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ON THE CONSISTENCY OF SUPERSYMMETRIC INSTANTON CALCULATION
IN CHIRAL GAUGE THEORIES

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ABSTRACT

Consistency of the algorithm of Amati and others for determining the properties of vacua in supersymmetric gauge theories, based on the instanton calculation at short distances, is further examined. When the summation over the degenerate vacua implicit in the instanton calculation is properly taken into account, the results for various condensates are found to be fully consistent in all cases studied. Our result strengthens the conclusion that supersymmetry is dynamically broken in some class of chiral gauge theories.

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1. - INTRODUCTION AND SUMMARY

Deduction of the properties of the ground states in asymptotically-free and supersymmetric gauge theories¹⁾⁻¹⁴⁾, is one of those cases in physics^{*)} where the system has such an amount of symmetries as to allow one to extract detailed dynamical information before one actually solves the theory. Great care is required of course in such a deduction, and in the present problem some debates have been under way concerning two different approaches, one based on the confinement picture^{2), 4), 7), 9), 10), 12)} and the other based on a Higgs picture^{5), 6), 8), 11), 13)}.

In the first of these approaches, constancy of certain Green's functions¹⁾ is exploited to relate^{2), 4), 7)} the short-distance effects computed by the instanton method to the properties of the vacua, characterized by various condensates. It is the purpose of this note to examine the consistency of this approach, in a general class of chiral gauge theories.

Extending the conclusions of the study in the massive supersymmetric QCD (SQCD)⁹⁾ and in a chiral SU(6) theory¹⁰⁾, we find that the determination of various condensates is fully consistent, among themselves and with the anomaly of Ref. 14)⁺). In our opinion, this confirms the correctness of the approach based on the confinement picture and, in particular, the occurrence of dynamical supersymmetry breaking in certain chiral SU(N) theories with odd N [SU(5) model of Refs. 4) and 5); for general N, see the last of Ref. 6) and below].

Before starting the actual analysis, let us make a few key observations. The first is that the clustering property of the Green's functions, used crucially in the approach in the confinement picture, holds for each single vacuum of the system. When the ground state is degenerate, care must be taken: clustering does not hold in general for vacuum expectation values of products of operators, summed over degenerate vacua.

*) Applications of current algebras in low-energy hadron physics are another such example.

+) This disagrees with the claims to the opposite effect recently made in Refs. 12) and 13). From the author's point of view, the conclusions of Ref. 12) are based on an improper interpretation of the instanton calculation. See the following footnote.

On the other hand, the instanton configuration in gauge theories respects much of the global symmetries associated with matter fields [e.g., $SU(N) \times SU(N) \times U_V(1)$ in QCD]. As a consequence, the instanton contribution to Green's functions must be interpreted as a quantity summed over the degenerated vacua⁷⁾ related by these symmetries. To apply the clustering on such Green's functions in a naïve manner would lead to inconsistencies^{*)}.

2. - SU(6) MODEL: UNBROKEN SUPERSYMMETRY AND WELL-DEFINED VACUA

For the purpose of illustration of how the summation over degenerate vacua implied by the instanton calculus can be effectively undone, let us first consider in detail the chiral SU(6) theory with matter fields belonging to four $\square$ and two $\bar{\square}$ multiplets, Φ_A^α and $A_{\alpha\beta}^a$ ($\alpha, \beta = 1 \dots 6$; $A = 1 \dots 4$; $a = 1, 2$). With the superpotential^{+) ,}

$$P = h_{AB,a} U_{AB,a} + h'_{abc} V_{abc} \quad (1)$$

the theory has a discrete symmetry, a Z_{12} , which is spontaneously broken to a Z_3 (see below): there are a finite number (four) of vacua (index of Witten) and supersymmetry cannot be spontaneously broken in this model^{")}.

Let us consider the following Green's functions (or the derivatives thereof):

$$G^{(1)} = \langle T U_{AB,a}(z_1) U_{CD,b}(z_2) V_{cde}(z_3) V_{fgh}(z_4) \rangle \quad (2)$$

*) What is called the intrinsic inconsistency in Ref. 12) seems to be of this nature.

+) Throughout, we have used the improved method of Ref. 12) to repeat the calculation (and to correct some of the errors). We also follow its notation, namely, $U_{AB,a} \equiv \Phi_A^\alpha \Phi_B^\beta A_{\alpha\beta}^a$; $V_{bcd} \equiv A_{\alpha_1\alpha_2}^b A_{\alpha_3\alpha_4}^c A_{\alpha_5\alpha_6}^d \epsilon^{\alpha_1 \dots \alpha_6}$, except for the gauge field composite, which we denote by $S \equiv (g^2/16\pi^2) \text{Tr} WW = 2 \cdot S$ [Ref. 12)]. Component fields are $\Phi = \phi + \sqrt{2}\chi\theta + \dots$; $A = \eta + \sqrt{2}\chi\theta + \dots$; $W = -i\lambda + \dots$.

") Any subsymmetry of SU(4) which is not broken by the superpotential due to a particular choice of $h_{AB,a}$, will remain a manifest symmetry of the vacua also, hence does not affect this argument.

$$G^{(2)} = (-) (\partial/\partial h_{CD,b}) \langle T S U_{AB,a} V_{cde} V_{fgh} \rangle, \quad (3)$$

$$G^{(3)} = (\partial^2/\partial h_{AB,a} \partial h_{CD,b}) \langle T S S V_{cde} V_{fgh} \rangle, \quad (4)$$

$$G^{(4)} = (-) (\partial/\partial h'_{cde}) \langle T S U_{AB,a} U_{CD,b} V_{fgh} \rangle, \quad (5)$$

$$G^{(5)} = (\partial^2/\partial h'_{cde} \partial h'_{fgh}) \langle T S S U_{AB,a} U_{CD,b} \rangle, \quad (6)$$

$$G^{(6)} = (\partial^2/\partial h_{CD,b} \partial h'_{fgh}) \langle T S S U_{AB,a} V_{cde} \rangle, \quad (7)$$

$$G^{(7)} = (-) (\partial^3/\partial h_{AB,a} \partial h_{CD,b} \partial h'_{cde}) \langle T S S S V_{fgh} \rangle, \quad (8)$$

$$G^{(8)} = (-) (\partial^3/\partial h_{CD,b} \partial h'_{cde} \partial h'_{fgh}) \langle T S S S U_{AB,a} \rangle, \quad (9)$$

$$G^{(9)} = (\partial^4/\partial h_{AB,a} \partial h_{CD,b} \partial h'_{cde} \partial h'_{fgh}) \langle T S S S S \rangle. \quad (10)$$

Unless otherwise indicated [as in (3') and (8')], arguments of the Green's functions are the lowest component of the chiral superfields. $G^{(1)} \sim G^{(9)}$ are thus constant (independent of x_i)¹⁾ and get non-vanishing instanton contribution at $h_{AB,a} = h'_{cde} = 0$. This latter property is desirable because our aim is to disentangle the effects of the "true" perturbation in h and h' from the effects of summation over degenerate vacua implied by the instanton calculation.

Since we deal with the full Green's functions we have the following formulae for the derivatives with respect to $h_{AB,a}$ and h'_{cde} ,

$$G^{(2)} = - \int d^6 z \langle T S(z_1) U_{AB,a}(z_2) U_{CD,b}(z_3) V_{cde}(z_3) V_{fgh}(z_4) \rangle, \quad (3')$$

$$G^{(7)} = \int d^6 z d^6 z' \langle T S(z_1) S(z_2) U_{AB,a}(z_3) U_{CD,b}(z_3) V_{cde}(z'_1) V_{fgh}(z'_2) \rangle, \quad (8')$$

etc. (By the non-renormalization theorem, the superpotential Eq. (1) is invariant under renormalization. Corrections of the D-term type may be present, but the formulae such as Eqs. (3') and (8') are correct for small h and h' .)

The single-instanton contribution can now be computed through the explicit construction of the fermion zero modes (and induced scalar modes) by using the improved method of Ref. 12). It gives, at $h = h' = 0$, for $G^{(1)} \sim G^{(9)}$ *),

$$\begin{aligned}
 G^{(1)} &= G^{(2)} = G^{(3)} = \dots = G^{(9)} \\
 &= c_0 \epsilon_{ABCD} \left\{ (\epsilon_{ac} \epsilon_{bf} \epsilon_{dg} \epsilon_{eh} + a \leftrightarrow b) \right. \\
 &\quad \left. + \text{perm. } (c,d,e) + \text{perm. } (f,g,h) \right\}
 \end{aligned} \tag{11}$$

($c_0 = \text{const. } \Lambda^{12}$).

The ϵ -structure of Eqs. (11) can be interpreted as due to the summation over the vacua related by $SL(4,C) \times SL(2,C)$ group. Note that, although the symmetry group of the theory with $h = h' = 0$ is $SU(4) \times SU(2)^{+)$, we are assuming that the degeneracy of the vacua is larger: its complex extension $SL(4,C) \times SL(2,C)$. This is indeed implied by the consideration of various $h, h' \rightarrow 0$ limits corresponding to different ways in which $h_{AB,a}$ and h'_{bcd} are let to zero. Analyses using the effective Lagrangians also support the existence of such a degeneracy¹⁵⁾.

These vacua are related (by definition) by

*) There are some errors in Eqs. (4.30) and (4.32) of Ref. 12).

+) $U(1)$ symmetries are not mentioned here because the summation over the $U(1)$ -related vacua does not affect the clustering property of $G^{(i)}$.

$$\begin{aligned} \langle U_{AB,a} \rangle_{\Omega(g,g')} &= U_A^A(g) U_B^{B'}(g) U_a^{a'}(g') \langle U_{A'B',a'} \rangle_{\Omega_0} \\ \langle V_{bcd} \rangle_{\Omega(g,g')} &= V_b^{b'}(g) V_c^{c'}(g) V_d^{d'}(g') \langle V_{b'c'd'} \rangle_{\Omega_0} \end{aligned} \quad (12)$$

where $U_A^A(g)$ and $V_b^{b'}(g')$ are respectively the (fundamental) representation of $SL(4,C)$ and $SL(2,C)$ and $|\Omega_0\rangle$ is some fixed reference vacuum.

As a consequence of their constancy, the Green's functions $G^{(i)}$ transform (via clustering) simply as a product of condensates. For instance, we have for $G^{(1)}$

$$\begin{aligned} G^{(1)}|_{\text{instanton}} &= \int dg dg' \langle T U_{AB,a} \dots V_{fgh} \rangle_{\Omega(g,g')} \\ &= \int dg dg' U_A^A(g) U_B^{B'}(g) \dots V_h^{h'}(g') \langle T U_{A'B',a'} \dots V_{f'g'h'} \rangle_{\Omega_0} \end{aligned} \quad (13)$$

where $\int dg$ ($\int dg'$) is the invariant $SL(4,C)(SL(2,C))$ group-space integration, normalized to unity.

While we are working within a semi-classical (non-perturbative) approximation with respect to the gauge coupling constant, it is a perturbative expansion with respect to the superpotential Eq. (1) which interests us here. Since the unperturbed ground state has a degeneracy related by the group, $SL(4,C) \times SL(2,C)$, an appropriate linear combination of these must be found which is the true ground state for infinitesimal but non-vanishing $h_{AB,a}$ and h_{cde}^* .

The classical theorem by Dashen¹⁶⁾ for the vacuum arrangement is unfortunately not very useful in theories with global supersymmetry (the potential being

*) This is much like the standard perturbation theory in a simplest quantum mechanical system with degenerate levels.

zero at the minima)*). Luckily enough, the identities of Ref. 14) provide the crucial information on the vacuum alignment in the presence of the superpotentials.

It is convenient to take the superpotential in the particular form,

$$h_{12,1} \neq 0, h_{34,2} \neq 0; h'_{111} \neq 0, h'_{222} \neq 0, \quad (14)$$

$$h_{AB,a} = h'_{cde} = 0 \quad \text{for other indices,}$$

first. In this case the identities of Ref. 14) read (in each of the true vacua),

$$\begin{aligned} -\langle S \rangle &= h_{12,1} \langle U_{12,1} \rangle = h_{34,2} \langle U_{34,2} \rangle \\ &= h'_{111} \langle V_{111} \rangle = h'_{222} \langle V_{222} \rangle, \end{aligned} \quad (15)$$

$$\langle U_{AB,a} \rangle = \langle V_{cde} \rangle = 0, \quad (15')$$

for other indices.

Let us form the following combination of $G_{AB\dots}^{(1)}$,

$$G_{\text{Singlet}}^{(1)} = \langle T \epsilon_{ABCD} \epsilon_{ac} \epsilon_{bf} \epsilon_{dg} \epsilon_{eh} U_{AB,a} U_{CD,b} V_{cde} V_{fgh} \rangle. \quad (16)$$

Since this quantity is invariant under the $SL(4,C) \times SL(2,C)$ transformation of the vacuum, the summation over $|\Omega_{g,g'}\rangle$ in Eq. (13) is trivial, $\int dg dg' = 1$; at the same time $|\Omega_0\rangle$ can be taken to be the "true" vacuum, $|\Omega_{t.v.}\rangle$. Thus,

*) This being so, the general background for the present work remains to be that given in Refs. 16) and 17).

$$G_{\text{singlet}}^{(1)} = \epsilon_{ABCD} \epsilon_{ac} \epsilon_{bf} \epsilon_{dg} \epsilon_{eh} \langle T U_{AB,a} U_{CD,b} V_{cde} V_{fgh} \rangle_{\text{true vac.}} \quad (17)$$

The left-hand side is found from the instanton result, Eq. (11), to be $24^2 \cdot 10 c_0$.

On the right-hand side of Eq. (17), one can use the clustering property at $|x_i - x_j| \rightarrow \infty$ (which is of course valid for Green's functions referring to a single vacuum), together with Eq. (15'), to get^{*}

$$G_{\text{singlet}}^{(1)} = -8 \langle T U_{12,1} U_{34,2} V_{111} V_{222} \rangle_{t.v.} \quad (18)$$

Thus, one finds ($C \equiv -720 c_0$)

$$\langle T U_{12,1} U_{34,2} V_{111} V_{222} \rangle_{t.v.} = C = \text{const. } \Lambda^{12}, \quad (19)$$

while

$$\langle T U_{AB,a} U_{CD,b} V_{cde} V_{fgh} \rangle = 0 \quad \text{unless} \quad \begin{cases} (AB,a), (CD,b) = (12,1) \text{ or } (34,2) \\ (cde), (fgh) = (111) \text{ or } (222) \end{cases} \quad (19')$$

Furthermore, the anomalies Eqs. (15) lead to

$$\langle T U_{12,1} U_{12,1} V_{111} V_{222} \rangle_{t.v.} = (h_{34,2} / h_{12,1}) C, \quad (20)$$

etc., and in fact, to the complete determination of the condensates,

$$-\langle S \rangle = \left(h_{12,1} h_{34,2} h'_{111} h'_{222} C \right)^{1/4} \cdot e^{2\pi i k/4} \quad (21)$$

(in the k^{th} vacuum, $k = 1 \dots 4$) together with Eqs. (15) and (15').

There is a four-fold degeneracy of the vacuum (= Witten's index) deriving from the spontaneous breakdown of the Z_{12} symmetry to a Z_3 . The summation over

^{*}) Since these Green's functions are constant, we identify here $\langle T U_{12,1} U_{34,2} V_{111} V_{222} \rangle_{t.v.}$ and $\langle T U_{34,2} U_{12,1} V_{111} V_{222} \rangle_{t.v.}$, etc.

these vacua implied in the instanton calculation explains why Green's functions other than $G^{(1)} \sim G^{(9)}$, say $\langle TSS \rangle$, $\langle TSSS \rangle$, etc., are vanishing at short distances (to all orders of h , h') but that there is no contradiction with the non-vanishing condensate Eq. (21) in the individual vacuum. (On the other hand, the summation over these four-fold degenerate ground states is such that it does not invalidate the clustering property of the Green's functions, $G^{(1)} \sim G^{(9)}$.)

The consistency check is now made as follows. From Eqs. (15), (15') and (21), one gets all Green's functions appearing in Eqs. (3)-(10), and takes the derivatives with respect to h and h' explicitly, for example,

$$G^{(2)} \Big|_{\text{true vacuum}} = C, \quad \text{if } (AB,a)(CD,b)(cde)(fgh) = (12,1)(34,2)(111)(222) \\ \text{or } (12,1) \leftrightarrow (34,2); (111) \leftrightarrow (222), \quad (22) \\ = 0 \quad \text{otherwise}.$$

The corresponding sum over the $SL(4,C) \times SL(2,C)$ -related vacua is obtained by tracing the steps used in Eqs. (16)-(19) backward. One thus finds ($c_0 = -C/720$)

$$\int dg dg' G^{(2)} \Big|_{\Omega(g,g')} = c_0 \epsilon_{ABCD} \left\{ (\epsilon_{ac} \epsilon_{bd} \epsilon_{de} \epsilon_{eh} + a \leftrightarrow b) \right. \\ \left. + \text{perm.}(c,d,e) + \text{perm.}(f,g,h) \right\}, \quad (23)$$

in full accord with the direct instanton result for $G^{(2)}$, Eq. (11).

In the same fashion, one can verify that the direct instanton results for $G^{(3)} \sim G^{(9)}$, Eqs. (11), are all consistent with the corresponding quantities referring to the "true" vacuum, inferred from Eq. (2), Eqs. (15), (15') and the clustering, after the summation over the $SL(4,C) \times SL(2,C)$ -related vacua.

Once consistency is found for the particular choice of the superpotential of the form of (14), the general case can be dealt with as follows. For a generic set of $h_{AB,a}$ and h'_{bcd} , it is possible (as can be shown by direct construction) to make an $SL(4,C) \times SL(2,C)$ transformation of the chiral fields Φ_A and A_a such that in the new basis the superpotential takes the form of (14). By applying then the inverse of such a transformation on Eqs. (15), (15'), (18)~(21), one finds the solution for the condensates in the general case. Since the combination appearing in Eqs. (16) and (17) is invariant under $SL(4,C) \times SL(2,C)$, consistency with

the instanton results, Eq. (11), is maintained^{*}).

We conclude that the determination of the vacuum properties in our SU(6) model based on the instanton calculus is fully consistent, internally and with the identities of Ref. 14). [This generalizes the findings of Ref. 10) in a simpler version of chiral SU(6) theory.]

3. - CHIRAL SU(N) THEORIES WITH EVEN N: RUNAWAY VACUA

Next, consider chiral SU(N) gauge theories with even N, $N \geq 8$. For definiteness, take $N = 8$ (the treatment of other cases and the conclusions are analogous). The matter fields are Φ_A^α and $A_{\alpha\beta}$ ($\alpha, \beta = 1 \dots 8$; $A = 1 \dots 4$) belonging to four $\square$ and one $\bar{\square}$ multiplets. The superpotential in this case is given by
 $(U_{AB} \equiv \Phi_A^\alpha \Phi_B^\beta A_{\alpha\beta})$

$$P = \int h_{AB} U_{AB} \quad (24)$$

where h_{AB} can be taken to be of the form,

$$\begin{aligned} h_{AB} &= -h_{BA} \neq 0, \text{ if } (AB) = (12) \text{ or } (34), \\ h_{AB} &= 0 \quad \text{otherwise} \end{aligned} \quad (25)$$

by an SU(4) transformation of Φ_A 's.

The constant Green's functions (or their h derivatives) which get contribution from an instanton at $h_{AB} = 0$, are ($V \equiv A_{\alpha_1\alpha_2} A_{\alpha_3\alpha_4} A_{\alpha_5\alpha_6} A_{\alpha_7\alpha_8} \epsilon^{\alpha_1 \dots \alpha_8}$),

$$H_{ABCD}^{(1)} = (-) \langle T S(z_1) S(z_2) S(z_3) U_{AB}(z_4) U_{CD}(z_5) V(z_6) \rangle, \quad (26)$$

^{*}) This argument works because all relations here involve chiral superfields only.

$$H_{ABCD}^{(2)} = (\partial/\partial h_{CD}) \langle T SSSS U_{AB} V \rangle, \quad (27)$$

$$H_{ABCD}^{(3)} = (-) (\partial^2/\partial h_{AB} \partial h_{CD}) \langle T SSSSS V \rangle. \quad (28)$$

The result of the instanton calculation is ($c'_0 = \text{const.} \neq 0$)

$$H_{ABCD}^{(i)} = c'_0 \cdot \epsilon_{ABCD} \quad (\text{all } i) \quad (29)$$

again to be interpreted as quantities summed over the vacua related by $SL(4, C)$.

In order to undo this summation and to get $H^{(i)}$ referring to the true vacuum, one needs to know the vacuum alignment. The anomalous and non-anomalous identities of Ref. 14) in this case read (assuming unbroken supersymmetry),

$$-\langle S \rangle = h_{12} \langle U_{12} \rangle = h_{34} \langle U_{34} \rangle, \quad (30)$$

$$-6\langle S' \rangle = h_{12} \langle U_{12} \rangle + h_{34} \langle U_{34} \rangle, \quad (31)$$

$$\langle U_{AB} \rangle = 0 \quad \text{for } (AB) \neq (12) \text{ or } (34). \quad (32)$$

By applying a procedure similar to Eq. (15) ~ Eq. (19), to Eqs. (30) and (32) and the instanton result for $H^{(1)}$, one finds

$$\langle SSS U_{12} U_{34} V \rangle_{\text{true Vac.}} = C (\propto \Lambda'^9), \quad (33)$$

$$\langle SSSS U_{12} V \rangle_{\text{t.v.}} = h_{34} C, \quad (34)$$

$$\langle SSSS U_{34} V \rangle_{\text{t.v.}} = h_{12} C, \quad (35)$$

$$\langle SSSSS V \rangle_{t.v.} = h_{12} h_{34} C \quad (36)$$

Equations (33)-(36) are again fully consistent with the direct instanton calculation of $H^{(2)}$ and $H^{(3)}$, Eq. (29) after a summation over the $SL(4,C)$ -related vacua.

Physical properties of the theory are, however, different from the $SU(6)$ case. The $SU(8)$ theory has not well-defined vacua [Eqs. (30), (31) and (33) in fact imply $\langle S \rangle = \langle U_{AB} \rangle = 0$; $\langle V \rangle = \infty$]. Such a pathological feature is not unexpected here, however, because of the existence of a perturbative flat direction ($\eta_{12} = \eta_{34} = \eta_{56} = \eta_{78} = a$) in the theory with $h \neq 0$.

4. - CHIRAL $SU(N)$ THEORIES WITH ODD N : DYNAMICALLY BROKEN SUPERSYMMETRY

Chiral $SU(N)$ theories with odd N appear to have a different type of the ground states: non-supersymmetric but well-defined vacua (see below). For $N = 9$, (the analysis and conclusions for the $N > 11$ cases are analogous), the matter fields are Φ_A^α and $A_{\alpha\beta}$ ($\alpha, \beta = 1 \dots 9$; $A = 1 \dots 5$). The superpotential can be chosen without losing the generality to be $P = h_{AB} U_{AB} \equiv h_{AB} \Phi_A \Phi_B$, with $h_{12} \neq 0$, $h_{34} \neq 0$; $h_{AB} = 0$ otherwise.

The analysis is done precisely as in the preceding cases, except that, with supersymmetry broken spontaneously, the identities of Ref. 14) are less restrictive. By making an ansatz^{*)} that the condensates $\langle U_{AB} \rangle$ are aligned with $h_{12} \neq 0$, $h_{34} \neq 0$ in this case also, one finds⁺⁾

$$\begin{aligned} \langle TSSU_{12}U_{34}Y_5 \rangle_{t.v.} &= (-1/h_{34}) \langle TSSSU_{12}Y_5 \rangle_{t.v.} \\ &= (-1/h_{12}) \langle TSSSU_{34}Y_5 \rangle_{t.v.} = (1/h_{12}h_{34}) \langle TSSSSY_5 \rangle_{t.v.} \quad (37) \\ &= C (\propto \Lambda^{21}) \end{aligned}$$

^{*)} The solution for the condensates, Eq. (37) below, obtained this way is consistent with the structure of the flat directions that occur when $h_{12} \rightarrow 0$ or $h_{34} \rightarrow 0$.

⁺⁾ $Y_A \equiv (g^2/32\pi^2) \epsilon_{i_1 \dots i_9} (WW)_{i_1 i_2 i_3}^j \dots A_{i_8 i_9} A_{j l} \Phi_A^l$.

Again, one arrives at a complete consistency with the three independent instanton results (at $h_{AB} = 0$),

$$\begin{aligned} \langle TSSSU_{AB}U_{CD}Y_E \rangle &= (\partial/\partial h_{CD}) \langle TSSSSU_{AB}Y_E \rangle \\ &= (\partial^2/\partial h_{AB}\partial h_{CD}) \langle TSSSSSSY_E \rangle = \epsilon_{ABCDE} (C/15), \end{aligned} \quad (38)$$

after the summation over the $SL(5, C)$ -related vacua.

Since this theory has no perturbative flat directions (for $h_{12} \neq 0$, $h_{34} \neq 0$), it is likely that any condensate of gauge-invariant composite operators (such as $\langle Y_A \rangle$) is finite⁶⁾. Thus, comparison between the non-vanishing Green's functions, Eq. (37), and the anomalous WT identities of Ref. 14)*),

$$\begin{aligned} \langle S \rangle + h_{12} \langle U_{12} \rangle &= \langle \{ \bar{Q}_\alpha, \bar{\psi}_1 \phi_1 / 2\sqrt{2} \} \rangle, \\ \langle S \rangle + h_{34} \langle U_{34} \rangle &= \langle \{ \bar{Q}_\alpha, \bar{\psi}_3 \phi_3 / 2\sqrt{2} \} \rangle, \\ 7\langle S \rangle + h_{12} \langle U_{12} \rangle + h_{34} \langle U_{34} \rangle &= \langle \{ \bar{Q}_\alpha, \bar{\chi}^2 / 2\sqrt{2} \} \rangle, \end{aligned} \quad (39)$$

($\bar{Q}$ being the supersymmetry generators) implies dynamical breaking of supersymmetry. [The same conclusion was reached in the last of Refs. 6) based on a Higgs-picture approach].

The theory has an exact $U(1)$ symmetry, with charges $(-20, -11, 22, 31)$ and -9 for $[\phi, \psi, \eta, \chi]$ and λ respectively, which is spontaneously broken. This means that there are continuous infinity of vacua in this theory. The occurrence of such a continuous degeneracy [in contrast with the $SU(6)$ theory considered earlier] appears to be necessary for dynamical supersymmetry breaking.

The fact that Green's functions with only S fields as their arguments are all vanishing at short distances (to all orders of h), is then interpreted as due

*) Component fields are denoted by $\Phi_A = \phi_A + \sqrt{2} \psi_A \theta + \dots$; $A = \eta + \sqrt{2} \chi \theta + \dots$; $W = -i\lambda + \dots$.

to the summation over these $U(1)$ -related vacua, and is compatible with the result, $\langle S \rangle \neq 0$, $\langle U_{12} \rangle \neq 0$, etc., in each vacuum.

5. - DISCUSSION

To summarize, the use of the clustering property on the chiral x -independent Green's functions computed by the instanton method at short distances, leads to the set of gauge-invariant condensates, which are self-consistent and in agreement with the identities of Ref. 14). Since we only rely on a standard consequence of asymptotic freedom and on the general principles (clustering) or supersymmetry identities (constancy of certain Green's functions), such an outcome is not really surprising. Though the fact that the results are consistent with the anomaly of Ref. 14) sounds somewhat mysterious, this cannot be regarded as unexpected either: the identities of Ref. 14) are combined consequences of anomalous (or non-anomalous) $U(1)$ identities and of supersymmetry, and the instanton definitely "knows" about them both.

What is non-trivially tested by our analysis is the smoothness of the behaviour of the chiral gauge theories at the small superpotentials, $h \approx 0$ (h being the generic coupling constant). Our analysis shows that the properties of the vacua in the theory with small $h \neq 0$ can be correctly computed in terms of the fermionic zero modes of the $h = 0$ theory, apart from the vacuum orientation that crucially depends on h (even at $h \approx 0$).

It is interesting that in contrast to the case of the chiral gauge theories discussed here [and in Ref. 10)], an analogous computation fails in SQCD^{7),*)}. In massive SQCD a consistent answer is obtained (even for small m) only if the non-zero fermionic modes are also taken into account⁹⁾.

In a theory with mass, such a discontinuous behaviour at $m \rightarrow 0$ is understandable^{7),9)} as due to the mass singularities, leading to corrections of the type, $\sim m^\ell (1/m)^\ell$, $\ell = 1, 2, \dots$. If so, in chiral theories where mass terms are forbidden by gauge invariance and where the expansion is made in cubic or quartic couplings, a smoother behaviour is expected. To be sure, there are $1/h$ factors arising from the propagators of the composite pseudo-Goldstone particles, and as

*) Inconsistency persists after our algorithm is applied on the results of Ref. 7) to undo the summation over the $SL(N, C) \times SL(N, C)$ -related vacua.

is well known^{16),17)}, they give rise to large ($O(h^0)$) corrections even in the $h \rightarrow 0$ limit, if one considers a perturbation around a wrong vacuum. But corrections are expected to be small if the perturbative expansion is made around the appropriate (true) vacuum. Our analysis fully confirms such expectations.

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