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CURVATURE CURVED TERMS
IN STRING THEORY EFFECTIVE ACTIONS

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Curvature cubed terms in string theory effective actions

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Abstract

Starting from the 3- and 4-point amplitudes we determine the R^3 -terms in the gravitational effective actions in the bosonic, heterotic and type II superstring theories. We prove the absence of unambiguous R^3 -terms (including the Gauss-Bonnet R^3 -invariant) in the heterotic and superstring cases. As a by-product, we check the presence of the square of the Lorentz Chern-Simons term in the heterotic string effective action and obtain the 3-loop contribution to the β -function of the bosonic sigma-model. We also discuss the structure of the field redefinition ambiguity in a gravitational effective action.

1. One possible way to understand compactification and phenomenology of string theories is to study a low-energy effective action (EA) for massless string modes [1-20]. In this paper we shall consider the gravitational EA in the three closed string theories: bosic, heterotic and type II super (we shall call them B, H- and S - theories for short). The structure of the leading R and R^2 -terms in the EA's is well-known at present [4-6,9-11,19]. There was a recent progress concerning R^4 -terms in the H- and S-theories [11-14]. As for the R^3 -terms it is generally believed that they are absent in the H- and S-theories (but are present in the B-theory, see, e.g., Zwiebach's second paper in ref. [9]). However [21] the claims against the R^3 -terms (see, e.g. [13,14]) which were based on the absence of k^6 -terms in the corresponding 3-point amplitudes cannot a priori rule out the Gauss-Bonnet R^3 -term [9,22] (which we shall denote G_3). Below we shall carry out a systematic study of the R^3 -terms analyzing also the 4-point amplitudes. There are only two independent R^3 -invariants which coefficients can be unambiguously fixed from the S-matrix: $I_1 = R_{\alpha\beta}^{\mu\nu} R_{\mu\nu}^{\alpha\beta} R_{\alpha\beta}^{\mu\nu}$ and G_3 , " R^3 " $\Rightarrow b_1 I_1 + b_2 G_3$. We shall find that while b_1 and b_2 are non-vanishing in the B-theory, they indeed both vanish in the H- and S-theories.

Let us call "unambiguous" the coefficients which can be uniquely fixed by comparing string-theory and field-theory S-matrices. As was pointed out in [10,11,19] the coefficients which change under field redefinitions in the EA are a priori ambiguous within the S-matrix approach. Such are, for example, the coefficients of the "Ricci terms", i.e. the terms containing the Ricci tensor. As we shall discuss below we may (partially) fix the ambiguity by choosing the coefficients of the Ricci terms as we like (see also sect.4).

2. There are five possible dimension 6 invariants ^{1/} which do not involve the Ricci tensor (see e.g. [22,23])

^{1/} There is also a number of other R^3 -contractions but they can be trivially reduced to (1) using the Ricci identity and symmetry properties.

$$I_1 = R^{\mu\nu}_{\alpha\beta} R^{\alpha\beta}_{\lambda\sigma} R^{\lambda\sigma}_{\mu\nu}, \quad I_2 = R^{\mu\nu}_{\alpha\beta} R^{\nu\lambda}_{\beta\sigma} R^{\lambda\mu}_{\sigma\alpha} \quad (1)$$

$$I_3 = R^{\alpha\nu}_{\mu\beta} R^{\beta\delta}_{\nu\lambda} R^{\lambda\mu}_{\delta\alpha}, \quad I_4 = R^{\mu\alpha}_{\nu\beta} R^{\mu\alpha}_{\delta\delta} R^{\nu\beta\delta\delta},$$

$$I_5 = R_{\lambda\mu\nu\rho} \partial^2 R^{\lambda\mu\nu\rho}, \quad R^{\mu\nu}_{\alpha\beta} = R^{\mu\nu}_{\dots\alpha\beta}, \quad R^{\lambda\mu}_{\lambda\alpha\beta} = \partial_\alpha \Gamma^{\lambda\mu}_{\lambda\beta} - \dots$$

They satisfy the following relations (in the last relation we omit total derivative and Ricci terms)

$$I_3 = I_2 - \frac{1}{4} I_1, \quad I_4 = \frac{1}{2} I_2, \quad I_5 = -I_1 - 4I_2 \quad (2)$$

Hence only two of them are independent. We choose these two to be I_1 and $G_3 = I_1 - 2I_2$.

To explain why the combination G_3 is special let us consider the "Gauss-Bonnet" invariants [9, 24]

$$\int d^D x \sqrt{g} \Omega_n = \int R^{q_1 q_2}_{\dots} \wedge R^{q_{2n-1} q_{2n}}_{\dots} \wedge e^{q_{2n+1}}_{\dots} \wedge e^{q_{2n+2}}_{\dots} \wedge \dots \wedge e^{q_{2n+1}}_{\dots} \wedge e^{q_{2n+2}}_{\dots} \wedge \dots \wedge e^{q_{2n+1}}_{\dots} \wedge e^{q_{2n+2}}_{\dots} \quad (3)$$

($\Omega_{2n/2}$ is the Euler number integrand). As is possible to check (see, e.g. [25])

$$\Omega_2 = R, \quad \Omega_4 = G_2, \quad \Omega_6 = 32 G_3 + (\text{Ricci terms}),$$

$$G_2 \equiv R^2_{\lambda\mu\nu\rho} - 4R^2_{\mu\nu} + R^2, \quad G_3 \equiv I_1 - 2I_2 \quad (4)$$

The expansion of $\int \Omega_n$ near a flat space ($e^0_0 = e_1, e^0_0 = dx, e^1_1 = 1/2 h^2 dx^2$) starts with $O(h^{n+1})$ -terms [24] ($\omega \sim O(h), R = d\omega + O(h^2)$).

$\int d\omega \wedge \dots \wedge d\omega \wedge e_0 \wedge \dots \wedge e_0 = 0$). Since G_3 differs from Ω_3 by Ricci terms its expansion in fact starts with $O(h^3)$ -terms (these terms do not contribute to the 3-point amplitude but as we shall discuss they produce exchange contributions to the 4-point amplitude).

Consider the most general lagrangian containing all possible independent curvature invariants,

$$\mathcal{L} = a_0 R + \alpha' (a_1 R^2_{\lambda\mu\nu\rho} + a_2 R^2_{\mu\nu} + a_3 R^2) + \alpha'^2 (b_1 I_1 + b_2 G_3 + b_3 R^{\mu\alpha}_{\nu\beta} R^{\nu\beta}_{\delta\delta} R^{\lambda\mu}_{\delta\delta} + \dots) \quad (5)$$

$$+ b_4 R_{\mu\nu\rho\lambda} R^{\mu\lambda} R^{\nu\rho} + \overset{-3-}{b_5} R_{\mu\nu} R^{\nu\lambda} R_\lambda^\mu + b_6 R_{\mu\nu} \partial^2 R^{\mu\nu} \\ + b_7 R_{\lambda\mu\nu\rho}^2 R + b_8 R_{\mu\nu}^2 R + b_9 R^3 + b_{10} R \partial^2 R) + O(\alpha'^3).$$

Making the general local field redefinition

$$g_{\mu\nu} \rightarrow g_{\mu\nu} + \alpha' (d_1 R_{\mu\nu} + d_2 g_{\mu\nu} R) + \alpha'^2 (d_3 R_{\mu\lambda\rho\sigma} R_{\nu}{}^{\lambda\rho\sigma} \\ + d_4 R_{\mu\alpha\beta\gamma} R^{\alpha\beta} + d_5 R_{\mu\lambda} R_\nu^\lambda + d_6 \partial^2 R_{\mu\nu} \quad (6) \\ + d_7 g_{\mu\nu} R_{\lambda\rho\alpha\beta}^2 + d_8 g_{\mu\nu} R_{\lambda\rho}^2 + d_9 g_{\mu\nu} R^2 + d_{10} g_{\mu\nu} \partial^2 R) + O(\alpha'^3),$$

we find that while the unambiguous coefficients a_0, a_1, b_1 and b_2 remain invariant, all other coefficients change. By a proper choice of d_1 and d_2 we can give a_2 and a_3 any desired values, e.g. the Gauss-Bonnet ones, $a_2 = -4a_1, a_3 = a_1$ (this choice will simplify the analysis of 4-point amplitude). Now we may use the freedom in $d_3, \dots, d_{10}$ in order to fix $b_3, \dots, b_{10}$. For example, we can make $b_3 = \dots = b_{10} = 0$. Since the field redefinition (6) does not change perturbative S-matrix it is possible to start with the EA in which $a_1, a_2, b_1, \dots, b_{10}$ have these particular values.

The low-energy expansion of a closed string theory EA has the following generic structure

$$S = -\frac{2}{\alpha'^2} \int d^D x \sqrt{g} \left\{ R + \gamma (\partial_\mu \phi)^2 - \frac{1}{12} \hat{H}_{\mu\nu\rho}^2 \right. \\ \left. + \alpha' e^{\gamma\phi} (a_1 G_2 + \dots) + \alpha'^2 e^{2\gamma\phi} (b_1 I_1 + b_2 G_3 + \dots) \right. \\ \left. + O(\alpha'^3 e^{3\gamma\phi}) \right\}, \quad \gamma = -\frac{4}{D-2}, \quad (7)$$

where $R = R^\lambda{}_{\mu\lambda\nu} g^{\mu\nu}$, $\alpha = g(\alpha')^{-1/8}$ and

$$\hat{H}_{\mu\nu\rho} = 3 \left\{ \partial_{[\mu} B_{\nu\rho]} + \alpha' c \omega_{[\mu}^{\alpha\beta} R_{\nu\rho]}^{\alpha\beta} + \dots \right\} \quad (8)$$

Dots in (7) stand for $\partial\phi$ - and $\hat{H}$ -dependent terms which we shall ignore (we also ignore gauge field dependent terms present in the heterotic string theory). In the

antisymmetric tensor field strength (8) we have included the first piece of the Lorentz Chern-Simons term ($\omega R - \frac{2}{3} \omega \omega \omega$) expected to be present in the heterotic string theory EA [4, 14].

The 3-point amplitudes in the B, H and S-string theories are [2, 26, 4]

$$\mathcal{H}_3 = \alpha \zeta_1^{\mu_1 \nu_1} \zeta_2^{\mu_2 \nu_2} \zeta_3^{\mu_3 \nu_3} N_{\mu_1 \mu_2 \mu_3} \tilde{N}_{\nu_1 \nu_2 \nu_3}, \quad (9)$$

$$N^{\mu_1 \mu_2 \mu_3} = k_1^{\mu_1} \delta^{\mu_2 \mu_3} + k_2^{\mu_2} \delta^{\mu_1 \mu_3} + k_3^{\mu_3} \delta^{\mu_1 \mu_2} + \frac{d'}{2} \varepsilon k_2^{\mu_2} k_3^{\mu_3} k_1^{\mu_1}$$

where $\tilde{N} = N(\varepsilon \rightarrow \tilde{\varepsilon})$, $\varepsilon_B = \tilde{\varepsilon}_B = \varepsilon_H = 1$ and $\varepsilon_S = \tilde{\varepsilon}_S = \tilde{\varepsilon}_H = 0$.

Using the expansions ($\partial_{\mu\nu} = \delta_{\mu\nu} + h_{\mu\nu}$)

$$R = \frac{1}{4} h_{\mu\nu} \square h_{\mu\nu} - \frac{1}{4} h_{\alpha\beta} (h_{\lambda\rho} \partial_\alpha \partial_\beta h_{\lambda\rho} + 2 \partial_\alpha h_{\lambda\rho} \partial_\beta h_{\lambda\rho}) + \dots + O(h''),$$

$$G_2 = -h_{\mu\nu} \partial_\mu \partial_\rho h_{\alpha\beta} \partial_\alpha \partial_\beta h_{\nu\rho} + \dots + O(h''), \quad (10)$$

$$I_1 = \partial_\rho \partial_\alpha h_{\beta\nu} \partial_\beta \partial_\lambda h_{\mu\alpha} \partial_\mu \partial_\nu h_{\lambda\rho} + \dots + O(h''), G_3 = \dots + O(h'')$$

where dots stand for terms which vanish when $h_{\mu\nu}$ is put on mass shell we can compute the 3-graviton amplitude for (7) and compare it with (9). Thus we fix the values of a_1 [9, 4] and b_1 in (5), (7)

$$a_{1B} = \frac{1}{4}, \quad a_{1H} = \frac{1}{8}, \quad a_{1S} = 0, \quad b_{1B} = \frac{1}{48}, \quad b_{1H} = b_{1S} = 0. \quad (11)$$

The value of the coefficient c in (8) can be found from the comparison of the Bhh-amplitudes [4] 2/

$$C_B = C_S = 0, \quad C_H = \frac{1}{4} \quad (12)$$

(note that $\omega^{\alpha\beta}_{\mu} = \partial_\alpha \partial_\beta h_{\mu} + O(h^2)$).

To determine b_2 (and to check that the ωR -term in (8) indeed forms the square with ∂B) it is necessary to compare 4-point amplitudes. It is sufficient to consider only terms in the 4-graviton amplitude in which the four polarization

2/ The vanishing of c for the B- and S-theories follows trivially from the anti-symmetry of $B_{\mu\nu}$, symmetry of $h_{\mu\nu}$ and left-right symmetry ($N = \tilde{N}$ in (7)) of these theories.

tensors are cyclically contracted with each other to form the matrix trace, e.g.

$\xi_{1,\mu\nu} \xi_{2,\nu\rho} \xi_{3,\rho\sigma} \xi_{4,\sigma\mu} = \xi_1 \xi_2 \xi_3 \xi_4$. The corresponding part of the string-theory 4-point amplitudes is [26,27,14]

$$\mathcal{A}_4 = -\frac{\alpha'^2 \alpha'^3}{256} \Gamma(s, t, u) \xi_1^{\mu_1 \nu_1} \xi_2^{\mu_2 \nu_2} \xi_3^{\mu_3 \nu_3} \xi_4^{\mu_4 \nu_4} K_{\mu_1 \dots \mu_4} \tilde{K}_{\nu_1 \dots \nu_4}, \quad (13)$$

$$\Gamma(s, t, u) \equiv \frac{\Gamma(-\frac{\alpha'}{4}s) \Gamma(-\frac{\alpha'}{4}t) \Gamma(-\frac{\alpha'}{4}u)}{\Gamma(1+\frac{\alpha'}{4}s) \Gamma(1+\frac{\alpha'}{4}t) \Gamma(1+\frac{\alpha'}{4}u)} = -\frac{64}{\alpha'^3 s t u} - 2 \frac{1}{2}(3) + O(\alpha'^3),$$

$$K^{\mu_1 \dots \mu_4} = u t \lambda(s) \delta^{\mu_1 \mu_2} \delta^{\mu_3 \mu_4} + u s \lambda(t) \delta^{\mu_1 \mu_3} \delta^{\mu_2 \mu_4} + s t \lambda(u) \delta^{\mu_1 \mu_4} \delta^{\mu_2 \mu_3},$$

$$\text{where } \tilde{K} = K(\lambda \rightarrow \tilde{\lambda}), \quad \lambda_B(x) = \tilde{\lambda}_B(x) = \lambda_H(x) = \frac{1}{1 + \frac{\alpha'}{4}x},$$

$$\tilde{\lambda}_H = \lambda_S = \tilde{\lambda}_S = 1, \quad x = s, t, u, \quad s = -(k_1 + k_2)^2, \quad t = -(k_1 + k_4)^2,$$

$$u = -(k_1 + k_3)^2, \quad k_i^2 = 0. \quad \text{The amplitude (13) can be rewritten as follows}$$

$$\mathcal{A}_4 = \frac{\alpha'^2}{2} \left[f(u, s, t) \xi_1 \xi_2 \xi_3 \xi_4 + f(t, s, u) \xi_1 \xi_2 \xi_4 \xi_3 \right.$$

$$\left. + f(s, u, t) \xi_1 \xi_3 \xi_2 \xi_4 \right] + \text{other structures}, \quad (14)$$

$$\text{where } f(u, s, t) \equiv f = \frac{1}{2} u \left[\lambda(s) \tilde{\lambda}(t) + \lambda(t) \tilde{\lambda}(s) \right], \quad \text{i.e.}$$

$$f_B = u + \frac{\alpha'}{4} u^2 + \frac{\alpha'^2}{16} (u^3 - u s t) + O(\alpha'^3), \quad (15)$$

$$f_H = u + \frac{\alpha'}{8} u^2 + \frac{\alpha'^2}{32} (u^3 - 2 u s t) + O(\alpha'^3), \quad (16)$$

$$f_S = u + O(\alpha'^3) \quad (17)$$

Here the k^2 -term represents the Einstein amplitude (see e.g. [18]), k^4 -term corresponds to the G_2 -term in (7) [19] and k^6 -term is relevant for our problem of determining b_2 .

There are several k^6 -contributions to the field theory (7) 4-graviton amplitude. First, there are contact h^4 -terms from I_1 , G_3 and $(\omega R)^2$ -terms. Next,

there are three types of graviton exchanges: $G_2 - G_2$ (with the both 3-vertices from the G_2 -term), $R - I_1$ and $R - G_3$ (with one 3-vertex from the Einstein term and one from I_1 or G_3). There are also the dilation ($G_2 - G_2$) and the antisymmetric tensor ($\omega R - \omega R$) exchanges but it is easy to see that they do not produce "irreducible" ζ^4 -structures (14) we are interested in. The relevant terms in the expansions of the curvature invariants are found after a lot of algebra to be

$$R = \dots - \frac{1}{4} h h \square h + \dots, \quad G_2 = -\frac{1}{2} \partial_\alpha h \partial_\alpha h \square h + \dots, \quad (18)$$

$$I_1 = -\frac{3}{2} \partial_\alpha \partial_\beta h \partial_\alpha \partial_\beta h \square h + \dots + 3(h \partial_\alpha \partial_\beta h \partial_\alpha \partial_\beta h \partial_\rho \partial_\rho h + \frac{1}{2} \partial_\alpha h \partial_\beta h \partial_\alpha \partial_\beta h \partial_\rho \partial_\rho h) + \dots, \quad (19)$$

$$G_3 = -\frac{3}{2} \partial_\alpha \partial_\beta h \partial_\alpha \partial_\beta h \square h + \dots + 3(h \partial_\alpha \partial_\beta h \partial_\alpha \partial_\beta h \partial_\rho \partial_\rho h + \frac{1}{2} \partial_\alpha h \partial_\beta h \partial_\alpha \partial_\beta h \partial_\rho \partial_\rho h + \frac{1}{4} \partial_\alpha h \partial_\beta h \partial_\rho \partial_\rho h \partial_\alpha \partial_\beta h) + \dots, \quad (20)$$

$$3 \omega_{\mu}^{ae} R_{\nu\rho}^{ae} \omega_{[\mu}^{mn} R_{\nu\rho]}^{mn} = \frac{1}{2} (\partial_\alpha h \partial_\beta h \partial_\rho \partial_\rho h \partial_\alpha \partial_\beta h - \partial_\alpha h \partial_\beta h \partial_\rho \partial_\rho h \partial_\alpha \partial_\beta h) + \dots \quad (21)$$

Here we have used the matrix notation $h \dots h \dots h = h_{\mu\nu} \dots h_{\lambda\mu} \dots h_{\lambda\mu}$, $h \dots h \dots h \dots h = h_{\mu\nu} \dots h_{\nu\lambda} \dots h_{\lambda\rho} \dots h_{\rho\mu}$. We have given only terms which give non-vanishing contributions to the relevant piece of the on-shell amplitude (14) (in particular, two of the three legs in the h^3 -terms are assumed to be on-shell). Given (18)-(21) it is straightforward to compute the h^4 -term in the generating functional $\mathcal{S}$ for the tree S-matrix in the field theory (7) with the action $S = \frac{1}{2} \int d^D x \{ -\frac{1}{2} h \square h + V_3(h) + V_4(h) \}$. Since all the relevant 3-vertices have the form

$$V_3 = J_{\mu\nu}(h) \square h_{\mu\nu}, \quad J_{\mu\nu} = \frac{1}{2} h_{\mu\lambda} h_{\lambda\nu}, \quad \text{we get simply } ^{3/}$$

$$\mathcal{S}^{(4)}[h_{in}] = \mathcal{X}^2 \int d^D x \{ V_4(h_{in}) + \frac{1}{2} J_{\mu\nu}(h_{in}) \square J_{\mu\nu}(h_{in}) \} \quad (22)$$

The total field-theory amplitude is given by (14) with

^{3/} Using (22) it is also easy to check that, e.g., the term $X = R_{\mu\alpha\beta\gamma} R_{\nu}^{\alpha\beta\gamma} R^{\mu\nu}$ in (5) gives vanishing contribution to the 4-point amplitude (contribution of the R - X -exchange exactly cancels the h^4 -contribution from X).

$$f(u, s, t) = u + \alpha' a_1 u^2 + \alpha'^2 [-3b_1 (u^3 - 4ust) - 3b_2 (u^3 - \frac{7}{2}ust) + \frac{1}{4}c^2 (u^3 - ust) + (a_1^2 + 3b_1 + 3b_2)(u^3 - 3ust)] + O(\alpha'^3) \quad (23)$$

where the first three α'^2 -terms are the contributions of the h^4 -vertices and the last is the contribution of the gravitational exchanges. Comparing (23) with (15), (16) and (17) we find, in addition to (11), (12),

$$b_{1B} = \frac{1}{24}, \quad b_{2H} = b_{2S} = 0 \quad (24)$$

3. Thus we have proved that there is no Gauss-Bonnet R^3 -term in the heterotic string and superstring theories. In the process we have also checked that the Lorentz Chern-Simons term indeed forms the square in sum with $\partial_\mu B_{\lambda\rho}$ in the heterotic string EA (this supplements the analysis of refs. [4,14]). To obtain these results within the $\mathcal{G}$ -model approach it would be necessary to do 3-loop computations^{4/}. The absence of unambiguous R^3 -terms in the superstring EA is in agreement with the known vanishing of 3-loop β -function in the $N=1$ supersymmetric $\mathcal{G}$ -model on Ricci-flat backgrounds [30,12]. Our results imply that the gravitational part of the 3-loop β -function in the heterotic $N=1/2$ supersymmetric $\mathcal{G}$ -model is given simply by the variation over $g_{\mu\nu}$ of the square of the Lorentz Chern-Simons term^{5/}.

Having found the R^3 -terms in the bosonic string theory EA it is now straightforward to write down the expression for the corresponding 3-loop β -function in the bosonic $\mathcal{G}$ -model (2-loop result was first found in [34]; 3-loop contribution was not known previously)^{6/}

$$\beta_{\mu\nu} = \alpha' R_{\mu\nu} + 2\alpha'^2 a_{1B} R_{\mu\alpha\beta\gamma} R_{\nu}{}^{\alpha\beta\gamma} - 3\alpha'^3 (b_{1B} + b_{2B}) \{ \partial_\rho R_{\lambda\mu\alpha\beta} \partial_\lambda R_{\nu\rho\alpha\beta} - R_{\mu\gamma\alpha\beta} R_{\rho\gamma\alpha\beta} R_{\nu\delta\epsilon\gamma} \} \quad (25)$$

^{4/} By now there is a strong evidence that the effective equations $\delta S/\delta g^{\mu\nu} = 0$, are equivalent to the conditions of conformal invariance of the corresponding sigma-model, i.e. to the vanishing of " β -functions" [28,5-8,10,31-33].

^{5/} At present only 2-loop β -functions were computed for this model [7,8,20].

^{6/} Thus we realize the suggestion [28] that it may be simpler to compute string tree amplitudes than loops in a $\mathcal{G}$ -model in order to determine the β -function (cf. also [11,12]).

$$+ 2 R_{\mu\alpha\beta\epsilon} R_{\lambda\alpha\beta\epsilon} R_{\mu\lambda\alpha\beta}] + (\mu \leftrightarrow \nu) \} + \\ + 3 \alpha'^3 b_{23} \{ [\partial_\rho R_{\lambda\alpha\beta\gamma} \partial_\lambda R_{\nu\alpha\beta\gamma} - R_{\lambda\alpha\beta\gamma} \partial_\lambda \partial_\rho R_{\mu\alpha\beta\gamma} \\ + R_{\mu\alpha\beta\lambda} R_{\rho\alpha\beta\epsilon} (3 R_{\nu\epsilon\beta\lambda} - R_{\nu\epsilon\beta\lambda})] + (\mu \leftrightarrow \nu) \} + O(\alpha'^4)$$

To obtain this expression we have computed $\delta S / \delta g^{\mu\nu}$ and omitted (i) terms with $g_{\mu\nu}$ (they are related to the dilaton "g-function" [8,10]) and (ii) α'^2 - and α'^3 -terms containing Ricci tensor (they are renormalization scheme dependent and in any case are irrelevant if we solve $\beta_{\mu\nu}=0$ perturbatively in α'). As discussed in [10,30] the off-shell relation between the effective equations and "g-functions" is $\frac{\delta S}{\delta \varphi^i} = \mathcal{R}_{ij} \beta^j$, where $\mathcal{R}(\varphi)$ transforms as a tensor under reparametrizations of the coupling constant space (under changes of a renormalization scheme). Perturbatively $\frac{\delta S}{\delta \varphi^i} = 0$ is thus equivalent to $\beta^i=0$ (the Ricci terms in $\delta S / \delta g_{\mu\nu}$ can be generated by multiplication of $\beta_{\mu\nu}$ by an appropriate $\mathcal{R}^{\mu\nu\alpha\beta}$ [10]).⁷¹

There is a simple argument explaining why b_2 should vanish in the superstring and heterotic string theories. The absence of G_3 from the S-theory EA follows trivially from the absence of α'^5 -terms in the corresponding 4-point amplitude (14), (17) (and the absence of exchanges because of $\alpha_{4,5}=0$). Consider the H-theory EA depending both on the metric and the gauge field. The G' -model analysis [5,8] suggests that when the gauge field is identified with the (properly modified) spin connection the H-theory EA should coincide with the S-theory one. This was checked on a number of terms [14]. If G_3 were present in the H-theory it would be impossible to cancel it by gauge field terms after identification $R \sim F$ since (as follows trivially from the structure of the 3-point amplitudes [4]) there is no F^3 - or $R^2 F$ -terms in the H-theory EA. The same argument implies the absence of the R^4 -Gauss-Bonnet term in the H-theory if it is absent in the S-theory (relevant F^4 and $R^2 F^2$

⁷¹ Note that in deriving (25) we have dropped in (5), (7) all ambiguous Ricci terms. Since these terms can be eliminated from the EA by a proper field redefinition, the terms that they generate (under differentiation over $g_{\mu\nu}$) in the g-function are renormalization scheme dependent.

terms already group with other R^4 -terms which contribute to the 4-point amplitude /13,14/).

The absence of the R^3 -terms in the heterotic and superstring EA's is consistent with arguments based on (linearized) $N=1$, $D=10$ supersymmetry [36]. Let us note also that it should be possible to derive the cancellation of the R^3 -terms in the H- and S-theories starting from the B-theory EA we have found and using the Kaluza-Klein approach of refs. [35].

4. We finish with remarks on the field redefinition ambiguity in a structure of the gravitational EA extending the discussion in ref. [10]. Consider a general EA (5) and make an arbitrary local field redefinition (6). The coefficients in (5) which remain invariant under this transformation can be unambiguously fixed from the S-matrix. A sufficient condition for a coefficient to be unambiguous is that the curvature invariant it multiplies should not be representable as a product of two non-trivial ($\neq \text{const } g_{\mu\nu}$) symmetric second rank tensors, $L_{\mu\nu} M^{\mu\nu}$. This condition is not, however, a necessary one. Whether a coefficient of the $L_{\mu\nu} M^{\mu\nu}$ -invariant in the EA is ambiguous (i.e. changes under a general redefinition of the metric) depends, for example, on the actual presence in the EA of at least one of the terms $L_{\mu\nu} g^{\mu\nu}$ or $M_{\mu\nu} g^{\mu\nu}$ 8/.

Let $\{\lambda_i\}$ be the full set of unambiguous coefficients in the EA and I_i be the corresponding curvature invariants. Then the EA has the following structure

$$S = \int d^D x \left\{ \sum_i \lambda_i I_i (g + Q(g)) + \sum_{n,m} \alpha_{nm} K_{n\mu\nu} K_m^{\mu\nu} \right\}, \quad (26)$$

where $Q_{\mu\nu} = \sum_n \nu_n K_{n\mu\nu}$ and $K_{n\mu\nu} (\neq \text{const } g_{\mu\nu})$

is the full set of covariant symmetric second-rank tensors ($K_{1\mu\nu} \sim R_{\mu\nu}$, $K_{2\mu\nu} \sim R_{\mu\nu}$, etc., see (6)). $\{\nu_n, \alpha_{nm}\}$ are the ambiguous coefficients of the theory. If

ν_n are taken completely arbitrary it could happen that without loss of generality

8/ For example, consider $X = \alpha_q' Q_q K_{\mu\nu} K^{\mu\nu}$, $K_{\mu\nu} = R_{\mu\alpha\beta\gamma} R_\nu^{\alpha\beta\gamma}$. X changes under $g_{\mu\nu} \rightarrow g_{\mu\nu} + \alpha_q' Q_q K_{\mu\nu}$ only if the coefficient Q_q in (5) is non-zero. Hence α_q' is ambiguous in the B-theory but is unambiguous in the S-theory.

lity all $\mathcal{E}_{nm}$ may be put equal to zero. If this were true we could put (by the redefinition $g \rightarrow Q(g) \rightarrow g$) all the ambiguous coefficients in the EA equal to zero and so disregard all the ambiguous structures $(R_{\mu\nu} Q^{\mu\nu}, R_{\mu\alpha\beta} R^{\alpha\beta}, R_{\nu}^{\alpha\beta} Q^{\mu\nu}, \text{ etc.})$ in analyzing higher order terms in the EA.^{9/}

Unfortunately, it appears that the above statement is not true. Consider the algebra $\{Q\}$ of all covariant rank 2 tensors constructed out of the curvature, covariant derivatives and $g_{\mu\nu}$. The monomials K_n form a linear basis in $\{Q\}$. Let $\{N_i\} \subset \{K_n\}$ be a set of all K_n 's which cannot be represented as $K_\ell^{\mu\alpha} K_m^{\nu\beta}$ for some ℓ and m ($N_1 = R_{\mu\nu}$, $N_2 = R_{\mu\nu}^{\alpha\beta}$, $N_3 = R_{\mu\nu}^{\alpha\beta} R^{\mu\nu}$, etc.). $\{N_i\}$ is the multiplicative basis in the algebra $\{Q\}$ since any K_n is a product of a number of N_i 's, $Q_{\mu\nu} = \sum_i \gamma_i N_i + \sum_{i,j} \gamma_{ij} N_i N_j + \dots = \sum_{i_1, \dots, i_p} \gamma_{i_1, \dots, i_p} N_{i_1} N_{i_2} \dots N_{i_p}$. Observing that a general EA, i.e. a sum of all possible curvature scalars, can be represented as a $g_{\mu\nu}$ -trace of an arbitrary covariant tensor $Q_{\mu\nu}$, we have: $S = \int d^4x \left\{ \sum_i \gamma_i N_i + \sum_{p>1} \sum_{i_1, \dots, i_p} \gamma_{i_1, \dots, i_p} N_{i_1} N_{i_2} \dots N_{i_p} \right\}$. It is easy to understand that γ_i and $N_{i\mu\nu}$ can be identified with λ_i and L_i in (26). In fact, $N_{i\mu\nu}$ cannot be equal to $L_{\mu\nu} M^{\mu\nu}$ since $L_{\mu\nu} = \sum_i \lambda_i N_{i\mu\nu} + \dots$ and $M_{\mu\nu} = \sum_j m_j N_{j\mu\nu} + \dots$. To see that all $\gamma_{i_1, \dots, i_p}$, $p>1$, are ambiguous, i.e. change under $g_{\mu\nu} \rightarrow g_{\mu\nu} + T_{\mu\nu}(g)$, $T_{\mu\nu} = \sum_p \sum_{i_1, \dots, i_p} d_{i_1, \dots, i_p} N_{i_1} N_{i_2} \dots N_{i_p}$, we note that $N_{i\mu\nu} \rightarrow N_{i\mu\nu} + g^{\mu\nu} \frac{\delta N_{i\mu\nu}}{\delta g_{\alpha\beta}} T_{\alpha\beta} - N_{i\mu\nu} T^{\mu\nu} + O(T^2)$, and so $\gamma_{i_1, \dots, i_p} \rightarrow \gamma_{i_1, \dots, i_p} - \sum_j \gamma_{i_1, \dots, i_p, j} d_{j, \dots, j_p} + \dots$. Though the number of $\{d_{i_1, \dots, i_p}\}$ is equal to the number of $\{\gamma_{i_1, \dots, i_p}\}$ the transformed value of $\gamma_{i_1, \dots, i_p}$ depends only on $d_{i_1, \dots, i_p}$, $q < p$ and hence it seems impossible to transform all $\{\gamma_{i_1, \dots, i_p}, p>1\}$ to zero by a proper choice of $\{d_{i_1, \dots, i_p}\}$.

9/

It is easy to see that it is always possible to transform away all the ambiguous terms which have the structure $K_n Q^{\mu\nu}$, where K_n is fixed and Q is arbitrary (e.g., the Ricci terms for $K_n = R_{\mu\nu}$, etc.). The required transformation $g \rightarrow g + T(g)$ is constructed order by order in ϵ . But at the same time this transformation generates also other structures: $K_m Q^{\mu\nu}$ with $m \neq n$ (e.g. in transforming away $R_{\mu\alpha\beta} R^{\alpha\beta} R^{\mu\nu}$ we generate $(R_{\mu\alpha\beta} R^{\alpha\beta})^2$, etc.). A non-trivial question is whether it is possible to transform away all such terms simultaneously.

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