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NEUTRINO EMISSION PROCESSES IN HYPERON-POPULATED NEUTRON STARS

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ABSTRACT

Several neutrino emission processes that can occur in neutron stars containing hyperons, nucleons, and electrons are examined in lowest order in the strong and weak interactions. These processes are of two types: neutrino pair bremsstrahlung processes of the form $AB + CD \rightarrow \nu\bar{\nu}$, where A,B,C,D denote baryons, and URCA processes of the form $AB + A'Ce\nu$. The processes are assumed to occur in a dense, degenerate environment subject to charge and beta equilibrium in which the electrons are extremely relativistic and the baryons non-relativistic. To obtain expressions for the emissivities, an independent particle description is employed in which the baryon Fermi energies and momenta are related through the introduction of effective masses, short-range correlations between the baryons are ignored, and the strong baryon-baryon interaction is approximated by a one pion exchange potential with coupling coefficients determined by symmetry considerations. Numerical results are then extracted for a number of neutron star scenarios characterized by fixed values of the fractional baryon populations and the electron to baryon ratio. In all scenarios considered, it is found that the URCA processes dominate the bremsstrahlung processes and that among the URCA processes, the most important processes are those involving nucleons alone, provided the nucleons in the interior are normal. If either the neutrons or protons are superfluid, however, then processes involving hyperons will contribute the largest emissivities, to such an extent that in stars with superfluid nucleons, the presence of hyperons will significantly accelerate the cooling. Possible improvements in the calculations and their consequences are briefly discussed in the conclusions.

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I. INTRODUCTION AND OVERVIEW

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The thermal history of neutron stars subsequent to their formation in supernova explosions has long been a subject of both observational and theoretical interest. Not only is neutron star cooling behavior interesting in its own right, but it also provides one of the most direct and sensitive probes of the dense matter in the interior of neutron stars. Different assumptions concerning the nature of this dense matter can yield radically different cooling curves in the first few thousand years after a neutron star is formed.

To date most of the observational work concerning neutron star cooling behavior has been focussed on the Crab and Vela pulsars. For the Crab pulsar an upper limit of 4.7×10^6 K on the effective surface temperature of the pulsar was established by the failure to observe unpulsed X-rays in an occultation experiment (Kolf et al. 1975). This limit was subsequently reduced to 3.2×10^6 K (Toor and Seward 1977) and then using the HRI detector aboard the Einstein observatory to 2.5×10^6 K (Harnden and Seward 1984). In contrast with the Crab pulsar, unpulsed, soft X-rays have been observed from the Vela pulsar (Harnden, Grant, and Seward 1985). If these unpulsed X-rays are thermal in origin, then the observed flux implies a Vela surface temperature of $\sim 1 \times 10^6$ K for a radius of 10 km; otherwise, this value must be regarded as an upper limit.

Values for the Crab and Vela pulsar surface temperatures can also be extracted indirectly from observations of Crab and Vela glitch phenomena on the basis of the vortex creep theory. The vortex creep theory provides an explanation for the change in the crust spin-down rate of a neutron star following a glitch assuming that the crust of the star and

crustal superfluid are coupled via the thermal creep of superfluid vortex lines pinned to the crustal nuclei (Alpar et al. 1984a). With this assumption the change in the spin-down rate can be related to a number of thermal creep relaxation times which characterize different pinning layers and which are all proportional to the internal temperature of the star. Thus, a fit to the post-glitch data obtained for the Crab and Vela pulsars yields values for the Crab and Vela interior temperatures, which can then be translated into effective surface temperatures. The values obtained in this manner are 1.6×10^6 K for the Crab surface temperature (Alpar, Nandkumar, and Pines 1985) and 3×10^5 K for the Vela temperature (Alpar et al. 1984b).

A point source of unpulsed, soft X-rays has also been observed in the supernova remnant RCW 103 (Tuohy and Garmire 1980). Assuming that these X-rays are thermal X-rays emitted from the surface of a hot neutron star, a surface temperature of $\sim 2 \times 10^6$ K is implied by the observed flux for a diameter of 22 km.

In addition to the observations of the Crab and Vela pulsars and the source in RCW 103, a number of searches have been conducted for possible point X-ray sources in the young supernova remnants Cas A, SN 1006, Tycho, and Kepler (Murray et al. 1979; Helfand, Chanan, and Novick 1980; Helfand 1981; Pye et al. 1981). In none of these supernova remnants was a point source of X-rays detected. If it is assumed nonetheless that the remnants do have neutron stars associated with them, then the absence of detected point sources in the remnants can be translated into upper limits on the effective surface temperatures of the presumed neutron stars. Of course, these upper limits do not carry the same weight as the

Crab, Vela, and RCW 103 observations, since the existence of neutron stars in the Cas A, SN 1006, Tycho, and Kepler remnants has not been established.

Early theoretical studies of neutron star cooling preceded the first observations by more than a decade (Chiu and Salpeter 1964; Bahcall and Wolf 1965; Finzi 1965; Tsuruta and Cameron 1966). In these early studies it was recognized that energy is most readily transported out of the stellar interior via thermal neutrino emission, since for interior temperatures $\lesssim 10^{10}$ K, the neutrino mean-free path exceeds the stellar radius. Within the so-called "standard" scenario, where the neutron star interior is assumed to consist only of neutrons, protons, and electrons in the normal state, the dominant neutrino emission mechanisms are the neutron URCA process,

and the inverse process

The neutrino emissivity from these processes was estimated with the help of nucleon-nucleon scattering data by a number of authors (Bahcall and Wolf 1965; Ellis 1965; Finzi 1965) and later calculated in a model incorporating one-pion exchange supplemented with a Fermi liquid treatment of short-range correlations (Friman and Maxwell 1979).

Two modifications in the standard cooling scenario are required by the existence of weak neutral currents and by the possibility that the nucleons in neutron star interiors are superfluid. The existence of weak neutral currents makes possible nucleon neutrino pair bremsstrahlung processes,

which can contribute significantly to the neutrino emissivity in neutron star interiors. These processes were first examined employing a Fermi liquid treatment of the strong interaction (Flowers, Sutherland, and Bond 1975) and later with the one-pion exchange interaction included (Firman and Maxwell 1979).

Nucleon superfluidity influences neutron star cooling through the introduction of gaps in the nucleon spectra on the nucleon Fermi surfaces. These gaps affect the cooling rates in two different ways. On the one hand, they modify the heat capacity of a neutron star in such a way that the star must dissipate heat at a higher interior temperature than it would otherwise. This accelerates the cooling of the star. On the other hand, the superfluid gaps severely restrict the phase space available to the outgoing nucleons in the neutrino emission processes and thereby diminish the stellar neutrino luminosity. Detailed studies of the URCA process in a superfluid environment (Wolf 1966; Itoh and Tsuneto 1972; Malone 1974) reveal that the neutrino emissivity is reduced exponentially when the interior nucleons are superfluid, which, of course, retards the cooling of the star. Which effect dominates in a given epoch of the star's history depends rather sensitively on both the interior conditions at the onset of the superfluid phase transition and on the nature of the superfluid phase (Malone 1974; Maxwell 1979; Tsuruta 1974, 1979).

Recently, a number of quantitative cooling calculations have been carried out within the standard scenario, in which a careful treatment of nucleon superfluidity is combined with an account of general relativistic effects and effects associated with the presence of large magnetic fields at the surfaces of neutron stars (Glen and Sutherland 1980; Van Riper and

Lamb 1981; Nomoto and Tsuruta 1981; Richardson et al. 1982). In two of the calculations (Nomoto and Tsuruta 1981; Richardson et al. 1982), the assumption of isothermality in the stellar core employed in previous work was relaxed and the full set of general relativistic structure equations solved. The results of these calculations indicate that the standard cooling scenario is consistent with or at worst marginally inconsistent with the surface temperatures observed for the Vela pulsar and the source in RCW 103 and with the upper limits established for the Crab pulsar and the hypothetical neutron stars in the Kepler and Tycho remnants. On the other hand, the upper limits associated with the Cas A and SN 1006 remnants appear to require more rapid cooling than the standard scenario is able to afford. Standard scenario cooling is also not consistent with the effective surface temperatures derived for the Crab and Vela pulsars on the basis of vortex creep theory (Alpar et al. 1984b; Alpar, Nandkumar, and Pines 1985).

Alternatives to standard scenario cooling hinge on the existence of exotic phases of matter in neutron star interiors. One possibility is a pion condensate (Sawyer and Scalapino 1972; Migdal 1973; Au and Baym 1974). Quite early it was recognized that the presence of pions will greatly accelerate the cooling of neutron stars via neutrino processes that involve only two degenerate nucleons instead of four as in the URCA and bremsstrahlung processes (Bahcall and Wolf 1965). Later work demonstrated that this result is not substantially affected by the fact that the pions will occur as a Bose condensate in neutron stars, rather than as free pions (Maxwell et al. 1977; Kiguchi 1977). The cooling of neutron stars will also be greatly accelerated if a phase transition to quark matter occurs in the stellar interior (Iwamoto 1980; Burrows 1980;

Duncan, Shapiro, and Wasserman 1983). In this case the relevant neutrino emission processes are quark beta decay and the corresponding electron capture process.

The neutrino emissivities associated with pion-condensed matter and quark matter are large enough that thermal radiation from neutron stars containing a significant amount of either type of matter would not be observable more than a few years after formation. This scenario would be consistent with upper limits established on neutron star surface temperatures; in fact, it would account for the lack of thermal radiation detected from the Cas A and SN 1006 supernova remnants. At the same time, however, it would rule out a black body interpretation of the observed unpulsed X-ray flux from the Vela pulsar and the RCW 103 source, and it would not be consistent with the Crab and Vela surface temperatures derived from vortex creep theory. In short, it seems that consistency with current observations requires more than one cooling scenario.

The possibility that neutron stars contain substantial numbers of hyperons could significantly influence the cooling rate in the standard scenario, perhaps accelerating it sufficiently to accommodate all observations within a single cooling scenario. This possibility was suggested originally on the basis of a Fermi gas treatment of dense baryon matter (Ambartsumyan and Saakyan 1960). This work revealed that at sufficiently large densities in neutron star interiors, it becomes energetically more profitable to maintain charge equilibrium with Σ^- hyperons than with electrons, which are extremely relativistic at neutron star densities. Subsequent studies included the effect of baryon-baryon interactions (Libby and Thomas 1969) and short-range correlations treated either

variationally (Pandharipande 1971; Pandharipande and Garde 1972; Bethe and Johnson 1974) or through a reaction matrix formalism (Moszkowski 1974). More recently, the equation of state of dense hyperonic matter has been calculated on the basis of relativistic mean field theory (Gao et al. 1981, Glendenning 1981, 1985).

The occurrences of hyperons in neutron star interiors makes possible a wide range of URCA and neutrino pair bremsstrahlung processes analogous to processes (1)-(4), but involving hyperons as well as nucleons. In the present work these processes are examined in some detail and expressions derived for the corresponding neutrino emissivities. The formalism employed is similar to that utilized by Friman and Maxwell (1979) and incorporates many of the same approximations. Among these are a non-relativistic, independent particle description of the participating baryons wherein all short-range correlations are ignored, and the relationship between the baryon Fermi energies and momenta is treated by means of effective masses. The strong baryon-baryon interaction is approximated by a one-pion exchange potential, which should be adequate to uncover the relative importance of various processes in neutron star cooling. In their work, Friman and Maxwell (1979) also considered the influence of short-range terms in the baryon-baryon interaction, parametrized using Fermi liquid parameters, and found their effect to be small. The relevant matrix elements are calculated in lowest order in the weak and strong interactions with recoil terms neglected. In this order, it is found that only the axial component of the weak interaction contributes to the matrix elements. Finally, a number of other approximations are made to facilitate the performance of the phase space integrals, including the neglect of the lepton momenta in the pion

propagator and the momentum conserving δ -function and replacement of the electron and baryon momenta by the corresponding Fermi momenta in the angular integrations.

In the next section the model employed to represent the interior of neutron stars is discussed in some detail. Briefly, we suppose that the matter there consists of a dense soup of relativistic, degenerate electrons and non-relativistic, degenerate baryons, all in the normal, as opposed to superfluid phase. The neutrino emission processes that can occur in such an environment and the diagrams contributing to these processes are examined in § III. Following a discussion of the relevant weak and strong interaction vertices in § IV, § V outlines the emissivity calculation for the various processes considered. Some numerical results are presented and discussed in § VI, while § VII contains a brief summary and conclusions.

II. THE NEUTRON STAR INTERIOR

It is generally believed that neutron stars originate in the aftermath of supernova explosions at interior temperatures $\gtrsim 10^{11}$ K and at interior densities exceeding that of nuclear matter. Initially, the interior temperature decreases very rapidly, regardless of the cooling scenario, and will drop below 10^{10} K within a short time after formation. At this temperature and at densities a few times nuclear matter density, electrons are both degenerate and highly relativistic, while baryons are degenerate but not relativistic.

The populations of the various fermion species in the stellar interior are specified by their Fermi momenta through the relation ($\hbar = c = 1$).

$$\rho_i = p_F^3(i)/3\pi^2 \quad (5)$$

where ρ_i denotes the number density of fermion species i . If the electrons are treated as completely relativistic and the baryons treated non-relativistically in an effective mass approximation, the corresponding Fermi energies are given by

$$E_F(e) = p_F(e) \quad (6)$$

$$E_F(i) = p_F^2(i)/2m_i^* \quad (7)$$

for electrons and baryons respectively, where m_i^* is the effective mass of baryon species i .

In the absence of hyperons, the conditions of charge equilibrium and β -decay equilibrium in the stellar interior require that

$$p_F(p) = p_F(e), \quad (8)$$

$$\mu_n = \mu_p + \mu_e. \quad (9)$$

If the chemical potentials in the last equation are approximated by the corresponding Fermi energies given by equations (6) and (7), we obtain near nuclear matter density,

$$p_F(n) \sim 340 \text{ MeV}/c, \quad (10)$$

$$p_F(p) = p_F(e) \sim 60 \text{ MeV}/c \quad (11)$$

so that $\mu_n \sim \mu_c \sim 60 \text{ MeV}$ and $\mu_p \sim 2 \text{ MeV}$ for $m_n^* = m_n$ and $m_p^* = m_p$.

From these relations it is evident that electrons, because of their extreme relativistic nature, are an energetically expensive way to maintain charge equilibrium. At densities below nuclear matter density, electrons (and perhaps muons) are the only source of negative charge, but as the density is increased, it eventually becomes more economical to replace the relativistic electrons with non-relativistic negatively charged hyperons. In the absence of pion condensates, the first such hyperons to appear are Σ^- hyperons. The relativistic mean field

calculations by Glendenning (1985) indicate that a significant Σ^- population may be present at matter densities, $\rho \sim 2.5 \rho_0$, where ρ_0 is the density of nuclear matter. At slightly higher densities, the Σ^- hyperons are accompanied by Λ hyperons and at much higher densities, by Σ^0 's, Σ^+ 's, and cascade particles. Depending upon the assumptions made regarding the baryon-baryon interaction, Glendenning finds that the Λ population is comparable to or exceeds the Σ^- population at densities in the range $\rho \gtrsim 2.4\text{--}5.8 \rho_0$. The populations of the other hyperon species, however, become significant only at much higher densities.

On the basis of these results, we suppose that the neutron star interior contains only Σ^- and Λ hyperons in addition to nucleons and electrons. The electrons are assumed to be degenerate and completely relativistic, the baryons degenerate and non-relativistic. It is assumed, furthermore, that the interior densities remain below those required for phase transitions to exotic forms of matter such as quark matter or pion condensed matter and that all the interior constituents are in the normal state; i.e., we do not consider any effects associated with baryonic superfluidity. The charge and β -decay equilibrium conditions then become

$$\rho_p = \rho_\Sigma + \rho_e \quad (12)$$

$$\mu_n = \mu_p + \mu_e \quad (13)$$

$$\mu_\Sigma = \mu_n + \mu_e \quad (14)$$

$$\mu_\Lambda = \mu_n \quad (15)$$

with the subscript " Σ " referring explicitly to the Σ^- hyperon.

III. NEUTRINO EMISSION PROCESSES

Single baryon decays, such as ordinary neutron decay, cannot occur in neutron star interiors because it is not possible to simultaneously

conserve energy and momentum in such decays in a highly degenerate environment. In order for neutrino emission to occur, a second baryon interacting strongly with the first must participate in the emission process. Such processes involving two strongly interacting baryons in the initial state are of two general types: URCA process,

$$A + B + C + B + e^- + \bar{\nu}, \quad (16)$$

and neutrino pair bremsstrahlung processes,

$$A + B + C + D + \nu + \bar{\nu}, \quad (17)$$

with the baryons A,B,C,D chosen consistent with charge conservation from among those assumed to exist in the stellar interior. The neutrinos emitted in these processes are thermal and once produced, non-interacting so that they do not participate in the equilibrium of the interior matter; the remaining particles, on the other hand, are in equilibrium with each other, as discussed in the previous section. Under these conditions, the inverse URCA processes

$$C + B + e^- + A + B + \nu \quad (18)$$

yield the same neutrino emissivities as the corresponding URCA processes and thus, need not be treated explicitly.

Since there is no strangeness changing weak neutral current in lowest order, the pair bremsstrahlung processes (17) must conserve strangeness. This restricts the possible weak interaction vertices to the vertices $n + n\bar{\nu}$, $p + p\bar{\nu}$, $\Sigma^- + \Sigma^-\bar{\nu}$, and $\Lambda + \Lambda\bar{\nu}$. The last vertex vanishes in the Weinberg-Salam theory since the Λ has both charge and isospin equal to zero. With these restrictions, the pair bremsstrahlung processes that can occur in the presence of neutrons, protons, Σ^- 's, and Λ 's can be divided into three classes. Processes of the first class have the form

$$B + B + B + \nu + \bar{\nu}, \quad (19)$$

where the single baryon B can be a neutron, proton, or Σ^- .

The diagrams contributing to processes (19) are illustrated in Figure 1. Here the solid lines designate either baryons or leptons, while the dashed lines represent meson exchanges that transfer momentum between the participating baryons in the directions indicated. Because the two initial baryons in each diagram are identical, only neutral meson exchange is possible, and all contributions to a particular process involve the same weak vertex. Among the four direct diagrams, labelled A,B,C,D in the figure, the baryon-baryon momentum transfers differ only by the momentum imparted to the thermal neutrino pair. Neglecting this momentum in comparison with the baryon momenta reduces the four-momentum transfers to the single expression

$$\vec{k} = \vec{p}_1 - \vec{p}_3. \quad (20)$$

The same approximation yields

$$\vec{k}' = \vec{p}_1 - \vec{p}_4, \quad (21)$$

for the momentum transfer in the four crossed contributions A',B',C',D', which must be included to make the matrix elements antisymmetric with respect to exchange of the final state baryons.

The second class of bremsstrahlung processes is characterized by the form

$$A + B + A + B + \nu + \bar{\nu} \quad (22)$$

with the pair of baryons (A,B) representing any of the choices (n,p), (Σ^- ,n), (Σ^- ,p), or (Σ^- , Λ). Note that the absence of a vertex $\Lambda \rightarrow \Lambda\nu\bar{\nu}$ excludes the choices (Λ ,n) or (Λ ,p). The relevant diagrams are depicted in Figure 2, where lines 1 and 3 refer to baryon A and lines 2 and 4 to baryon B. In contrast with class I processes, both neutral and charged

meson exchange can occur in class II processes. The diagrams with neutral meson exchange, labelled A,B,C,D, are characterized by the same ordering of the baryons in the final state as in the initial state; whereas in the charged meson exchange contributions, this ordering is reversed. Neglecting the neutrino momenta yields equations (20) and (21) for the momentum transfers associated with the two sets of diagrams. It should be observed that not all diagrams in Figure 2 contribute to all class II processes. Since a strong interaction model limited to one pion exchange is adopted here, diagrams requiring an exchange of strangeness between the baryons are not allowed. Thus, in processes involving a Σ -nucleon pair, only neutral meson exchange is possible. The absence of a weak $\Lambda \rightarrow \Lambda\nu\bar{\nu}$ vertex also limits the contributions to the process $\Lambda\Lambda \rightarrow \Lambda\Lambda\nu\bar{\nu}$ to diagrams F and G with lines 1 and 3 referring to the Σ^- and 2 and 4 to the Λ .

The remaining bremsstrahlung processes

$$\Sigma^- p + \Lambda\nu\bar{\nu} \quad (23)$$

$$\Lambda n + \Sigma^- p\nu\bar{\nu} \quad (24)$$

involve a charge transfer between the participating baryons. In the first process, the contributing diagrams are just those labelled F,G,H in Figure 2 with lines 1,2,3,4 referring to the Σ^- ,p,n, Λ respectively; whereas in the second process, the diagrams contributing are E,F,G with the identifications $1 \rightarrow n$, $2 \rightarrow \Lambda$, $3 \rightarrow \Sigma^-$, and $4 \rightarrow p$. For both processes, the momentum transfer is given by equation (21).

In contrast with the weak neutral current, the weak charged current has both strangeness conserving ($\Delta S=0$) and strangeness changing ($\Delta S=1$) contributions in lowest order. However, in the Cabibbo theory, the $\Delta S=1$ decays are suppressed by a factor $\tan^2\theta_c \sim 0.054$ in the squared matrix

elements relative to the $\Delta S=0$ decays and thus, should not be very important in the determination of the overall neutrino luminosity of a neutron star. For this reason the $\Delta S=1$ decays need not be considered further. The relevant $\Delta S=0$ weak decay vertices consist of the neutron decay vertex, $n \rightarrow p e \bar{\nu}$, and the two $\Delta S=0$ Σ^- decay vertices, $\Sigma^- \rightarrow \Sigma^0 e \bar{\nu}$ and $\Sigma^- \rightarrow \Lambda e \bar{\nu}$.

Like the pair bremsstrahlung processes, the strangeness conserving URCA processes occurring in the presence of neutrons, protons, Σ^- hyperons, and Λ hyperons can be grouped in three classes. The first class consists of the two processes,

$$nn + n p e^- \bar{\nu} \quad (25)$$

$$\Sigma^- \Sigma^- \rightarrow \Sigma^- \Lambda e^- \bar{\nu}, \quad (26)$$

with identical baryons in the initial state. Assuming that the strong interaction is adequately represented by one pion exchange, each process has six contributions: three direct contributions and three crossed contributions included to conform with the antisymmetry requirement in the initial state. These contributions are illustrated in Figure 3 with line 3 corresponding to the proton and Λ in (25) and (26) respectively. Note that in a one pion exchange model for the strong interaction, there is no diagram with the weak vertex attached to line 4 since this would require the existence of an isospin non-conserving $\pi\Lambda\Lambda$ vertex.

In the diagrams of Figure 3, the baryon momentum transfers all involve the momentum imparted to the electron, which is of order $p_F(e)$, the electron Fermi momentum. While $p_F(e)$ is not negligible in all cases, it is generally much smaller than the baryon Fermi momenta, so that neglect of the electron momentum in the momentum transfers associated with URCA processes is not an unreasonable approximation. With its

adoption and with the momentum of the thermal neutrino also neglected, the momentum transfers in the three direct diagrams (A,B,C) and the three crossed diagrams (A',B',C') just reduce to the simple bremsstrahlung expressions, equations (20) and (21) respectively.

Processes with identical baryons in the final state,

$$np \rightarrow p p e^- \bar{\nu} \quad (27)$$

$$\Sigma^- \Lambda \rightarrow \Lambda \Lambda e^- \bar{\nu}, \quad (28)$$

make up the second class of URCA processes. The contributions to the first reaction, depicted in Figure 4 with line 1 corresponding to the neutron, are analogous to those of Figure 3 except that the crossed diagrams involve interchange of the final state baryons rather than the initial state baryons. Because a pion cannot couple to two Λ lines, use of the one pion exchange model for the strong interaction restricts the contributions to the second process to those involving charge exchange (C and C'). For both processes, neglect of the lepton momenta yields equations (20) and (21) for the momentum transfers associated with the direct and crossed diagrams respectively.

The third class of URCA processes consists of processes involving a nucleon and a hyperon in both the initial and final states. The three possibilities consistent with the restrictions imposed on the strong and weak interactions are

$$\Sigma^- n \rightarrow \Sigma^- p e^- \bar{\nu} \quad (29)$$

$$\Sigma^- n \rightarrow \Lambda n e^- \bar{\nu} \quad (30)$$

$$\Sigma^- p \rightarrow \Lambda p e^- \bar{\nu}. \quad (31)$$

For the first two processes, the contributing diagrams are just the direct diagrams (A,B,C) in Figure 3 with lines 1 and 3 referring to the neutron and proton in (29) and to the Σ^- and Λ in (30). The direct

diagrams in Figure 4 with the identifications $1 \rightarrow \bar{\nu}$ and $3 \rightarrow \Lambda$ make up the contributions to the third process. In all cases the momentum transfer is given by equation (20).

IV. INTERACTION VERTICES

In lowest order the URCA and pair bremsstrahlung processes each involve a weak interaction whereby neutrinos are produced and a strong interaction through which momentum is exchanged between the interacting baryons. The weak interaction is mediated by a weak vector boson with a mass much larger than the momentum transfer to the leptons and thus, can be adequately approximated by the single vertex.

$$\mathcal{L}_w = 1/\sqrt{2} \, G \, \bar{f}_\mu \not{q}^\mu, \quad (32)$$

with q^μ and $\bar{f}_\mu$, the weak lepton and baryon currents, given by

$$\not{q}^\mu = \bar{u}(q_1) \gamma^\mu (1 + \gamma_5) v(q_2) \quad (33)$$

$$\bar{f}_\mu = \bar{u}(p_1) \gamma_\mu (g_V - g_A \gamma_5) u(p_2) \quad (34)$$

in a standard notation. For non-relativistic baryons, (34) reduces to

$$\bar{f}_\mu = \chi_1^\dagger (g_V \delta_{\mu 0} + g_A \delta_{\mu i} \sigma_i) \chi_2, \quad (35)$$

where χ_i denotes a two-component Pauli spinor.

Within the Cabibbo theory, the charged current vector and axial vector coupling strengths can be estimated using SU(3) symmetry relations. For the neutral current coupling strengths, we employ SU(3) symmetry relations in the Weinberg-Salam model (Weinberg 1972). The resulting expressions for g_V and g_A (at $q^2=0$) are given in Table I in terms of the reduced symmetric and antisymmetric coupling constants, D and F respectively, the Cabibbo angle θ_c , and the Weinberg angle θ_w .

At the densities and temperatures typical of neutron star interiors, the single most important contribution to the strong interaction between

degenerate nucleons is that with the longest range, the one pion exchange (OPE) interaction. Although shorter range meson exchanges, such as ρ -meson exchange, may not be negligible quantitatively, especially at higher densities, calculations based on the OPE interaction alone should be sufficiently reliable to discern the relative importance of various neutrino processes and to yield at least a semi-quantitative estimate of the overall stellar neutrino luminosity. This conclusion is supported by the work of Frieman and Maxwell (1979), who explicitly examined the effect of ρ -meson exchange and short-range correlations on the nucleon URCA and pair bremsstrahlung emissivities [process (1), (3), and (4)].

In processes involving hyperons, strange mesons may be exchanged between the baryons. However, since the lightest meson with strangeness, the K-meson, has a mass more than 3 1/2 times that of the pion, strange meson exchange should be less important than pion exchange. Thus, in the case of strange baryons as well as nucleons, we suppose that the strong interaction is adequately represented by the OPE contribution alone.

In momentum space, the OPE interaction that connects baryons A and B in the initial state with baryons C and D in the final state has the general form

$$V_{12}(k_\mu) = \bar{v}_2^{B\dagger} D(k_\mu)^\dagger (\not{m}_\mu^2 - \epsilon)^{-1} \not{V}_1^{A\dagger} C^\mu(k_\mu), \quad (36)$$

where $k_\mu = (k_0, \vec{k})$ is the four-momentum transferred from the pion emission vertex (labelled 1) to the absorption vertex (labelled 2), and $\epsilon = k_0^2 - |\vec{k}|^2$. To construct the relativistic vertex functions, V_1 and $v_2^\dagger$, we employ the pseudovector form of the pion-baryon interaction Lagrangian and introduce the baryon annihilation and creation operators, a and $a^\dagger$. The resulting expression for the emission vertex function is

$$V_1^{A+C\pi}(k_\mu) = f(A+C\pi) \bar{u}(p_C) \gamma_5 \gamma^\mu u(p_A) k_\mu a_1^{\dagger(C)} a_1(A). \quad (37)$$

Applying the non-relativistic reduction to this expression and to the corresponding expression for the absorption vertex and neglecting the energy transfer k_0 in the pion propagator then yields the non-relativistic baryon-baryon potential used in the present work,

$$V_{12}(\vec{k}) = - \frac{f(A+C\pi)}{m_\pi} \frac{f(D+B\pi)}{m_\pi} \frac{\vec{\sigma}_1 \cdot \vec{k} \vec{\sigma}_2 \cdot \vec{k}}{k^2 + m_\pi^2} \quad (38)$$

The coupling strengths appearing here can each be expressed as a product of an isospin-reduced coupling strength, common to all vertices involving the same isospin multiplets, and an isospin vector coupling coefficient. In particular,

$$f(A+C\pi) = g_{AC\pi} (I_C I_C^Z I_\pi^Z |I_A I_A^Z), \quad (39)$$

where I_A and I_A^Z are the total isospin and isospin projection of baryon A and I_π^Z is the pion isospin projection. Using the SU(3) symmetry relations among vertices involving two octet baryons and one octet meson, the reduced strengths can, in turn, be parametrized in terms of an overall strength g and the SU(3) ratio $\alpha = D/(D+F)$. For the relevant multiplet combinations, we obtain the expressions

$$g_{NN\pi} = \sqrt{3} g \quad (40)$$

$$g_{\Sigma\Sigma\pi} = 2\sqrt{2} g(1-\alpha) \quad (41)$$

$$g_{\Sigma\Lambda\pi} = 2/\sqrt{3} g\alpha \quad (42)$$

$$g_{\Lambda\Sigma\pi} = -2 g\alpha, \quad (43)$$

which, when multiplied by the appropriate vector coupling coefficients according to equation (39), yield the strong interaction coupling strengths compiled in Table 2.

V. NEUTRINO EMISSIONS

In addition to the interaction vertices, the matrix elements for the various neutrino processes involve an intermediate baryon propagator. We employ the same expression for this propagator that was used by Friman and Maxwell (1979), namely

$$iD(\vec{p} + \vec{q}, E_p \pm \omega) = \pm i/\omega, \quad (44)$$

where $\vec{q}$ and $\vec{p}$ are the lepton and external baryon momenta, ω is the lepton energy, and the sign is chosen positive or negative according to whether the external baryon is outgoing or incoming. For bremsstrahlung processes, $\omega = \omega_\nu + \omega_\nu$, while for URCA processes, $\omega = \omega_e + \omega_\nu \equiv \omega_e$ since $\omega_e \gg \omega_\nu \sim kT$. As pointed out by Friman and Maxwell (1979), use of (44) amounts essentially to the neglect of recoil terms in the propagator.

The matrix elements constructed with equation (44) are independent of the vector component of the weak current in lowest order. To illustrate this result for the class I bremsstrahlung processes, consider the contributions from diagrams A and B in Figure 1. For diagram A, we obtain using (32), (35), (38), and (44)

$$M_I^A(\vec{k}) = \frac{G}{\sqrt{2}} \left(\frac{f(B+B\pi^0)}{m_\pi} \right)^2 \chi_3^\dagger (g_V^{B \rightarrow B} \delta_{\mu 0} + g_A^{B \rightarrow B} \delta_{\mu i} \sigma_i) \vec{\sigma} \cdot \vec{k} \chi_1 \frac{1}{\omega} \frac{1}{k^2 + m_\pi^2} \chi_4^\dagger \vec{\sigma} \cdot \vec{k} \chi_2 \not{k}^\mu, \quad (45)$$

where χ_i denotes the Pauli spinor associated with line i , $\not{k}^\mu$ is the weak lepton current given by (33), and B identifies the baryon species involved. Since the corresponding contribution from diagram B differs from this only in the order of the interactions and the sign of the baryon propagator, the weak vector terms in the two contributions cancel. Similar pairwise cancellations occur among the other six diagrams in

Figure 1 and among the eight contributions to the class II bremsstrahlung matrix elements shown in Figure 2. For the class III bremsstrahlung processes and the URCA processes, the cancellation involves up to three diagrams at a time and depends upon the particular algebraic relationships that exist among the various coupling coefficients displayed in Tables 1 and 2.

By contrast, the axial vector matrix elements do not cancel among the different diagrams. Using the Pauli spin matrix identities,

$$i\vec{\sigma}(\vec{\sigma}\cdot\vec{k}) + i(\vec{\sigma}\cdot\vec{k})\vec{\sigma} = 2\ i\vec{k} \quad (46)$$

$$i\vec{\sigma}(\vec{\sigma}\cdot\vec{k}) - i(\vec{\sigma}\cdot\vec{k})\vec{\sigma} = 2\ \vec{\sigma}\times\vec{k}, \quad (47)$$

and noting that $\sigma_i\delta_i = -\vec{\sigma}\cdot\vec{k}$ in our metric, the summed axial matrix elements can be expressed as

$$M(\vec{k}, \vec{k}') = \frac{M_1(\vec{k})}{k^2 + m_\pi^2} + \frac{M_2(\vec{k}')}{k'^2 + m_\pi^2} \quad (48)$$

with

$$M_1(\vec{k}) = \frac{\chi}{\omega} \left\{ [a_1 \chi_3^\dagger \chi_1\ i\vec{k}\cdot\vec{k} + a_2 \chi_3^\dagger \vec{\sigma}\times\vec{k}\ \chi_1\cdot\vec{k}] \chi_4^\dagger \vec{\sigma}\cdot\vec{k}\ \chi_2 + \chi_3^\dagger \vec{\sigma}\cdot\vec{k}\ \chi_1 [a_3 \chi_4^\dagger \chi_2\ i\vec{k}\cdot\vec{k} + a_4 \chi_4^\dagger \vec{\sigma}\times\vec{k}\ \chi_2\cdot\vec{k}] \right\} \quad (49)$$

$$M_2(\vec{k}') = \frac{\chi}{\omega} \left\{ [b_1 \chi_4^\dagger \chi_1\ i\vec{k}'\cdot\vec{k}' + b_2 \chi_4^\dagger \vec{\sigma}\times\vec{k}'\ \chi_1\cdot\vec{k}'] \chi_3^\dagger \vec{\sigma}\cdot\vec{k}'\ \chi_2 + \chi_4^\dagger \vec{\sigma}\cdot\vec{k}'\ \chi_1 [b_3 \chi_3^\dagger \chi_2\ i\vec{k}'\cdot\vec{k}' + b_4 \chi_3^\dagger \vec{\sigma}\times\vec{k}'\ \chi_2\cdot\vec{k}'] \right\} \quad (50)$$

where

$$\gamma = \frac{G}{f^2} \left(\frac{g}{m_\pi} \right)^2, \quad (51)$$

and $\vec{k}$ and $\vec{k}'$ are given by equations (20) and (21). For the bremsstrahlung processes, $\omega = \omega_y$, and $a_1 = a_3 = 0$. The remaining coefficients are given in Table 3 in terms of the coupling parameters α , D , and F defined in § IV and the quantity $\delta \equiv D+F$. For the URCA

processes, $\omega \equiv \omega_c$ and we define

$$\delta_c = (D+F) \cos\theta_c \quad (52)$$

$$\zeta_{\pm} = \sqrt{8/3} [(1-\alpha) D \pm \alpha F] \cos\theta_c \quad (53)$$

$$\eta = 4[(1-\alpha)D + 1/3 \alpha F] \cos\theta_c, \quad (54)$$

yielding the expressions collected in Table 4 for the coefficients in equation (49). The corresponding coefficients in equation (50) are related to these through the prescriptions: $b_1 = -a_3$, $b_2 = -a_4$, $b_3 = -a_1$, $b_4 = -a_2$ for class I URCA processes; $b_i = -a_i$, $i = 1-4$ for class II URCA processes; and $b_i = 0$, $i = 1-4$ for class III URCA processes.

The various matrix elements must now be squared and summed over the spins of the participating fermions. To perform the spin sums, we evaluate the baryon traces of the squared matrix elements and contract with the relativistic lepton trace,

$$\text{Tr}(\ell_j^\dagger \ell_i) = 8(q_{1i}q_{2j} + q_{2i}q_{1j} + (\omega_1\omega_2 - \vec{q}_1\cdot\vec{q}_2)\delta_{ij} + i\epsilon_{ijab} q_1^a q_2^b), \quad (55)$$

where $\vec{q}_1, \vec{q}_2$ and ω_1, ω_2 are the lepton momenta and energies, and ϵ_{ijab} is the completely antisymmetric tensor of rank 4. If the same approximation is made in the momentum conservation relation as was made in equations (20) and (21) for the momentum transfers — i.e., if the lepton momenta are neglected compared with the baryon momenta — the terms in the lepton trace involving the lepton momenta will all vanish in the phase space integrations and need not be considered further. The surviving term involving the product of lepton energies yields spin-summed squared matrix elements of the form

$$\sum_{\text{spins}} |M(\vec{k}, \vec{k}')|^2 = 128 \gamma^2 \frac{\omega_1}{\omega^2} \frac{\omega_2}{\omega^2} f(\vec{k}, \vec{k}') \quad (56)$$

with $\omega = \omega_1 + \omega_2$ and

$$\mathcal{F}(\vec{k}, \vec{k}') = C_1 \frac{k^2}{(k^2 + m_\pi^2)} + C_2 \frac{(k'^2)}{(k'^2 + m_\pi^2)} + C_3 \frac{k^2 k'^2 - C_4 (\vec{k} \cdot \vec{k}')^2}{(k^2 + m_\pi^2)(k'^2 + m_\pi^2)}, \quad (57)$$

where the coefficients in the last expression are tabulated in Tables 5

and 6 for the bremsstrahlung and URCA processes, respectively. In

Table 6 the auxiliary quantities are defined by

$$h_1 = 2/3 D^2 \cos^2 \theta_c \quad (58)$$

$$h_2 = 4(1-\alpha)^2 F^2 \cos^2 \theta_c \quad (59)$$

$$h_3 = 4/3 \alpha(1-\alpha) DF \cos^2 \theta_c \quad (60)$$

$$d_\pm = 3(1-\alpha)^4 + 2/3 \alpha^2(1-\alpha)^2 \pm 1/9 \alpha^4. \quad (61)$$

We must now integrate the product of the spin-summed squared matrix element and the total neutrino energy over the available phase space subject to the constraints of energy conservation,

$$E_1 + E_2 = E_3 + E_4 + \omega_1 + \omega_2 \quad (62)$$

and momentum conservation (with $\vec{q}_1$ and $\vec{q}_2$ set equal to zero),

$$\vec{p}_1 + \vec{p}_2 = \vec{p}_3 + \vec{p}_4. \quad (63)$$

For the bremsstrahlung processes, defining the Fermi-Dirac factor,

$$\mathcal{L}_{\nu\nu} = \frac{1}{S} n_1 n_2 (1-n_3)(1-n_4), \quad (64)$$

where S is the number of identical baryons in the final state and

$$n_i = \left[1 + e^{[(E_i - E_F(i))/KT]} \right]^{-1} \quad (65)$$

with K equal to the Boltzmann constant, we have

$$\begin{aligned} \epsilon_{\nu\nu} = & \frac{1}{\mathcal{H}} \int \prod_{i=1}^4 \frac{d^3 p_i}{(2\pi)^3} \frac{d^3 q_1}{(2\pi)^3} \frac{d^3 q_2}{2\omega_1 (2\pi)^3 2\omega_2} (2\pi) \delta(E_{\text{final}} - E_{\text{init}}) \\ & \times (2\pi)^3 \delta(\vec{p}_{\text{final}} - \vec{p}_{\text{init}})(\omega_1 + \omega_2) \mathcal{L}_{\nu\nu} \sum_{\text{spins}} |M(\vec{k}, \vec{k}')|^2. \end{aligned} \quad (66)$$

The corresponding expression for the URCA processes is identical to this except that the URCA Fermi-Dirac factor,

$$\mathcal{L}_{\text{URCA}} = \frac{1}{S} n_1 n_2 (1-n_3)(1-n_4)(1-n_e), \quad (67)$$

has to be inserted in place of $\mathcal{L}_{\nu\nu}$, and the sum of neutrino and anti-neutrino energies replaced by the neutrino energy alone.

The integrals over the neutrino angles in equation (66) can be performed immediately since the integrand does not depend on the directions of the neutrino momenta. To perform the baryon angular integrations, we introduce auxiliary energy integrals via the insertion,

$$1 = \int dE_i \delta(E_i - E_F(i)), \quad (68)$$

replace the energy δ -functions with δ -functions in the momentum magnitudes using equation (7),

$$\delta(E_i - E_F(i)) = m_i^*/p_F(i) \delta(p_i - p_F(i)), \quad (69)$$

and make the approximation,

$$\int d^3 p_i dE_i \delta(p_i - p_F(i)) \rightarrow \int d^3 p_i \delta(p_i - p_F(i)) \int dE_i, \quad (70)$$

which is valid for momenta close to the Fermi surface. This yields

$$\epsilon_{\nu\nu} = \frac{128}{\mathcal{H}} \gamma^2 \mathcal{L}_{\nu\nu} \int \prod_{i=1}^4 \frac{d^3 p_i}{(2\pi)^3} \frac{m_i^*}{p_F(i)} \delta(p_i - p_F(i)) \delta(\vec{p}_{\text{final}} - \vec{p}_{\text{init}}) \mathcal{F}(\vec{k}, \vec{k}') \quad (71)$$

with $\mathcal{F}$ defined by equation (57) and

$$\mathcal{L}_{\nu\nu} = \int \prod_{i=1}^4 \frac{dE_i d\omega_1 d\omega_2}{\omega_1 + \omega_2} \frac{\omega_1^2 \omega_2^2}{\omega_1 + \omega_2} \mathcal{L}_{\nu\nu} \delta(E_{\text{final}} - E_{\text{init}}). \quad (72)$$

For the URCA processes an analogous procedure yields the same expression but with

$$\mathcal{L}_{\text{URCA}} = \int \prod_{i=1}^4 \frac{dE_i d\omega_1 d\omega_2}{\omega_1 + \omega_2} \mathcal{L}_{\text{URCA}} \delta(E_{\text{final}} - E_{\text{init}}) \quad (73)$$

in place of $\mathcal{L}_{\nu\nu}$.

The energy integrals here can be carried out using the equilibrium relations, equations (13)-(15), with the chemical potentials in those relations approximated by the corresponding Fermi energies. Introducing the dimensionless variables,

$$x_1 = (E_1 - E_F(1))/KT, \quad 1 = 1, 2 \quad (74)$$

$$x_1 = (E_F(1) - E_1)/KT, \quad 1 = 3, 4 \quad (75)$$

$$x_5 = (E_F(e) - \omega_e)/KT \quad (76)$$

$$y = \omega/KT \quad (77)$$

with $\omega = \omega_1 \omega_2$ in (72) and $\omega = \omega_v$ in (73) and extending the limits of the x-integrations to $\pm\infty$ then leads to the approximate expressions,

$$I_{v\bar{v}} = \frac{1}{S} \frac{(KT)^8}{30} \int_0^\infty dy y^4 \int_{-\infty}^\infty \prod_{i=1}^4 \frac{dx_i}{1+e^{x_i}} \delta\left(\sum_{i=1}^4 x_i - y\right) \quad (78)$$

$$I_{URCA} = \frac{1}{S} (KT)^8 \int_0^\infty dy y^3 \int_{-\infty}^\infty \prod_{i=1}^5 \frac{dx_i}{1+e^{x_i}} \delta\left(\sum_{i=1}^5 x_i - y\right) \quad (79)$$

which are correct up to terms of order $\exp(-E_F(i)/KT)$ and can be evaluated analytically using a technique described by Baym and Pethick (1978). We finally obtain

$$I_{v\bar{v}} = \frac{1}{S} \frac{41}{14175} \frac{(2\pi)^8}{128} (KT)^8 \quad (80)$$

$$I_{URCA} = \frac{1}{S} \frac{11513}{120960} \frac{(2\pi)^8}{256} (KT)^8. \quad (81)$$

To perform the momentum integrals, we first reexpress the function $\tilde{f}(\vec{k}, \vec{k}')$ in terms of $\vec{k}$ and the baryon momenta through use of the

$$\vec{k} \cdot \vec{k}' = 1/2(p_1^2 + p_2^2 - p_3^2 - p_4^2) \quad (82)$$

$$k'^2 = p_1^2 + p_2^2 + k^2 - 2(\vec{p}_1 - \vec{p}_2) \cdot \vec{k} - 2\vec{p}_1 \cdot \vec{p}_2, \quad (83)$$

which follow from (20), (21), and the momentum conservation relation,

(63), make the insertion

$$1 = \int d^3k \delta(\vec{k} + \vec{p}_3 - \vec{p}_1), \quad (84)$$

and note that in equation (71) the baryon momenta are all constrained to the corresponding Fermi surfaces by the δ -functions in the momentum magnitudes. Carrying out the baryon momentum integrals and the momentum transfer angular integrals then eliminates the momentum conserving δ -function and the δ -function in (84) and fixes the angles θ_1 and θ_2 between $\vec{k}$ and the incoming momenta, $\vec{p}_1$ and $\vec{p}_2$. For the bremsstrahlung processes, after inserting (80) for $I_{v\bar{v}}$ and observing that $\vec{k} \cdot \vec{k}' = 0$ for those bremsstrahlung processes that depend on both $\vec{k}$ and $\vec{k}'$ (class I and II processes), we obtain the result

$$e_{v\bar{v}} = \frac{\gamma^2 m_N^5}{2\pi\hbar} \frac{1}{S} \frac{82}{14175} \prod_{i=1}^4 \left(\frac{m_i}{m_N}\right) (KT)^8 \left[C_1 \tilde{f}_1(k_{min}, k_{max}) + C_2 \tilde{f}_1(k'_{min}, k'_{max}) + C_3 \tilde{f}_2(k_{min}, k_{max}) \right], \quad (85)$$

with

$$\tilde{f}_1(k_1, k_2) = \int_{k_1}^{k_2} \frac{dk}{m_N} \frac{k^2}{k^2 + m_\pi^2} \left[1 - \frac{m_\pi^2}{\sqrt{t_1^2 - t_2^2}} \right]^2 \quad (86)$$

$$\tilde{f}_2(k_1, k_2) = \int_{k_1}^{k_2} \frac{dk}{m_N} \frac{k^2}{k^2 + m_\pi^2} \left[1 - \frac{m_\pi^2}{\sqrt{t_1^2 - t_2^2}} \right] \quad (87)$$

where

$$t_1(k) = p_F^2(3) + p_F^2(4) - k^2 + m_\pi^2 - 2p_F(1) p_F(2) \cos\theta_1 \cos\theta_2 \quad (88)$$

$$t_2(k) = 2p_F(1) p_F(2) \sin\theta_1 \sin\theta_2 \quad (89)$$

and

$$\cos\theta_1 = (p_F^2(1) - p_F^2(3) + k^2)/2p_F(1)k \quad (90)$$

$$\cos\theta_2 = -(p_F^2(2) - p_F^2(4) + k^2)/2p_F(2)k. \quad (91)$$

The corresponding expression for the URCA processes is

$$\epsilon_{URCA} = \frac{\gamma_{m_N}^2}{2\pi\hbar} \frac{1}{S} \frac{11513}{120960} \prod_{i=1}^4 \left(\frac{m_i^*}{m_N} \right) (KT)^8 [C_1 \mathcal{F}_1(k_{\min}, k_{\max}) + C_2 \mathcal{F}_1(k_{\min}, k_{\max}) + C_3 \mathcal{F}_2(k_{\min}, k_{\max}) + C_4 \mathcal{F}_3(k_{\min}, k_{\max})] \quad (92)$$

with

$$\mathcal{F}_3(k_1, k_2) = (\vec{k} \cdot \vec{k}')^2 \int_{k_1}^{k_2} dk \frac{1}{m_N} \frac{1}{k^2 + m_\pi^2} \frac{1}{\sqrt{t_1^2 - t_2^2}}, \quad (93)$$

where $\vec{k} \cdot \vec{k}'$ is given by (82) with $p_i = p_F(i)$. In both emissivity expressions the requirement that the angles θ_1 and θ_2 be real fixes the limits of the k -integrals; in particular,

$$k_{\min} = \max(|p_F(1) - p_F(3)|, |p_F(2) - p_F(4)|) \quad (94)$$

$$k_{\max} = \min(p_F(1) + p_F(3), p_F(2) + p_F(4)) \quad (95)$$

$$k'_{\min} = \max(|p_F(1) - p_F(4)|, |p_F(2) - p_F(3)|) \quad (96)$$

$$k'_{\max} = \min(p_F(1) + p_F(4), p_F(2) + p_F(3)) \quad (97)$$

The remaining integrations over k can all be performed analytically, but since the resulting expressions are somewhat unwieldy, they have been collected in an appendix.

VI. NUMERICAL RESULTS

In this section we present some numerical results that have been obtained with the formulae developed in the previous section. These results are meant to be of an exploratory nature rather than quantitatively precise; the goal is to provide an indication of the importance of various neutrino processes involving nucleons and hyperons and to determine to what extent these processes depend upon the make up of the neutron star interior. With this goal in mind, we have selected several interior scenarios within which the neutrino emissivities are evaluated. In each scenario the fractional baryon population, defined by

$$Y_i = \rho_i / \rho_b, \quad i = n, p, \Sigma^-, \text{ or } \Lambda, \quad (98)$$

where ρ_b is the total baryon density, and the electron to baryon ratio,

$$Y_e = \rho_e / \rho_b \quad (99)$$

are fixed subject to the constraints of baryon number conservation,

$$\sum_i Y_i = 1 \quad (100)$$

and charge neutrality,

$$Y_e = Y_p - Y_\Sigma. \quad (101)$$

In reality, of course, the baryon and electron fractions are density dependent, so that the description of a real neutron star interior would require a combination of the scenarios considered here.

The particular values of Y_i , $i = n, p, \Sigma$, and Λ , and Y_e assumed in the different scenarios are listed in Table 7. In the first two scenarios only nucleons and electrons are included. Scenario 1 represents the standard scenario employed in most cooling calculations to date where the bulk of the baryon population consists of neutrons with small, equal admixtures of protons and electrons. As pointed out by Glendenning (1985), the small charged particle populations in this scenario reflect an incomplete treatment of the charge symmetry energy in nuclear matter. Correction of this deficiency leads to substantially enhanced proton and electron populations, as represented by scenario 2. In scenarios 3 and 4 hyperons are included in addition to nucleons and electrons. Scenario 3 assumes a rather moderate admixture of hyperons, while scenario 4 with a 50% hyperon admixture represents an extreme case. Because of the charge neutrality constraint, equation (101), the occurrence of Σ^- particles in neutron star interiors permits a particular proton population to be maintained with a smaller electron fraction. This is especially evident in a comparison of the electron fractions in scenarios 2 and 4.

To reduce the emissivity expressions of the previous section to numerical form necessitates choices for the various interaction parameters defined in § 4 and for the baryon effective masses defined by equation (7). For the weak interaction parameters we adopt the values $D = 0.756$, $F = 0.477$, and $\cos\theta_c = 0.973$ (Bourquin et al. 1983), which yield charged current coupling strengths in accord with the empirical strengths (see, e.g., Kubodera, Kohyama, and Kim 1985). The strong interaction parameters are fixed by requiring that the expressions for $f(n + p\pi^-)$ and $f(\Sigma^- + \Sigma^0\pi^-)$ given in Table 2 agree with the average empirical values (Dumbrajs et al. 1983). This yields $g = 0.998$ and $\alpha = 0.617$. Combining the value for g with the overall weak coupling strength, $G = 1.164 \times 10^{-5} \text{ GeV}^{-2}$ (Dumbrajs et al. 1983) and introducing the quantity $T_9 = T/10^9 \text{ K}$ permits equations (85) and (92) for the bremsstrahlung and URCA emissivities to be reexpressed in the convenient numerical forms

$$\epsilon_{\nu\bar{\nu}} = 1.224 \times 10^{20} \frac{1}{S} \prod_{i=1}^4 \left(\frac{m_i^*}{m_N} \right) \left[C_1 \int_1^{\infty} \int_1^{\infty} (k_{\min}, k_{\max}) + C_2 \int_1^{\infty} \int_1^{\infty} (k_{\min}^1, k_{\max}^1) \right. \\ \left. + C_3 \int_1^{\infty} \int_1^{\infty} (k_{\min}, k_{\max}) \right] T_9^8 \frac{\text{ergs}}{\text{cm}^3\text{-sec}} \quad (102)$$

$$\epsilon_{\text{URCA}} = 2.014 \times 10^{21} \frac{1}{S} \prod_{i=1}^4 \left(\frac{m_i^*}{m_N} \right) \left[C_1 \int_1^{\infty} \int_1^{\infty} (k_{\min}, k_{\max}) + C_2 \int_1^{\infty} \int_1^{\infty} (k_{\min}^1, k_{\max}^1) \right. \\ \left. + C_3 \int_1^{\infty} \int_1^{\infty} (k_{\min}, k_{\max}) + C_4 \int_1^{\infty} \int_1^{\infty} (k_{\min}, k_{\max}) \right] T_9^8 \frac{\text{ergs}}{\text{cm}^3\text{-sec}}, \quad (103)$$

where S is 2 for the class I bremsstrahlung processes and class II URCA processes and 1 for all other processes. Numerical values for the C -coefficients, obtained from the expressions in Tables 5 and 6 using the values adopted for D , F , $\cos\theta_c$, and α , are tabulated in Table 8. Explicit expressions for the $\int$ -integrals appear in the appendix.

Choosing numerical values for the baryon effective masses is a more difficult task in that these effective masses are generally density dependent and at typical neutron star densities could vary significantly from the bare masses. Rather than address this difficult problem here, however, we bypass it by simply equating the effective masses with the bare masses. Such a procedure ought to be adequate for discerning the relative importance of different contributions to the total neutrino emissivity in line with the general philosophy outlined at the beginning of this section. Alternative choices for the effective masses can be easily accommodated through appropriate substitutions in equations (102) and (103).

Results for the emissivities obtained in the two scenarios without hyperons are summarized in Table 9 for $T_9 = 1$ and two different densities. Results for other temperatures are related to these through an overall factor T_9^8 , as seen in equations (102) and (103). Since all the processes considered here exhibit the same temperature dependence, any conclusions drawn regarding their relative significance will not depend on the particular temperature adopted.

For both scenarios the URCA processes are far more important than the bremsstrahlung processes, a result which could have been predicted from the magnitudes of the overall coefficients in equations (102) and (103). In scenario 1 with its relatively small proton fraction, the process $nn + n\bar{\nu}$ dwarfs all other processes, accounting for more than 90% of the total emissivity. In scenario 2 the enhanced proton fraction has two consequences: first, it increases the emissivity from the $nn + n\bar{\nu}$ process by about 50%; second, it leads to significant contributions from the other nucleon URCA processes, $np + p\bar{\nu}$. Together these two

effects yield a total emissivity in scenario 2 that is nearly double that of scenario 1.

With the inclusion of hyperons we obtain the emissivities tabulated in Table 10 for $T_9 = 1$ and the same two densities. As for the scenarios without hyperons, the URCA processes dominate the bremsstrahlung processes when hyperons are included. It is also apparent that the purely nucleonic processes are generally more important than processes involving hyperons, a result which probably has its origin in the larger value of the axial coupling strength in neutron decay as compared with Σ^- decay. In support of this supposition, we note that the most important process involving hyperons, $\Sigma n \rightarrow \Sigma p e \bar{\nu}$, also involves an $n \rightarrow p$ conversion.

The most important process in both hyperon scenarios is the process $nn \rightarrow npe \bar{\nu}$. In scenario 3 the emissivity from the other nucleon URCA process, $np \rightarrow ppe \bar{\nu}$ is about 40% of the $nn \rightarrow npe \bar{\nu}$ emissivity, while the emissivity from the hyperon process, $\Sigma n \rightarrow \Sigma p e \bar{\nu}$, ranges from 10 to 15% of the $nn \rightarrow npe \bar{\nu}$ emissivity, depending upon the density. Other hyperon processes of some significance include the processes $\Sigma n \rightarrow \Lambda n e \bar{\nu}$, $\Sigma p \rightarrow \Lambda p e \bar{\nu}$, and $\Sigma \Sigma \rightarrow \Sigma \Lambda e \bar{\nu}$. The last of these is the only purely hyperonic process that is non-negligible. Altogether, processes involving hyperons contribute about 20% of the total emissivity in scenario 3, which is roughly 25% larger than the emissivity in scenario 2.

As compared with scenario 3, the emissivity from the dominant process, $nn \rightarrow npe \bar{\nu}$, is about 15 to 20% smaller in scenario 4 so that the ratio $e(np \rightarrow ppe \bar{\nu})/e(nn \rightarrow npe \bar{\nu})$ is increased somewhat. By contrast, the emissivities from the hyperon processes are all substantially larger in scenario 4. The $\Sigma n \rightarrow \Sigma p e \bar{\nu}$ emissivity, for example, is almost a quarter of the $nn \rightarrow npe \bar{\nu}$ emissivity in this scenario, and the summed emissivities

from all hyperon processes make up more than 30% of the total. However, the increased hyperon contribution is offset by the reduced $nn \rightarrow npe \bar{\nu}$ emissivity so that the total emissivities in scenarios 3 and 4 are nearly equal.

The density dependence of the emissivities is illustrated in Figures 5 and 6 for several of the more important URCA processes in scenarios 3 and 4. As the figures reveal, the emissivities obtained with fixed baryon number fractions and effective masses exhibit a smooth, monotonic dependence on the density that is quite weak. It should be emphasized, however, that these results do not incorporate the density dependence of the effective masses, which, in view of the strong effective mass dependence exhibited in equations (102) and (103), could be more important than the density dependence associated with the $\int$ -integrals.

VII. SUMMARY AND CONCLUSIONS

In this work several neutrino emission processes expected to occur in neutron stars containing hyperons have been examined. As a basis for the calculations, we supposed the neutron star interior to consist of a dense, degenerate soup of extremely relativistic electrons and non-relativistic neutrons, protons, Σ^- 's, and Λ 's subject to the constraints of local beta and charge equilibrium. The neutrino emissivities were then evaluated for a number of scenarios characterized by fixed baryon and electron number fractions. In every scenario considered, the most important emission processes were found to be the nucleon URCA processes, $nn \rightarrow npe \bar{\nu}$ and $np \rightarrow ppe \bar{\nu}$, which even in the most extreme hyperon scenario, together account for two-thirds of the total neutrino emissivity. Thus, if all baryons in the interior are in the normal state, the presence of

hyperons will not significantly affect the overall neutrino luminosity of the star.

The situation is quite different, however, if either the neutrons or protons in the interior are superfluid, since in that case both of the nucleon URCA processes will be suppressed exponentially and the neutrino flux must be fed by other processes. In a star containing hyperons, one or more of the processes $n + \Lambda n\bar{\nu}$, $\bar{p} + \Lambda p\bar{\nu}$, and $\Sigma + \Sigma n\bar{\nu}$ can occur, depending upon which nucleons are superfluid. In the event that both neutrons and protons are superfluid in the interior, only the last process will occur with a significant rate. If hyperons are not present, the only possibilities are the nucleon bremsstrahlung processes, $nn + nn\bar{\nu}$ and $pp + pp\bar{\nu}$, again depending upon which nucleons are superfluid. According to Table 10, these processes have emissivities that are generally an order of magnitude below those from the nucleon URCA processes. Consequently, in stars containing superfluid nucleons, the presence of hyperons will substantially enhance the cooling rate due to neutrino emission from the interior.

As discussed previously, the calculations reported here were intended mainly to give an idea of the relative magnitudes of various processes and as such have entailed a number of simplifying approximations. To make the results more quantitative, a number of improvements are necessary. Among these is a better description of the strong interaction. The OPE potential employed here adequately describes only the long range part of the baryon-baryon interaction and by itself is probably too attractive. A more realistic baryon-baryon interaction would have to incorporate other meson exchanges and could alter our results significantly. Friman and Maxwell (1979) found, for example, that inclusion of

ρ -meson exchange reduces the calculated emissivities by partially quenching the strong tensor component of the OPE potential. Other meson exchanges are probably less significant, but it should be noted that inclusion of K-meson exchange makes possible additional contributions to those processes that involve both nucleons and hyperons.

A second improvement concerns the treatment of the incoming and outgoing baryon states in the various processes. In an environment as dense as the interior of a neutron star, the independent particle approximation may not be adequate for quantitative calculations but have to be replaced by some ansatz that includes short range correlations. The effect of such correlations was also considered by Friman and Maxwell (1979). Their results indicate that correlations reduce the calculated emissivities in a more or less uniform way that does not depend strongly on the density, the assumed form of the correlations, or the particular process considered.

A better interaction model and the inclusion of short range correlations are probably both necessary to achieve quantitatively reliable results, but neither modification is likely to alter our conclusions regarding the relative importance of various processes or their density dependences in a major way. A more realistic treatment of the baryon effective masses, however, could have qualitative consequences. On the one hand, a strong density dependence in the effective masses would manifest itself in the density dependence of the calculated emissivities, as mentioned in the previous section. On the other hand, significant differences in the ratio of the effective mass to the bare mass for different baryons would modify the emissivities from different processes in different ways, thereby affecting their relative magnitudes. Finally,

it is the effective masses that determines to a large extent whether the nucleons in the neutron star interior are superfluid or normal.

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APPENDIX — MOMENTUM TRANSFER INTEGRALS

The momentum transfer integrals defined by equations (86), (87), and (93) are most easily performed after making the substitution

$$\frac{k^2}{k^2 + m_\pi^2} = 1 - \frac{m_\pi^2}{k^2 + m_\pi^2}. \quad (104)$$

With this substitution, evaluation of the first integral is straightforward, yielding

$$f_1(k_1, k_2) = \frac{\delta k}{m_N} + \frac{m_\pi}{m_N} \left[\frac{m_\pi k}{k^2 + m_\pi^2} - 3 \tan^{-1} \left(\frac{k}{m_\pi} \right) \right] \Big|_{k_1}^{k_2} \quad (105)$$

with $\delta k = k_2 - k_1$. To evaluate f_2 and f_3 , we introduce the auxiliary integral

$$f(k_1, k_2) = \int_{k_1}^{k_2} \frac{dk}{k^2 + m_\pi^2 \sqrt{t_1^2 - t_2^2}}, \quad (106)$$

in terms of which f_2 and f_3 can be written

$$f_2(k_1, k_2) = \frac{1}{m_N} \int_{k_1}^{k_2} \frac{k^2 dk}{k^2 + m_\pi^2} - \frac{m_\pi^2}{m_N} \int_{k_1}^{k_2} \frac{dk}{\sqrt{t_1^2 - t_2^2}} + \frac{m_\pi^4}{m_N} f(k_1, k_2) \quad (107)$$

$$f_3(k_1, k_2) = \frac{(p_1^2 + p_2^2 - p_3^2 - p_4^2)^2}{4m_N} f(k_1, k_2) \quad (108)$$

where equation (82) with $p_i = p_F(i)$ has been used in the second expression. The first two integrals in (107) yield

$$\int_{k_1}^{k_2} \frac{k^2 dk}{k^2 + m_\pi^2} - \frac{m_\pi^2}{m_N} \int_{k_1}^{k_2} \frac{dk}{\sqrt{t_1^2 - t_2^2}} = \delta k - m_\pi \tan^{-1} \left(\frac{k}{m_\pi} \right) \Big|_{k_1}^{k_2}$$

$$+ \frac{1}{2} m_\pi \sin^{-1} \left(\frac{u - 2m_\pi^2 k^2}{\Omega} \right) \Big|_{k_1}^{k_2} \quad (109)$$

with

$$u = (p_1^2 - p_2^2)(p_3^2 - p_4^2) + (p_1^2 + p_2^2 + p_3^2 + p_4^2)m_\pi^2 + m_\pi^4 \quad (110)$$

and

$$\Omega = [u^2 + 4p_2^2(p_1^2 - p_3^2)m_\pi^2]^{1/2} \quad (111)$$

where again $p_i \equiv p_F(i)$. The integral defined by (106) is somewhat more complicated but can still be performed analytically. Introducing the quantities

$$\Delta = p_2^2(p_1^2 - p_3^2) - um_\pi^2 - m_\pi^6 \quad (112)$$

$$d = \Delta / (k^2 + m_\pi^2), \quad (113)$$

we obtain

$$f(k_1, k_2) = \frac{1}{2\sqrt{|\Delta|}} \sin^{-1} \frac{u + 2m_\pi^4 + 2d}{\Omega} \left| \frac{k_2}{k_1} \right| \quad (114)$$

for $\Delta < 0$ and

$$f(k_1, k_2) = -\frac{1}{2\sqrt{\Delta}} \ln \left(u + 2m_\pi^4 + 2d + 2[d^2 + (u + 2m_\pi^4)d - m_\pi^2 \Delta]^{1/2} \right) \left| \frac{k_2}{k_1} \right| \quad (115)$$

for $\Delta > 0$.

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TABLE 1
WEAK INTERACTION VERTICES

Vertex	g_V	g_A
$n + n\bar{\nu}$	$-1/2$	$-1/2(D+F)$
$p + p\bar{\nu}$	$1/2 - 2 \sin\theta_w$	$1/2(D+F)$
$\Sigma^- + \Sigma^-\bar{\nu}$	$-1 + 2 \sin\theta_w$	$-F$
$n + p e^- \bar{\nu}$	$\cos\theta_c$	$(D+F) \cos\theta_c$
$\Sigma^- + \Lambda e^- \bar{\nu}$	0	$\sqrt{2/3} D \cos\theta_c$
$\Sigma^- + \Sigma^0 e^- \bar{\nu}$	$\sqrt{2} \cos\theta_c$	$\sqrt{2} F \cos\theta_c$

TABLE 2
STRONG INTERACTION VERTICES

Vertex	$f(A + C\pi)$	Vertex	$f(A + C\pi)$
$n + n\pi^0$	$-g$	$\Sigma^0 + \Sigma^-\pi^+$	$-2g(1-\alpha)$
$p + p\pi^0$	g	$\Sigma^- + \Lambda\pi^-$	$2/\sqrt{3} g\alpha$
$n + p\pi^-$	$\sqrt{2} g$	$\Sigma^0 + \Lambda\pi^0$	$2/\sqrt{3} g\alpha$
$p + n\pi^+$	$-\sqrt{2} g$	$\Lambda + \Sigma^0\pi^0$	$2/\sqrt{3} g\alpha$
$\Sigma^- + \Sigma^-\pi^0$	$-2g(1-\alpha)$	$\Lambda + \Sigma^-\pi^+$	$-2/\sqrt{3} g\alpha$
$\Sigma^- + \Sigma^0\pi^-$	$2g(1-\alpha)$		

TABLE 3
COEFFICIENTS IN BREMSSTRAHLUNG MATRIX ELEMENTS

Process	a_2	a_4	b_1	b_2	b_3	b_4
$nn+n\nu\bar{\nu}$	$-\delta$	$-\delta$	0	δ	0	δ
$pp+pp\nu\bar{\nu}$	δ	δ	0	$-\delta$	0	$-\delta$
$\Sigma\Sigma+\Sigma\Sigma\nu\bar{\nu}$	$-8F(1-\alpha)^2$	$-8F(1-\alpha)^2$	0	$8F(1-\alpha)^2$	0	$8F(1-\alpha)^2$
$np+np\nu\bar{\nu}$	δ	$-\delta$	2δ	0	-2δ	0
$\Sigma n+\Sigma n\nu\bar{\nu}$	$-4F(1-\alpha)$	$-2\delta(1-\alpha)$	0	0	0	0
$\Sigma p+\Sigma p\nu\bar{\nu}$	$4F(1-\alpha)$	$-2\delta(1-\alpha)$	0	0	0	0
$\Lambda n+\Lambda n\nu\bar{\nu}$	0	0	$\frac{4}{3} Fa^2$	$-\frac{4}{3} Fa^2$	$-\frac{4}{3} Fa^2$	$-\frac{4}{3} Fa^2$
$\Sigma p+\Lambda n\nu\bar{\nu}$	0	0	$\sqrt{8/3} Fa$	$-\sqrt{8/3} Fa$	$-\sqrt{8/3} \delta\alpha$	0
$\Lambda n+\Sigma p\nu\bar{\nu}$	0	0	$\sqrt{8/3} \delta\alpha$	0	$-\sqrt{8/3} Fa$	$-\sqrt{8/3} Fa$

TABLE 4

COEFFICIENTS IN URCA MATRIX ELEMENTS

Process	a_1	a_2	a_3	a_4
$nn+npe\nu$	$2\delta_c$	0	$-2\delta_c$	$2\delta_c$
$\Sigma\Sigma+\Sigma\Lambda e\nu$	$2\zeta_+(1-\alpha)$	$2\zeta_-(1-\alpha)$	$-\sqrt{2/3} n\alpha$	$\sqrt{2/3} n\alpha$
$np+ppe\nu$	$-2\delta_c$	0	$2\delta_c$	$2\delta_c$
$\Sigma\Lambda+\Lambda\Lambda e\nu$	0	0	$\frac{4}{3} \sqrt{\frac{2}{3}} Da^2 \cos\theta_c$	$\frac{4}{3} \sqrt{\frac{2}{3}} Da^2 \cos\theta_c$
$\Sigma n+\Sigma pe\nu$	$4\delta_c(1-\alpha)$	0	$-n$	n
$\Sigma n+\Lambda ne\nu$	ζ_+	ζ_-	$-\sqrt{8/3} \delta_c \alpha$	$\sqrt{8/3} \delta_c \alpha$
$\Sigma p+\Lambda pe\nu$	$-\zeta_+$	$-\zeta_-$	$\sqrt{8/3} \delta_c \alpha$	$\sqrt{8/3} \delta_c \alpha$

TABLE 5

COEFFICIENTS IN SQUARED BREMSSTRAHLUNG MATRIX ELEMENTS

Process	C_1	C_2	C_3	C_4
NN+NN $\nu\bar{\nu}$	δ^2	δ^2	δ^2	$-3\delta^2$
$\Sigma\Sigma+\Sigma\Sigma\nu\bar{\nu}$	$64F^2(1-\alpha)^4$	$64F^2(1-\alpha)^4$	$64F^2(1-\alpha)^4$	$-192F^2(1-\alpha)^4$
np+np $\nu\bar{\nu}$	δ^2	$1/2\delta^2$	$2\delta^2$	$-2\delta^2$
$\Sigma N+\Sigma N\nu\bar{\nu}$	$(8F^2+2\delta^2)(1-\alpha)^2$	0	0	0
$\Sigma\Lambda+\Sigma\Lambda\nu\bar{\nu}$	0	$8/3F^2\alpha^4$	0	0
$\Sigma p+\Lambda n\nu\bar{\nu}$	0	$(2F^2+2/3\delta^2)\alpha^2$	0	0
$\Lambda n+\Sigma p\nu\bar{\nu}$	0	$(2F^2+2/3\delta^2)\alpha^2$	0	0

Note: N denotes either n or p.

TABLE 6

COEFFICIENTS IN SQUARED URCA MATRIX ELEMENTS

Process	C_1	C_3	C_4
nn+npe $\bar{\nu}$	$4\delta_C^2$	δ_C^2	$-3\delta_C^2$
$\Sigma\Sigma+\Sigma\Lambda e\bar{\nu}$	$4\{3(1-\alpha)^4+1/3\alpha^4\}h_1+4\alpha^2h_2$ + $4(2\alpha-1)h_3$	$4d_{-}h_1$ $-16/3\alpha^2(h_2+h_3)$	$-8d_{+}h_1$ $+16/3\alpha^2(h_2+h_3)$
np+ppe $\bar{\nu}$	$4\delta_C^2$	δ_C^2	$-3\delta_C^2$
$\Sigma\Lambda+\Lambda\Lambda e\bar{\nu}$	$4/3\alpha^4h_1$	$-4/9\alpha^4h_1$	$-8/9\alpha^4h_1$
$\Sigma n+\Sigma pe\bar{\nu}$	$4(1-\alpha)^2\delta_C^2+2\alpha^2h_1+3h_2+6h_3$	0	0
$\Sigma N+\Lambda Ne\bar{\nu}$	$2\alpha^2\delta_C^2+3(1-\alpha)^2h_1-h_3$ + $2F^2\alpha^2\cos^2\theta_C$	0	0

Note: $C_2 = 0$ for the processes $\Sigma n + \Sigma pe\bar{\nu}$ and $\Sigma N + \Lambda Ne\bar{\nu}$. For the other processes, $C_2 = C_1$.

TABLE 7

BARYON AND ELECTRON NUMBER FRACTIONS

Scenario	Y_e	Y_n	Y_p	Y_Σ	Y_Λ
1	0.05	0.95	0.05	—	—
2	0.20	0.80	0.20	—	—
3	0.15	0.55	0.25	0.10	0.10
4	0.05	0.25	0.25	0.20	0.30

TABLE 8

NUMERICAL VALUES OF COEFFICIENTS IN SQUARED MATRIX ELEMENTS

Process	C_1	C_2	C_3	C_4
NN + NN $\nu\bar{\nu}$	1.520	1.520	1.520	-4.561
$\Sigma\Sigma + \Sigma\Sigma\nu\bar{\nu}$	0.313	0.313	0.313	-0.940
np + np $\nu\bar{\nu}$	1.520	0.760	3.041	-3.041
$\Sigma N + \Sigma N\nu\bar{\nu}$	0.713	0	0	0
$\Sigma\Lambda + \Sigma\Lambda\nu\bar{\nu}$	0	0.088	0	0
$\Sigma p + \Lambda n\nu\bar{\nu}$	0	0.559	0	0
nn + npe $\bar{\nu}$	5.757	5.757	1.439	-4.318
$\Sigma\Sigma + \Sigma\Lambda e\bar{\nu}$	0.456	0.456	-0.351	0.135
np + ppe $\bar{\nu}$	5.757	5.757	1.439	-4.318
$\Sigma\Lambda + \Lambda\Lambda e\bar{\nu}$	0.070	0.070	-0.023	-0.046
$\Sigma n + \Sigma pe\bar{\nu}$	2.144	0	0	0
$\Sigma N + \Lambda Ne\bar{\nu}$	1.311	0	0	0

Note: N denotes either n or p.

TABLE 9

EMISSIVITY RESULTS IN SCENARIOS WITHOUT HYPERONS

Process	Scenario 1		Scenario 2	
	$\epsilon(\rho=\rho_0)$	$\epsilon(\rho=5\rho_0)$	$\epsilon(\rho=\rho_0)$	$\epsilon(\rho=5\rho_0)$
nn + nn $\bar{\nu}$	1.03	2.27	0.93	2.10
pp + pp $\bar{\nu}$	0.13	0.46	0.39	1.05
np + np $\bar{\nu}$	0.56	1.54	1.15	2.88
nn + npe $\bar{\nu}$	41.73	90.27	61.24	137.74
np + ppe $\bar{\nu}$	2.00	5.18	17.78	43.34
Total	45.44	99.72	81.50	187.12

Note: $\rho_0 = 0.145 \text{ fm}^{-3}$ (nuclear matter density). The emissivities are given in units of $10^{20} \text{ ergs/cm}^3$ for $T_9 = 1$.

TABLE 10

EMISSIVITY RESULTS IN SCENARIOS WITH HYPERONS

Process	Scenario 1		Scenario 2	
	$\epsilon(\rho=\rho_0)$	$\epsilon(\rho=5\rho_0)$	$\epsilon(\rho=\rho_0)$	$\epsilon(\rho=5\rho_0)$
nn + nn $\bar{\nu}$	0.75	1.76	0.46	1.19
pp + pp $\bar{\nu}$	0.46	1.18	0.46	1.18
np + np $\bar{\nu}$	1.19	3.03	1.00	2.65
$\Sigma n + \Sigma n\bar{\nu}$	0.14	0.39	0.22	0.57
$\Sigma p + \Sigma p\bar{\nu}$	0.14	0.39	0.22	0.57
$\Sigma \Sigma + \Sigma \Sigma\bar{\nu}$	0.13	0.39	0.21	0.57
$\Sigma \Lambda + \Sigma \Lambda\bar{\nu}$	0.02	0.07	0.04	0.11
$\Sigma p + \Lambda n\bar{\nu}$	0.10	0.28	0.18	0.46
nn + npe $\bar{\nu}$	58.63	137.76	45.63	115.05
np + ppe $\bar{\nu}$	22.25	54.86	22.75	57.37
$\Sigma n + \Sigma p\bar{\nu}$	6.65	18.96	10.92	28.07
$\Sigma n + \Lambda n\bar{\nu}$	3.81	10.97	7.14	17.83
$\Sigma p + \Lambda p\bar{\nu}$	3.80	10.94	7.13	17.78
$\Sigma \Sigma + \Sigma \Lambda\bar{\nu}$	3.21	8.64	4.93	11.98
$\Sigma \Lambda + \Lambda n\bar{\nu}$	0.25	0.71	0.46	1.12
Total	101.62	250.61	101.94	256.96

Note: $\rho_0 = 0.145 \text{ fm}^{-3}$ (nuclear matter density). The emissivities are given in units of $10^{20} \text{ ergs/cm}^3$ for $T_9 = 1$.

FIGURE CAPTIONS

1. Class I bremsstrahlung diagrams.
2. Class II bremsstrahlung diagrams.
3. Class I URCA diagrams.
4. Class II URCA diagrams.
5. Neutrino emissivities obtained in scenario 3 for selected URCA processes.
6. Neutrino emissivities obtained in scenario 4 for selected URCA processes.

Fig. 2

Fig. 1

Fig. 3

Fig. 4

Fig. 5

Fig. 6