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LATTICE CALCULATION OF THE $K^+ \rightarrow \pi^+\pi^0$ AMPLITUDE
IN THE QUENCHED APPROXIMATION^{*)}

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A B S T R A C T

We compute the matrix element: $\lambda_{LL} = \langle K^0 | \bar{d}\gamma_\mu(1-\gamma_5)s \cdot \bar{d}\gamma^\mu(1-\gamma_5)s | K^0 \rangle$ on a $10^3 \times 20$ lattice, $\beta = 6.0$, in the quenched approximation, and for six values of the pseudo-scalar meson mass ranging from 0.50 to 0.75 GeV. We observe a smooth dependence from m_p^2 , as expected from chiral perturbation theory. Extrapolating to the chiral limit and using the Wilson coefficient of continuum QCD, we obtain the $K^+ \rightarrow \pi^+\pi^0$ amplitude: $A = (8 \pm 3)10^{-8}$ to be compared to the experimental value: $A = 3.7 \times 10^{-8}$. The ratio of λ_{LL} to its vacuum-saturation value is: $B = 0.7 \pm 0.3$, at a momentum scale of the order of the inverse lattice spacing: $a^{-1} = 1.6 \pm 0.2$ GeV. We calculate also the matrix element $\lambda_{LR} = \langle K^0 | \bar{d}\gamma_\mu(1-\gamma_5)s \cdot \bar{d}\gamma^\mu(1+\gamma_5)s | K^0 \rangle$, which exhibits the expected chiral behaviour and is approximately consistent with its vacuum-saturation value.

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Non-leptonic weak amplitudes pose a fascinating challenge to lattice QCD. Theoretical analysis shows¹⁾ that the $\Delta I = 3/2$ amplitudes in K decay can be computed without great complications, unlike $\Delta I = 1/2$ amplitudes which are affected by severe ultra-violet divergences. First results concerning the matrix elements of four-fermion operators between pseudoscalar meson states have already been reported²⁾⁻⁴⁾.

In this paper, we present a Monte Carlo calculation of the $K^+ \rightarrow \pi^+\pi^0$ amplitude on a $10^3 \times 20$ lattice, with $\beta = 6.0$ and in the quenched approximation. As a side result, we have determined the $K^0 - \bar{K}^0$ matrix element of a $\Delta S = 2$ operator involving left-handed and right-handed currents. The latter result is of interest in that it provides a further check of the chiral behaviour of the lattice matrix elements.

Using the Wilson short-distance expansion, the effective, $\Delta I = 3/2$, $\Delta S = -1$ Hamiltonian can be written according to⁵⁾:

$$H_{3/2} = \frac{G \sin \theta_c \cos \theta_c}{2\sqrt{2}} C_4(\mu) O_4(\mu) \quad (1)$$

with μ a subtraction point and:

$$O_4 = \bar{s} \gamma_\mu (1-\gamma_5) d \bar{u} \gamma_\mu (1-\gamma_5) u + \\ + \bar{s} \gamma_\mu (1-\gamma_5) u \bar{u} \gamma_\mu (1-\gamma_5) d - \\ - \bar{s} \gamma_\mu (1-\gamma_5) d \bar{d} \gamma_\mu (1-\gamma_5) d \quad (2)$$

First-order $SU(3) \times SU(3)$ chiral perturbation theory relates⁶⁾ the $K^+ \rightarrow \pi^+\pi^0$ matrix element of $H_{3/2}$ to the $K^0 \rightarrow \bar{K}^0$ matrix element of an appropriate $\Delta S = 2$ operator which belongs to the same 27-plet of $SU(3)$, according to the formula^{*)}:

$$\langle \pi^+\pi^0 | H_{3/2} | K^+ \rangle = \frac{G \sin \theta_c \cos \theta_c}{2\sqrt{2}} C_4(\mu) \frac{3}{4} \frac{m_K^2 - m_\pi^2}{F_\pi} \frac{\lambda_{LL}}{m_K^2} \quad (3)$$

where $F_\pi = 93$ MeV and:

$$\lambda_{LL} = \langle K^0 | O_{LL}(\mu) | \bar{K}^0 \rangle \quad (4)$$

*) Here and in the following we refer to the Feynman invariant matrix elements.

$$O_{LL} = \bar{d} \gamma_\mu (1 - \gamma_5) s \bar{d} \gamma_\mu (1 - \gamma_5) s \quad (5)$$

According to chiral perturbation theory, λ_{LL}/m_K^2 in Eq. (3) has to be computed in the chiral limit, $m_K^2 = 0$, and Eq. (3) holds up to terms of higher order in m_K^2 . The numerical calculation described below gives:

$$\frac{\lambda_{LL}}{m_K^2} \Big|_{\text{chiral limit}} = (0.05 \pm 0.02) \text{ GeV}^2 \quad (6)$$

The evaluation of the Wilson coefficient C_4 is entirely conventional. At $\mu = 1.6$, which is appropriate for our lattice, and for $\Lambda_{\text{QCD}} = 0.2 \text{ GeV}$, we obtain:

$$C_4(1.6 \text{ GeV}) = 0.5 \quad (7)$$

Inserting (6) and (7) into Eq. (3), we obtain^{*}:

$$\frac{\langle \pi^+ \pi^0 | H_{3/2} | K^+ \rangle}{m_K} \Big|_{\text{theory}} = (8 \pm 3) \cdot 10^{-8} \quad (8)$$

to be compared with the experimental result:

$$\frac{\langle \pi^+ \pi^0 | H_{3/2} | K^+ \rangle}{m_K} \Big|_{\text{expt}} = 3.7 \cdot 10^{-8} \quad (9)$$

Equation (6) can also be compared to the so-called vacuum saturation result. We find:

$$B(\mu = 1.6 \text{ GeV}) \equiv \frac{\lambda_{LL}}{m_K^2} \left(\frac{16}{3} F_K^2 \right)^{-1} = 0.7 \pm 0.3 \quad (10)$$

($F_K \sim 1.25 F_\pi$). With further, more restrictive assumptions, λ_{LL} can be related to the $K_L - K_S$ mass difference⁷⁾. We find λ_{LL} to be positive, which corresponds to $m_L > m_S$, as observed.

The agreement between Eqs. (8) and (9) can be considered to be rather good, given the fact that this is a calculation from first principles. We stress that our result is independent of the value of the subtraction point μ , unlike what

^{*}) We have used: $G_F^2 = 1.023 \times 10^{-5}$; $\sin \theta_c = 0.213$.

happens in other approximation schemes, at least if one is close enough to the continuum limit. It will be very interesting to see if the agreement persists or improves at larger values of β , where the impact of lattice artefacts is reduced, and beyond the quenched approximation.

We describe now our calculation. We have used the quark propagators computed in Ref. 8), where the chiral properties of lattice QCD have been studied. The propagators are computed for 15 gauge field configurations, at three values of the Wilson parameter K :

$$K = 0.1515 ; 0.1530 ; 0.1545 \quad (11)$$

This allows six possible quark mass combinations in the pseudoscalar density correlation function:

$$G_5(t) = \sum_{\vec{x}} \langle P_5(\vec{x}, t) P_5(0) \rangle \quad (12)$$

$$P_5 = \bar{q}_1 \gamma_5 q_2$$

We have determined the pseudoscalar and vector meson masses by fitting to the appropriate correlation functions the single particle expression, e.g.:

$$G_5^{SP}(t) = Z_5 \cosh \left[m_p \left(t - \frac{T}{2} \right) \right] \quad (13)$$

in the range:

$$6 \leq t \leq 14 \quad (14)$$

The one-particle propagator gives a very good description of $G_5(t)$ in the range (14). In the same range, one-particle saturation is also a good approximation when the four-fermion operator is inserted^{*)}.

The matrix element λ_{LL} , Eq. (4) can be obtained²⁾ from the three-point correlation function :

*) The wider range $3 \leq t \leq 17$ was used in Ref. 8) with a two-cosine fit to $G_5(t)$. The range (14) is more appropriate to reduce the systematic errors when dealing with three-point functions as in the present case. Meson masses and the lattice spacing obtained with (14) are consistent with those found previously.

$$G(t) = \sum_{\vec{x}, \vec{y}, y_0} \langle P_5(\vec{x}, t) \hat{O}_{LL}(\vec{y}, y_0) P_5(0) \rangle \quad (15)$$

fitting it to the one-particle approximation^{*)}:

$$G^{SP}(t) = - \frac{\lambda_{LL}}{2m_p} Z_5 \left(t - \frac{T}{2}\right) \sinh \left[m_p \left(t - \frac{T}{2}\right)\right] + \\ + Z \cosh \left[m_p \left(t - \frac{T}{2}\right)\right] \quad (16)$$

To reduce the statistical fluctuations, it is actually more convenient²⁾ to fit Eq. (15) with the equivalent expression :

$$G^{SP}(t) = - \frac{\lambda_{LL}}{2m_p} \frac{Z_5}{Z_0} \left(t - \frac{T}{2}\right) G_0(t) + \\ + Z' G_5(t) \quad (17)$$

where:

$$G_0(t) = \sum_{\vec{x}} \langle A_0(\vec{x}, t) P_5(0) \rangle \\ G_0^{SP}(t) = Z_0 \sinh \left[m_p \left(t - \frac{T}{2}\right)\right] \quad (18)$$

and A_μ is the local axial current, $\bar{q}_1 \gamma_\mu \gamma_5 q_2$. Finally, the operator $\hat{O}_{LL}$ in Eq. (15) is:

$$\hat{O}_{LL} = O_{LL} + \delta O_{LL} \quad (19)$$

where O_{LL} is the bare operator defined in Eq. (5) and δO_{LL} the perturbative correction computed in Refs. 9) and 4) [for a detailed discussion, see Ref. 1)].

We present in Fig. 1 the results for λ_{LL} as a function of the pseudoscalar mass-squared $(m_p)^2$ (full circles). For comparison, the two points obtained in Ref. 2) are reported (open circles) and seen to be consistent.

The chiral behaviour ($\lambda_{LL} \propto m_p^2$, for $m_p^2 \rightarrow 0$) is very well supported by our results. The full line in Fig. 1 represents a quadratic fit to our data:

^{*)} The factor $(2m_p)^{-1}$ was missing in Ref. 2). This has little numerical consequences, since $2m_p \sim 1$ there. We have reintroduced this factor when comparing with our results.

$$\lambda_{LL} a^4 = A_1 (am_p)^2 + A_2 (am_p)^4 \quad (20)$$

Combining with:

$$a^{-1} = (1.6 \pm 0.2) \text{ GeV} \quad (21)$$

we find:

$$\begin{aligned} a^{-2} A_1 &\equiv \frac{\lambda_{LL}}{m_p^2} \Big|_{\text{chiral limit}} = (0.05 \pm 0.02) \text{ GeV}^2 \\ A_2 &= 0.07 \pm 0.05 \end{aligned} \quad (22)$$

which leads to the results stated in Eqs. (8) and (10). The errors given are purely statistical, and have been obtained as discussed in Ref. 8). A few comments are in order.

- (i) The $(m_p)^4$ correction in Eq. (20) is of the size expected in chiral perturbation theory: at the kaon mass, it corresponds to a 30% correction. It would be interesting to see if the same holds true in a calculation with dynamical fermions, where terms of the order of $m_p^4 \ln(m_p^2)$ may become important¹⁰⁾.
- (ii) The subtraction δO_{LL} is computed with the perturbation theory, so that the correct chiral behaviour of $\hat{O}_{LL}$ is expected to appear only at sufficiently large values of β . Previous experience¹¹⁾ and our results about the normalization constants of the chiral currents⁸⁾ indicate that $\beta = 6.0$ is barely sufficient to suppress non-perturbative effects. A new calculation with $\beta > 6.2$ would be quite illuminating, and it would allow for the extension of the calculation to smaller values of $(m_p)^2$.

It is also interesting to compute the matrix element of the L-R operator:

$$\begin{aligned} O_{LR} &= \bar{d} \gamma_\mu (1 - \gamma_5) s \bar{d} \gamma_\mu (1 - \gamma_5) s \\ \lambda_{LR} &= \langle K^0 | O_{LR} | \bar{K}^0 \rangle \end{aligned} \quad (23)$$

Including the perturbative correction $\delta O_{LR}^{(9),4)}$, we obtain the results reported in Fig. 2. The solid line is the best linear fit to our data:

$$\lambda_{LR} a^4 = A'_1 + A'_2 (m_P a)^2 \quad (24)$$

and we find:

$$\begin{aligned} \lambda_{LR} \Big|_{\substack{\text{chiral} \\ \text{limit}}} &\equiv a^{-4} A'_1 = - (0.14 \pm 0.08) \text{GeV}^4 \\ A'_2 &= - 0.004 \pm 0.02 \end{aligned} \quad (25)$$

λ_{LR} does not vanish for $m_P^2 \rightarrow 0$, as expected, and is compatible with the vacuum saturation result:

$$(\lambda_{LR})_{vac} = - \frac{8}{3} \frac{F_K^2 m_K^2}{m_S} \simeq - 0.10 \text{GeV}^4 \quad (26)$$

for $m_S = 150 \text{ MeV}$.

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FIGURE CAPTIONS

Fig. 1 The $K_0 - \bar{K}_0$ matrix element of $\hat{O}_{LL}$ as a function of the kaon mass-squared. All quantities are in lattice units. Full dots: this work; open dots: Ref. 2). The curve represents the quadratic fit, Eq. (20) of text.

Fig. 2 The $K_0 - \bar{K}_0$ matrix element of $\hat{O}_{LR}$. The line is the linear fit, Eq. (24) of text.

- Figure 1 -

- Figure 2 -