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PROTON-PROTON BREMSSTRAHLUNG AND INFORMATION ON OFF-SHELL ASPECTS OF THE
NUCLEON-NUCLEON FORCE

Harold W. Fearing

TRIUMF
4004 Wesbrook Mall, Vancouver, B.C., Canada V6T 1W5

Recent developments in the theory of nucleon-nucleon bremsstrahlung and
their implications for our knowledge of the off-shell nucleon-nucleon
force are reviewed.

INTRODUCTION

At the level of conventional nuclear physics, the nucleon-nucleon (N-N) force is probably the most fundamental quantity which one must understand. By now a huge quantity of elastic scattering data defines the elastic phases, and the on-shell force can be parameterized quite successfully via a nonrelativistic potential. We now understand the long and medium range force microscopically in terms of meson exchanges, and have made a start, though not yet quantitatively successful, in describing the short range force in terms of quarks.

Information about off-shell aspects of the N-N force has been much harder to come by, even though it enters in a variety of processes such as quasielastic (p,2p) reactions, nuclear matter calculations, three body wavefunctions, as well as nucleon-nucleon bremsstrahlung (ppγ). In all of these except ppγ however, there are strongly interacting spectator particles which complicate tremendously the interpretation of such processes. Thus ppγ has long been considered the simplest and the cleanest method of learning about the off-shell force. Perhaps as a reflection of this, it has been the subject of a special session or a review talk at many of the previous few-body conferences in this series.

In the past year or so there have been major new developments which make it valuable to again review the status of ppγ. First a new experiment,¹ carried out at TRIUMF, has obtained for the first time an extensive set of analyzing power data. This gives a new type of information to test against theory. Second, and to some extent in anticipation of this new data, there has been a new calculation² using for the first time some of the modern, theoretically based potentials and including a number of corrections which had not been included in previous calculations. It is thus the purpose of this paper to review these new developments, particularly with regard to the theory and what it tells us about the off-shell N-N force.

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THEORETICAL APPROACHES

There have been two major approaches to a description of ppy. The simplest is the soft photon approximation (SPA)^{3,4} which is based on an expansion of the N-N amplitude about an on-shell point via the Low theorem. One writes the ppy amplitude as $M_{\text{ppy}} = A/k + B + Ck$ and the cross section as $d\sigma/d\Omega = A^2/k + 2AB + (B^2 + 2AC) + \dots$ where k is the photon momentum and where A and B are known functions of the kinematics and of the on-shell information. This approach is simple and easily gauge invariant and relativistically invariant. It however contains no off-shell information (which is buried in the unknown C) except to the extent that it disagrees with data.

Thus it was a surprise to find that the SPA results agreed with all data existing prior to the new TRIUMF experiment at least as well as did results from other approaches^{4,5}. This was true in particular for the 200 MeV cross section data from the original TRIUMF experiment,⁵ where older potential model calculations indicated that off-shell effects should be important.

This rather puzzling situation, together with the fact that there were very few calculations of analyzing power which could be compared with the data being taken motivated another look at the second approach, a standard potential model. The aim was to use some of the modern theoretically based potentials, such as Paris⁶ or Bonn,⁷ which had not been done before, to see if the difficulties could be traced to the potential and, at the same time, to include as many of the possible corrections as one could.

In the potential model approach one assumes a non relativistic N-N potential and an electromagnetic interaction. From these one can calculate directly the bremsstrahlung, though for realistic (non-local) potentials there are complications in obtaining a conserved electromagnetic current. Given a potential there are two possible methods of solution, each of which has its advantages and disadvantages. One can solve the Schrödinger equation in coordinate space to obtain the exact two-nucleon scattering wavefunctions and then use these to calculate matrix elements of the electromagnetic interaction. The alternative approach, which we have used, is to solve the Lippmann-Schwinger equation in momentum space to obtain the N-N t-matrix half off shell and then combine this with propagator and electromagnetic vertex functions to get the amplitude. In this approach the t-matrix is given by two-nucleon matrix elements of the following operator:

$$T = t(E_f) \frac{1}{E_f - H_0 + i\epsilon} V_{\text{em}} + V_{\text{em}} \frac{1}{E_i - H_0 + i\epsilon} t(E_i) + t(E_f) \frac{1}{E_f - H_0 + i\epsilon} V_{\text{em}} \frac{1}{E_i - H_0 + i\epsilon} t(E_i)$$

The first two terms correspond to the single scattering pieces of Fig. 1a and

Fig. 1. Single scattering (a) and double scattering (b) contributions to ppy.

the last to the double scattering contribution of Fig. 1b. V_{em} is the electromagnetic operator at the photon-nucleon vertex, H_0 is the free two-proton Hamiltonian, either relativistic or non-relativistic. The two-nucleon t-matrices, $t(E_i)$ and $t(E_f)$, need to be evaluated half off shell. For the hard photon kinematics of most interest E_i and E_f are quite different so that the t-matrix is needed at the beam energy E_i when the photon is emitted after the strong scattering and at a much lower energy E_f when the photon is emitted first. This is in contradistinction to the SPA where all of the elastic information is evaluated at some intermediate average kinematic situation.

In the static limit the electromagnetic operator is

$$V_{em} = \frac{-e}{2\pi m \sqrt{k}} \{ \vec{p}_1 \cdot \vec{\epsilon} - \frac{1}{2} \mu_p \sigma^{(1)} \cdot \vec{k} \times \vec{\epsilon} \}$$

summed over the two nucleons. Here p_1 is the nucleon momentum, μ_p is the proton total magnetic moment, and ϵ is the photon polarization vector. In our calculations we have also included a number of relativistic corrections coming from the expansion of the relativistic vertex in powers of p/m , including those used by Liou and Sobel,⁸ some additional higher order terms, and a two-body term coming from the required relativistic invariance of an interaction with a loosely bound system.⁹ These corrections, an extension of the relativistic spin corrections (RSC) of Liou and Sobel, are the most important numerically of the effects we include.

The half-shell N-N t-matrices can be written as a product of the on-shell t-matrix and a real half-shell function as $t_L(E, Q_{off}, Q_{on}) = f_L(Q_{off}, Q_{on}) t_L(E, Q_{on}, Q_{on})$. By substituting this into the partial wave momentum space Lippmann-Schwinger equation one obtains an integral equation which can be solved for $f_L(Q_{off}, Q_{on})$, from which one can obtain both on- and off-shell t-matrices.

We have obtained results for three different potentials, Paris,⁶ Bonn,⁷ and extended Reid soft core (ERSC),¹⁰ none of which have been used in ppy calculations before. The first two of these are state-of-the-art theoretically based potentials which fit the elastic data very well. (For the Bonn potential the f 's were actually obtained by interpolation from a table of t-matrices.) The ERSC

potential is the old Reid soft core potential extended to higher partial waves by Day,¹⁰ and thus is phenomenological, but still fits the data reasonably well. In each case partial waves through $J=5$ were included. One pion exchange was used for the partial waves $J=6$ to $J=20$ and some Coulomb corrections were included. The t -matrices were evaluated in the two-nucleon center of mass, but were then transformed and rotated using the proper relativistic transformations to the frame and axis of the overall system.

So far only the single scattering terms have been included in the calculation, though an explicit evaluation of the double scattering term is underway. This term has been calculated only at lower energies,¹¹ where it seems to be small but probably non negligible. It is also intimately related to the question of gauge invariance. One can show that the leading term in k of the double scattering term is just that needed to make the single scattering terms gauge invariant to that order¹² and that furthermore this term can be calculated uniquely using soft photon techniques. More important for practical purposes is the fact that this term is proportional to the total center of mass momentum. Thus by working in the center of mass we both suppress the contribution of the double scattering term and make the result of the other terms gauge invariant to this order. One can also show that the single scattering terms can be made gauge invariant to all orders by adding a term proportional to the center of mass momentum,¹³ but this term is now not unique. Thus our result is strictly gauge invariant, so that there should be no gross errors resulting from the absence of the cancellations usually enforced by gauge invariance. However there will be terms, gauge invariant in themselves, and of order k and higher, which have been left out. Some of these terms will be included when the full double scattering term is added explicitly. There is in addition another class of gauge terms which are very hard to calculate, and which have never been included. These come from the momentum dependence of the potential. Some can be obtained by a minimal substitution, but the problem of getting a conserved two-nucleon current which is consistent with a given potential and at the same time physically correct is quite subtle and not yet completely solved.¹⁴

Thus the present calculation differs from previous ones in that it uses modern potentials and contains a large number of corrections which have not all been included before. It is next of interest to see how the results compare with the new data and then what has been learned about the off-shell force.

RESULTS AND COMPARISON WITH DATA

Figure 2 shows some of our results compared with the new TRIUMF data for the analyzing power A_y in the kinematic situation which is most off shell. Some additional examples were shown in the preceding paper.¹ The agreement with data is rather good. It is also good in those cases corresponding to soft photon kinematics where the SPA results and those of potential models are essentially identical.

Fig. 2. Analyzing power, A_y , and cross section results obtained using Bonn (solid line), Paris (dashed line), and ERSC (dotted line) potentials as compared with the SPA (short dashed line) and the data of Ref. 1. The dot-dashed line is an on-shell approximation obtained by taking $f=1$ in the Bonn potential calculation.

Fig. 3. Results for cross section and analyzing power obtained by setting individual partial waves to zero in the Bonn potential calculation. S: 1S_0 ; P: 3P_0 , 3P_1 , 3P_2 - 3F_2 ; D: 1D_2 .

The exciting feature of these results is that they show for essentially the first time unequivocal evidence for effects not given by the on-shell SPA and which are presumably related to the off-shell nature of the N-N force. Furthermore the modern theoretically based potentials such as Bonn and Paris seem to do a good job describing the data, thus increasing our confidence in their correctness both on and off shell. The agreement of earlier 200 MeV data⁵ with SPA rather than potential models must in retrospect have been related to experimental difficulties with the absolute normalization of the cross section. An unexpected result is the similarity in the predictions from all of the potentials, though all three are basically sums of Yukawas which fit on-shell data, and thus in fact all have very similar structures.

OFF-SHELL INFORMATION

It is easy to see that off-shell effects are important in obtaining the above

results as it is easy within the context of a potential model calculation to make a purely on-shell approximation. One simply puts the half-shell function $f(Q_{\text{off}}, Q_{\text{on}}) = 1$, its on-shell value. Results in this limit are also shown in Fig. 2 and are clearly ruled out by the data. Thus we conclude that some sort of off-shell effect is crucial in fitting the data for A_y .

To understand the sensitivity to such effects and why the potential results are all similar one must look in more detail. For the kinematics of the new TRIUMF experiment $0 < Q_{\text{on}} < 1.9 \text{ fm}^{-1}$, $Q_{\text{off}} < 2.5 \text{ fm}^{-1}$, and the difference $|Q_{\text{off}} - Q_{\text{on}}|$ is always less than about 2 fm^{-1} with 1.5 fm^{-1} being a more typical value. Not all partial waves contribute equally as can be seen from Fig. 3 where results obtained by dropping one partial wave at a time are plotted. The 1S_0 wave is not too important for A_y and important only near the endpoints for the cross section, which is unfortunate as it is the one which reflects most strongly the differences among existing potentials. The P waves (3P_0 , 3P_1 , and 3P_2 - 3F_2) dominate, with the 3P_0 being less important than the others.

Thus one needs to look at the off-shell behavior of the S and P waves in the region of off-shell momenta $\pm 2 \text{ fm}^{-1}$ about the on-shell point. Figure 4 shows some of these half-shell functions for the various potentials. The differences are largest of course for the S waves. For the dominant P waves however the three potentials are really very similar off shell as well as on shell. Differences of this order in the off-shell behavior result in the variations seen in A_y in Fig. 2 which are clearly not distinguishable within the errors of the existing experiments. For reference Fig. 4 shows also the half-shell functions for simple one pion exchange, which are quite different from those obtained from the potentials.

One can examine more exotic potentials, such as one based on quantum chromodynamics proposed by Kukulin et al.,¹⁵ which has a strongly attractive central region instead of the usual repulsive core, supports forbidden bound states, and leads to a node in the deuteron wavefunction. Here it contributes only in S wave however, where its half-shell function, as shown in Fig. 4, is quite different from those of the other potentials. Fig. 5 shows the results for A_y when the Paris 1S_0 amplitude is replaced by the one obtained from this potential. Here the similarity to Paris everywhere except for the cross section at the endpoints is a consequence of the fact that the S wave is a small contributor, so that even major changes have little effect. One can also try making variations in the off-shell behavior of other partial waves. Two examples are shown also in the Fig. 5. For them the 3P_1 half-shell functions have been multiplied by factors $1 + \alpha \Delta Q$ and $1 + \alpha |\Delta Q|$ which are linear in $\Delta Q = Q_{\text{off}} - Q_{\text{on}}$ and which lead to about a factor of two change in the functions f at $\Delta Q = 1.5 \text{ fm}^{-1}$. Such changes, though less drastic than those given for the S wave by the Kukulin potential¹⁵ can probably already be ruled out by the data.

Thus it appears that at least in a qualitative way the present results constrain the off-shell behavior of the half-shell function to some extent

Fig. 4. Half-shell function for the potentials as in Fig. 2. The dot-dashed curve is from simple one-pion exchange and the short-dashed curve in the 1S_0 state is from the Kukulin potential of Ref. 15.

Fig. 5. A_y and cross section results obtained with some different half-shell functions compared with the standard result from the Paris potential (solid line). Dashed curves correspond to changes in the 3P_1 waves as described in the text. Dotted curve uses the Kukulin potential for the 1S_0 partial wave and Paris for the others.

for the dominant P waves, but to a much lesser extent for the S wave since it is a small contributor. It is hard however to translate these results into constraints on the coordinate space potential since the half-shell function is related to the potential only through complicated and energy dependent integral relations. Thus one cannot easily impose the necessary constraints such as preserving the one pion exchange tail or the deuteron wave function.

SUMMARY AND OUTLOOK

There have been major advances in ppy since the last time it was summarized at one of these meetings. Now analyzing power data is available¹ for the first time and soon there will be cross section results over a wide kinematic range.¹ New calculations have been made using modern theoretically based potentials and including a large number of corrections.² These results fit the data for A_y and for the first time there is conclusive evidence for contributions from off-shell, non soft photon terms. Apparently modern potentials such as Bonn and Paris are approximately correct off shell as well as on shell. One can rule out dramatic variations in off-shell behavior of the dominant P waves but will require much more detailed experiments to distinguish very similar potentials such as Bonn and Paris. Alternatively, to separate such potentials one might look for special kinematic situations, other spin observables, or perhaps processes such as npy, to find greater sensitivity to the S wave where the potentials differ most.

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