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BARYOGENESIS AND ENTROPY GENERATION IN A SUPERSTRING MODEL

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ABSTRACT

We work out details of a previously proposed scenario for baryogenesis in a superstring model, based on the Affleck-Dine mechanism exploiting the decays of large expectation values for squark and slepton fields after inflation. We show how the coefficients of the relevant quartic scalar operators can acquire the necessary imaginary parts. We examine possible sources of entropy among the gauge-singlet scalar and fermion fields including the gravitino, as well as the low-energy $SU(2)_L \times U(1)^2 \rightarrow SU(2)_L \times U(1)_Y \rightarrow U(1)_{em}$ phase transitions. We argue that none of these dilute substantially the baryon-to-entropy ratio, which can easily be in the range preferred by cosmology. This means, in particular, that the Polonyi and gravitino problems are solved.

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The general requirements¹⁾ for cosmological baryogenesis have been known for almost 20 years: interactions that violate baryon number, violate CP, and are not in thermal equilibrium. The advent of grand unified theories (GUTs) with their baryon-non-conserving interactions revived interest in this possibility and spawned many scenarios for baryogenesis. A favoured possibility was the out-of-equilibrium decay²⁾ of some heavy particle such as a gauge or Higgs boson. The first cloud on the horizon of this scenario arose from inflation. Baryogenesis had to occur after reheating at the end of the inflationary epoch, and phenomenological bounds on density perturbations³⁾ and on the gravitino abundance⁴⁾ in supersymmetric GUTs tended to push the reheating temperature below plausible values of the GUT particle masses.

The troubled out-of-equilibrium decay scenario became truly untenable with the advent of superstring-inspired models⁵⁾. In these, topology specifies the set of low-mass [$m \ll O(m_P)$: $m_P \equiv (8\pi G_N)^{-\frac{1}{2}} = 2.4 \times 10^{18} \text{ GeV}$] particles and their interactions. In order to accommodate the observed stability of the proton, baryon-number-violating gauge bosons and combinations of Yukawa coupling must be exorcised from the effective low-energy theory by topological fiat⁵⁾. Only particles with masses $O(m_P)$ could have interactions violating baryon number, and these are too heavy to be produced after inflation in any four-dimensional field-theoretical model^{*)}.

An alternative scenario for baryogenesis, proposed by Affleck and Dine⁷⁾ previously in the context of conventional supersymmetric GUTs, is the adiabatic decay of large vacuum expectation values of squark and slepton fields ϕ in a cold Universe after inflation. It is expected that the low-energy effective theory contains in general baryon-number-violating quartic scalar operators $O_4 \equiv \phi^P (\phi^*)^{4-P}$: $p = 0, 1, 2$ with coefficients $O(\tilde{m}^2/m_P^2)$, where $\tilde{m} = O(100 \text{ GeV to } 1 \text{ TeV})$ is the scale of supersymmetry breaking in the effective low-energy theory. The renormalizable supersymmetric terms in the effective low-energy potential have in general certain flat directions in field space. Large vacuum expectation values $\langle 0 | \phi | 0 \rangle = \phi_0 = O(m_P)$ may develop in any of these directions during inflation. Then, if the energy density after inflation is not larger than $O(\tilde{m}^2 m_P^2)$, the Universe is cold and dominated by $\tilde{m}^2 |\phi|^2$ terms in the low-energy effective potential, and $\langle 0 | O_4 | 0 \rangle \sim \phi_0^4 = O(m_P^4)$. As the Universe expands, the fields ϕ

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*) So far, no plausible mechanism for inflation in a superstring model has been exhibited⁶⁾, but that is another story.

relax adiabatically, conserving baryon number. Eventually, the Universe thermalizes at some low temperature which may be $O(10^2 \text{ to } 10^4) \text{ GeV}$, and the coherent squark and slepton fields ϕ transfer to particles the baryon number which is present in the vacuum if $\langle 0 | \phi | 0 \rangle \neq 0$ and the coefficient of O_4 in the low-energy effective potential has a non-zero imaginary part.

We have argued in a previous paper⁸⁾ that this mechanism could well solve the puzzle of baryogenesis in superstring models. We catalogued the gauge-invariant quartic scalar operators O_4 which could appear in the effective low-energy theory, verified that the D-terms in the potential can vanish in some direction: $\langle 0 | \phi | 0 \rangle$ and $\langle 0 | O_4 | 0 \rangle \neq 0$, and argued that the F-terms in the potential could also vanish in a direction where some $\langle 0 | O_4 | 0 \rangle \neq 0$. In this paper we work out further details of such an Affleck-Dine scenario^{7),9),8)}, paying attention to the following points:

- We show how loop diagrams can generate an imaginary part for the coefficient of some operator O_4 , even if this is not present in leading order.
- We discuss the cosmological fate of the weakly-coupled gauge-singlet fields which appear in superstring-inspired models⁵⁾. We show that their scalar and fermionic components, as well as the gravitino, decay without reheating the Universe or generating excess entropy. Thus the Polonyi and gravitino problems are solved.
- We also discuss the possibility of excess entropy generation during the phase transitions in the low-energy gauge theory. The minimal low-energy gauge group is $SU(3)_C \times SU(2)_L \times U(1)^2$ with rank 5⁵⁾, leading us to expect two phase transitions: $SU(2)_L \times U(1)^2 \rightarrow SU(2)_L \times U(1)_Y$ and $SU(2)_L \times U(1)_Y \rightarrow U(1)_{em}$ respectively. We show that in a class of models discussed previously¹⁰⁾, the earlier of these phase transitions is second order and does not generate any entropy, while the second is either a weakly first order transition which increases the entropy by $O(10)$ at most, or is second order if the supersymmetry breaking scale is plausibly large.

Taken together, these results indicate that a cosmologically interesting baryon-to-entropy ratio can be generated in a superstring model, and that it is not diluted significantly by any of the possible sources of entropy which were the bane of other cosmological models¹¹⁾.

The starting point for baryogenesis à la Affleck-Dine is a quartic scalar

operator $O_4 = \phi^P (\phi^*)^{4-P}$. In a previous paper⁸⁾, we have catalogued all such operators with $p = 0, 1$ or 2 which are composed of conventional squark and slepton fields and/or the scalar components of the additional light colour-triplet fields present in 27 representations of E_6 . We have also indicated which of these operators are invariant under the $SU(3)_C \times SU(2)_L \times U(1)_Y$ of the Standard Model combined with some additional $U(1)_Y$, in E_6 . In this paper we specialize to the particular $U(1)_Y$, which appears in the minimal rank-5 subgroup of E_6 obtainable by Hosotani symmetry breaking¹²⁾ in a Calabi-Yau manifold, namely that for which the conventional 5 and 10 representations of $SU(5)$ have

$$Y'(\underline{5}) = -\frac{1}{2} Y'(\underline{10}) \quad (1)$$

Consultation of the Table in our previous paper⁸⁾ reveals that only the following $p = 2$ operators are invariant under $SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)_{Y'}$:

$$\left. \begin{aligned} & LQ u^{c*} d^{c*}, \quad (c u^c \bar{c} \bar{c}^*, LQ u^{c*} D^{c*}, \bar{H} Q u^{c*} d^{c*}, \bar{H} Q u^{c*} D^{c*} \\ & Q D H^* u^{c*}, \quad L d^c \bar{c}^* D^*, \bar{L} D^c \bar{c}^* D^*, \bar{H} d^c \bar{c}^* D^*, \bar{H} D^c \bar{c}^* D^* \\ & d^c d^c H^* \bar{c}^*, \quad d^c D^c H^* \bar{c}^*, \quad D^c D^c H^* \bar{c}^* \end{aligned} \right\} \quad (2)$$

The first of these operators, namely $O_4 \equiv LQ u^{c*} d^{c*}$, was already considered by Affleck and Dine⁷⁾, who pointed out that the following combination of squark and slepton vacuum expectation values:

$$\left. \begin{aligned} & \langle 0 | u_3^c | 0 \rangle = a, \quad \langle 0 | u_1 | 0 \rangle = -v, \quad \langle 0 | S_2^c | 0 \rangle = a \\ & \langle 0 | \mu | 0 \rangle = v, \quad \langle 0 | b_1^c | 0 \rangle = e^{i\theta} \sqrt{v^2 + a^2} \end{aligned} \right\} \quad (3)$$

corresponded to a flat direction for the D-terms of the Standard Model, and would give this operator a non-zero vacuum expectation value:

$$\langle 0 | O_4 | 0 \rangle = -v^2 a^{*2} \quad (4)$$

We pointed out in our previous paper that the vacuum expectation values (3) still corresponded to a flat direction of the D-terms even after those for the additional $U(1)$ in Eq. (1) were included, and could be generated by De Sitter fluctuations during an inflationary epoch⁹⁾. Calculating the coefficient of the operator O_4 in the low-energy effective theory with broken supersymmetry we expect

$$L_{\text{eff}} \supset c_4 O_4 \quad : \quad c_4 = \hat{c} \left(\frac{\tilde{m}}{m_P} \right)^2 \quad (5)$$

where $\tilde{m}$ is the scale of supersymmetry breaking in the low-energy observable sector, which we expect to be $O(m_{1/2}^*)$, and $\hat{c}$ is unknown, but expected to be real in leading order and $O(1)$ at most^{*}.

The vacuum expectation values (3) violate CP spontaneously, yielding $\text{Im}\langle 0 | c_4 O_4 | 0 \rangle \neq 0$, if $\arg(a) \neq \arg(v)$. We cannot be sure that such phases will be generated in general, and so look for intrinsic CP violation in the form of an imaginary part of the coefficient c_4 . Affleck and Dine⁷⁾ suggested that this might arise from the one-loop diagram of Fig. 1, through a CP-violating phase for the gaugino mass $m_{1/2}$. However, we expect^{10),13)} $\arg(m_{1/2}) = 0$ in leading order, and so must look for another source of $\text{Im}c_4$ at the one-loop level. One-loop wave-function renormalization diagrams like those in Fig. 2 contribute to the renormalization group equation for c_4 terms of the form

$$\begin{aligned} \frac{d}{dt} c_{c^*d^*}^{ab} \supset & \\ \frac{1}{32\pi^2} \left[Z_{a'}^a(L) c_{c^*d^*}^{a'b} + Z_{b'}^b(Q) c_{c^*d^*}^{ab'} \right. & \\ \left. + (Z_{c'}^c(u^c))^* c_{c'^*d^*}^{ab} + (Z_{d'}^d(d^c))^* c_{c^*d'^*}^{ab} \right] & \end{aligned} \quad (6)$$

where we have exhibited explicitly the generation indices of the four scalar fields: $L, Q, u^c, d^c \equiv \phi$. The wave-function renormalization factors $Z(\phi)$ are given by

$$Z_{a'}^a(\phi) = \sum_{\lambda} \lambda^{aef} (\lambda^{a'ef})^* \quad (7)$$

where the sum over Yukawa couplings λ runs through all those involving the fields ϕ . The $Z(\phi)$ are Hermitian matrices: their off-diagonal elements have non-zero phases, and $Z(L) \neq Z(Q) \neq Z(u^c) \neq Z(d^c)$ in general. A priori, the wave-function renormalizations (6) can generate

$$\text{Arg } c_4 = O\left(\frac{\lambda^2}{32\pi^2}\right) \ln \frac{m_x}{m_w} = O\left(\frac{\lambda^2}{\lambda}\right) : \alpha_{\lambda} \equiv \frac{\lambda^2}{4\pi} \quad (8)$$

^{*}) It might be $O(\Lambda_c/m_P)^{2m}$: $m > 0$, where Λ_c is the scale of gaugino condensation in the hidden sector.

providing intrinsic CP violation in the coefficient c_4 of the $\Delta B \neq 0$ operator O_4 .

We now analyze the cosmological evolution of the ϕ fields, whose energy density is

$$\rho_\phi \approx \tilde{m}^2 \phi^2 \quad (9)$$

where ϕ takes the initial value $\phi_0 \sim m_p$ after inflation. Thereafter, the scalar fields ϕ evolve according to their zero temperature classical equation of motion

$$\ddot{\phi} + 3H\dot{\phi} + \tilde{m}^2\phi = V'(\phi) \quad (10)$$

In a matter-dominated Universe, the solution⁷⁾ to (10) is $\phi \propto t^{-1} \sin \tilde{m}t$, while the baryon number density stored in the oscillations is just

$$n_B \approx (\ln c_4) \tilde{m} \phi^2 \quad (11)$$

so that $n_B \propto t^{-2} \propto R^{-3}$, where R is the cosmological scale factor. In a radiation-dominated Universe, $\phi \propto t^{-3/4} \sin \tilde{m}t$ so that again $n_B \propto t^{-3/2} \propto R^{-3}$. Thus, in both cases the baryon number density per comoving volume is conserved. The ϕ fields eventually decay when the decay rate $\Gamma_\phi \sim (\alpha/\pi)^2 \tilde{m}^3/\phi^2$ equals the expansion rate $H \sim \rho^{1/2}/m_p$. As we will see shortly, due to the presence of weakly-coupled massive fields, the ϕ fields never actually dominate the expansion rate. Nevertheless, we find that the baryon asymmetry is not very different from the previous analyses.

Next we turn to the weakly-coupled gauge-singlet fields which could a priori generate a large amount of entropy which might drown the baryon density^{14), 11)}. Among the gauge-singlet fields which must appear in the low-energy effective theory⁵⁾, those which have been most studied¹⁵⁾ are the Y field, whose vacuum expectation value fixes the gauge couplings (called in some references S) and the X field, whose vacuum expectation value fixes the overall scale of the compactified dimensions (called in some references T). The physical spectrum includes the two spin-zero components in each of the Y and X supermultiplets, the $\tilde{Y}$ spin- $\frac{1}{2}$ fermion, and the gravitino. We adopt the general belief that supersymmetry is broken by gaugino condensation¹⁶⁾ in the hidden sector, and assume a superpotential for the Y field of the form

$$\frac{P(Y)}{m_P^3} = c + h \exp(-z) : z \equiv \frac{\sigma Y}{m_P} : \sigma \equiv \frac{8\pi^2}{C_2(G')} \quad (12)$$

where $h = O(1)$ and G' is the subgroup¹⁷⁾ of the hidden sector gauge group which condenses. The scale of condensation in the hidden sector is

$$\frac{\Lambda_c}{m_P} = \left(\frac{m_P \sigma}{z X_R} \right)^{1/2} \exp(-\frac{1}{3}z) \quad (13)$$

and the resulting gravitino mass is

$$\frac{m_{3/2}}{m_P} = \left(\frac{h^2 \sigma^2 m_P^3}{4 X_R^3} \right)^{1/2} e^{-z} \quad (14)$$

The fermion and spin-zero Y field masses are related to $m_{3/2}$ (14) by

$$m_{\tilde{Y}} = 2z m_{3/2}, \quad m_{Y_{1,2}} = |2z \pm 1| m_{3/2} \quad (15)$$

Since Y_R determines the gauge coupling: $Y_R = m_P/g^2$ and $\sigma \gg 1$ in interesting cases¹³⁾, we expect $|z| \gg 1$ and hence $m_{\tilde{Y}}, m_{Y_{1,2}} > m_{3/2}$ in general. The overall scale of these masses (13) and (14) is set by the vacuum expectation value of X_R which we expect¹⁸⁾ to be $O(1)$, but we have no complete model of the potential for X . We do know, however, that radiative corrections depend¹⁹⁾ on X_R :

$$V_{RC}(X_R)/m_P^4 = \frac{m_P^4}{2\pi^2 Y_R^2} \exp(-\frac{4z}{3}) \frac{\ln(2X_R/m_P)}{4X_R^2} \quad (16)$$

The curvature of this term in the potential provides an order-of-magnitude lower bound on $m_{X_R}^2$:

$$\frac{m_{X_R}^2}{m_P^2} \gtrsim \frac{O(1)}{4\pi^2} \exp(-\frac{4z}{3}) \quad (17)$$

which is useful in the subsequent analysis of entropy generation. We have the general prejudice that $\tilde{m} \ll m_{3/2}$, $m_{\tilde{Y}}, m_{Y_{1,2}} \sim \Lambda_c^3/m_P^2 \ll m_P$, and the lower bound (17) on $m_{X_R}^2$ is comparable to our best estimate¹³⁾ of $m_{3/2}, m_{\tilde{Y}}$.

How do the $\tilde{Y}$, $Y_{1,2}$, X and gravitino fields affect the baryon-to-entropy ratio? Since it turns out that scalar fields produce the most important effect, we begin the discussion with their evolution: the most important of all is the

lightest of the X and $Y_{1,2}$ fields, which we call for the sake of argument Y. Just as the squark and slepton fields after inflation are evolving towards their global minima at $\phi = 0$, the Y field will also evolve according to a similar classical equation of motion, with

$$\rho_Y \simeq m_Y^2 Y^2 \quad (18)$$

It is relatively straightforward to show that during their evolution

$$\rho_Y/\rho_\phi \simeq (m_Y/\tilde{m})^2 \gg 1 \quad (19)$$

so that the expansion rate is dominated by the Y fields. The Y oscillations decay when

$$\Gamma_Y \sim m_Y^3/m_P^2 \sim H \sim (m_Y Y)/m_P \quad (20)$$

where Γ_Y is the decay rate for the Y field, i.e., when the magnitude of Y is

$$Y_D \sim m_Y^2/m_P \quad (21)$$

Since the ϕ fields have not yet decayed, the Universe becomes dominated by the relativistic decay products of the Y field, and we must ask when they thermalize. Their interaction rate is given by

$$\begin{aligned} \Gamma &\sim n_Y \sigma \sim m_Y Y_D^2 (R_Y/R)^3 (\alpha^2/m_Y^2) (R/R_Y)^2 \\ &\sim \alpha^2 (m_Y^3/m_P^2) (R_Y/R) \end{aligned} \quad (22)$$

where the number density of relativistic particles initially is $\sim n_Y$ and σ is their elastic scattering cross-section. R_Y is the scale factor when $Y = Y_D$ and the powers of R/R_Y correspond respectively to the decrease in the density of relativistic particles with respect to a comoving volume and to the redshifts of the particles' kinetic energies which modify their cross-sections $\sigma \sim \alpha^2/E^2$. The energy density in radiation is now

$$\rho_r = m_Y^2 Y_D^2 (R_Y/R)^4 \sim (m_Y^6/m_P^2) (R_Y/R)^4 \quad (23)$$

so that the expansion rate becomes

$$t^{-1} \sim H \sim \rho_r^{1/2}/m_p \sim (m_Y^3/m_p^2)(R_Y/R)^2 \quad (24)$$

Thus thermalization occurs when t^{-1} (24) $\sim \Gamma$ (22), i.e., when

$$\Gamma t \sim \alpha^2 (R/R_Y) \sim 1 \quad (25)$$

which occurs at $R_T \equiv R_Y/\alpha^2$.

We can now solve for the value of ϕ at which the ϕ oscillations cease. After Y decay, we can write ϕ in terms of R_Y :

$$\phi^2 \simeq \left(m_Y^{1/2} / \tilde{m}^{3/2} m_p^2 \right) (R_Y/R)^3 \quad (26)$$

If $m_Y^3 < (\alpha/\pi) \tilde{m} m_p^2$, ϕ decays when $\Gamma_\phi \sim H$ (24), i.e., at R_ϕ given by

$$R_\phi \simeq \left[\left(\frac{\alpha}{\pi} \right)^{3/5} \frac{\tilde{m}^{9/10} m_p^{4/5}}{m_Y^{1/10}} \right]^{-1} R_Y \quad (27)$$

corresponding to

$$\phi_D \sim \left(\tilde{m}^{3/5} m_Y^{1/5} m_p^{1/5} \right) \left(\frac{\alpha}{\pi} \right)^{3/5} \quad (28)$$

We recall that the baryon number stored in the oscillations was (11) $n_B \sim \text{Im} c_4 \tilde{m} \phi^2$, so that after ϕ reaches ϕ_D at R_ϕ :

$$n_B \sim \text{Im} c_4 \left(\frac{\alpha}{\pi} \right)^{6/5} \tilde{m}^{1/5} m_Y^{3/5} m_p^{2/5} (R_\phi/R)^3 \quad (29)$$

or

$$n_B \sim \text{Im} c_4 \left(m_Y^{1/2} / m_p^2 \tilde{m}^{1/2} \right) (R_Y/R)^3 \quad (30)$$

When $R \sim R_T$, $\rho_r \sim (m_Y^6/m_p^2) \alpha^8$ so that the entropy in thermalized radiation is

$$S \sim \left(m_Y^{9/2} / m_p^{3/2} \right) \alpha^6 \quad (31)$$

and the final baryon-to-entropy ratio^{*)} is

$$n_B/s \simeq \text{Im } c_4 \left(\frac{m_Y}{m_P^{1/2} \tilde{m}^{1/2}} \right) \quad (32)$$

This result holds when $m_Y^3 < (\alpha/\pi) \tilde{m} m_P^2$ as we expect. If $m_Y^3 > (\alpha/\pi) \tilde{m} m_P^2$ then the ϕ fields decay after the energies of the relativistic decay products of the Y field have redshifted away to insignificance, and the energy density is dominated by the coherent ϕ oscillations as considered previously^{7),8)}. In this case^{*)}

$$n_B/s \simeq \text{Im } c_4 \left(\frac{\alpha}{\pi} \right)^{1/3} \left(\frac{m_P}{\tilde{m}} \right)^{1/6} \quad (32')$$

This result offers us a lower bound on the mass of the Y field. There will be no significant dilution of the baryon-to-entropy ratio below $O(\text{Im } c_4)$ if

$$m_Y \gtrsim (\tilde{m} m_P)^{1/2} \quad (33)$$

as advocated in Ref. 13), while there will be dilution by $O(10^4)$ if $m_Y \sim 10^6 \text{ GeV}^{20)}$.

Decays of coherent oscillations in the X field can be treated in a similar way. However, as pointed out earlier, we are only interested in the lightest of all the scalar fields, because the oscillations of the more massive fields decay first into radiation. The energy density in relativistic particles redshifts faster than that in the oscillations in the lightest field. As a result, at the time when the lightest field's oscillations decay, the ratio of their energy densities is $\rho_H/\rho_L \sim (m_L/m_H)^{4/3} < 1$. Thus we can neglect the effects of all but the lightest fields.

It is now a fairly simple task to show that the gravitinos and $\tilde{Y}$ fermions do not dilute the asymmetry (32) any further. Before decay, the ratio of the gravitino energy density to ρ_Y is simply

$$\rho_{3/2}/\rho_Y \sim \frac{m_{3/2}}{m_Y^{1/2} m_P^{1/2}} \ll 1 \quad (34)$$

if we assume that the initial abundance of gravitinos was restored after

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*) As correctly pointed out by Linde⁹⁾, n_B/s can never exceed unity. In cases where formulae (32) and (32') would appear to give $n_B/s > 1$, in fact reheating after ϕ decay augments the entropy so that $n_B/s \lesssim 1$. See also footnote on p.4 for one way in which n_B/s might be suppressed.

inflation, while any dilution due to inflation⁴⁾ would only decrease their relevance. In addition, gravitinos decay when

$$\Gamma_{3/2} \sim m_{3/2}^3 / m_P^2 \sim H \sim \frac{m_Y^3}{m_P^2} \left(\frac{R_Y}{R} \right)^2 \quad (35)$$

i.e., when

$$\frac{R_{3/2}}{R_Y} \sim \left(\frac{m_Y}{m_{3/2}} \right)^{3/2} > 1 \quad (36)$$

and one can easily show again that gravitino decays produce non-thermalized radiation with an energy density

$$\rho_{r(3/2)} \sim \left(m_{3/2}^{1/2} m_Y / m_P^{5/2} \right) \left(\frac{R_Y}{R_{3/2}} \right)^3 \quad (37)$$

so that when $R = R_{3/2}$

$$\frac{\rho_{r(3/2)}}{\rho_r} \sim \frac{m_Y}{m_P^{1/2} m_{3/2}^{1/2}} \ll 1 \quad (38)$$

Thus the radiation in gravitino decay products is clearly negligible compared to the pre-existing radiation from Y decays. It is easy to check analogously that $\tilde{Y}$ decays also provide a negligible contribution to the radiation density.

We turn finally to the possibility of entropy generation during low-energy phase transitions. There are two of these in the minimal model¹⁰⁾ adopted here: $SU(2)_L \times U(1)_Y \times U(1)_Y \rightarrow SU(2)_L \times U(1)_Y$ and $SU(2)_L \times U(1)_Y \rightarrow U(1)_{em}$. The first of these is caused by the development of a vacuum expectation value x for an $SU(2)_L$ singlet field N , while the latter arises from the development of vacuum expectation values v and $\bar{v}$ for the two Higgs doublets H and $\bar{H}$. Since the development of both v and $\bar{v}$ requires a Yukawa mixing term

$$\lambda H \bar{H} N : \langle 0 | N | 0 \rangle = x \Rightarrow (\lambda x) H \bar{H} \quad (39)$$

the scale of the first transition is presumably not many orders higher than the second. However, phenomenological bounds on neutral currents¹⁰⁾ as well as primordial nucleosynthesis²¹⁾ tell us that x must be at least several times larger than v and $\bar{v}$, which justifies treating the two phase transitions separately.

We believe that these two phase transitions arise from the logarithmic evolution of soft supersymmetry breaking parameters¹⁰⁾. This scenario is reminiscent of the Coleman-Weinberg²²⁾ model for $SU(2)_L \times U(1)_Y$ gauge symmetry

breaking, which was shown to generate considerable entropy²³⁾. Does the same happen here? The effective potential at zero temperature for $x = 0(Q_1)$, where the soft supersymmetry-breaking N scalar mass-squared m_N^2 vanishes, takes the approximate form

$$V_0(x) \simeq \frac{3k^2}{8\pi^2} (2m_D^2(Q_1) + A_k^2(Q_1)) x^2 \ln \frac{x}{Q_1} + \frac{25}{120} g_E^2(Q_1) x^4 \quad (40)$$

In Eq. (40), k is the DD^cN Yukawa coupling, m_D^2 is the D scalar mass-squared and we estimate $m_D \sim 1.8m_{\frac{1}{2}}$, A_k is the supersymmetry-breaking coefficient of the trilinear term kDD^cN in the effective scalar potential, and g_E is the $U(1)'$ gauge coupling¹⁰⁾. The finite-temperature corrections to (40) take the approximate form

$$V_T(x) \simeq \frac{T^2}{24} \{ 3(2\lambda^2 + 3k^2) + 5g_E^2 \} x^2 \quad (41)$$

Following Witten²³⁾, we can use (41) in the interesting domain of x if we also replace the logarithm in (40) by $\ln T/Q$. To analyze entropy generation during the formation of $\langle 0|N|0 \rangle \equiv x \neq 0$, we first determine the preferred value of x by minimizing $V_0(x)$ and define the reheating temperature $T_{R1} \equiv |V(0) - V(x)|^{1/4}$. Then we determine the critical temperature T_{c1} from the condition $\partial^2/\partial x^2 (V_0(x) + V_T(x))|_{x=0} = 0$. If we found $T_{R1} > T_{c1}$, we would conclude that the entropy increased by a factor $\Delta_1 = (T_{R1}/T_{c1})^3$. In fact, we find that for values of k , Q_1 , A_k and λ favoured by model calculations¹⁰⁾, T_{R1} is always smaller than T_{c1} and this phase transition is always second order. Two representative examples are

$$k = \begin{cases} 0.25 \\ 0.55 \end{cases}, Q_1 = \begin{cases} 8.9 \times 10^6 \text{ GeV} \\ 2.4 \times 10^9 \text{ GeV} \end{cases}, A_k = \begin{cases} -3.5 m_{\frac{1}{2}} \\ -3.0 m_{\frac{1}{2}} \end{cases}, T_R = \begin{cases} 1.0 m_{\frac{1}{2}} \\ 2.4 m_{\frac{1}{2}} \end{cases}, T_c = \begin{cases} 2.0 m_{\frac{1}{2}} \\ 3.8 m_{\frac{1}{2}} \end{cases} \quad (42)$$

almost independently of the value of λ .

We now make a similar analysis of the $SU(2)_L \times U(1)_Y \rightarrow U(1)_{em}$ phase transition. Here the zero-temperature potential in the neighbourhood of Q_2 : $m_H^2(Q_2) = 0$ is approximately

$$V_0(v, \bar{v}) = \frac{3h^2}{8\pi^2} (2m_{\tilde{Q}}^2 + A_h^2) v^2 \ln \frac{v}{Q_2} + \frac{m_H^2}{H} \bar{v}^2 + 2x\lambda A_\lambda v \bar{v} \\ + \lambda^2 \left[x^2 (v^2 + \bar{v}^2) + v^2 \bar{v}^2 \right] + \left(\frac{g_1^2 + \frac{3}{5}g_2^2}{8} \right) (v^2 - \bar{v}^2)^2 + \frac{g_E^2}{120} (5x^2 - 4v^2 - \bar{v}^2)^2 \quad (43)$$

where h is the $HQ_3U_3^c$ coupling responsible for the t -quark mass which we take to be 0.25 corresponding to $m_t \sim 40$ GeV, A_λ is the corresponding trilinear coefficient which we calculate to be $-3.0m_{\frac{1}{2}}$, and we estimate $m_{\tilde{Q}}^2 \sim (1.8m_{\frac{1}{2}})^2$, $m_H^2 \sim 0.45 m_{\frac{1}{2}}^2$, $x \sim 2m_{\frac{1}{2}}$ and $\lambda A_\lambda \sim -0.11 m_{\frac{1}{2}}^{10}$. For definiteness, we also take $Q_2 \sim m_{\frac{1}{2}}$, but our results are not sensitive to this choice. The finite-temperature potential for v , which is the trigger for the phase transition, is

$$V_T(v) = \frac{T^2}{24} \left\{ 3(\lambda^2 + 3h^2) + 12 \left(\frac{g_1^2}{4} + \frac{3g_2^2}{4} + \frac{1}{15}g_E^2 \right) \right\} v^2 \quad (44)$$

As before, we first find the minimum of $V_0(v, \bar{v})$ and define $T_{R_2} \equiv [V_0(0,0) - V(v, \bar{v})_{\min}]^{1/4}$. Then we find T_{c_2} from the condition $\partial^2/\partial v^2(V_0 + V_T)|_{v=\bar{v}=0} = 0$, replacing the logarithm in (43) by $\ln(T/Q_2)$. Our results for different representative values of λ are shown below

λ	$v/m_{1/2}$	$\bar{v}/m_{1/2}$	$m_{1/2}$	$T_{R_2}/m_{1/2}$	$T_{c_2}/m_{1/2}$	$\Delta_2 = (T_{R_2}/T_{c_2})^3$	} (45)
0.15	1.3	0.75	117 GeV	0.7	0.4	6	
0.23	0.8	0.25	210 GeV	0.47	0.4	1.6	
0.26	0.6	0.2	277 GeV	0.39	0.38	1.08	
0.28	0.4	0.13	416 GeV	0.28	0.39	2nd order	
0.30	0	0	no phase transition				

We see that the factor of entropy increase Δ_2 is never more than six, and the phase transition is second order for the larger values of $m_{\frac{1}{2}}$ which are cosmologically preferred²¹⁾. Thus the previous experience²³⁾ with the Coleman-Weinberg²²⁾ phase transition is not repeated.

We conclude that our current knowledge and understanding of the gauge-singlet and non-singlet fields in the effective low-energy theory coming from the

superstring do not reveal any large source of entropy which could dilute the previous estimates (32) and (32') by more than a factor of 10. Moreover, we have seen that $\text{Arg } c_4$ could well be as large as $O(\alpha)$ (8). The baryon number of the Universe is certainly not embarrassingly small!

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FIGURE CAPTIONS

Fig. 1 : Diagram contributing to $\text{Im}c_4$ if the gaugino masses $m_{\tilde{V}}$, $m_{\tilde{X}}$ have CP-violating phases⁷⁾.

Fig. 2 : Diagrams contributing to $\text{Im}c_4$ if the Yukawa couplings λ have CP-violating phases.

Fig. 1

Fig. 2