CHIRAL ACTIONS FOR MINIMAL $N = 2$ SUPERGRAVITY

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A B S T R A C T

We construct chiral superfields, chiral densities, and chiral Lagrangians in curved $N = 2$ superspace. Using these results, we explicitly calculate the Lagrangian for a recently discovered minimal supergravity multiplet. The gauged version of this theory with a local $SO(2) \times SO(2)$ invariance describes on-shell the truncation of $N = 4$ supergravity with gauged $SU(2) \times SU(2)$. Finally, we discuss the coupling of vector and tensor multiplets to $N = 2$ supergravity.

1. Introduction

In a preceding paper [1] a minimal $N = 2$ supergravity multiplet $(32+32)$ with a local $SO(2)$ invariance was found. In spite of its irreducibility, it contains a physical vector multiplet where the pseudoscalar is replaced by an antisymmetric tensor. The multiplet and its transformation laws were derived from a geometrical theory in conventional extended superspace. It was shown that the transformation laws describe on-shell the truncation of a $N = 4$ supergravity theory with an antisymmetric tensor potential [2]. However, an off-shell lagrangian for the minimal $N = 2$ multiplet was still missing. It is the purpose of the present paper to fill this gap.

In sect. 2 a short derivation of the $N = 2$ vector and tensor multiplets in curved superspace is given. These are the basic off-shell representations of $N = 2$ supersymmetry. Both of them can be described by chiral superfields, which are analyzed in sect. 3. It is shown that a chiral superfield with $U(1)$ weight $w = 2$ is reducible to either a vector or a tensor superfield. Then we define chiral densities in $U(2)$ superspace and construct a special chiral density $\mathcal{E}$. The integral of a chiral density over x and Θ is a candidate for a chiral action. We find that there are two kinds of gauge invariant chiral lagrangians. The first one is the Yang-Mills lagrangian and the second one describes the coupling of a tensor and an Abelian vector multiplet.

In sect. 4 we use these results to construct a lagrangian for the minimal supergravity multiplet. We calculate its component form and derive the equations of motion. Then we try to generalize this lagrangian to the one-parameter class of supergravity multiplets given in appendix A of ref. [1]. It turns out, however, that a parameter n like in $N = 1$, $16+16$ supergravity [3, 4] does not exist in the case $N = 2$. Another extension of the minimal lagrangian leads to a $N = 2$ supergravity theory with gauged $SO(2) \times SO(2)$. Like in the corresponding $N = 4$ theory with gauged $SU(2) \times SU(2)$ [5], the scalar potential is unbounded from below.

Finally we discuss the coupling of the tensor multiplet to $N = 2$ supergravity. The free multiplet can be coupled to minimal supergravity and the improved multiplet (including a mass term) even to conformal supergravity [6]. The self-interactions that have been found in flat superspace [7, 8] can in general not be coupled to $N = 2$ supergravity.

Our conventions are the same as in the preceding paper [1].

2. The N = 2 vector and tensor multiplets in curved superspace

The basic off-shell representations of N = 2 supersymmetry are the vector multiplet [9] and the tensor multiplet [10]. They are also the basic ingredients to build chiral actions since chiral superfields can be decomposed into these two multiplets. In the following we give a short derivation of the vector and the tensor multiplet in curved superspace. The superspace geometry will be kept as general as possible, i. e. the structure group is $SL(2, \mathbb{C}) \times U(2)$ and the torsion and curvature are subject to the natural constraints only.

2.1. Vector multiplet

The N = 2 vector multiplet is described by a Lie algebra valued one-form gauge potential $A = dz^{\underline{M}} A_{\underline{M}}$. The field strength

$$F = dA + AA \quad (1)$$

satisfies the Bianchi identity

$$\tilde{D}F = 0 ,$$

$$\oint_{\underline{CBA}} (\tilde{D}_{\underline{C}} F_{\underline{BA}} + T_{\underline{CB}}^{\underline{D}} F_{\underline{DA}}) = 0 , \quad (2)$$

where $\tilde{D}$ is the full gauge covariant derivative.

The Yang-Mills potential A contains a highly reducible representation of supersymmetry and has therefore to be restricted by covariant constraints. The N = 2 Yang-Mills constraints are [9]

$$\begin{aligned} F_{\beta\alpha}^{BA} &= 4 \varepsilon_{\beta\alpha} \varepsilon^{BA} \overline{W} , & F_{\beta A}^{B\alpha} &= 0 , \\ F_{\dot{\beta}B\alpha A} &= 4 \varepsilon_{\dot{\beta}\alpha} \varepsilon_{BA} W . \end{aligned} \quad (3)$$

The superfield W has the U(1) weight $w(W) = 2$. The solution of the Bianchi identities (2) subject to these constraints gives

$$\begin{aligned} F_{\beta\alpha\dot{\alpha}}^B &= -2i \varepsilon_{\beta\alpha} \tilde{D}_{\dot{\alpha}}^B \overline{W} , \\ F_{\dot{\beta}B\alpha\dot{\alpha}} &= -2i \varepsilon_{\dot{\beta}\alpha} \tilde{D}_{\dot{\alpha}B} W , \end{aligned}$$

$$\begin{aligned}
 F_{\beta\dot{\beta}\alpha\dot{\alpha}} &= \varepsilon_{\beta\dot{\alpha}} f_{(\beta\alpha)} - \varepsilon_{\beta\alpha} f_{(\dot{\beta}\dot{\alpha})} , \\
 f_{\beta\alpha} &= \frac{1}{4} [\tilde{\mathcal{D}}_{\beta}^A, \tilde{\mathcal{D}}_{\alpha A}] W - 4W V_{\beta\alpha} - 4\bar{W} W_{\beta\alpha} , \\
 f_{\beta\dot{\alpha}} &= -\frac{1}{4} [\tilde{\mathcal{D}}_{\beta A}, \tilde{\mathcal{D}}_{\dot{\alpha}}^A] \bar{W} - 4\bar{W} \bar{V}_{\beta\dot{\alpha}} - 4W \bar{W}_{\beta\dot{\alpha}} . \quad (4)
 \end{aligned}$$

Moreover, we obtain the conditions

$$\begin{aligned}
 \tilde{\mathcal{D}}_A^{\dot{\alpha}} W &= \tilde{\mathcal{D}}_{\dot{\alpha}}^A \bar{W} = 0 , \\
 (\frac{1}{2} [\tilde{\mathcal{D}}^{\alpha A}, \tilde{\mathcal{D}}_{\alpha}^B] - 8\bar{R}^{AB}) W &= (\frac{1}{2} [\tilde{\mathcal{D}}_{\dot{\alpha}}^A, \tilde{\mathcal{D}}^{\dot{\alpha} B}] - 8R^{AB}) \bar{W} . \quad (5)
 \end{aligned}$$

Now the independent component fields of the Yang-Mills potential can be determined. The transformation law $\delta A = -\tilde{\mathcal{D}}\Lambda$ allows to choose the gauge

$$A_M| \sim (ia_m, 0, 0) \quad (6)$$

for the $\theta = \bar{\theta} = 0$ components. The remaining component fields are defined by

$$\begin{aligned}
 W| &= A , \quad \bar{W}| = \bar{A} , \\
 \tilde{\mathcal{D}}_{\alpha}^A W| &= \lambda_{\alpha}^A , \quad \tilde{\mathcal{D}}_A^{\dot{\alpha}} \bar{W}| = \bar{\lambda}_A^{\dot{\alpha}} , \\
 (\frac{1}{2} [\tilde{\mathcal{D}}^{\alpha A}, \tilde{\mathcal{D}}_{\alpha}^B] - 8\bar{R}^{AB}) W| &= N^{AB} . \quad (7)
 \end{aligned}$$

They form the vector multiplet

$$(a_m, \lambda_{\alpha}^A, A | N^{AB}) \quad (8)$$

with 8 bosonic and 8 fermionic degrees of freedom.

The lagrangian of the vector multiplet in flat superspace is [9]

$$\mathcal{L}_{YM} \sim \int d^4\theta \operatorname{tr} (WW) + h.c. . \quad (9)$$

The generalization to curved superspace will be given later.

2.2. Tensor multiplet

The $N = 2$ tensor multiplet is described by a two-form gauge potential $B = \frac{1}{2} dz^{\underline{M}} dz^{\underline{N}} B_{\underline{NM}}$ with the transformation law

$$\delta B = d\omega , \quad (10)$$

where ω is a one-form gauge parameter. The field strength

$$G = dB ,$$

$$G_{\underline{CBA}} = \oint_{\underline{CBA}} (\mathcal{D}_{\underline{C}} B_{\underline{BA}} + T_{\underline{CB}}^{\underline{D}} B_{\underline{DA}}) \quad (11)$$

satisfies the Bianchi identity

$$dG = 0 ,$$

$$E^{\underline{A}} E^{\underline{B}} E^{\underline{C}} E^{\underline{D}} (\mathcal{D}_{\underline{D}} G_{\underline{CBA}} + \frac{3}{2} T_{\underline{DC}}^{\underline{E}} G_{\underline{EBA}}) = 0 . \quad (12)$$

The proper constraints on the field strength G are [11]

$$\begin{aligned} G_{\underline{\gamma}\underline{\beta}\underline{\alpha}} &= 0 , \\ G_{\underline{\gamma}\underline{\beta}\underline{a}}^{CB} &= G_{\underline{C}\underline{B}\underline{a}}^{\dot{\gamma}\dot{\beta}} = 0 , \\ G_{\underline{\gamma}\underline{B}\underline{a}}^{C\dot{\beta}} &= 2i (G_a)_{\underline{\gamma}}^{\dot{\beta}} G^C_B , \quad G^A_A = 0 . \end{aligned} \quad (13)$$

G^A_B is a hermitian superfield with vanishing $U(1)$ weight. We remark that the constraints on $G_{\underline{\gamma}\underline{\beta}\underline{a}}$ are not independent but a consequence of $G_{\underline{\gamma}\underline{\beta}\underline{\alpha}} = 0$ [12]. The solution of the Bianchi identities (12) subject to the constraints (13) yields

$$\begin{aligned} G_{\underline{\gamma}\underline{\beta}\underline{\alpha}\underline{\alpha}}^C &= -\frac{2}{3} \varepsilon_{\dot{\beta}\underline{\alpha}} \sum_{\underline{\beta}\underline{\alpha}} \varepsilon_{\underline{\gamma}\underline{\beta}} \mathcal{D}_{\underline{\alpha}}^B G^C_B , \\ G_{\underline{\gamma}\underline{C}\underline{\beta}\underline{\alpha}\underline{\alpha}} &= -\frac{2}{3} \varepsilon_{\underline{\beta}\underline{\alpha}} \sum_{\underline{\beta}\underline{\alpha}} \varepsilon_{\underline{\gamma}\underline{\beta}} \bar{\mathcal{D}}_{\underline{\alpha}B} G^B_C , \end{aligned}$$

$$\begin{aligned} G_{\gamma\delta\beta\dot{\alpha}\dot{\alpha}} &= 2i (\varepsilon_{\gamma\alpha} \varepsilon_{\delta\dot{\beta}} \tilde{G}_{\beta\dot{\alpha}} - \varepsilon_{\gamma\dot{\beta}} \varepsilon_{\delta\alpha} \tilde{G}_{\alpha\dot{\beta}}), \\ \tilde{G}_{\alpha\dot{\alpha}} &= -G^A_B \tilde{U}^B_{\alpha\dot{\alpha}A} - \frac{1}{12} [\mathcal{D}^A_\alpha, \bar{\mathcal{D}}_{\dot{\alpha}B}] G^B_A, \end{aligned} \quad (14)$$

and the condition

$$\mathcal{D}^{(C}_\gamma G^{BA)} = \bar{\mathcal{D}}^{(C}_{\dot{\gamma}} G^{BA)} = 0. \quad (15)$$

The independent component fields of the two-form potential can now be defined as follows:

$$\begin{aligned} B_{\underline{MN}}| &\sim \begin{pmatrix} t_{mn} & 0 \\ 0 & 0 \end{pmatrix}, \quad G^A_B| = C^A_B, \\ \mathcal{D}^B_\alpha G^A_B| &= 3\chi^A_\alpha, \quad \bar{\mathcal{D}}^{\dot{\alpha}}_B G^B_A| = 3\bar{\chi}^{\dot{\alpha}}_A, \\ \mathcal{D}^\alpha_A \mathcal{D}^B_\alpha G^A_B| &= 12F, \quad \bar{\mathcal{D}}^A_\alpha \bar{\mathcal{D}}^{\dot{\alpha}}_B G^B_A| = 12\bar{F}. \end{aligned} \quad (16)$$

They form the tensor multiplet

$$(t_{mn}, C^A_B, \chi^A_\alpha | F) \quad (17)$$

with 8+8 degrees of freedom. On-shell these fields describe the $N = 2$ scalar (hyper-) multiplet [13].

Like the vector multiplet, the tensor multiplet is also part of a chiral superfield. To derive an explicit relation, we observe that a solution to the constraints (13) is just given by the Yang-Mills constraints (3):

$$\begin{aligned} B^{BA}_{\beta\alpha} &= 4\varepsilon_{\beta\alpha} \varepsilon^{BA} \bar{\phi}, \quad B^{B\dot{\alpha}}_{\beta A} = 0, \\ B_{\dot{\beta}B\dot{\alpha}A} &= -4\varepsilon_{\dot{\beta}\dot{\alpha}} \varepsilon_{BA} \phi. \end{aligned} \quad (18)$$

The prepotential ϕ has the $U(1)$ weight $w(\phi) = 2$. From (11) we then obtain

$$\begin{aligned}
B_{\beta\alpha\dot{\alpha}}^B &= -2i \varepsilon_{\beta\alpha} \bar{\mathcal{D}}_{\dot{\alpha}}^B \bar{\Phi} , \\
B_{\beta B\alpha\dot{\alpha}} &= 2i \varepsilon_{\beta\dot{\alpha}} \mathcal{D}_{\alpha B} \Phi , \\
B_{\beta\dot{\beta}\alpha\dot{\alpha}} &= \varepsilon_{\beta\dot{\alpha}} b_{(\beta\alpha)} + \varepsilon_{\beta\alpha} b_{(\beta\dot{\alpha})} , \\
b_{\beta\alpha} &= -\frac{1}{2} (\mathcal{D}_{\beta}^A \mathcal{D}_{\alpha A} - 8 V_{\beta\alpha}) \Phi - 4 \bar{\Phi} W_{\beta\alpha} , \\
b_{\beta\dot{\alpha}} &= \frac{1}{2} (\bar{\mathcal{D}}_{\beta A} \bar{\mathcal{D}}_{\dot{\alpha}}^A + 8 \bar{V}_{\beta\dot{\alpha}}) \bar{\Phi} - 4 \Phi \bar{W}_{\beta\dot{\alpha}} ,
\end{aligned} \tag{19}$$

as well as the conditions

$$\begin{aligned}
\bar{\mathcal{D}}_A^{\dot{\alpha}} \Phi &= \mathcal{D}_{\alpha}^A \bar{\Phi} = 0 , \\
G^{AB} &= -\frac{1}{4} (\mathcal{D}^{\alpha A} \mathcal{D}_{\alpha}^B - 8 \bar{R}^{AB}) \Phi - \frac{1}{4} (\bar{\mathcal{D}}_{\dot{\alpha}}^A \bar{\mathcal{D}}^{\dot{\alpha} B} - 8 R^{AB}) \bar{\Phi} .
\end{aligned} \tag{20}$$

These conditions are invariant under the gauge transformation

$$\delta \Phi = \Lambda , \tag{21}$$

where Λ satisfies

$$\begin{aligned}
\bar{\mathcal{D}}_A^{\dot{\alpha}} \Lambda &= 0 , \\
(\mathcal{D}^{\alpha A} \mathcal{D}_{\alpha}^B - 8 \bar{R}^{AB}) \Lambda + (\bar{\mathcal{D}}_{\dot{\alpha}}^A \bar{\mathcal{D}}^{\dot{\alpha} B} - 8 R^{AB}) \bar{\Lambda} &= 0 .
\end{aligned} \tag{22}$$

Roughly speaking, the tensor multiplet is a chiral superfield minus a vector multiplet.

To conclude this section, we consider lagrangians for the tensor multiplet in flat superspace. We start with the coupling to an abelian vector multiplet, which is given by [14]

$$\mathcal{L} \sim \int d^4\theta \Phi W + h.c. , \tag{23}$$

where

$$\bar{D}^{\dot{\alpha}}_A W = 0, \quad D^{\alpha A} D_{\alpha}^B W = \bar{D}^A_{\dot{\alpha}} \bar{D}^{\dot{\alpha} B} \bar{W}. \quad (24)$$

The coupling to non-abelian gauge fields is not possible.

The lagrangian of a self-interacting tensor multiplet has the form (23), where W is now a function of G^A_B and its covariant derivatives subject to the conditions (24). The most general self-interaction in flat superspace is given by [8]

$$W = a(G^A_B) \bar{F} - a_{IA}{}^B \bar{\chi}^A_{\dot{\alpha}} \bar{\chi}^{\dot{\alpha}}_B, \quad (25)$$

where a is a real function of G^A_B satisfying the Laplace equation

$$a_{IA}{}^B{}_{IB}{}^A = 0. \quad (26)$$

(Here $\bar{\chi}^{\dot{\alpha}}_A = \frac{1}{3} \bar{D}^{\dot{\alpha}}_B G^B_A$, $\bar{F} = \frac{1}{4} \bar{D}^A_{\dot{\alpha}} \bar{\chi}^{\dot{\alpha}}_A$, and $a_{IA}{}^B = \partial a / \partial G^A_B$.) The two SU(2) covariant cases are the free tensor multiplet [10],

$$W = \bar{F}, \quad (27)$$

and the improved tensor multiplet [15, 7, 6, 14],

$$W = G^{-1} \bar{F} - G^{-3} G^{AB} \bar{\chi}^A_{\dot{\alpha}} \bar{\chi}^{\dot{\alpha}}_B, \quad (28)$$

where $G = (G^A_B G^B_A)^{1/2}$. In the case of the improved multiplet it is possible to add a mass term $W = m$ [14]. On-shell this theory describes a massive $N = 2$ scalar multiplet.

3. Chiral superfields, chiral densities, and chiral lagrangians

The construction of superfield lagrangians using a chiral basis of superspace has been successful in $N = 1$ supergravity [16, 17, 4]. In this section we generalize the results of ref. [4] to $N = 2$ supergravity, still keeping the geometry of superspace as general as possible. In the component formalism, similar results were obtained in ref. [18].

3.1. Chiral superfields

Chiral superfields in U(2) superspace are subject to the condition

$$\bar{\mathcal{D}}_A^{\dot{\alpha}} \phi = 0 . \quad (29)$$

Their component fields can be defined as

$$\phi^{(n) A_n \dots A_1}_{\alpha_n \dots \alpha_1} = [\mathcal{D}_{\alpha_n}^{A_n} \dots \mathcal{D}_{\alpha_1}^{A_1}] \phi | \quad (n=0, \dots, 4). \quad (30)$$

Here the brackets denote the completely antisymmetrized product (with normalization factor). The transformation laws of the component fields under supergravity transformations are

$$\delta \phi^{(n) A_n \dots A_1}_{\alpha_n \dots \alpha_1} = (\mathcal{J}_B^{\beta} \mathcal{D}_{\beta}^B + \bar{\mathcal{J}}_{\dot{\beta}}^{\dot{B}} \bar{\mathcal{D}}_{\dot{B}}^{\dot{\beta}}) [\mathcal{D}_{\alpha_n}^{A_n} \dots \mathcal{D}_{\alpha_1}^{A_1}] \phi | . \quad (31)$$

Like in the case N = 1 [4], these transformation laws can be written in the short form

$$\delta \phi = \eta^{\underline{M}} D_{\underline{M}} \phi + w(\phi) L \phi , \quad (32)$$

where

$$\phi = \sum_{n=0}^4 \frac{1}{n!} \Theta_{A_1}^{\alpha_1} \dots \Theta_{A_n}^{\alpha_n} \phi^{(n) A_n \dots A_1}_{\alpha_n \dots \alpha_1} . \quad (33)$$

The parameters $\eta^{\underline{M}}$ and L are, up to order Θ^2 , given by

$$\begin{aligned} \eta^m &= 2i \Theta_A \bar{\epsilon}^m \bar{\mathcal{J}}^A + (\Theta^2)_{AB} (\bar{\mathcal{J}}^A \bar{\epsilon}^n \bar{\epsilon}^m \bar{\psi}_n^B) - (\Theta^2)^{\alpha\beta} (\bar{\epsilon}^n \bar{\mathcal{J}}_A)_{\alpha} (\bar{\epsilon}^m \bar{\psi}_n^A)_{\beta} + \dots , \\ \eta_M^{\underline{M}} &= \mathcal{J}_M^{\underline{M}} + (\Theta^2)_{AB} \left(\frac{2}{3} \mathcal{J}_M^{\underline{M}} \bar{R}^{AB} - \frac{8}{3} \delta_M^A \mathcal{J}_C^{\underline{M}} \bar{R}^{CB} - \frac{4}{3} \delta_M^A \mathcal{J}_{\alpha}^B V^{\alpha\bar{M}} \right. \\ &\quad \left. + \frac{1}{2} \delta_M^A \bar{\mathcal{J}}_{\alpha}^B V^{\mu\bar{\alpha}} - \bar{\mathcal{J}}_{\alpha M} \tilde{V}^{\mu\bar{\alpha}AB} \right) + (\Theta^2)^{\alpha\beta} \left(-\frac{4}{3} \delta_{\alpha}^{\underline{M}} \mathcal{J}_{\beta}^A \bar{R}_{AM} \right. \\ &\quad \left. + \frac{2}{3} \mathcal{J}_M^{\underline{M}} V_{\alpha\beta} - \frac{8}{3} \delta_{\alpha}^{\underline{M}} \mathcal{J}_M^{\gamma} V_{\gamma\beta} + \frac{3}{2} \delta_{\alpha}^{\underline{M}} \bar{\mathcal{J}}_M^{\dot{\alpha}} V_{\beta\dot{\alpha}} \right) - \frac{1}{2} \eta^m \psi_m^{\underline{M}} + \dots , \end{aligned}$$

$$\begin{aligned}
 L = & -(\Theta_A \bar{\sigma}^a \bar{J}^B) \tilde{U}_a^A{}_B + (\Theta^2)_{AB} \left[-\frac{1}{4} (\bar{J}^{\dot{\alpha}A} \mathcal{D}^{\alpha C} + \bar{J}^{\dot{\alpha}C} \mathcal{D}^{\alpha A}) \tilde{U}_{\alpha\dot{\alpha}C}^B \right. \\
 & + \frac{i}{2} (\bar{J}^A \bar{\sigma}^m \sigma^a \bar{\psi}_m^C) \tilde{U}_{aC}^B \left. \right] + (\Theta^2)^{\alpha\beta} \left[-\frac{1}{3} \bar{J}_A \mathcal{D}_\alpha^B \tilde{U}_{\beta\dot{\alpha}B}^A \right. \\
 & \left. - \frac{i}{2} (\sigma^m \bar{J}_A)_\alpha (\sigma^a \bar{\psi}_m^B)_\beta \tilde{U}_{aB}^A \right] + \dots \quad (\Theta = \bar{\Theta} = 0) . \quad (34)
 \end{aligned}$$

The geometrical meaning of the new Θ -variables was clarified in ref. [19]. Some useful formulas for products of Θ 's are collected in the appendix.

In contrast to the case $N = 1$, a chiral $N = 2$ superfield with $U(1)$ weight $w = 2$ is reducible. There are two possible reductions. The first one is given by eq. (5) and leads to the Θ -expansion

$$\begin{aligned}
 W = & A + \Theta_A^\alpha \lambda_\alpha^A - \frac{1}{4} (\Theta^2)_{AB} (N^{AB} + 8 A \bar{R}^{AB} |) \\
 & - \frac{i}{2} (\Theta^2)^{\alpha\beta} (\sigma^{mn})_{\alpha\beta} (\mathcal{F}_{mn} + \psi_{mA} \sigma_n^A \bar{\lambda}^A - \bar{\psi}_m^A \sigma_n^A \lambda_A \\
 & - i \psi_{mA} \psi_n^A \bar{A} + i \bar{\psi}_m^A \bar{\psi}_{nA} A) + 2 (\Theta^2)^{\alpha\beta} (A V_{\alpha\beta} | + \bar{A} W_{\alpha\beta} |) + \dots , \quad (35)
 \end{aligned}$$

where

$$\mathcal{F}_{mn} = \partial_m a_n - \partial_n a_m - i [a_m, a_n] . \quad (36)$$

We shall call W a (non-abelian) vector superfield.

The second reduction is given by the gauge transformation (21, 22), which allows to set

$$\phi | = 0 , \quad \mathcal{D}_\alpha^A \phi | = 0 , \quad \mathcal{D}^{\alpha A} \mathcal{D}_\alpha^B \phi | = \bar{\mathcal{D}}_\alpha^A \bar{\mathcal{D}}^{\alpha B} \bar{\phi} | . \quad (37)$$

In this gauge the Θ -expansion of ϕ reads

$$\begin{aligned}
 \phi = & \frac{1}{2} (\Theta^2)_{AB} C^{AB} + \frac{1}{2} (\Theta^2)^{\alpha\beta} (\sigma^{mn})_{\alpha\beta} t_{mn} \\
 & + \frac{1}{3} (\Theta^3)_A^\alpha \left[4 \chi_\alpha^A + i (\sigma^m \bar{\psi}_m^B)_\alpha C^A{}_B - i (\sigma^k \bar{\sigma}^{mn} \bar{\psi}_k^A)_\alpha t_{mn} \right]
 \end{aligned}$$

$$\begin{aligned}
 & + \frac{1}{3} \Theta^4 \left[F + \frac{1}{3} \bar{R}^{AB} C_{AB} \right] - \frac{1}{4} (\bar{\psi}_m^A \bar{\psi}^{mB}) C_{AB} - \frac{1}{3} V^{\alpha\beta} (G^{mn})_{\alpha\beta} t_{mn} \Big| \\
 & - \bar{W}_{\alpha\beta} (\bar{G}^{mn})^{\alpha\beta} t_{mn} \Big| + \frac{1}{4} (\bar{\psi}_k^A \bar{G}^{mn} \bar{\psi}_A^k) t_{mn} \Big] . \quad (38)
 \end{aligned}$$

We shall call Φ a tensor superfield. The fact that its Θ -expansion starts only at order Θ^2 considerably simplifies the calculation of certain products of chiral superfields. Namely, if one of the factors is a tensor superfield, the Θ^3 - and Θ^4 -components of the remaining superfields do not enter at all.

3.2. Chiral densities

Chiral densities in $U(2)$ superspace are defined by the transformation law

$$\delta \Delta = (-)^m D_{\underline{M}} (\eta^{\underline{M}} \Delta) + w(\Delta) L \Delta , \quad (39)$$

where

$$\Delta = \sum_{n=0}^4 \frac{1}{n!} \Theta_{A_1}^{\alpha_1} \dots \Theta_{A_n}^{\alpha_n} \Delta^{(n)}_{\alpha_n \dots \alpha_1} A_n \dots A_1 . \quad (40)$$

The components of Δ have the $U(1)$ weight $w(\Delta^{(n)}) = w(\Delta) - n + 4$.

A special chiral density $\mathcal{E}$ with $U(1)$ weight $w(\mathcal{E}) = -4$ can be constructed from the lowest component

$$\mathcal{E}^{(0)} = e = \det e_m^a . \quad (41)$$

Up to order Θ^2 , we find

$$\begin{aligned}
 \frac{1}{e} \mathcal{E} = & 1 + i \Theta_A G^m \bar{\psi}_m^A + (\Theta^2)_{AB} \left(\frac{16}{3} \bar{R}^{AB} \right) - \bar{\psi}_m^A \bar{G}^{mn} \bar{\psi}_n^B \\
 & + (\Theta^2)^{\alpha\beta} \left(-\frac{8}{3} V_{\alpha\beta} \right) - \frac{1}{2} (G^{mn})_{\alpha\beta} \bar{\psi}_m^A \bar{\psi}_{nA}^{\beta} + \dots . \quad (42)
 \end{aligned}$$

3.3. Chiral lagrangians

Given the chiral density $\mathcal{E}$, a vector superfield W , and a tensor superfield Φ , one can construct three chiral densities with vanishing $U(1)$ weight: $\mathcal{E}WW$, $\mathcal{E}\Phi W$, and $\mathcal{E}\Phi\Phi$. The integrals of these densities over x and Θ are chiral actions, provided that they are invariant under the gauge transformation of Φ . This condition is satisfied for the first and second, but not for the third integral.

Thus there are two kinds of chiral lagrangians. The first one is the Yang-Mills lagrangian

$$\mathcal{L}_{YM} \sim \int d^4\Theta \mathcal{E} \text{tr}(WW) + h.c. . \quad (43)$$

(The integral is defined such that $\int d^4\Theta \Theta^4 = 1$.) This generalizes the lagrangian (9) to curved superspace.* A Yang-Mills coupling via the Chern-Simons form like in $N = 1$ supergravity [20, 4] is not possible in the case $N = 2$ [12].

The second set of chiral lagrangians has the form

$$\mathcal{L} = \int d^4\Theta \mathcal{E} \Phi W + h.c. , \quad (44)$$

where W is an abelian vector superfield. In flat superspace this reduces to (23). Inserting (42), (38), and (35) into (44), we obtain explicitly

$$\begin{aligned} \mathcal{L} = & \frac{1}{3} e \left[AF + \bar{A}\bar{F} - \frac{1}{4} C_{AB} N^{AB} + 2 C_{AB} (A\bar{R}^{AB} + \bar{A}R^{AB}) \right] \\ & - \frac{1}{2} \varepsilon^{\kappa\ell mn} t_{\kappa\ell} \partial_m a_n + \chi^A \lambda_A + \bar{\chi}_A \bar{\lambda}^A + i(\psi_m^A \bar{\sigma}^m \bar{\chi}^A) \bar{A} \\ & + i(\bar{\psi}_m^A \bar{\sigma}^m \chi_A) A + \frac{i}{2} (\psi_m^A \bar{\sigma}^m \bar{\lambda}^B + \bar{\psi}_m^A \bar{\sigma}^m \lambda^B) C_{AB} \\ & - (\psi_m^A \bar{\sigma}^{mn} \psi_n^B) \bar{A} C_{AB} - (\bar{\psi}_m^A \bar{\sigma}^{mn} \bar{\psi}_n^B) A C_{AB}] . \end{aligned} \quad (45)$$

The superconformal version of this lagrangian was first given in ref. [21]. The gauge transformation $\delta t_{mn} = \partial_m \omega_n - \partial_n \omega_m$ changes $\mathcal{L}$ only

* More general couplings of vector multiplets to $N = 2$ supergravity were given in refs. [22, 23].

by a divergence since $\partial_k (e \varepsilon^{k\ell mn}) = 0$. It is remarkable that this gauge invariance holds not only in flat superspace but also in a general supergravity background.

4. The supergravity lagrangian

Using the results of the preceding section, it is now straightforward to construct a lagrangian for the minimal supergravity multiplet derived in ref. [1]. We simply take the lagrangian (44, 45) and identify t_{mn} and a_m with the auxiliary fields t'_{mn} and v_m of the supergravity multiplet. As a consequence, we have to replace $\phi \rightarrow \phi'$, $G^A_B \rightarrow \varepsilon^A_B$, $W \rightarrow -R$, etc.. Thus the supergravity lagrangian reads

$$\mathcal{L}_{SG} = 6 \int d^4\theta \mathcal{E} \phi' R + h.c. . \quad (46)$$

Its component form can be calculated from (45) using

$$\begin{aligned} \frac{1}{2} \mathcal{D}^{\alpha A} \mathcal{D}_{\alpha}^A R &= -\frac{1}{2} R_{ab}{}^{ab} + \frac{1}{8} e^{-U} (f^{\alpha\beta} \bar{f}_{\alpha\beta} + f_{\alpha\beta} \bar{f}^{\alpha\beta}) \\ &- \frac{i}{4} (T^{\alpha A} \mathcal{D}_{\alpha\dot{\alpha}} \bar{T}_A^{\dot{\alpha}} + \bar{T}_A^{\dot{\alpha}} \mathcal{D}_{\alpha\dot{\alpha}} T^{\alpha A}) - \frac{1}{4} (\mathcal{D}^a U) \mathcal{D}_a U + \frac{1}{4} e^{-2U} \tilde{G}^a \tilde{G}_a \\ &+ 16 R \bar{R} - \tilde{U}_a{}^{AB} \tilde{U}_{AB}{}^a - \frac{1}{2} \mathcal{D}^a \mathcal{D}_a U - \frac{1}{2} (T^{\alpha A} t_{\alpha A} + \bar{T}_{\dot{\alpha} A} \bar{t}^{\dot{\alpha} A}) \\ &- \frac{1}{2} e^{-U} T^{\alpha A} \bar{T}_A^{\dot{\alpha}} \tilde{G}_{\alpha\dot{\alpha}} - \frac{1}{2} T^{\alpha A} \bar{T}_B^{\dot{\alpha}} \tilde{U}_{\alpha\dot{\alpha} A}{}^B - \frac{3}{8} T^A T^B \bar{T}_A \bar{T}_B \end{aligned} \quad (47)$$

and

$$\begin{aligned} & -\frac{1}{2} R_{ab}{}^{ab} | - i (\bar{\psi}_m^A \bar{\sigma}^m)^\alpha t_{\alpha A} | - i (\bar{\sigma}^m \psi_{mA})^{\dot{\alpha}} \bar{t}_{\dot{\alpha}}^A | \\ &= -\frac{1}{2} \mathcal{R} + \frac{1}{4} \varepsilon^{k\ell mn} (\bar{\psi}_k^A \bar{\sigma}_\ell \psi_{mnA} - \psi_{kA} \sigma_\ell \bar{\psi}_{mn}^A) \\ &- \frac{1}{8} e^{-C/2} (\bar{\psi}_m^A \bar{\psi}_{nA}) (\sigma^{mn})^{\alpha\beta} f_{\alpha\beta} | + \frac{1}{8} e^{-C/2} (\psi_{mA} \psi_n^A) (\bar{\sigma}^{mn})_{\dot{\alpha}\dot{\beta}} \bar{f}^{\dot{\alpha}\dot{\beta}} | \\ &+ \frac{1}{2} (\bar{\psi}_m^A \bar{\sigma}^{mn} \bar{\psi}_n^A) M + \frac{1}{2} (\psi_m^A \sigma^{mn} \psi_n^A) \bar{M} \\ &- \frac{i}{2} \varepsilon^{k\ell mn} (\psi_{kA} \sigma_\ell \bar{\psi}_m^B) \tilde{U}_{nB}^A | , \end{aligned}$$

$$\mathcal{R} = e_a{}^m e_b{}^n R_{mn}{}^{ab} \Big| . \quad (48)$$

After a partial integration we obtain

$$\begin{aligned} \frac{1}{e} \mathcal{L}_{SG} = & -\frac{1}{2} \mathcal{R} + \frac{1}{4} \varepsilon^{klmn} (\bar{\psi}_k^A \bar{\sigma}_l \psi'_{mnA} - \psi_{kA} \bar{\sigma}_l \bar{\psi}'_{mn}{}^A) \\ & + \frac{1}{4} e^{-C} F_{ab} \bar{F}^{ab} \Big| - \frac{i}{4} (\bar{\varphi}_A \bar{\sigma}^m D'_m \varphi^A + \varphi^A \bar{\sigma}^m D'_m \bar{\varphi}_A) \\ & - \frac{1}{4} (\mathcal{D}_a U) \mathcal{D}^a U \Big| + \frac{1}{4} e^{-2C} \tilde{G}^a \tilde{G}_a \Big| \\ & + \varepsilon^{klmn} \hat{V}_k \partial_l t'_{mn} - \frac{1}{9} (2M\bar{M} + \hat{b}_a{}^{AB} \hat{b}_{AB}{}^a) \\ & + \frac{1}{8} e^{-C/2} \varepsilon^{abcd} (\bar{\psi}_a^A \bar{\sigma}_b \varphi_A + i \bar{\psi}_a^A \bar{\psi}_{bA}) F_{cd} \Big| \\ & - \frac{1}{8} e^{-C/2} \varepsilon^{abcd} (\psi_{aA} \bar{\sigma}_b \bar{\varphi}^A + i \psi_{aA} \psi_b^A) \bar{F}_{cd} \Big| \\ & + \frac{1}{2} (\psi_{aA} \bar{\sigma}^{ab} \varphi^A + \bar{\psi}_a^A \bar{\sigma}^{ab} \bar{\varphi}_A) \mathcal{D}_b U \Big| \\ & - \frac{3}{8} e^{-C} (\varphi^A \bar{\sigma}_a \bar{\varphi}_A) \tilde{G}^a \Big| - \frac{i}{4} e^{-C} (\psi_{aA} \varphi^A - \bar{\psi}_a^A \bar{\varphi}_A) \tilde{G}^a \Big| \\ & - \frac{3}{16} \varphi^A \varphi^B \bar{\varphi}_A \bar{\varphi}_B + \frac{3}{16} i (\psi_{mA} \varphi^A - \bar{\psi}_m^A \bar{\varphi}_A) (\varphi^B \bar{\sigma}^m \bar{\varphi}_B) \\ & + \frac{1}{4} (\psi_{mA} \bar{\sigma}^{mn} \varphi^A) \psi_{nB} \varphi^B + \frac{1}{4} (\bar{\psi}_m^A \bar{\sigma}^{mn} \bar{\varphi}_A) \bar{\psi}_n^B \bar{\varphi}_B \\ & - \frac{1}{8} \varepsilon^{klmn} [\psi_{kA} \psi_l^A (\bar{\psi}_m^B \bar{\sigma}_n \varphi_B) - \bar{\psi}_k^A \bar{\psi}_{lA} (\psi_{mB} \bar{\sigma}_n \bar{\varphi}^B)] . \quad (49) \end{aligned}$$

Here we used the definitions

$$\begin{aligned}
 \hat{V}_m &= V_m - \frac{1}{2} e_m^a \tilde{U}_a - \frac{1}{16} (\varphi^A \sigma_m \bar{\varphi}^A), \\
 \hat{b}_{\alpha\dot{\alpha}B}^A &= b_{\alpha\dot{\alpha}B}^A + \frac{3}{2} (\varphi_\alpha^A \bar{\varphi}_{\dot{\alpha}B} - \frac{1}{2} \delta_B^A \varphi_\alpha^C \bar{\varphi}_{\dot{\alpha}C} + \frac{1}{2} \epsilon_{AB} \varphi_\alpha^C \bar{\varphi}_{\dot{\alpha}C}), \\
 \psi'_{mn}{}^\alpha &= D'_m \psi_n{}^\alpha - D'_n \psi_m{}^\alpha, \\
 D'_m \varphi_\alpha^A &= \partial_m \varphi_\alpha^A - \omega_{m\alpha}{}^\beta \varphi_\beta^A - i \hat{V}_m \varphi_{\alpha A}. \quad (50)
 \end{aligned}$$

The various $\theta = \bar{\theta} = 0$ components are given by

$$\begin{aligned}
 F_{ab}| &= e_a^m e_b^n \left[\partial_m a_n + i \partial_m a'_n + \frac{1}{2} e^{C12} (i \psi_{mA} \sigma_n \bar{\varphi}^A + \psi_{mA} \psi_n^A) - (m \leftrightarrow n) \right], \\
 \bar{F}_{ab}| &= e_a^m e_b^n \left[\partial_m a_n - i \partial_m a'_n + \frac{1}{2} e^{C12} (i \bar{\psi}_m^A \sigma_n \varphi_A + \bar{\psi}_m^A \bar{\psi}_{nA}) - (m \leftrightarrow n) \right], \\
 \partial_a U| &= e_a^m (\partial_m C - \frac{1}{2} \psi_{mA} \varphi^A - \frac{1}{2} \bar{\psi}_m^A \bar{\varphi}_A), \\
 \tilde{G}^a| &= \frac{1}{2} \epsilon^{a\ell mn} \left[\partial_\ell t_{mn} + 2 (a_\ell \partial_m a_n + a'_\ell \partial_m a'_n) - e^C (\psi_{\ell A} \sigma_{mn} \varphi^A + \bar{\psi}_\ell^A \bar{\sigma}_{mn} \bar{\varphi}_A) + i e^C \psi_{\ell A} \sigma_m \bar{\psi}_n^A \right], \\
 \tilde{U}^a| &= \frac{1}{4} \epsilon^{a\ell mn} (\partial_\ell t'_{mn} + i \psi_\ell^A \sigma_m \bar{\psi}_n^A). \quad (51)
 \end{aligned}$$

The correct normalization of the kinetic terms of $\mathcal{L}_{SG}$ can be achieved by replacing $\varphi_{\alpha} \rightarrow \sqrt{2} \varphi_{\alpha}$, $C \rightarrow \sqrt{2} C$, and $t_{mn} \rightarrow \sqrt{2} t_{mn}$. (We shall not do this here in order to avoid the factors $\sqrt{2}$.) The kinetic terms of the vector fields, however, have the wrong sign. This means that on-shell a duality transformation on the vector fields has to be performed. We remark that a similar situation occurs in the corresponding $SU(4)$ supergravity theory [24, 2], which is related to the $SO(4)$ theory [25] by duality transformations.

The variation of $\mathcal{L}_{SG}$ with respect to M , $\hat{b}_{aB}^A$, and $\hat{v}_m$ gives $\bar{M} = 0$, $\hat{b}_{aB}^A = 0$, and $\tilde{U}_a + \frac{1}{8} (\varphi^A \epsilon_a \bar{\varphi}^A) = 0$. Thus the field equations are

$$R = \bar{R} = 0 ,$$

$$\tilde{U}_{\alpha\dot{\alpha}B} + \frac{1}{2} T_{\alpha}^A \bar{T}_{\dot{\alpha}B} - \frac{1}{4} \delta_B^A T_{\alpha}^C \bar{T}_{\dot{\alpha}C} = 0 . \quad (52)$$

These superfield equations contain all the other equations of motion, too. They correspond with eq. (62) of ref. [1].

In our previous paper [1] it was shown that the transformation laws of the minimal $N = 2$ multiplet describe on-shell the truncation of a $N = 4$ supergravity theory [2]. We have not compared the lagrangians because our conventions are completely different from those of [2]. We have checked, however, that the $N = 1$ limit of (49) is the lagrangian for the 16+16 supergravity multiplet with $n = \frac{1}{4}$ [4].

In appendix A of ref. [1] it was shown that there exists a whole class of minimal supergravity multiplets parameterized by a real number m . We shall now try to construct a supergravity lagrangian for $m \neq 0$. ($m = 0$ is the case discussed before.) Since the lagrangian should contain the term $\mathcal{L}_{SG} \sim e \epsilon^{k\ell mn} v_k \partial_{\ell} t'_{mn}$, it must again have the form (46). The component expression, however, is different from (49). In particular, the curvature scalar picks up the factor e^{mC} . This factor can be absorbed by a rescaling of the vielbein, but the rescaling also changes the parameter m into $m' = 0$. We conclude that there exists no supergravity lagrangian with canonical kinetic terms for $m \neq 0$. This differs from $N = 1$ supergravity, where the lagrangian for the 16+16 multiplet contains a real parameter n [3, 4].

At last we consider the following extension of the supergravity lagrangian (46):

$$\mathcal{L} = 6 \int d^4 \Theta \mathcal{E} \phi' (R + g W_0) + h.c. = \mathcal{L}_{SG} + \mathcal{L}_g . \quad (53)$$

g is a complex parameter with (mass) dimension 1 and W_0 is the vector superfield appearing in the constraints (17) of ref. [1]. The component form of $\mathcal{L}_g$ can be calculated from (45) if one replaces $W \rightarrow g W_0$ and $a_m \rightarrow g(a'_m + i a_m) + \bar{g}(a'_m - i a_m)$. The result is

$$\begin{aligned} \mathcal{L}_g = & g e \left[\frac{4}{3} e^{C/2} \bar{M} - \varepsilon^{\kappa\ell mn} (a'_\kappa + i a_\kappa) \partial_\ell t'_{mn} \right. \\ & \left. - i e^{C/2} (\bar{\psi}_m^A \bar{\epsilon}^m \varphi^A) + 2 e^{C/2} (\bar{\psi}_m^A \bar{\epsilon}^{mn} \bar{\psi}_n^A) \right] + h.c. . \end{aligned} \quad (54)$$

The variation of the entire lagrangian $\mathcal{L}$ with respect to t'_{mn} yields

$$\partial_{[m} \hat{v}_{n]} = (g + \bar{g}) \partial_{[m} a'_{n]} + i(g - \bar{g}) \partial_{[m} a_{n]} . \quad (55)$$

This equation relates the gauge field $\hat{v}_m$ of the local $SO(2)$ symmetry to the physical vector fields a_m and a'_m . Hence we obtain a $N = 2$ supergravity theory with gauged $SO(2) \times SO(2)$. The coupling constants are the real and the imaginary part of g . Elimination of the auxiliary field M produces the potential term

$$\mathcal{L}_{pot} = 8 |g|^2 e \cdot e^C . \quad (56)$$

Like in the corresponding $N = 4$ theory with gauged $SU(2) \times SU(2)$ [5], the scalar potential is unbounded from below.

5. Matter coupling

The $N = 2$ scalar (hyper-) multiplet consists of four real scalars and two Weyl spinors [13]. In conventional superspace three off-shell representations of these fields have been found: the tensor multiplet with 8+8 degrees of freedom [10], the relaxed hypermultiplet (56+56) [26],

and the first-order multiplet (136+136) [27]. The common problem of all these off-shell versions is that their self-interactions and couplings to other multiplets are restricted.

In the following we discuss the coupling of the tensor multiplet to supergravity. The general form of the lagrangian is

$$\mathcal{L}_\phi \sim \int d^4\Theta \mathcal{E} \phi W + h.c. , \quad (57)$$

where W is a vector superfield depending on

$$G^A_B, \bar{\chi}^{\dot{\alpha}}_A = \frac{1}{3} \bar{\mathcal{D}}^{\dot{\alpha}}_B G^B_A, \quad \bar{F} = \frac{1}{4} \bar{\mathcal{D}}^{\dot{\alpha}}_A \bar{\chi}^{\dot{\alpha}}_A. \quad (58)$$

We shall call W a kinetic superfield (multiplet). The covariant derivatives of the superfields (58) are

$$\begin{aligned} \mathcal{D}^A_\alpha G_{BC} &= \sum_{BC} \delta^A_B \chi_{\alpha C}, \quad \bar{\mathcal{D}}^{\dot{\alpha}}_A G^{BC} = \sum_{BC} \delta^B_A \bar{\chi}^{\dot{\alpha} C}, \\ \mathcal{D}^A_\alpha \bar{\chi}^{\dot{\alpha}}_B &= -\delta^A_B (\tilde{G}_{\alpha\dot{\alpha}} + G^C_D \tilde{U}_{\alpha\dot{\alpha}C}) - (i\mathcal{D}_{\alpha\dot{\alpha}} + 2U_{\alpha\dot{\alpha}}) G^A_B, \\ \bar{\mathcal{D}}^{\dot{\alpha}}_A \bar{\chi}^{\dot{\beta}}_B &= \varepsilon^{\dot{\alpha}\dot{\beta}} \varepsilon_{AB} \bar{F} + 4\bar{V}^{\dot{\alpha}\dot{\beta}} G_{AB} + 2\varepsilon^{\dot{\alpha}\dot{\beta}} \sum_{AB} R_{AC} G^C_B, \\ \mathcal{D}^A_\alpha \bar{F} &= 2i\mathcal{D}_{\alpha\dot{\alpha}} \bar{\chi}^{\dot{\alpha} A} - 2G_{BC} \mathcal{D}^C_\alpha R^{BA} + 2G^{AB} \mathcal{D}^C_\alpha R_{CB} \\ &\quad + 2G^{AB} \bar{\mathcal{D}}^{\dot{\alpha}}_B U_{\alpha\dot{\alpha}} + 4W_{\alpha\beta} \chi^{\beta A} - 4R^{AB} \chi_{\alpha B} + 3U_{\alpha\dot{\alpha}} \bar{\chi}^{\dot{\alpha} A}, \\ \bar{\mathcal{D}}^{\dot{\alpha}}_A \bar{F} &= -\frac{4}{3} \bar{\mathcal{D}}^{\dot{\beta}}_B (\bar{V}^{\dot{\alpha}\dot{\beta}} G_{BA}). \end{aligned} \quad (59)$$

Using these equations it is easy to see that the self-interactions (25) can in general not be coupled to $N = 2$ supergravity (not even to minimal supergravity). Therefore we consider here only the two $SU(2)$ covariant cases, the free and the improved tensor multiplet.

The free tensor multiplet can be coupled to the minimal supergravity multiplet found in ref. [1]. The kinetic superfield is

$$W = \bar{F} \quad (60)$$

like in flat superspace (27). A generalization of (60) to $U(2)$ superspace with only natural constraints is not possible.

In contrast to the free multiplet, the improved tensor multiplet can be coupled to a general supergravity background [6]. The kinetic superfield is

$$W = G^{-1}(\bar{F} + 2 G^{AB} R_{AB}) - G^{-3} G^{AB} \bar{\chi}_{\dot{\alpha}A} \bar{\chi}_{\dot{\alpha}B}^{\dot{\alpha}}, \quad (61)$$

where $G = (G^A_B G^B_A)^{1/2}$. In flat superspace this reduces to (28). It is a tedious calculation to show that the expression (61) really satisfies the conditions (5) of a vector superfield. The mass term for the improved multiplet in curved superspace is given by

$$W = m W_0, \quad (62)$$

where W_0 is the vector superfield appearing in the constraints (17) of ref. [1].

More general supergravity-matter couplings were constructed in ref. [23]. However, the scalar multiplets have an off-shell central charge, which would require an additional bosonic coordinate in superspace [28]. The most general on-shell couplings were given in ref. [29].

6. Conclusion

In this and the preceding paper [1] we have presented a minimal $N = 2$ supergravity theory (32+32) with a local $SO(2)$ invariance. Its most interesting property is that it describes the $N = 2$ part of a $N = 4$ supergravity theory [2]. We have shown that the $SO(2)$ can be gauged with two independent coupling constants. This is a truncation of $N = 4$ supergravity with gauged $SU(2) \times SU(2)$ [5].

The $N = 1$ limit of the minimal $N = 2$ theory is the 16+16 multiplet with $n = \frac{1}{4}$ [4], resp. $n = -\frac{1}{2}$ [3]. In appendix A of ref. [1] we have shown that the $N = 2$ analogues of the $N = 1$ multiplets exist for all other values of n , too. It turns out, however, that a $N = 2$ supergravity lagrangian can be constructed only for one value of the parameter.

The matter couplings are also subject to much stronger restrictions than in the case $N = 1$. This is due to the fact that a minimal off-shell

representation of the $N = 2$ scalar multiplet requires either an anti-symmetric tensor or a central charge. In the preceding section we have given the coupling of the free and the improved tensor multiplet to $N = 2$ supergravity.

Finally we remark that the no-go theorem for partial breaking of $N = 2$ supergravity [30], which is based on the superconformal tensor calculus, does not necessarily apply to the minimal 32+32 multiplet. It is an interesting question if this theory coupled to matter could lead to exceptional $N = 2$ supergravities like the one given in ref. [31].

Appendix

We give some useful formulas for products of the new Θ -variables introduced in sect. 3.1. First we define

$$\begin{aligned} (\Theta^2)_{AB} &= \Theta_A^\alpha \Theta_{\alpha B}, \quad (\Theta^2)^{\alpha\beta} = \Theta_A^\alpha \Theta^{\beta A}, \\ (\Theta^3)_A^\alpha &= (\Theta^2)_{AB} \Theta^{\alpha B} = -(\Theta^2)^{\alpha\beta} \Theta_{\beta A}, \\ \Theta^4 &= (\Theta^3)_A^\alpha \Theta_\alpha^A. \end{aligned} \quad (63)$$

The various products of Θ 's are then given by

$$\begin{aligned} \Theta_A^\alpha \Theta_B^\beta &= -\frac{1}{2} \varepsilon^{\alpha\beta} (\Theta^2)_{AB} - \frac{1}{2} \varepsilon_{AB} (\Theta^2)^{\alpha\beta}, \\ \Theta_A^\alpha (\Theta^2)_{BC} &= \frac{1}{3} \sum_{BC} \varepsilon_{AB} (\Theta^3)_C^\alpha, \\ \Theta_A^\alpha (\Theta^2)^{\beta\gamma} &= -\frac{1}{3} \sum_{\beta\gamma} \varepsilon^{\alpha\beta} (\Theta^3)_A^\gamma, \\ \Theta_A^\alpha (\Theta^3)_B^\beta &= -\frac{1}{4} \varepsilon^{\alpha\beta} \varepsilon_{AB} \Theta^4, \\ (\Theta^2)_{AB} (\Theta^2)_{CD} &= \frac{1}{6} (\varepsilon_{AC} \varepsilon_{BD} + \varepsilon_{AD} \varepsilon_{BC}) \Theta^4, \\ (\Theta^2)^{\alpha\beta} (\Theta^2)^{\gamma\delta} &= -\frac{1}{6} (\varepsilon^{\alpha\gamma} \varepsilon^{\beta\delta} + \varepsilon^{\alpha\delta} \varepsilon^{\beta\gamma}) \Theta^4, \\ (\Theta^2)_{AB} (\Theta^2)^{\alpha\beta} &= 0. \end{aligned} \quad (64)$$

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