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UNIFICATION OF THE POINCARÉ GROUP WITH BRST
AND PARISI-SOURLAS SUPERSYMMETRIES

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ABSTRACT

The principles of quantum mechanics are used to derive the second-quantized field theory from the classical point particle. The fields of the field theory inevitably depend on two extra bosonic and two extra anticommuting co-ordinates. Previous treatments have used incorrect choices to fix the gauge for reparametrization invariance. The second-quantized BRST action is invariant under the supergroup $\text{IOSp}(D, 2/2)$ which contains the Poincaré group as well as Parisi-Sourlas supersymmetries. One of the extra bosonic co-ordinates is the remnant for the point particle of a string length.

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In a series of papers, starting from the Virasoro conditions and culminating in Ref. 1), a gauge-invariant formulation of the free open bosonic string was found. To incorporate the many supplementary fields required in order that the Virasoro conditions follow from an action, we were led to introduce two anticommuting co-ordinates, which coincide with those found in the first-quantized BRST approach of Kato and Ogawa²⁾. It also has the remarkable feature that the second-quantized gauge-invariant field theory makes use of the first-quantized BRST charge, Q , the action^{1),3)} being of the form $\chi Q \chi$, with gauge transformation $\delta \chi = Q \Lambda$.

In extending this work to the interacting theory, we^{4),5)} and the Kyoto group⁶⁾ joined open strings by their end points. We thus recovered the beautiful picture of the splitting and joining of strings found in the light-cone theory⁷⁾. The apparent price was that while the string lengths in the light-cone are related to p^+ , in the covariant theory the string lengths had to be introduced by hand. Despite this, it was shown⁵⁾ that the on-shell scattering amplitude for three strings did not contain any dependence on the string lengths. The method of proof indicated that any on-shell tree amplitude was independent of the string lengths and coincided with the Koba-Nielsen amplitude.

In fact, the technical reason for the introduction of the string lengths, at least in the work of Refs. 3) and 4), was the need to solve the so-called duality problem. The introduction of string lengths, however, has the disadvantage of giving an infinite degeneracy, corresponding to the value of the string length, α .

A further point of concern is that in the light-cone theory the $p^+ > 0$ and $p^+ < 0$ correspond to the strings being outgoing and incoming respectively. This property was carried⁴⁾ over to the string lengths in the gauge-covariant theory. However, to actually implement this, rather than state it in an ad hoc way one requires a light-cone-like time, τ . In the usual loose discussion of the quantization of the point particle, one indeed loses the parameter τ of the string path and its associated invariance. We observed that this loss of invariance in the process of quantization is unusual and presumably should not occur in a correct treatment⁸⁾.

Given that we require two additional bosonic co-ordinates, the string lengths and τ , we have to explain how they disappear from physical processes. The obvious candidate is a Parisi-Sourlas⁹⁾ type of symmetry. In fact, such a symmetry was apparent in the Hamiltonian of the free theory for the non-zero modes¹⁾

$\sum_{n=1}^{\infty} n (\alpha_n^+ \alpha_n + \beta_n^+ \bar{\beta}_n + \bar{\beta}_n^+ \beta_n)$, together with its effect on the on-shell counting of states, where one finds that 24 (light-cone count) $= 26-2 =$ number of bosonic-number of fermionic dimensions. This symmetry is also present in the interaction vertex [See Eqs. (1) and (32) of Ref. 4)]. It is obvious that the disappearance of α - and τ -dependence from physical processes is due to an extension of this Parisi-Sourlas mechanism.

An alternative formulation of the interacting open bosonic string was given in Ref. 3). This formulation joins strings at their mid-points and so does not need string lengths. There appear, however, to be difficulties in this formulation with closed bosonic strings.

In this paper we demonstrate that the above discussion is indeed what happens. To do this, we need look no further than the point particle and derive its second-quantized formulation according to the usual rules of quantum mechanics, taking care to adopt a correct gauge-fixing procedure and not neglect boundary condition considerations. We show that the point particle $x^\mu(\tau)$ ($\mu = 0, 1$ to $D-1$) naturally lives in a space which has $D+2$ bosonic co-ordinates and two anticommuting co-ordinates. The extra co-ordinates disappear from physical quantities by the Parisi-Sourlas mechanism as a result of gauge-fixing a local symmetry in the second-quantized theory. The latter local symmetry originates from the reparametrization symmetry of the world line in the classical point particle theory. In fact, one finds that BRST and anti-BRST symmetries are none other than two of the Parisi-Sourlas symmetries.

There are innumerable discussions in the literature of the first quantization of the classical point particle which has an action that can be written in the form

$$A = \int_0^T d\tau [p_\mu \dot{x}^\mu - e^\omega (p_\mu^2 + m^2)] \quad (1)$$

This action is invariant under the following local transformations

$$\delta x^\mu = 2e^\omega \varepsilon p^\mu, \delta p^\mu = 0, \delta \omega = \bar{e}^\omega \frac{d(\varepsilon e^\omega)}{d\tau} \quad (2)$$

which are generated by the Hamiltonian. However, it was pointed out¹⁰⁾ many

years ago that this invariance only held if $\epsilon(0) = \epsilon(T) = 0$, and consequently the gauge $\lambda = e^\omega = \text{constant}$ is not allowed. Use of this gauge has been made in string theory, in Refs. 2) and 11). The first paper to carry out correctly the first quantization of the free point particle is given in Ref. 12). We now repeat some of the steps given in this reference, taking a slightly different gauge, namely $\dot{\omega} = 0$, which is more convenient for the second quantization, to be discussed shortly. It is easily found that the corresponding BRST action is given by¹²⁾

$$H = \int_0^T dt [p_\mu \dot{x}_\mu - e^\omega (p_\mu^2 + m^2) + b \dot{\omega} - \dot{\bar{c}} \rho - \bar{\rho} \dot{c} + \bar{\rho} \rho e^\omega] \quad (3)$$

and is invariant under

$$\begin{aligned} \delta x^\mu &= \lambda \lambda c e^\omega p^\mu, \quad \delta p^\mu = 0, \quad \delta \omega = e^\omega \lambda \rho, \quad \delta \rho = 0 \\ \delta \bar{c} &= -\lambda e^\omega b + \lambda c \bar{\rho} e^\omega, \quad \delta c = \lambda c e^\omega \rho \\ \delta \bar{\rho} &= e^\omega \lambda (p_\mu^2 + m^2 - \bar{\rho} \rho), \quad \delta b = -\lambda e^\omega \{ c (p_\mu^2 + m^2 - \bar{\rho} \rho) + \rho b \} \end{aligned} \quad (4)$$

These transformations are generated in the sense of $\delta \cdot = [\cdot, \lambda Q]$ by the BRST charge which is given by

$$Q = e^\omega c (p_\mu^2 + m^2 - \bar{\rho} \rho) + e^\omega \rho b \quad (5)$$

The Hamiltonian can be read off from Eq. (3) and is

$$\mathcal{H} = e^\omega (p_\mu^2 + m^2 - \bar{\rho} \rho) \quad (6)$$

It is given by

$$\mathcal{H} = - \{ Q, \bar{\rho} \} \quad (7)$$

and obviously commutes with Q . One can also consider the anti-BRST charge which is given by

$$\bar{Q} = e^\omega \{ \bar{c} (p_\mu^2 + m^2 - \bar{\rho} \rho) + \bar{\rho} b \} \quad (8)$$

and obeys the relations

$$Q^2 = \bar{Q}^2 = \{Q, \bar{Q}\} = 0, \quad \mathcal{H} = \{\bar{Q}, P\} \quad (9)$$

In the above, x^μ , p^μ , c , p and ω are Hermitian while $\bar{c}$ and $\bar{p}$ are antiHermitian. We now first-quantize the above system by the usual rule

$$\{, \}_{\pm} \rightarrow \frac{1}{i\hbar} [,]_{\pm} \quad (10)$$

both for commutation and anticommutation relations. The resulting relations can be represented by the following operator replacements:

$$\begin{aligned} x^\mu &\rightarrow x^\mu, \quad c \rightarrow c, \quad \bar{c} \rightarrow \bar{c}, \quad \omega \rightarrow \omega \\ p^\mu &\rightarrow -i \frac{\partial}{\partial x^\mu}, \quad p \rightarrow i \frac{\partial}{\partial \bar{c}}, \quad \bar{p} \rightarrow -i \frac{\partial}{\partial c}, \quad b \rightarrow -i \frac{\partial}{\partial \omega} \end{aligned} \quad (11)$$

Upon making these replacements, we find that

$$\mathcal{H} = e^\omega \left(-\frac{\partial^2}{\partial x^2} + m^2 - \frac{\partial}{\partial c} \frac{\partial}{\partial \bar{c}} \right) \quad (12)$$

and

$$\begin{aligned} Q &= e^\omega c \left(-\partial_m^2 + m^2 - \frac{\partial}{\partial c} \frac{\partial}{\partial \bar{c}} \right) + e^\omega \frac{\partial}{\partial \bar{c}} \frac{\partial}{\partial \omega} \\ \bar{Q} &= e^\omega \bar{c} \left(-\partial_m^2 + m^2 - \frac{\partial}{\partial c} \frac{\partial}{\partial \bar{c}} \right) - e^\omega \frac{\partial}{\partial c} \frac{\partial}{\partial \omega}. \end{aligned} \quad (13)$$

We now consider the second-quantized BRST theory, the action for which should lead to a Schrödinger equation that tells us that the time development in τ is generated by the Hamiltonian $\mathcal{H}$. We consider a function $\chi(z, \tau)$ of the co-ordinates $z = (x^\mu, \omega, c)$ and proper time τ . The action is

$$\int dz d\tau \quad \chi^* \left(i \frac{\partial}{\partial \tau} - \mathcal{H} \right) \chi \quad (14)$$

Since $\mathcal{H}$ and $d/d\tau$ commute with Q and $\bar{Q}$, we observe the symmetries

$$\delta \chi = i \wedge \varphi \chi, \quad \delta \bar{\chi} = i \bar{\wedge} \bar{\varphi} \bar{\chi} \quad (15)$$

where $\wedge$ and $\bar{\wedge}$ are constant anticommuting antiHermitian parameters.

The reader recognizes that $\mathcal{H}$ is an $\text{IOSp}(D-1, 1/2)$ invariant operator if $\mu = 0, \dots, D-1$. However, our experience with light-cone theories leads us to identify $e^{-\omega} = \alpha$ and $\chi^*[\dots, \lambda] = \chi[\dots, -\lambda]$. The co-ordinate α is the remnant of the string length for the point particle and is the momentum conjugate to y^- . Introducing the Fourier transform

$$\chi(\alpha) = \frac{\alpha}{2\pi} \int_{-\infty}^{\infty} dy^- e^{i\alpha y^-} \tilde{\chi}(y^-) \quad (16)$$

where $\tilde{\chi}(y^-)$ is a real function, the action of Eq. (14) becomes

$$\int dx^\mu d\bar{c} dc dy^- d\tau \tilde{\chi} \left(\eta^{\hat{\mu}\hat{\nu}} \frac{\partial}{\partial y^{\hat{\mu}}} \frac{\partial}{\partial y^{\hat{\nu}}} + m^2 - \frac{\partial}{\partial c} \frac{\partial}{\partial \bar{c}} \right) \chi \quad (17)$$

where

$$y^{\hat{\mu}} = x^\mu \quad \text{for } \mu = \hat{\mu} = 0, 1, \dots, D-1 \quad (18)$$

and

$$y^- \equiv y^{D-1} - y^D, \quad y^+ \equiv \tau \equiv y^{D-1} + y^D \quad (19)$$

This action, which is, of course, the same as that of Eq. (14), is invariant under the group $\text{IOSp}(D, 2/2)$. Let us denote the generators of this group by

$$J^{\mu\nu}, J^{+\mu}, J^{-\mu}, J^{+-}, J^{A\mu}, J^{A+}, J^{A-}, J^{AB} \quad (20)$$

where $A, B = 1, 2$ refer to the c and $\bar{c}$ co-ordinates respectively.

It is instructive to evaluate (14) in component fields. Using

$$\chi = \bar{\Psi} + c \Theta + \bar{c} \bar{\Theta} + c \bar{c} \psi \quad (21)$$

we find it becomes

$$\int d\tau^u d\tau d\omega \left[\bar{\Psi}^* i \frac{d}{d\tau} \Psi + \bar{\Theta}^* i \frac{d}{d\tau} \Theta - \bar{\Psi}^* e^{-\omega} (-\partial^2 + m^2) \Psi - e^{\omega} \bar{\Theta}^* (-\partial^2 + m^2) \Theta - e^{\omega} \psi^* \psi + h.c. \right] \quad (22)$$

Making a similar replacement in Eq. (17), we recognize an action discussed by Parisi and Sourlas⁹⁾ which has the symmetry

$$\begin{aligned} \delta \Psi &= 2 \varepsilon^{\hat{\nu}} \frac{\partial}{\partial y^{\hat{\nu}}} \Theta, \quad \delta \bar{\Psi} = \varepsilon^{\hat{\nu}} y^{\hat{\nu}} \bar{\Theta} \\ \delta \Theta &= 0, \quad \delta \bar{\Theta} = -(\varepsilon^{\hat{\nu}} y^{\hat{\nu}} \Psi + 2 \varepsilon^{\hat{\nu}} \frac{\partial}{\partial y^{\hat{\nu}}} \bar{\Psi}). \end{aligned} \quad (23)$$

where $\varepsilon_{\hat{\nu}}$ is an anticommuting parameter. These are the symmetries $J^{A\mu}$ when we also include a similar transformation with Θ and $\bar{\Theta}$ interchanged.

Of course, in the light-cone formalism, in which the action of Eq. (14) is written, the generators involving the $\pm$ co-ordinates take on a different form. In particular, one finds that

$$J^{c-} = \frac{c}{\alpha} (p_{\mu}^2 + m^2 - \bar{p} p) - \frac{1}{\alpha} p y^- = \bar{Q} \quad (24)$$

Similarly, we recognize $J^{\bar{c}-}$ to be generated by $\bar{Q}$. Consequently, the BRST and anti-BRST transformations appear as two of the Parisi-Sourlas generators which are contained in $IOSp(D, 2/2)$ and are therefore contained in the same group as the Poincaré group.

The three generators $J^{(AB)}$ generate transformations only in the $c, \bar{c}$ sector which are

$$c \rightarrow ec + f\bar{c}, \quad \bar{c} \rightarrow g\bar{c} + hc. \quad (25)$$

and preserve the form $c\bar{c}$. Consequently, we find that $eg-fh = 1$ and so they generate the group $SU(1,1)$.

Having specified the BRST action, we must also give the physical-state-defining condition, which is $Q\chi = 0$ as well as demanding that χ have zero ghost number. We note that on fields with zero ghost number the action of Q and $\bar{Q}$ is identical, namely in components we find that

$$\square \Psi + \Psi - \frac{\partial \Psi}{\partial \omega} = 0, \quad \square \equiv (-\partial^2 + m^2) \quad (26)$$

However, any states of the form $Q\eta$ will have zero scalar product with themselves and all physical states. Hence, as usual, we can in effect neglect such spurious states. Consequently, we regard any two states χ and χ' to be equivalent if

$$\chi' = \chi + QN \quad (27)$$

and from each equivalence class we may choose a specific representative. We note that if N is to have the correct ghost number then it is of the form $N = \bar{c}\eta$, and in terms of component fields we find that

$$\Psi' = \Psi + e^{\frac{\omega}{\partial \omega}} \eta, \quad \Psi' = \Psi + e^{\omega} (-\partial^2 + m^2) \eta. \quad (28)$$

Using η , we may set ψ' to zero, leaving the remaining choice to be η subject to $\square \eta = 0$. However, from Eq. (26), Ψ' is also subject to $\Psi' = 0$. At first sight it would seem that one could also remove Ψ' , but this is prevented by a subtlety due to boundary conditions. The action is only defined if all the fields, and thus η , vanish when $\omega = \pm \infty$ (or when $\alpha = 0$ and ∞). It is not then possible to use the first of Eqs. (28) to set $\Psi' = 0$; however, we can set the ω (i.e., α) dependence of Ψ' to be constant, i.e., $\partial \Psi' / \partial \omega = 0$, or indeed any prescribed dependence on ω .

The final result of this analysis is that we have one field Ψ which is a function of x^μ and τ and satisfies $\square \Psi = 0$. The equation of motion then sets

$(d/d\tau)\Psi = 0$, and we recover the usual result, namely one scalar particle. Note that, were it not for the above subtlety, then the cohomology of Q would be trivial and the theory without content.

It is important to stress that although the BRST action of Eq. (14) is $\text{IOSp}(D, 2/2)$ -invariant, the physical-state-defining condition is not. As we have seen above, the physical states have no τ -dependence, but still some arbitrary but prescribed α (or y^-) dependence. Hence, an extension of the argument of Eq. (9) of Ref. 9) is needed to get rid of this α -dependence in physical amplitudes. The right-hand side of this equation now reads in our notation

$$\int d^{D-2} x dy^- dy^+ d\bar{c} dc F(\gamma_i x + Y^- y^+, x^2 + y^- y^+ + c\bar{c}) \quad (29)$$

Going Euclidean on the $y^- y^+$ variables, one writes

$$y^- = r e^{-i\theta}, \quad y^+ = r e^{i\theta}. \quad (30)$$

In (29), the angular θ integral then in effect sets y^- to zero, and the r and $c\bar{c}$ integrals cancel each other out as in Ref. 9). Setting y^- to zero implies in Fourier-transformed language that the physical amplitudes are α -independent, and the α -dependence of the external state wave functions is indeed irrelevant.

For the free point particle it only remains to give the gauge-covariant action from which the BRST action of Eq. (14) can be found by quantization. Consider the action

$$\int dx^\mu \frac{d\alpha}{d\tau} d\bar{c} dc d\tau \left\{ \chi_1^* i \frac{d}{d\tau} \chi_1 - \chi_2^* \phi \chi_1 - \chi_1^* \phi \chi_2 \right\} \quad (31)$$

where

$$\chi_1^*(\dots, \alpha) = \chi(\dots, -\alpha). \quad (32)$$

and similarly for χ_2 . This action possesses the local gauge symmetry

$$\delta \chi_1 = \phi \wedge, \quad \delta \chi_2 = i \frac{d}{d\tau} \wedge \quad (33)$$

$$\delta \chi_1^* = -\Lambda^* Q, \quad \delta \chi_2^* = -\Lambda^* i \frac{d}{d\tau} \quad (34)$$

where $\Lambda[\dots, \alpha]^* = \Lambda[\dots, -\alpha]$. While χ_1 has ghost number zero, χ_2 and Q have ghost number +1 and -1 respectively. Consequently, we have the component expansions

$$\chi_1 = \bar{\Psi}_1 + c \bar{c} \Psi_1, \quad \chi_2 = \bar{c} \Psi_2, \quad \Lambda = \bar{c} \Theta \quad (35)$$

and the resulting field equations are

$$\begin{aligned} -i \frac{d}{d\tau} \bar{\Psi}_1 + e^\omega \frac{\partial \Psi_2}{\partial \omega} &= 0, \quad -i \frac{\partial \Psi_1}{\partial \tau} + e^\omega (-\partial^2 + m^2) \Psi_2 = 0 \\ (-\partial^2 + m^2) \bar{\Psi}_1 + \Psi_1 - \frac{\partial \Psi_1}{\partial \omega} &= 0 \end{aligned} \quad (36)$$

while the gauge invariances are

$$\delta \Psi_1 = (-\partial^2 + m^2) e^\omega \Theta, \quad \delta \bar{\Psi}_1 = e^\omega \frac{\partial \Theta}{\partial \omega}, \quad \delta \Psi_2 = +i \frac{\partial \Theta}{\partial \tau} \quad (37)$$

Let us first recover classically the correct count, that is, one scalar field. Any legitimate gauge choice of course works, but to be definite let us begin by using Θ to set $\phi_1 = 0$ with the Θ -invariance remaining provided $\Box \Theta = 0$. The field equations imply that $(-\partial^2 + m^2) \phi_2 = 0$ and we may then set $\phi_2 = 0$, while now the residual invariance is $(-\partial^2 + m^2) \Theta = 0$ and $(d/d\tau) \Theta = 0$. Finally, recalling our previous discussion on the ω (i.e., α) boundary condition, we may set $\bar{\Psi}_1$ to be independent of ω . Then we are left with $\bar{\Psi}_1$ subject to $(-\partial^2 + m^2) \bar{\Psi}_1 = 0$.

We now BRST-quantize the action of Eq. (31) utilizing the gauge choice $\phi_2 = \phi_1$. The corresponding action in component fields is

$$\begin{aligned} \int dx^\mu d\omega d\tau \{ & \bar{\Psi}_1^* (i \frac{\partial \Psi_1}{\partial \tau} - e^\omega (-\partial^2 + m^2) \Psi_2) + \Psi_1^* (i \frac{d}{d\tau} \bar{\Psi}_1 - e^\omega \frac{\partial \Psi_2}{\partial \omega}) \\ & - \Psi_2^* e^\omega ((-\partial^2 + m^2) \bar{\Psi}_1 + \Psi_1 - \frac{\partial \Psi_1}{\partial \omega}) + b^* (-\Psi_1 + \Psi_2) + b (\Psi_1^* + \Psi_2^*) \\ & + \bar{\Theta}^* (-e^\omega (-\partial^2 + m^2) + i \frac{d}{d\tau}) \Theta + \Theta^* (i \frac{d}{d\tau} - e^\omega (-\partial^2 + m^2)) \bar{\Theta} \} \end{aligned} \quad (38)$$

where the ghost and antighost are also denoted by Θ and $\bar{\Theta}$ respectively. The corresponding BRST transformations are

$$\begin{aligned}\delta \psi_1 &= e^\omega (-\partial^2 + m^2) \lambda \Theta, \quad \delta \bar{\psi}_1 = e^\omega \frac{\partial}{\partial \omega} (\lambda \Theta), \quad \delta \psi_2 = -i \frac{\partial}{\partial \tau} (\lambda \Theta) \\ \delta \bar{\Theta}^* &= -\lambda b^*, \quad \delta \Theta = \delta b = 0.\end{aligned}\tag{39}$$

Upon implementing the constraint $\psi_1 + \psi_2 = 0$ by integrating over b , we recover in ω , i.e., α , space the action of Eq. (14). The BRST transformations of Eq. (39) become, upon elimination of b , the BRST transformations of Eq. (15), upon suitable redefinition of $\Psi_1, \psi_2, \bar{\Theta}$ and Θ so as to fit them in χ .

To go to y -space, one uses the transformations

$$\begin{aligned}\chi_1(\dots, \alpha) &= \frac{\alpha}{2\pi} \int_{-\infty}^{+\infty} dy^- e^{i\alpha y^-} \tilde{\chi}_1(\dots, y^-) \\ \chi_2(\dots, \alpha) &= \frac{\alpha}{2\pi} \int_{-\infty}^{+\infty} dy^- e^{i\alpha y^-} \hat{\chi}_2(\dots, y^-).\end{aligned}\tag{40}$$

It is important to note that the second-quantized gauge-invariant action of Eq. (31) is not $\text{IOSp}(D, 2/2)$ -invariant. Although it leads to the $\text{IOSp}(D, 2/2)$ -invariant BRST action, the physical state conditions which follow from the gauge-invariant action in the usual way are not $\text{IOSp}(D, 2/2)$ -invariant. In this way, one has not simply acquired a light-cone formulation in a higher dimension.

A correct application of the principles of quantum mechanics has inevitably led us to a formulation of second-quantized field theory that has two extra bosonic and two extra fermionic co-ordinates. One of the extra bosonic co-ordinates is the point-particle remnant of the string lengths introduced into covariant string theory in Refs. 4)-6). This is yet another example where string theory has provided the motivation to discover new phenomena in ordinary quantum field theory. Here, we find the unification of the Poincaré group with Parisi-Sourlas and BRST supersymmetries. This is the first time that Parisi-Sourlas symmetry appears in particle field theory, allowing a unification between a space-time group and the somewhat ad hoc BRST symmetry. It is possible that this work will provide a natural framework for previous attempts at such unifications¹³⁾.

By including space-time symmetries in the same group as symmetries of the two-dimensional world sheet acting on τ and σ (via the string length), we can

hope to understand why so many features of strings (supersymmetry, anomalies, BRST....) could be understood either from the space-time or from the world-sheet points of view.

The extension of the ideas of this paper to the bosonic string is straightforward, and will be presented elsewhere. One finds in particular that the world sheet metric cannot be set equal to a constant, and that one extra degree of freedom cannot be gauged away. This remaining degree of freedom is none other than the ω given in this paper for the point particle. As well as ω and τ , one finds one additional anticommuting mode over and above those required in Ref. 2). The BRST and gauge-invariant actions have the same generic form as those of Eqs. (14) and (31) of this paper. A detailed treatment of the supersymmetric case will also be given.

While this work was in progress, we received the paper of Ref. 14). It also discusses the group $\text{IOSp}(D, 2/2)$. As far as we can understand, it seems that the postulates which it makes are quite different from the derived results given in our paper. This is particularly apparent in the absence of τ from the action and the different status of the x^- and $c\bar{c}$ co-ordinates which play a vital rôle in our work.

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