UA2: \quad m_W = 81.2 \pm 1.1 \text{ (stat)} \pm 1.3 \text{ (syst)} \text{ GeV} \quad [164] \]

The systematic error is largely determined by the uncertainty of the absolute energy scale of the calorimetric devices employed for the measurement of the electron energy. This uncertainty is inherent in the present designs of the calorimeters, and cannot be improved substantially. Calorimetric upgrades of both detectors are under way. The chief aim of UA2 is to obtain a 4π closure of their calorimeter, utilizing the same technique of scintillator sampling calorimetry as used before, thereby accepting the present limitations. The UA1 Collaboration have embarked on constructing a new calorimeter based on a ‘warm liquid’ as sampling medium, which offers the potential of much reduced uncertainty in absolute calibration.

Adding statistical and systematic errors quadratically we obtain an average W mass value

\[ m_W = 81.8 \pm 1.5 \text{ GeV}. \]

At the energies of the CERN $\bar{p}p$ collider, W production is still dominated by $q\bar{q}$ annihilation involving at least one valence quark. The production and subsequent leptonic decay of a $W^+$ and $W^-$ is shown in Fig. 46. For $V-A$ coupling, the W spin is oriented along the $\bar{p}$ direction of flight. The $e^+$ will be emitted preferentially along the $\bar{p}$ direction, and the $e^-$ along the $p$ direction (‘charge asymmetry’). The direction of the charged lepton in the W centre-of-mass frame is expected to be $\propto (1 \pm \cos \theta^*)^2$ where $\theta^*$ is the angle between the $e^+$ ($e^-$) and the $\bar{p}$ direction. If the sign of the lepton is not measured,

![Fig. 46](image)

\text{Fig. 46.} $V-A$ fermion helicity configuration in W production and leptonic decay.

![Fig. 47](image)

\text{Fig. 47} \quad a) \text{ The UA1 measurement of } Q \cos \theta^*, \text{ where } Q = +1 (-1) \text{ for } e^+ (e^-), \text{ and } \theta^* = 0 \text{ along the } \bar{p} \text{ beam, and } b) \text{ the UA2 measurement of } |\cos \theta^*|.
as is the case in the UA2 detector, the averaged angular distribution will be \( \propto 1 + \cos^2 \theta^* \). Figure 47 shows the measured angular distributions which are consistent with the expectation. The more precise UA1 measurement rules out any spin of the W other than 1.

### 7.2 Electroweak aspects of the Z

The event sample of \( Z \rightarrow e^+e^- \) is essentially free of background, and hence the tight criteria on electron identification, which are necessary in the \( W \rightarrow e\nu \) decay, can be somewhat relaxed. Figure 48 shows the invariant mass distribution of high-mass \( e^+e^- \) pairs as measured in the UA1 and UA2 detectors. The cluster of events from \( Z \) production and decay is well separated from background from misidentified two-jet events.

![Graph of invariant mass distribution](image)

**Fig. 48** Invariant mass distribution of high-mass \( e^+e^- \) pairs as measured: a) by UA1 and b) by UA2.

The \( Z \) mass is obtained from the fit of a Breit-Wigner resonance shape, distorted by the experimental resolution. The resulting mass values are:

- **UA1**: \( m_Z = 93.0 \pm 1.4 \) (stat) \( \pm 3.0 \) (syst) GeV \[163]\n- **UA2**: \( m_Z = 92.5 \pm 1.3 \) (stat) \( \pm 1.5 \) (syst) GeV \[164]\n
As in the W case, the systematic error is dominated by the uncertainty of the absolute energy scale of the calorimeters.

Adding statistical and systematic errors quadratically we obtain an average \( Z \) mass value

\[
m_Z = 92.6 \pm 1.7 \text{ GeV}.
\]

### 7.3 \( \sin^2 \theta_W \) from the W and Z masses

The W and Z masses are measured to 2\% accuracy. As outlined in Section 1.3, electroweak radiative corrections must be taken into account when comparing with predictions of the Electroweak Theory at this level of precision.

Including \( O(\alpha) \) electroweak radiative corrections in the OS scheme (Section 1.3) the W and Z masses are predicted to be \[167\]

\[
m_W = \frac{\pi \alpha}{G_F \sin^2 \theta_W (1 - \Delta r) \sqrt{2}}
\]

\[
m_Z = \frac{\pi \alpha}{G_F \sin^2 \theta_W \cos^2 \theta_W (1 - \Delta r) \sqrt{2}}.
\]

The radiative correction \( \Delta r \) is

\[
\Delta r = 0.0696 \pm 0.0020,
\]

for a top-quark mass \( m_t = 36 \) GeV and a Higgs mass \( m_H = m_Z \); \( \Delta r \) is only slightly dependent on \( m_t \) and \( m_H \) for 'reasonable' masses much below 1 TeV. The bulk of the radiative correction \( \Delta r \) is caused by the renormalization of \( \alpha \), which is evaluated at \( Q^2 = 0 \), up to \( Q^2 = m_W^2 \), i.e. from the QED
renormalization of the electric charge due to vacuum polarization. Inserting the values of $\alpha$, $G_F$, and the measured masses, we obtain for $\sin^2 \theta_w$:

\[
\begin{align*}
UA1: & \quad \sin^2 \theta_w = 0.214 \pm 0.006 \text{ (stat)} \pm 0.015 \text{ (syst)} \quad [163] \\
UA2: & \quad \sin^2 \theta_w = 0.226 \pm 0.005 \text{ (stat)} \pm 0.008 \text{ (syst)} \quad [164]
\end{align*}
\]

Adding again statistical and systematic errors quadratically we obtain an average $\sin^2 \theta_w$ from the W and Z masses:

\[
\sin^2 \theta_w = 0.223 \pm 0.008.
\]

So far, it has been assumed that the $g$ parameter, $g = m_W^2/(m_Z^2 \cos^2 \theta_w)$ is equal to 1. Defining $\sin^2 \theta_w$ from the W mass, we obtain from the measured W and Z masses:

\[
\begin{align*}
UA1: & \quad g = 1.026 \pm 0.037 \text{ (stat)} \pm 0.019 \text{ (syst)} \\
UA2: & \quad g = 0.996 \pm 0.033 \text{ (stat)} \pm 0.009 \text{ (syst)}.
\end{align*}
\]

The weighted average is

\[
\bar{g} = 1.008 \pm 0.026,
\]

in agreement with 1.

8. ELECTROWEAK PHYSICS ASPECTS AT LEP/SLC AND HERA

8.1 Precision tests of the Electroweak Theory in $e^+e^-$ annihilation

Many aspects of the Electroweak Theory have been tested in ‘low-energy’ NC reactions. Agreement between predicted and various measured NC coupling constants has been found. Values of $\sin^2 \theta_w$ as determined from different NC reactions agree within 10%. The average $\sin^2 \theta_w$ from low-energy NC reactions, $\sin^2 \theta_w = 0.228 \pm 0.006$, agrees within 5% with the average value from the W and Z masses, $\sin^2 \theta_w = 0.223 \pm 0.008$. In a few years we expect to do much better.

Our strategy of testing the Electroweak Theory at a new level of precision might look as follows (we assume that electroweak radiative corrections are applied throughout): check the consistency of $\sin^2 \theta_w$ as determined from different $e^+e^-$ annihilation reactions with the precise value which will be obtained from the Z mass. In case these analyses lead to different values of $\sin^2 \theta_w$, this might signify ‘new physics’.

We do not know at which level of precision new physics will show up. We take here as an interesting level of precision the level of genuine weak $O(\alpha)$ radiative corrections, following the line of Altarelli et al. [168]. Such corrections change $\sin^2 \theta_w$ typically by 0.001 (0.5% of $\sin^2 \theta_w$). For comparison, the QED radiative corrections from vacuum polarization change $\alpha$ from 1/137 at $Q^2 = 0$ to 1/128 at $Q^2 = m_W^2$, causing a 7% change of $\sin^2 \theta_w$ as determined from the W and Z masses. Today, we only sense the overall electroweak corrections at the 2–3σ level. Hence we aim at improving today’s precision by a factor of 5 at the $e^+e^-$ colliders LEP and SLC.

From a scan of the line shape of the Z resonance, the mass $m_Z$ can be measured to $\pm 20$ MeV provided the centre-of-mass energy of the machine will be known to $\pm 10$ MeV. An error $\Delta m_Z$ translates into an error of $\sin^2 \theta_w$ according to $\Delta \sin^2 \theta_w = \Delta m_Z/150$ GeV. For $\Delta m_Z = 20$ MeV we obtain $\Delta \sin^2 \theta_w = 0.00013$. To appreciate this experimental precision we note that the uncertainty on $m_H$ within 10 GeV $< m_H < 1$ TeV causes an uncertainty of $\pm 0.0025$ in the determination of $\sin^2 \theta_w$ from a given value of $m_Z$ [169], see Fig. 49.

Unfortunately, the precision of $\sin^2 \theta_w$ from $m_Z$ is unparalleled by other processes. The reactions which come close in their potential of high precision are the measurements of the left–right
The dependence of $\sin^2 \theta_w = 1 - \frac{m_t^2}{m_Z^2}$ on $m_Z$ for different values of $m_H$ and $m_t$. 

asymmetry $A_{LR}$, of the forward–backward asymmetry (or: ‘charge asymmetry’) $A_{FB}$, and of the $\tau$-polarization asymmetry $A_{POL}$. In Born approximation, these asymmetries are related, at the $Z$ pole, to the NC coupling constants as follows:

$$A_{LR}(m_Z) = \frac{\sigma_L - \sigma_R}{\sigma_L + \sigma_R} = \frac{2P g_{\nu e} g_{\nu e}}{(g_{\nu e}^2 + g_{\nu e}^2)}$$

$$A_{FB}(m_Z) = \frac{\sigma^F - \sigma^B}{\sigma^F + \sigma^B} = \frac{3g_{\nu e} g_{\nu e} g_{\tau \tau}}{[(g_{\tau \tau}^2 + g_{\tau \tau}^2)(g_{\tau \tau}^2 + g_{\tau \tau}^2)]}$$

$$A_{POL}(m_Z) = 2g_{\nu \mu} g_{\tau \tau} (g_{\tau \tau}^2 + g_{\tau \tau}^2).$$

The given expression for $A_{LR}$ refers to the collision of $e^-$ with longitudinal polarization $P$ with unpolarized $e^-$; for $A_{FB}$ to the collision of unpolarized $e^+$ and $e^-$; as for $A_{POL}$, the expression refers to an average over all production angles. The asymmetry $A_{POL}$ is independent of beam polarization.

Of these possibilities, $A_{LR}$ is the most promising: since it is independent of the final-state fermion type, all final states can be used, significantly enhancing the statistical precision. The price to pay is to have longitudinal beam polarization, with well-known polarization $P$. Figure 50 (from [170]) shows which accuracy one might achieve under favourable circumstances: for $\Delta P/P = 1\%$ at $P = 0.45$, and $10^6 Z$ decays, $\Delta \sin^2 \theta_w$ is as low as 0.0004, for $\sin^2 \theta_w = 0.225$. The SLC at SLAC will have longitudinal $e^-$ polarization as an option. The prospects for longitudinal beam polarization in LEP at CERN are dim for the time being, for financial and technical reasons.

The forward–backward asymmetry $A_{FB}$ with unpolarized beams depends on the type of final-state fermion. Experimentally the easiest reaction is $e^+e^- \rightarrow \mu^+\mu^-$. The asymmetry $A_{FB}$ is then proportional to $g_{\nu e} g_{\nu e}$, which is very close to zero and hence makes it difficult to obtain a precise measurement of $\sin^2 \theta_w$. Accumulating $10^4 e^+e^- \rightarrow \mu^+\mu^-$ events, and with careful control of systematic errors, one might achieve $\Delta \sin^2 \theta_w = 0.002$ (the choice of Nature of a $\sin^2 \theta_w$ very close to 0.25, where $g_{\nu e} = g_{\nu e} = 0$ renders $A_{FB}$ insensitive to $\sin^2 \theta_w$). Longitudinal beam polarization would significantly increase the sensitivity.
The asymmetry $A_{\text{pot}}$ is an easy measurement. One selects $e^+e^- \to \tau^+\tau^-$ events (low-multiplicity events), and measures the $\pi$ momentum spectrum from $\tau \to \pi \nu$ decay. In this two-body decay, the $\pi$ momentum vector is correlated with the direction of the $\tau$ polarization vector. Hence in the laboratory frame, the $\pi$ momentum spectrum exhibits a slope with momentum which is a measure of the $\tau$ polarization. Accumulating $10^5$ $e^+e^- \to \tau^+\tau^-$ events, and with careful control of background from other $\tau$ decay modes, one might achieve $\Delta \sin^2 \theta_W = 0.002$.

We conclude that data taking at the Z resonance with unpolarized beams will provide significant tests of QED radiative corrections but not of weak radiative corrections. The weak correction level is only accessible with longitudinal beam polarization.

### 8.2 Z width and neutrino counting

Perhaps one of the most important measurements at LEP/SLC is to ‘count’ the number of fermion generations with neutrinos with mass less than $m_Z/2$. The total Z width is $\Gamma_{\text{tot}} \approx 2.8$ GeV. It can be measured from the line shape with an error of $\Delta \Gamma_{\text{tot}} \approx 20$ MeV. This experimental error is small compared to the expected increase of the Z width by 180 MeV per new neutrino generation. The experiment requires the comparison of the experimentally measured width with the one predicted by the Electroweak Theory, including QCD corrections in the quark sector.

Another, less model-dependent, method to count the number of neutrino generations is to measure the cross-section for radiative Z production with subsequent decay into neutrinos:

$$e^+e^- \to Z \gamma \to \nu\bar{\nu}$$

The experimental signature is a single photon in the final state. The experiment is best carried out at centre-of-mass energies somewhat above the Z resonance. The photon energy spectrum then develops a peak, separated from the bremsstrahlung spectrum. The photon energy spectra at various centre-of-mass energies are shown in Fig. 51.

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{photon_energy_spectrum.png}
\caption{The photon energy spectrum from $e^+e^- \to \nu\bar{\nu}$, for different values of $\sqrt{s}$ (from Ref. [167]).}
\end{figure}
A fourth neutrino generation, for example, would increase the cross-section for $e^+e^- \rightarrow \bar{\nu}\nu\gamma$ by 30%. Hence the cross-section should be measured to 5% or better. The chief problem is to evaluate correctly the background from radiative Bhabha scattering, $e^+e^- \rightarrow e^+e^-\gamma$, where both the $e^+$ and $e^-$ remain at small scattering angles and thus escape detection. This background problem seems to be under control [171].

8.3 Higgs particle search

Higgs particles are theoretically needed ‘bêtes noires’ of the Electroweak Theory (Jarlskog [172]). They are the origin of the masses of the fermions and vector bosons. The minimal version of the theory requires the physical existence of one neutral scalar Higgs particle. Its couplings are well defined but not its mass. Nobody has yet seen this particle. Is this because $m_H$ is too large?

A recent survey of the situation has been given by Maiani [173]. In short, $m_H$ is bound within the range

$$15 \text{ MeV} < m_H < 1 \text{ TeV}.$$ 

The lower bound of 15 MeV is derived from the absence of long-range forces in nuclei, and is considered safe. A more stringent lower bound from the requirement of vacuum stability in the presence of higher corrections, $m_H > 7 \text{ GeV}$, is subject to some theoretical uncertainty. An upper bound is 1 TeV, where the width of the Higgs particle becomes so large that it can no longer be considered a well-defined particle.

At the $Z$ pole, or in the energy range of LEP II ($\sqrt{s} \leq 190 \text{ GeV}$), searches for the Higgs particle can be carried out via the processes

$$e^+e^- \rightarrow H + Z \ (Z \rightarrow e^+e^- , \mu^+\mu^-)$$

$$(f \bar{f}) \rightarrow H + \gamma \ (\text{radiative toponium decay}).$$

The experiments are difficult because the branching ratios are very low. However, the expected machine luminosities and cross-sections are such that conclusive results should emerge on the existence of the predicted physical Higgs particle, provided $m_H \leq 100 \text{ GeV}$.

8.4 Electron–proton scattering at large $Q^2$

The ep collider HERA, currently under construction at DESY, is scheduled to start physics operation in 1990. For the first time, collisions of 30 GeV electrons with 820 GeV protons will be observed, at a typical $Q^2 \sim 10^6 \text{ GeV}^2$ ($Q_{\text{max}}^2 = 1 \times 10^5 \text{ GeV}^2$). The typical processes will be deep inelastic NC scattering, $e + p \rightarrow e + X$, and CC scattering, $e + p \rightarrow \nu + X$.

At the level of the quark constituents, the NC process is the same as which is studied at $Q^2 \sim 10^{-8} \text{ GeV}^2$ by parity violation experiments in atomic transitions, at $Q^2 \sim 1 \text{ GeV}^2$ in the $e^+d$ scattering experiment at SLAC, and at $Q^2 \sim 100 \text{ GeV}^2$ in the $\mu^+\bar{\nu}$ scattering experiment at CERN. The cross-section consists of three terms:

$$\sigma_{\text{NC}} = \sigma_{\text{em}} + \sigma_{\text{int}} + \sigma_{\text{weak}},$$

where $\sigma_{\text{em}} \propto \alpha^2/Q^4$, and $\sigma_{\text{weak}} \propto G_F^2/(1 + Q^2/m_\pi^2)$. The weak-electromagnetic interference cross-section, $\sigma_{\text{int}} \propto \alpha G_F^2/Q^2 (1 + Q^2/m_\pi^2)$, will be of the same order as the electromagnetic cross-sections at $Q^2 = m_H^2 \sim 10^4 \text{ GeV}^2$ (see Fig. 52, from [174]). In order to disentangle the various contributions, longitudinal polarization of the electron (positron) beam will prove important.

An important aspect of the research at HERA will be the investigation of the Lorentz structure of the electroweak currents at very large $Q^2$. In particular, the question of the possible appearance at large $Q^2$ of right-handed weak currents will be addressed. These currents, required by left-right symmetric gauge groups such as SU(2) $_L \otimes$ SU(2)$_R \otimes$ U(1), are presumably mediated by a heavy intermediate ‘right-handed’ boson $W_R$.  

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We know from the measurement of the asymmetry parameter $\xi$ in muon decay that the mass of such a right-handed W is larger than about 400 GeV provided the associated right-handed neutrino has a mass less than 10 MeV (Section 5.1). The search for right-handed CCs in $\nu N$ scattering is not limited by the neutrino mass, but is sensitive only to the mixing angle between the physical right-handed W and its associated heavy mass-eigenstate (Section 5.2). These limitations fall apart at HERA. Figure 53 (from [174]) shows the expected change of cross-section of CC ep scattering, for various masses of a second, right-handed, W as a function of $Q^2$. Right-handed W’s can be detected up to masses of $\sim 500$ GeV. Analogously, a similar mass range for additional heavy Z’ bosons can be explored in the NC ep scattering. However, the latter physics can be studied at LEP II as well, while the study of right-handed CC processes involving neutrinos can only be done at HERA.

9. CONCLUSION

The ‘low-energy’ NC experiments are all consistent with the Electroweak Theory, and give an average value $\sin^2 \theta_w = 0.228 \pm 0.006$. The properties of the W and Z bosons are equally consistent with the Electroweak Theory, with an average value $\sin^2 \theta_w = 0.223 \pm 0.008$. This agreement is truly impressive. There seems no other way than Nature knowing about the Electroweak Theory.

But, how well has the theory been tested experimentally? The equality of $\sin^2 \theta_w$ from the W and Z masses on the one hand, and from (chiefly) $\nu N$ scattering on the other hand, is tested to 5% precision only. The internal consistency of the various low-energy NC experiments is tested to 10% precision only. Most of the 30 or so NC experiments whose results are quoted in support of the Electroweak Theory give moral support only, and do not allow stringent quantitative checks to be made. Last, but not least is the problem: Where is the Higgs particle?

Our conclusion is that the phenomenology of the Electroweak Theory is tested to be correct at the 5% level. Surprises might be in store when going to significantly higher precision.

Meanwhile we take an optimistic practical attitude and take note of the currently best value of $\sin^2 \theta_w = 1 - m_W/m_Z$, obtained by averaging the result from the W and Z masses, and the result from the low-energy NC experiments:

$$\sin^2 \theta_w = 0.226 \pm 0.005 \ .$$
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