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Quarkonium energy splittings in
perturbative QCD

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Abstract:

A $q\bar{q}$ -potential model, derived from perturbative QCD and including one- and two-loop contributions, is applied to calculate energy splittings for heavy quarkonium states. Calculations are performed in momentum space using analytical partial-wave projections. The two-loop contributions give a significant improvement over the first-order results. The results are consistent with experimental data.

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It is well known that the energy levels of heavy quarkonia can be described well by QCD-like potential models, which include a Coulombic part for short distances and a (linear) confinement potential.¹ The Coulombic part of the potential is associated with one-gluon-exchange (OGE) with inclusion of loop contributions to give a running coupling constant $\alpha_s(q^2)$. One approach, which has recently been employed to calculate energy splittings in the $t\bar{t}$ -system², uses the experimental information about $\alpha_s(q^2)$ to obtain an OGE-potential, makes a Fourier transform and adds other potential terms in coordinate space to describe the known quarkonia data. It is not clear if this is a consistent procedure, particularly if one takes into account that $\alpha_s(q^2)$ from second-order renormalization-group equations implies a linear potential behaviour at large distances³. As a consequence it is interesting to consider to what extent a purely QCD-derived potential can describe quarkonium spectra without additional terms.

To avoid a possible inaccuracy from the Fourier transform, we want to calculate the corresponding spectra directly in momentum space by using a partial-wave projection method. While the Fourier transform for the relevant potentials has to be performed numerically, the partial-wave projection to S-waves can be done analytically. Details of the formalism will be discussed below. We will mainly concentrate on the discussion of energy level splittings rather than on the actual meson masses, because these data are relatively independent from the quark masses and depend only on the potential.

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The quarkonium potential derived from perturbative QCD^{3,4} (using SU(3) and $N_f=3$ for the number of quark flavours), which includes one- and two-loop contributions, can be written as

$$V(q^2) = -\frac{4\pi}{3} \frac{\alpha_s(q^2)}{q^2} \quad (1)$$

$$\alpha_s(q^2) = \frac{12\pi}{27 \ln q^2/\Lambda^2} \left[1 - \frac{64}{81} \frac{\ln |\ln q^2/\Lambda^2|}{\ln q^2/\Lambda^2} \right] \quad (2)$$

which is valid for sufficiently large momentum transfer. For the QCD scale parameter we use $\Lambda=0.4$ GeV, which is in agreement with limits imposed by experimental data³.

In our calculation we will replace the four-momentum q by the corresponding three-dimensional momentum transfer and project the potential to the S-wave subspace, giving

$$V(p, p') = \int d^3x P_0(\mathbf{x}) V(\vec{q}^2) \quad (3)$$

where $\vec{q} = \vec{p} - \vec{p}'$; $\vec{q}^2 = \vec{p}^2 + \vec{p}'^2 - 2\vec{p}\vec{p}' \cdot \mathbf{x}$.

The S-wave bound states due to the above potential are then obtained by solving

$$\det [1 - G_0(E)V] = 0 \quad (4)$$

on a discrete grid of momenta.

Here we use the Green's operator of the relativistic Lippmann-Schwinger equation⁵ for $G_0(E)$.

The first term to the potential equ. (1), which includes one-loop contributions, is given by

$$V(p, p') = \frac{8}{27p p'} \left[\ln \left| \ln \left(\frac{p-p'}{\Lambda} \right)^2 \right| - \ln \left| \ln \left(\frac{p+p'}{\Lambda} \right)^2 \right| \right] \quad (5)$$

Equ. (5) contains logarithmic singularities at $p=p'$ and $|p \pm p'| = \Lambda$, the first of which is the well-known on-shell singularity of the Coulomb potential in momentum space⁶. Fortunately, these singularities are integrable, but one has to use a rather high-dimensional matrix in equ. (4) (100x100 in our calculation). On the other hand the Coulomb problem can be solved analytically and provides a good test for the formalism.

We want to emphasize that the results from the one-loop contribution to the potential are essentially parameter-free (apart from the quark masses and Λ).

The second contribution to the potential is more difficult and ambiguous because of a non-integrable singularity at $q^2 = \Lambda^2$. Its regularization introduces a cut-off parameter and the results are not so well determined as before. As this singularity appears at a rather small momentum transfer, the regularized potential will nevertheless be consistent with the large- q^2 behaviour of QCD.

To describe the cut-off procedure, let us write $t = \ln q^2/\Lambda^2$. The singularity then appears at $t=0$. Now we introduce a cut-off parameter t_0 such that whenever $|t| < t_0$, we will replace

$$t^{-2} \ln |t| \longrightarrow t_0^{-2} \ln |t|$$

in equ. (2). This procedure allows us to perform the integration in equ. (3) again analytically. As the result is rather lengthy, we will indicate the required formulae

$$\begin{aligned} \int dt t^{-2} \ln |t| &= -t^{-1} (1 + \ln |t|) \\ \int dt \ln |t| &= t (\ln |t| - 1) \end{aligned} \quad (6)$$

The integration bounds are $\ln\left(\frac{p^2 + p'^2}{\Lambda^2}\right)^2$. $t_0 \rightarrow 0$ gives the singular result again, while $t_0 \rightarrow \infty$ completely suppresses the two-loop contributions. The first-order potential is independent of t_0 . In our calculation we used $t_0 = 0.8$, which was chosen by comparing with experimental quarkonia data. Consequently the results are not considered as a parameter-free prediction from QCD, but rather demonstrate consistency of perturbative QCD with heavy quarkonium data.

Our results for the energy splittings between the low-lying S-wave states of the $c\bar{c}$ -, $b\bar{b}$ - and $t\bar{t}$ -system are given in Table I. The table shows results from the first- and second-order potential and also gives a comparison with experimental data⁷. The quark masses were chosen to roughly reproduce the mass of the ground state in each system.

Our results are consistent with qualitative expectations. Let us first consider the one-loop contribution. If compared with experimental data, the agreement improves with increasing quark mass and decreasing bound state radius. Only for the $t\bar{t}$ -system, however, the results appear to be realistic⁸, while the $c\bar{c}$ -interaction is very badly described by the one-loop contribution. The second-order contribution gives a remarkable improvement especially for the $c\bar{c}$ - and $b\bar{b}$ -system. While it increases the energy splittings in these systems, the opposite effect appears for the low-lying $t\bar{t}$ -states. This seems quite interesting, because it indicates a transition from the non-perturbative to the perturbative region of QCD. The effect of the second-order contribution can be compared with the behaviour of the running coupling constant of equ. (2). Here we can see,

that this contribution to the potential is positive (repulsive) for large momentum transfer, while its sign changes at a critical value given by $\ln(q_0^2/\Lambda^2) = 1$. As a consequence we can conclude that the small momentum transfer components of the potential dominate in $c\bar{c}$ - and $b\bar{b}$ -states, while for the $t\bar{t}$ -system the higher momentum transfer components are more important. Considering the comparison with experimental data, the inclusion of the second-order potential extends the validity of the potential to smaller quark masses and higher excited states, which means to larger distances.

The $t\bar{t}$ -system may be of a special interest, because of the relatively weak dependence on the cut-off parameter t_0 and the fact that the model-independent first-order potential already gives realistic results.

Buchmüller and Tye³ have pointed out that the $t\bar{t}$ -spectrum would distinguish between different potential models, which give very similar results for the $c\bar{c}$ - and $b\bar{b}$ -spectra. The reason is that all of these potentials have a very similar behaviour for distances between 0.1 and 1 fm, which is the relevant range for these systems. They differ, however, in their predictions for the $t\bar{t}$ -system. In particular, for phenomenological power-law and logarithmic potentials⁹ the 2a-1s splitting in t-quarkonium is considerably smaller than for QCD-like potentials.¹⁰ Clearly experimental information about t-quarkonium is therefore very important and would provide a more direct test of QCD.

Although in Table I we have compared our results with those from a typical representative of QCD-like potentials⁸, our results including the second-order contribution are in better agreement with those of the phenomenological potentials. Our calculation

with different values of Λ and t_0 show that this is not a peculiar feature of the chosen parameters. For smaller values of Λ , the splitting is even smaller and rather independent of t_0 . A smaller value of Λ would also increase the validity of perturbative QCD, so that higher orders would have a smaller effect on the energy splittings.

Actually it would be possible in our calculation to get a better agreement with QCD-like potentials by increasing t_0 sufficiently. Note that $t_0 \rightarrow \infty$ would give the first-order results. However, such a procedure seems inconsistent, because it worsens the results for $c\bar{c}$ - and $b\bar{b}$ -states while simultaneously extending the regime of perturbative QCD. As a consequence, it would not be possible to restore e.g. the $b\bar{b}$ -results by higher order contributions. As a consequence we consider our above results as a realistic estimate from QCD. These results may be related to the appearance of logarithmic terms in the running coupling constant.

In addition to the results shown above, we have also calculated the energy splittings of flavoured quarkonium states, like $t\bar{t}$ -, $c\bar{c}$ - and $b\bar{b}$ -states. The results for these systems are given in Table II. It can be seen that the results are qualitatively similar to those given in Table I. For the ground state of the $b\bar{c}$ -system we obtain $M(b\bar{c}) = 6.40$ GeV, which is in general agreement with results from different calculations.¹¹

To conclude, we have calculated quarkonium energy splittings directly in a momentum space calculation using analytical expressions for the partial-wave projected potentials. We have applied the OGE-potential derived from QCD, which includes

one- and two-loop contributions. Relativistic kinematics are included in our calculation. The results show an interesting behaviour, which can qualitatively be understood by our knowledge about the convergence behaviour of QCD. We have compared results from first-order with those from second-order contributions and found that the range of applicability was considerably increased by the inclusion of the two-loop contributions. The results make it plausible that further improvement could be achieved with higher-order contributions. They also demonstrate that heavy quarkonium states are very important to test QCD in its perturbative regime.

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Table I

Quarkonium energy splittings in MeV

	$c\bar{c}$ ($m_c=1870\text{MeV}$)		$b\bar{b}$ ($m_b=5190\text{MeV}$)		$t\bar{t}$ ($m_t=35.1\text{GeV}$)	
	$t_0=\infty$	exp	$t_0=\infty$	exp	$t_0=\infty$	0.8 Ref.8
E(21)	55	493	345	525	773	549
E(32)		50	28	272	317	272
E(43)				40	128	197
E(54)						158

Table II

Quarkonium energy splittings in MeV
($t_0=0.8$)

	$b\bar{c}$	$t\bar{c}$	$t\bar{b}$
E(21)	612	691	539
E(32)	26	120	324
E(43)		24	105
E(54)			32