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$N = 1$ SUPERSYMMETRIC TRUNCATIONS
AND THE SUPERSTRING LOW-ENERGY EFFECTIVE THEORY

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ABSTRACT

The question of discarding heavy fields in $N = 1$ supersymmetric theories is analyzed. The analysis is applied to the compactification of the $E_8 \times E_8$ superstring.

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In an interacting field theory we are sometimes interested in the couplings of a subset of the fields. The best known example is a theory with heavy (H) and light (L) fields in which we want to find the effective action for the light fields, $S_{\text{eff}}(L)$. In a first approximation, integrating the heavy fields out to find $S_{\text{eff}}(L)$ amounts to solving the field equation for H as a function of L and substituting back into the original action. However, in general, it is very difficult to follow this procedure as in the case of higher dimensional theories which, upon compactification, yield an infinite tower of massive modes. Instead of integrating the heavy fields out what is usually done is to perform a truncation by setting them to zero in the action.

This truncation is not always consistent since $H(L)=0$ need not be a solution of the H field equation. However, if the truncated Lagrangian agrees with the effective one up to terms suppressed by inverse powers of the heavy mass, the inconsistency is not relevant for the low-energy world.

In this note we investigate these features for a theory that has $N=1$ supersymmetry. After reviewing some aspects of the consistency of the truncation procedure for an arbitrary theory, and in particular for Kaluza-Klein type theories¹, we consider the $N=1$ supersymmetric case where we conclude that the correct low energy couplings of the light fields are obtained by setting $H=0$ in the superpotential rather than in the lagrangian itself. Similar results are also valid for $N=1$ supergravity.

Finally, we apply our conclusions to the compactification of the $E_8 \times E_8$ superstring^{2,3}.

In a first approximation, the criterion for a consistent truncation involving the heavy fields is expressed by the condition that they cannot appear linearly in the action, otherwise the field equation:

$$\partial S / \partial H = 0 \quad (1)$$

will not have $H(L)=0$ as a solution and there will be effective interactions among the L fields obtained by substituting $H(L)$ into the original action. These effective interactions can be seen to come from tree diagrams with H internal lines.

In the Kaluza-Klein context, the condition has a similar statement as we now explain. Every higher-dimensional field can be expanded in terms of harmonics $Y_{nl}(y)$ of the internal manifold:

$$\Phi(x,y) = \sum_{n,l} \phi_{nl}(x) Y_{nl}(y) \quad (2)$$

where the index n labels the mass eigenvalues and l is a degeneracy index. The heavy fields ($\phi_{nl}, n>0$) will not appear linearly in the action if the integrals:

$$\int dy \sqrt{g(y)} Y_{nl} \prod_l (Y_{0l})^{p_l}$$

vanish for all values of n, l and powers p_l . These integrals would be

the coefficients of HL^p -like terms. Since the harmonics $Y_{nl}(y)$ form a complete set, the integrals vanish if and only if the zero modes Y_{0l} close under multiplication, which in turn is true if and only if the zero modes are constant or covariantly constant.

The inconsistency of setting the heavy Kaluza-Klein modes to zero is not taken as a serious problem because usually the truncation is a good approximation at energies below the compactification scale⁴. However, this is not true in general if there are dimension $d < 4$ couplings between heavy and light fields (like HL^2). In this case the couplings may be very strong ($\sim M_H$) and the heavy fields will induce effective interactions which are not suppressed by inverse powers of M_H ; the truncation then cannot be a good low-energy approximation.

Let us investigate these issues if we have the further information that the four-dimensional theory has an $N=1$ supersymmetry⁵. For a renormalizable globally supersymmetric theory, the superpotential W is a cubic function of the chiral superfields. If we consider heavy ($\bar{H}_i$) and massless ($\bar{L}_\alpha$) superfields which we will assume have already been shifted by their (supersymmetric) VEV's, the most general superpotential is:

$$\begin{aligned}
 W = & \frac{1}{2} M_{ij} \bar{H}_i \bar{H}_j + g_{i\alpha\beta} \bar{H}_i \bar{L}_\alpha \bar{L}_\beta + g_{ij\alpha} \bar{H}_i \bar{H}_j \bar{L}_\alpha + g_{ijk} \bar{H}_i \bar{H}_j \bar{H}_k \\
 & + g_{\alpha\beta\gamma} \bar{L}_\alpha \bar{L}_\beta \bar{L}_\gamma
 \end{aligned} \tag{3}$$

Since in the superfield formulation the action is linear in W , it is inconsistent to set the heavy superfields to zero if $\partial W / \partial \bar{H}_i |_{\bar{H}=0} - g_{i\alpha\beta} \bar{L}_\alpha \bar{L}_\beta \neq 0$. Notice that such a truncation is also inconsistent with supersymmetry as implied by the transformation law of the fermionic component of $\bar{H}_i$:

$$\delta_\xi \psi_i = -\sqrt{2} \xi (\partial W / \partial H_i)^* + \dots \quad (4)$$

which does not vanish when $H_i=0$ where H_i is the scalar component of $\bar{H}_i$.

The scalar potential, without the D^2 -terms, is:

$$V = |\partial W / \partial L_\alpha|^2 + |\partial W / \partial H_i|^2 \quad (5)$$

Solving the field equation for H_i we find:

$$H_i = -M^{-1}_{ij} g_{j\alpha\beta} L_\alpha L_\beta + \mathcal{O}(1/M^2) \quad (6)$$

Therefore, integrating out the heavy fields at tree level we find:

$$V_{\text{eff}}(L_\alpha) = |\partial W / \partial L_\alpha|^2 |_{H=0} + \mathcal{O}(1/M) \quad (7)$$

From equation (5) we also obtain the truncated potential:

$$V_{\text{trunc}}(L_\alpha, H_i=0) = |g_{i\alpha\beta} L_\alpha L_\beta|^2 + |\partial W / \partial L_\alpha|^2 |_{H=0} \quad (8)$$

Clearly, V_{trunc} does not agree with V_{eff} up to $\mathcal{O}(1/M)$ terms. Note

however that if we truncate the superpotential, the resulting potential will correctly give V_{eff} up to $\mathcal{O}(1/M)$. This follows because W , being a dimension three operator, cannot have L-H couplings with dimensionful coefficients, other than those that would give a mass to L_α . Therefore W_{trunc} differs from W_{eff} up to $\mathcal{O}(1/M)$ and the full lagrangian will have the same behaviour. We then conclude that even though the truncation is inconsistent for $\partial W / \partial H_i|_{H=0} \neq 0$, the correct low-energy theory can always be obtained by truncating the superpotential.

In N=1 supersymmetry coupled to Yang Mills, the low-energy theory is found by setting the $\bar{H}_i$ to zero in the superpotential and dropping the massive vector superfields that might result from spontaneous gauge symmetry breaking⁶.

In N=1 supergravity coupled to matter⁷, the truncation is consistent if the Kähler potential K , the superpotential W and the chiral function f_{AB} do not have terms linear in the discarded fields. As in global supersymmetry, this follows either from the superspace expression for the action or from the transformation law of the fermionic components of the discarded superfields. In order to get the effective theory for the massless fields we can set the heavy fields to zero in the functions mentioned above. These truncated functions will agree with the effective ones, obtained by integrating the heavy fields, up to $\mathcal{O}(1/M_H)$ terms. For instance:

$$K_{\text{trunc}}(L) = K(L, H=0) \quad (9a)$$

$$K_{\text{eff}}(L) = K(L, H=0) + \mathcal{O}(1/M_H) \quad (9b)$$

However, due to the non-renormalizability of the theory, $K(L, H=0)$ includes terms that are suppressed by inverse powers of the Planck mass. Therefore, the $\mathcal{O}(1/M_H)$ terms cannot be neglected (since by assumption $M_H < M_P$). The conclusion is that only the renormalizable interactions of the massless fields can be reliably deduced by a truncation. The situation is improved when some underlying symmetries can be used to restrict the form of the effective theory for the massless fields.

The ideas discussed so far can be applied to the compactification of the $E_8 \times E_8$ superstring² on Calabi-Yau spaces³. In a Calabi-Yau manifold, zero modes are determined by the (1,1) and (1,2) harmonic forms, the number of which is given by b_{11} and b_{12} respectively. In general, $b_{11} \geq 1$ and $b_{12} \geq 0$, but only the (1,1) Kähler form is covariantly constant. Therefore, it is consistent to discard all the heavy modes, while keeping all the zero modes, only for a manifold with $b_{11}=1$ and $b_{12}=0$. In such a manifold, setting the heavy fields to zero in the D-10 action⁸ and integrating over the compact dimensions corresponds, at tree level, to the dimensional reduction performed by Witten⁹. In this case, the functions K and W can be obtained in two different ways. The D-4

potential V can be derived from the non-space-time-derivative bosonic terms in the ten-dimensional Lagrangian, K and W can then be read off from the potential⁹. Alternatively^{10,11}, K and W can be found directly from the fermionic terms that give rise to the four-dimensional gravitino mass term $e^{K/2} W \bar{\psi}_\mu \gamma^{\mu\nu} \psi_\nu$. Either procedure gives the same expression for K and W , as expected for a consistent truncation. This model leads to only one generation of quarks and leptons.

Compactification on more general spaces leads to models with a larger number of generations, thus it is of considerable interest to find the low-energy effective action for the massless fields in this case¹¹. However, as we explained above, setting the heavy modes to zero in the Lagrangian is not permissible. In fact, it can be checked explicitly that the potential V obtained by truncating the ten-dimensional action displays inconsistencies similar to those in equation (8). Therefore, in order to get an approximate low-energy theory for the massless modes we must follow the procedure advocated in this note and determine the functions K , W and f_{AB} in which the heavy fields have been set to zero. For K and W this is achieved by truncating the term that reduces to the D=4 gravitino mass term. Similarly, f_{AB} is obtained by truncating the gauge kinetic terms. The expressions deduced in this manner are correct up to $\mathcal{O}(1/M_c)$ and can be improved by using symmetry arguments¹¹.

We can similarly analyze superstring compactifications on orbifolds¹², in which the holonomy group is not $SU(3)$ but the point

group P of the orbifold. The covariantly constant modes correspond then to P singlets. Therefore, there are truncations like those performed in ref. (13) which are consistent and lead to models with more than one generation.

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REFERENCES

- 1) M.J. Duff, B.E.W. Nilsson and C.N. Pope , Phys. Rep. **130** (1986) 1.
M.J. Duff, B.E.W. Nilsson, C.N. Pope and N.P. Warner, Phys. Lett. **149B** (1984) 90.
M.J. Duff and C.N. Pope, Nucl. Phys. **B255** (1985) 355.
M.J. Duff, "Kaluza-Klein Konsistency", CERN-TH preprint no. 4296/85. (1985).
- 2) D.J. Gross, J. Harvey, E. Martinec and R. Rohm, Nucl. Phys. **B256** (1985) 253.
- 3) P. Candelas, G. Horowitz, A. Strominger and E. Witten, Nucl. Phys. **B258** (1985) 46.
- 4) E. Witten, in "Particles and Fields 2", Proceedings of the Banff Summer Institute on Particles and Fields, edited by A.Z. Capri and A.N. Comal (1981).
S. Weinberg, in "The Santa Fe Meeting", edited by T. Goldman and M. M. Nieto, World Scientific (1985).
- 5) M. Dine and W. Fischler, Nucl. Phys. **B227** (1983) 477.
- 6) S. Dimopoulos and H. Georgi, Nucl. Phys. **B193** (1981) 150.
- 7) E. Cremmer, S. Ferrara, L. Girardello and A. van Proeyen, Nucl. Phys. **B212** (1983) 413 ; J. Bagger, Nucl. Phys. **B211** (1983) 302.
- 8) A.H. Chamseddine, Nucl. Phys. **B185** (1981) 403; E. Bergshoeff, M. de Roo and P van Nieuwenhuizen, Nucl. Phys **B195** (1982) 97; G. Chapline and N. Manton, Phys. Lett **120B** (1983) 105.
- 9) E. Witten, Phys. Lett. **155B** (1985) 151.

- 10) J. P. Derendinger, L. E. Ibañez and H. P. Nilles, Nucl. Phys. **B267** (1986) 365.
- 11) C. P. Burgess, A. Font and F. Quevedo, Nucl. Phys. **B272** (1986) 661.
- 12) L. Dixon, J. Harvey, C. Vafa and E. Witten, Nucl. Phys. **B261** (1985) 678.
- 13) S. Ferrara, C. Kounnas and M. Porrati, "General dimensional reduction of ten-dimensional supergravity and the superstring", UCB-UCLA preprint UCB-PTH-86/5, UCLA/86/TEP/8 (1986).