CERN-TH.4554/86
UMN-TH-586/86

COSMIC STRINGS AND INFLATION

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ABSTRACT

We examine the compatibility of inflation with the cosmic string theory for galaxy formation. There is a general conflict between having sufficient string tension to effect galaxy formation, and reheating after inflation to a high enough temperature that strings may form in a thermal phase transition. To escape this conflict, we propose a class of models where the inflaton is coupled to the string-producing field. The strings are formed late in inflation as the inflaton rolls towards its zero-temperature value. A large subset of these models have a novel large-scale distribution of galaxies that is fractal, displays biasing without dynamics or feedback mechanisms, and contains voids.

CERN-TH.4554/86
UMN-TH-586/86
September 1986

1. - INTRODUCTION

Theories of galaxy formation have reached the point where it is necessary to postulate the existence of some unobserved form of matter or energy in order to reconcile observational constraints on the early Universe with the observed properties and large-scale distribution of galaxies. We can divide most models into three simple categories: those based on particles with low-velocity dispersion (cold dark matter)¹⁾ in which structure forms hierarchically, those based on particles with high-velocity dispersion (hot dark matter)²⁾ in which structure first appears as gravitationally-collapsing sheets with typical transverse dimensions of the order of tens of megaparsecs, and those based on cosmic strings in which matter is accreted onto topological singularities in a classical background field³⁾.

Models based on hot dark matter face a number of observational difficulties⁴⁾⁻⁶⁾. These include a recent formation time for galaxies^{4),5)}, large predicted (and unobserved) aggregations of hot gas⁵⁾ and the prediction that galaxies should be found exclusively in sheets and their intersections⁶⁾.

Models based on cold dark matter are quite successful at reproducing the observed small-scale clustering of matter, and the properties of individual galaxies¹⁾. On the other hand, these models have a problem in explaining the existence of any structure on scales greater than about 10 megaparsecs. There is now a considerable body of evidence in favour of such structure⁷⁾⁻⁹⁾. Two of the most conspicuous examples are the void in Boötes⁷⁾ with an apparent radius of about $30h^{-1}$ mpc and the Perseus supercluster⁹⁾, a filamentary association of clusters with a long axis of about $200h^{-1}$ mpc. These features appear to be difficult to explain in the context of the cold dark matter picture of galaxy formation^{*}). In the hot dark matter picture they are expected, but other concomitant phenomena are not observed. An even greater difficulty for cold dark matter is the recent observation of large-scale peculiar velocities¹¹⁾. If confirmed, these data would conclusively rule out the picture of cold dark matter with standard biasing and a scale-free spectrum of density perturbations¹²⁾.

A third model relies on the possibility of cosmic strings to seed galaxy formation³⁾. In this model, spontaneous symmetry breaking in the early Universe

^{*}) However, comparisons of simulations with the data do not show a clear contradiction¹⁰⁾.

gives rise to linear topological defects in a background field. The tension in these strings causes them to contract and create separated loops as the Universe expands. These loops then act as seed masses for the formation of galaxies and clusters. Although this model¹³⁾ produces galaxies and clusters in a reasonable manner^{*)}, there are some difficulties here as well. First, there seems to be no a priori reason why there should be large regions devoid of any suitable galaxy seeds, i.e., voids. Second, while the filamentary nature of an object like the Perseus supercluster would seem to argue in favour of strings, the correlation length for a string is never greater than the event horizon. There would be no reason for a series of large loops to be strung out linearly. Third, high peculiar velocities present some difficulties for cosmic strings¹⁴⁾, although they are not nearly so problematic as they are for cold dark matter.

A fourth, and perhaps the most serious, problem is the conceptual one of reconciling cosmic strings with the large-scale homogeneity and isotropy of the Universe as deduced from observations of the microwave background. In Big Bang theories of the Universe, the only known solution to the very large-scale problems is to appeal to inflation¹⁵⁾. Although specific proposals for inflation are certainly incorrect, many general features of the paradigm are likely to remain in any successful model. Among these is the prediction of scalar and tensor density fluctuations¹⁶⁾. The amplitude of these fluctuations depends on the expansion rate of the Universe during the inflationary epoch. Experimental upper limits on these amplitudes place upper limits on the expansion rate during inflation¹⁷⁾. This, in turn, places an upper limit on T_R , the reheating temperature after inflation. If T_R is smaller than T_c , the critical temperature for the phase transition during which cosmic strings form, then the said string will not be present in the Universe except on unobservably large scales. In general, one expects the critical temperature to be related to the string tension. For strings to be important for galaxy formation, a relatively large string tension is required, and through the chain of reasoning outlined above we find a conflict between inflation and cosmic strings.

In Section 2 of this paper we examine the argument in more detail and look at possible loopholes. The most promising loophole involves abandoning the thermal phase transition and replacing it with a phase transition driven by coupling

*) Although the number density and correlation functions of galaxies and clusters form a consistent picture, there are a number of unanswered questions concerning the details of how matter accretes onto the seed loops. We shall not address these questions in this paper.

the string field with the inflationary fields. In Section 3, we construct a rather ad hoc particle physics model as a proof of existence of this type of model. Finally, it turns out that these models may produce a different spectrum of strings than in the standard thermal phase transition. In Section 4, we give a qualitative description of how we expect the large-scale distribution of galaxies to look in such a model, and how we have inadvertently addressed the first three problems mentioned above.

2. - THE REHEATING CONSTRAINT

We wish to demonstrate a conflict between inflation and the cosmic string model for galaxy formation. The crucial concept we need from inflation is T_R , the maximum temperature to which the Universe reheats after inflation. For strings the crucial feature is that the strings form in a thermal phase transition as the Universe cools from $T > T_c$ to $T < T_c$. It should be obvious that to reconcile inflation with cosmic strings we require $T_R > T_c$. If this were not so, then the strings would exist only on unobservably large scales, i.e., they are inflated away.

We begin by making an estimate for T_c , but we will return to this issue later. In order that cosmic strings influence galaxy formation, one requires that the string tension μ be in the range^{13),18)}

$$G\mu \approx 1-4 \times 10^{-6} \quad (1)$$

where $G = 1/M_{Pl}^2$ is Newton's constant and $M_{Pl} = 1.22 \times 10^{19}$ GeV is the Planck mass. In general μ must be calculated, but one expects $\mu \approx v^2$, where v is the vacuum expectation value associated with the field, ϕ , which embodies the strings. Further, one expects $T_c \sim v \sim \sqrt{\mu}$. So for the moment we will take $T_c \approx 10^{16}$ GeV and require $T_R > 10^{16}$ GeV to make inflation and cosmic strings compatible.

Aside from the practical problem of constructing inflationary models with $T_R > 10^{16}$ GeV, there are bounds on T_R coming from (i) graviton production^{19),17)}, which implies that $T_R < 6 \times 10^{16}$ GeV, and (ii) the production of primordial density perturbations¹⁷⁾, which implies that $T_R < 2 \times 10^{16}$ GeV. These limits come from requiring that fluctuations in the microwave background are below the observational upper limits. Because of the similarity between these bounds on T_R and our estimate of T_c , and because we feel these limits are conservative, we have re-

examined the bound on T_R derived from scalar density perturbations. First we derive a conservative bound in the spirit of Ref. 17), and then we give what we feel is a more realistic bound on T_R that takes into account some of the more general aspects of building models of inflation.

Inflation predicts a spectrum of curvature fluctuations with amplitude¹⁶⁾

$$\epsilon = \frac{H^2}{\pi^{3/2} \dot{\psi}} \quad (2)$$

where $\dot{\psi}$ is the time derivative of the inflaton field ψ . Equation (2) is to be evaluated at the time the length scale in question leaves the horizon during inflation. During inflation, the Hubble parameter is given by $H^2 = 8\pi\Lambda/3M_{Pl}^2$, where Λ is the energy density of the inflaton field. Even if the energy is thermalized with maximum efficiency, conservation of energy ensures that the reheat temperature is less than

$$T_R \leq \left(\frac{90}{8\pi^3 g_*} \right)^{1/4} (M_{Pl} H)^{1/2} \quad (3)$$

where g_* is the number of relativistic degrees of freedom that share in the reheating. The last input is to use the equation of motion for ψ . Late in inflation, $\dot{\psi} = -(V'/3H)$, where $V' = \partial V/\partial\psi$ and V is the scalar potential associated with the inflaton. We integrate $\dot{\psi}$ from the time the length scale corresponding to the observational limits on the microwave background left the horizon during inflation, t_μ , until the end of inflation, $\int(d\psi/V') = -\int(dt/3H)$. Under quite general conditions, during this period $V'_\mu < V'$ and $H_\mu > H$. We then obtain the constraint

$$V'_\mu < \frac{3\sigma H^2}{\chi_\mu} \quad (4)$$

where σ is the vacuum expectation value of ψ after inflation and $\chi_\mu = Ht_\mu$ is the amount of inflation that occurs after the microwave scales leave the horizon. Combining Eqs.(2) -(4), we find

$$T_R < \left(\frac{3.35 \epsilon}{g_*^{1/2} \chi_\mu} \right)^{1/2} M_{Pl} \left(\frac{\sigma}{M_{Pl}} \right)^{1/2} \quad (5)$$

Taking $\epsilon = 10^{-4}$ and $\chi_\mu = 50$ for small-scale anisotropies, we find a bound $T_R < 2.6 \times 10^{-3} M_{Pl}$.

This bound on T_R assumes $g_* = 1$ and $\sigma = M_{Pl}$, which are conservative. We have also assumed instant reheating and taken no account of the shape of the potential, which can only lower the bound on V'_μ in Eq. (4). Let us leave $\sigma = M_{Pl}$. Certainly if $\sigma > M_{Pl}$ we must confront quantum gravity face to face; however, setting $\sigma < M_{Pl}$ is not particularly compelling either. For g_* we must at least take $g_* > 4$ for the scalar and gauge degrees of freedom associated with the string. Alternatively, we might demand that g_* also include the light degrees of freedom of the standard $SU(3) \times SU(2) \times U(1)$ model where $g_* = O(100)$. To study the effects of the shape of the potential, we have studied two cases popular with model builders

$$V_1(\psi) = \frac{\lambda}{4} \left[\psi^4 \ln(\psi^2/\sigma^2) + \frac{1}{2} (\sigma^4 - \psi^4) \right] \quad (6a)$$

$$V_2(\psi) = \mu^4 \left(1 - \psi^3/\sigma^3 \right)^2 \quad (6b)$$

For the Coleman-Weinberg case (6a)²⁰⁾, the reduction in V'_μ is

$$\psi_\mu / 2\sigma = \frac{1}{2} \left[\frac{a}{\ln a^{-1/2}} \right]^{1/2}$$

with

$$a = \frac{\pi}{4\chi_\mu} \frac{\sigma^2}{M_{Pl}^2}$$

For $\chi_\mu = 50$, the reduction factor is 0.042. For case (6b)²¹⁾, motivated by supersymmetry, the reduction in V'_μ is

$$\frac{\psi_\mu}{\sigma} = \frac{\sigma^2}{M_{Pl}^2} \frac{8\pi}{6\chi_\mu} \simeq .084$$

It is apparent that integrating the roll of ϕ reduces V' by at least a factor of 10 for semirealistic models of inflation. (This reduces T_R by a factor of $\sqrt{10}$.) Combining this with our considerations of g_* (reducing T_R by another factor of $\sqrt{10}$), a more realistic, but still conservative, bound on T_R is

$$T_R \lesssim 2.6 \times 10^{-4} M_{Pl} \quad (7)$$

So far we have supposed that reheating takes place in less than a Hubble time. Properly, we should consider the rate at which the inflaton field decays, Γ_D . If $\Gamma_D > H$, then reheating is fast and $T_R \sim (HM_{Pl})^{\frac{1}{2}}$. If $\Gamma_D < H$, then after the inflatons have all decayed the temperature is $T_D \sim (\Gamma_D M_{Pl})^{\frac{1}{2}}$; however, the maximum temperature of the radiative decay products is higher $T_R \sim [\Gamma_D H M_{Pl}^2]^{1/4}$. Still, if $\Gamma_D < H$, we cannot hope to saturate the bound on T_R . If we consider the two potentials in Eq. (6), then in both cases we find that $m_\phi \sim H$ where m_ϕ is the mass of the inflaton at its minimum. Under the most optimistic circumstances, we expect that $\Gamma_D \sim f^2 m_\phi$, where f is some coupling of the inflaton to matter fields. It seems likely that Γ_D is substantially less than H , and the bound on T_R is even tighter than that in Eq. (7).

At this point it should be clear that it is very difficult to obtain reheat temperatures as high as 10^{16} GeV. Certainly no model proposed so far has this feature. If the critical temperature is really $T_c = 10^{16}$ GeV, then the cosmic string theory for galaxy formation is incompatible with inflation. It is now time to re-examine the argument that $T_c \sim \sqrt{\mu} \sim v$. There are two ways one might make $T_c < \sqrt{\mu}$. First, we might try to make $\mu \gg v^2$. To see if this is reasonable, we have examined the string tension in a simple U(1) model with the Lagrangian

$$\mathcal{L} = \int d^4x \quad \frac{1}{4} F^2 + \frac{1}{2} |D_\mu \phi|^2 + V(\phi) \quad (8)$$

and $V(\phi) = (\lambda/4)(\phi^2 - v^2)^2$, $D_\mu = \partial_\mu - ieA_\mu$ and $F^{\mu\nu}$ is the gauge field strength. The string tension can then be expressed as an integral of the energy over the transverse dimensions of the string. Making an axially symmetric ansatz²³⁾ for the fields with adjustable parameters, we have varied the parameters to minimize the energy. In this way, we obtain upper limits to the string tension. From dimensional analysis we can write $\mu = v^2 f_1(\kappa)$ where $\kappa = \lambda/e^2$. From our variational calculation we find

$$f_1(\kappa) < \frac{\pi}{2} (1 + 1.8 \kappa) \quad ; \quad \kappa < 1 \quad (9a)$$

$$f_1(\kappa) < \frac{\pi}{4} \left(1 + 2 \ln \left(\frac{2\kappa}{3} \right) \right) \quad ; \quad \kappa \gg 1 \quad (9b)$$

Evidently, it is not possible to make $\mu \gg v^2$. It is, however, possible to make $T_c \gg v$ under some circumstances. We must be careful to define what we mean by T_c . Usually T_c is defined to be the temperature at which the symmetric phase becomes unstable as T is lowered. In the case of a first-order transition, $\phi = 0$ is locally stable but globally unstable, while for a second-order transition $\phi = 0$ becomes locally unstable. In our case we are interested in restoring the symmetry, so we want T_c defined by when the asymmetric phase becomes unstable as T increases. Roughly speaking, T_c is determined by balancing the finite temperature corrections to the potential and the zero temperature effective mass at $\langle \phi \rangle = v$,

$$(\lambda + g^2) T_c^2 - m_\phi^2 = 0 \quad (10)$$

where the terms λT^2 and $g^2 T^2$ come from the one-loop temperature corrections due to scalar and vector loops respectively. Here g is the gauge coupling constant and usually $m_\phi^2 \approx \lambda v^2$. We can see that

$$T_c^2 \approx \frac{\lambda}{\lambda + g^2} v^2 \quad (11)$$

It would appear that one can make T_c arbitrarily small by making $\lambda \ll g^2$. In a non-supersymmetric model, one is prohibited from making $\lambda \lesssim g^4$ because of radiative corrections. In that case, $T_c \sim gv$. To make $T_c \ll v$ requires very small gauge couplings. Since in grand unified theories (GUTs) $g \sim 0.3$, we may conclude that strings resulting from the breaking of a GUT symmetry are in conflict with inflation. In a supersymmetric model, one may treat λ as a free parameter and dial²⁴⁾ it to zero if one is so inclined. However, one should note that in supersymmetric models, arguments based on gravitino production²⁵⁾ limit $T_R \leq 3 \times 10^{10} m_{3/2}$, where $m_{3/2}$ is the gravitino mass. For $m_{3/2} = 100$ GeV, this would imply $T_c \approx 3 \times 10^8$ GeV or $\lambda \approx 10^{-16}$. Finally, we note that for superstring theories $m_{3/2}$ is expected to be large²⁶⁾, $O(10^{12}$ GeV), so there is no gravitino constraint on T_R . One should also note, however, that in superstring theories, it is in general quite difficult to produce cosmic strings^{27),28)}. Flux tubes serving the rôle of cosmic strings can be formed²⁸⁾, but only if $T_R \gtrsim 10^{16}$ GeV.

The constraint we have developed so far is valid for strings that form as vortex lines in local gauge theories. The situation changes slightly for global symmetry breaking²⁹⁾. In this case the effective string tension contains a logarithmic factor $\ln(vR) \approx 120$ where R is the typical separation of strings, taken to be the horizon size. As a result, the naive critical temperature can be

an order of magnitude smaller. We note that a theory of galaxy formation based on global strings has not been worked out in any detail, so these last remarks may not be relevant to cosmology. Another possibility is that strings form as flux tubes in a confining gauge theory³⁰⁾. Such strings are formed at a temperature scale $T \sim \sqrt{\sigma} \sim \Lambda$, where Λ is the confinement scale of the theory. Hence, these strings are incompatible with our current knowledge of inflation.

3. - FORMING STRINGS DURING INFLATION

At this point we have a fairly solid argument that a cosmic string theory of galaxy formation, where the strings form in a thermal phase transition, is in conflict with inflation. Certainly no inflationary model that we know of is capable of forming strings in this way. So, if one is truly enamoured of cosmic strings, one is faced with giving up inflation or with finding another way to make strings. We feel that any solution to the large-scale problems of the Universe will embody inflationary features. Instead, we propose an alternative model for string formation that involves coupling the string field ϕ to the inflaton ψ . We suggest that for small values of ψ the symmetric phase of ϕ is preferred, but as ψ rolls towards its minimum at σ , the symmetric phase is destabilized and strings form.

The basic model that we will look at is of the form $V = V_1 + V_2$, where

$$V_1 = \frac{\lambda}{4} (\phi^2 + \psi^2 - a\psi^2)^2 \quad (12a)$$

$$V_2 = \mu_I^4 \left(1 - \frac{\psi^3}{\sigma^3} \right)^2 \quad (12b)$$

In choosing the inflationary potential V_2 we are motivated by supersymmetry considerations²¹⁾, since we will be considering small couplings. So we set $\sigma \sim M_{Pl}$. The parameter μ_I sets the scale of the inflationary potential. The Hubble constant during inflation is $H^2 \approx (8\pi/3)(\mu_I^4/M_{Pl}^2)$. From considerations of density fluctuations we require $\mu_I < 5 \times 10^{-4} M_{Pl}$. The detailed nature of V_2 should not change our conclusions.

The form of V_1 is designed to implement string formation late in inflation. It is understood that ϕ is a complex field. The effective mass of the ϕ field at $\phi = 0$ is

$$m_\phi^2 = \lambda (v^2 - a\psi^2) \quad (13)$$

Early in inflation, ϕ is small and $m_\phi^2 > 0$. If $a\sigma^2 > v^2$, then by the time inflation ends $m_\phi^2 < 0$ and the $U(1)$ symmetric phase will be unstable.

When strings form is determined by the evolution of the ϕ field³¹⁾. As long as $m_\phi^2 > H^2$, the ϕ field fluctuates around $\phi = 0$ with $\langle \phi^2 \rangle \propto H^4/m_\phi^2$. For $|m_\phi^2| < H^2$, ϕ behaves like a massless complex field. Crudely, $\phi(x)$ undergoes a two-dimensional random walk of step size H with one step per time $t \sim 1/H$. Accordingly $\langle \phi^2 \rangle \sim H^3 t$. More properly, the results of evolving ϕ during this epoch are described by giving the power spectrum $\phi_k^2 = H^2/k^3$, and specifying that ϕ_k is distributed according to a Gaussian. Finally, when $-m_\phi^2 > H^2$, the evolution of ϕ is dominated by the potential. Once this happens, the phase of ϕ is preserved and ϕ rolls radially away from $\phi = 0$. On the other hand, before the time when $-m_\phi^2 = H^2$, fluctuations in ϕ may significantly alter the phase and cause ϕ to cross the point $\phi = 0$. Since the strings are to be identified with loci where $\phi = 0$, strings are continually being formed as long as $-m_\phi^2 < H^2$. However, as soon as $-m_\phi^2 > H^2$, the loci $\phi = 0$ are fixed. We therefore identify the epoch when strings form by the appropriate value of ψ ,

$$\psi_s^2 = \frac{1}{a} \left(\frac{H^2}{\lambda} + v^2 \right) \quad (14)$$

Now, let ϕ_g be the value of ϕ when length scales relevant to galaxy formation leave the horizon. Then, if $\phi_g > \psi_s$, i.e., ψ_s precedes ϕ_g , strings cannot act as seeds for galaxy formation. However, if the converse occurs, strings will be present on galactic scales and may seed the formation of galaxies. To do so, the string tension for this model,

$$\mu = a\sigma^2 - v^2 \quad (15)$$

should be about $10^{-6} M_{Pl}^2$. The value of ϕ_g is determined by integrating $\dot{\phi} = -V'/3H$; $V' = \partial V_2/\partial \phi$ and is found to be $\phi_g = (4\pi/3\chi_g)\sigma^3/M_{Pl}^2$.

We now have three constraints on the parameters of our model (λ , a , v , μ_1 and σ) from adiabatic density fluctuations, $\phi_s > \phi_g$, and having an appropriate

string tension. To make our analysis consistent there is one additional constraint on the potential, and that is that the string sector should not interfere with the onset of inflation,

$$\lambda v^4 < \mu_I^4 \quad (16)$$

It is not difficult to satisfy these four relations. For example, take $\sigma = M_{Pl}$, $\mu_I = 10^{-4} M_{Pl}$ and ($v = 0$, $a = 10^{-6}$, $\lambda \lesssim 10^{-7}$) or ($v = 10^{-3} M_{Pl}$, $a = 2 \times 10^{-6}$, $\lambda \lesssim 10^{-4}$). In Fig. 1 we show the various constraints for other parameter choices. The constraints on λ , v , μ_I are nearly independent of a and σ . Given that we satisfy the constraints, there are two qualitatively different ways for the network of strings to manifest itself depending upon the value of m_ϕ^2 at ϕ_g . If $m_\phi^2 > H^2$ when a given scale leaves the horizon, then the network of strings on that scale should look just like it does in the thermal phase transition case³⁾. The correlation length for the strings is always the horizon size. After a given scale leaves the horizon, the correlations on that scale are washed out by subsequent fluctuations. This is not surprising since the fluctuations ($\delta\phi \sim H$) are larger than the RMS value of the field $\langle \phi^2 \rangle^{\frac{1}{2}} \sim H^2/m_\phi$.

However, if $|m_\phi^2| < H^2$ when that scale leaves the horizon, then we get qualitatively different results. There are correlations on all scales. As mentioned earlier, these correlations are determined by the power spectrum $\phi_k^2 = H^2/k^3$. We will explore the effects of these correlations on large-scale structure in the next section.

Whether or not $|m_\phi^2| < H^2$, or vice versa, depends on the parameters of our model. Given that $\phi_s > \phi_g$, if $m_\phi^2(\phi_g) < 0$ (i.e., $v^2 < a\phi_g^2$) we always find $|m_\phi^2| < H^2$. Since $a\sigma^2 \sim 10^{-6} M_{Pl}^2$ and $\phi_g^2 \sim 10^{-2}\sigma^2$, this condition is tantamount to asking for $v^2 \sim 10^{-8}$, where a priori we would allow $v^2 \sim 10^{-6} M_{Pl}^2$. On the other hand, if $m_\phi^2(\phi_g) > 0$, ($v^2 > a\phi_g^2$), then we find a critical value for λ , $\lambda_c = H^2/(v^2 - a\phi_g^2)$. If $\lambda < \lambda_c$ we will have the new distribution of strings. For $v^2 = 10^{-6} M_{Pl}^2$, $a = 2 \times 10^{-6}$, $\mu_I = 10^{-4} M_{Pl}$, $\sigma = M_{Pl}$, we find $\lambda_c \approx 10^{-9}$. This is a considerably smaller value of λ than was derived from the consistency constraint, $\lambda \sim 10^{-4}$. In Fig. 1 we show λ_c as a function of v^2 and μ_I . λ_c is nearly independent of a and σ . Models with small v^2 ($\lesssim 10^{-8} M_{Pl}^2$) will always exhibit the new string structure, but models with large v^2 will only exhibit the new structure if λ is much smaller than necessary.

Before proceeding, we ask if it is really possible to construct a model of the kind we propose. Because we are requiring couplings like λ to be very small,

the model must be derived in the context of supersymmetry, otherwise it will be necessary to impose fine tuning conditions to cancel the radiative corrections to $\lambda\phi^4$. Although it is not trivial, one can write down a model which possesses the desired properties of a potential like Eq. (12). The model is derived from the following superpotential (in units where $M_{Pl}/\sqrt{8\pi} = 1$):

$$F = \mu^2 \left[\eta_1 (\psi^3 - 1 + \phi^4) + \eta_2 (\phi^2 - a\psi^2) \right] \quad (17)$$

where η_1 and η_2 are gauge singlet chiral supermultiplets and $\phi^2 = \phi_1^2 + \phi_2^2$ for two Higgs supermultiplets ϕ_1 and ϕ_2 . In the no-scale supergravity models³²⁾, the scalar potential is given by

$$V = \sum_i \left| \frac{\partial F}{\partial \phi_i} \right|^2 + \left| \frac{\partial F}{\partial \eta_i} \right|^2 + \left| \frac{\partial F}{\partial \psi} \right|^2 \quad (18)$$

One can show that at high temperatures the origin is a stable minimum. One can also show the mass terms for η_{1R} , η_{1I} , η_{2R} , η_{2I} , ϕ_{1I} , ϕ_{2I} and ψ_I are all positive ($\psi = \psi_R + i\psi_I$, etc.), and hence are all trapped at the origin at low temperatures. At $\eta_1 = \eta_2 = \phi_{1I} = \phi_{2I} = \psi_I = 0$, the potential is given by

$$V = \mu^4 \left[(\psi^3 - 1)^2 - a^2 \psi^4 + \phi^8 - \phi^4 + 2\psi^3 \phi^4 - 2a\psi^2 \phi^2 \right] \quad (19)$$

Qualitatively, this potential has the same form as Eq. (12) with $v = 0$, $\lambda = (\mu/M_{Pl})^4$. Inflation occurs identically to that in Eq. (12b), and ϕ picks up a vacuum expectation value $\approx a\phi^2$, so we need $a = 2.5 \times 10^{-5}$. In this case, strings are formed via de Sitter fluctuations as

$$m_\phi^2 = \mu^4 \left[56\phi^4 - 12\phi^2 + 24\psi^3\phi^2 - 4\psi^2\psi^2 \right] < H^2 = \frac{\mu^4}{3} \quad (20)$$

Clearly one should not take the model presented in Eq. (17) too seriously. For our purposes, it is sufficient to note that it serves as a proof of existence for the general type of potential necessary to reconcile strings and inflation in the context of supersymmetry. We note that the above model cannot be formulated in the context of superstring models. [Indeed, a model of inflation has also yet to be formulated in that context³³⁾.] An optimistic viewpoint would be to say that this is an example of a theory which embodies a bold new approach to the problem of large-scale structure (see next section). On the other hand, one could look

at this as being representative of the depths to which one must go in order to dredge up a model where cosmic strings and inflation are compatible.

4. - STRUCTURE IN THE NEW MODEL

In this section we look at the large-scale structure which results from the models described in Section 3. For the case when $m_{\phi}^2(\psi_g) > H^2$, the distribution of strings on galactic and larger scales should be the same as in the usual thermal phase transition picture, so we will concentrate on the $m_{\phi}^2(\psi_g) < H^2$ case, comparing our results to the usual case where appropriate. We will assume that the small-scale details of galaxy formation are similar to the usual scenario, i.e., that galaxies form around loops of strings that act as seed masses^{13),18)}. The source of the seed loops is not direct generation from the large-scale network of strings; rather, the large-scale network is supposed to spawn "parent" loops which then fragment into "daughter" loops which act as seeds. Thus the short-range correlations are determined by parent/daughter fragmentation. We will suppose that the same thing happens in our picture so that the success of the usual string model in fitting the galaxy/galaxy and cluster/cluster correlations is preserved. However, the large-scale structure in our model will be different. The distribution of parent loops depends on the large-scale distribution of strings, and this is different in our model.

In the usual picture the large-scale distribution always looks the same, i.e., it is scale-free, in a way that depends upon the dynamics of the string network. The distribution is a random walk network of strings with step size of the order of the horizon length. As the horizon size increases, parent loops pinch off the network so that short wavelength oscillations of string separate from the network. We emphasize that strings are everywhere; any horizon volume at any time is likely to contain long string and parent loops. This is a situation that does not seem conclusive to forming large voids as in Boötes, or structures like the Pisces-Perseus filament which are much larger than the horizon sizes associated with galaxy and cluster formation.

In our model the primordial distribution of strings has a great deal of structure on large scales. To study this, we have used a simple analytic technique. We model the string as the zeros of a complex field ϕ . This is motivated by our picture of the field as originally centred around the origin and subsequently evolving away, but preserving its original, local phase. Previous models for string formation involve random spatial fluctuations of ϕ about a mean of

zero. In this case, the power spectrum is flat. A high-frequency cut-off, k_c , can be imposed by the observation that features too small to cause galaxy formation can be ignored. On the other hand, if the fluctuations in the field are not generated locally but are quantum fluctuations from an epoch of inflation, then the power spectrum $|\phi(k)| \propto k^{-3}$. This spectrum diverges logarithmically at small k . The model has an explicit cut-off at scales that leave the horizon during inflation when $m_\phi^2 = H^2$. For most parameter choices this length scale is much greater than our current horizon. Therefore we will impose a low-frequency cut-off k_h , where k_h^{-1} is comparable to the present horizon, and a background field value ϕ_b generated by longer wavelength modes.

The first issue we study is that of the density of strings in space. This is simply related to the probability $P(\epsilon)$ of a string passing within a distance ϵ of a randomly-chosen point. For $k_c(\epsilon) \ll 1$

$$P(\epsilon) = \pi \epsilon^2 L/V \quad (21)$$

where L/V is the total string length per volume. Consider a point with a given value of ϕ and $\nabla\phi$. One can always do a gauge transformation so that ϕ lies along the real axis ($\phi = \phi_R + i\phi_I$). Now define a spatial unit vector in the direction of $\nabla\phi_I$, $\hat{e}_I = \nabla\phi_I / |\nabla\phi_I|$. Then the condition that a string pass within ϵ of the point is

$$\epsilon \left| \hat{e}_I \times \nabla\phi_R \right| > \phi \quad (22)$$

Since ϕ and $\nabla\phi$ have independent Gaussian distributions, we may compute

$$P(\epsilon) = \frac{1}{3} \left(\frac{\epsilon \sigma_g}{\sigma} \right)^2 e^{-\frac{|\phi_b|^2}{2\sigma^2}} \quad (23)$$

where σ and σ_g are the standard deviations of $|\phi_g|$ and $|\nabla\phi|$ respectively. Comparing Eqs. (21) and (23), we see that

$$L/V = \frac{1}{3\pi} \left(\frac{\sigma_g}{\sigma} \right)^2 e^{-\frac{|\phi_b|^2}{2\sigma^2}} \quad (24)$$

The values of σ and σ_g are given by integrating over the power spectra appropriately normalized: $\sigma^2 = \int k^2 \phi_k^2 dk$, $\sigma_g^2 = \int k^4 \phi_k^2 dk$. For the standard model ϕ_k^2 is a constant, so

$$L/V = K_c^2 / 5\pi \quad (25)$$

For our model $\phi_k^2 = a/k^3$ and

$$L/V = \frac{K_c^2}{6\pi \ln(K_c/K_h)} e^{-\frac{|\phi_b|^2}{2\sigma^2}} \quad (26)$$

We see that in our model the mean density of string is reduced from the standard model. The reduction due to the logarithmic factor is of order 9. This is probably not a serious difficulty for galaxy formation. The reduction due to the exponential factor may be arbitrarily large depending on ϕ_b . However, there are two reasons why this should not concern us. First, given our cut-offs, σ is given by $\sigma^2 = a \ln K_c/K_h$; but ϕ_b^2 is not greatly different in value, since ϕ_b is due to the long wavelength fluctuations $\langle \phi_b^2 \rangle \approx a \ln K_h/\tilde{k}$, where $\tilde{k}$ is the scale for which $m_\phi^2 = H^2$, so a relatively small value of $\langle \phi_b^2 \rangle / 2\sigma^2$ is not unreasonable. The second point is that, given an equal probability of being on any galaxy in the Universe, the median observable reduction from the exponential is a factor of 0.5. This is perhaps the strangest consequence of our model. We propose that the Universe, far from being filled with galaxies, is a wasteland of inhospitably homogeneous gas surrounding islands of galaxies where life and cosmology are possible.

Having satisfied ourselves that there is enough string in our model to explain the number of galaxies, we turn to the question of how they are distributed. First, we argue that the large-scale distribution should be nearly scale-free (a fractal), i.e., the distribution of objects of size ℓ at separation r depends only on the dimensionless ratio r/ℓ and not on r or ℓ independently³⁴⁾. In fact, there is a logarithmic dependence on scale through the behaviour of σ . More seriously, the scale-free nature of the distribution is only apparent when viewed over a sufficiently large volume. Since we can only see out to the horizon, the scale-free nature of the distribution will be lost as $k \rightarrow k_h$. For example, suppose in our volume of the Universe $\phi_b^2 = 2a$, where a is the normalization of the power spectrum. Then there will be no structures at all for $k \lesssim 10k_h$; for scales $k > 10k_h$ the distribution will become more and more fractal-like as the memory of ϕ_b is lost.

Next we look at clustering and voids. Suppose the field has a specified value ϕ_0 at a position x_0 . Then the expectation value of the field is given by

$$\langle \phi(x) \rangle = \alpha(|x-x_0|) \phi_0 \quad (27)$$

where $\alpha(0) = 1$ and $\alpha \rightarrow 0$ as $|x-x_0| \rightarrow \infty$. It is straightforward to show that

$$\alpha(r) = \frac{\int |\phi_k|^2 \frac{\sin kr}{kr} k^2 dk}{\int |\phi_k|^2 k^2 dk} \quad (28)$$

The distribution of $\phi(x)$ around this mean value is a Gaussian with a width $\sigma_*^2(r)$ which goes to zero at $r = 0$ and relaxes to σ^2 as $r \rightarrow \infty$. We see that

$$\sigma_*^2(r) = \sigma^2(1 - \alpha^2) \quad (29)$$

Since the probability of a string is proportional to the statistical weight of the state $\phi(x) = 0$, the expected density of strings around x_0 is given by

$$D_s(r) = \frac{\sigma_g^2}{3\pi\sigma^2(1-\alpha^2)} e^{-\frac{\alpha^2|\phi_0|^2}{2\sigma^2(1-\alpha^2)}} \quad (30)$$

For the standard model, α is

$$\alpha(r) = \frac{3}{(k_c r)^3} \left[\sin(k_c r) - k_c r \cos(k_c r) \right] \quad (31)$$

For our model we obtain

$$\alpha(r) = \frac{1}{\ln(k_c/k_h)} \int_{k_h r}^{k_c r} \frac{\sin y}{y^2} dy \quad (32)$$

We note that for the interesting case where $k_c^{-1} \ll r \ll k_h^{-1}$, Eq. (31) reduces to

$$\alpha(r) \approx \frac{-3}{(k_c r)^2} \cos(k_c r) \quad (33)$$

whereas Eq. (32) becomes

$$\alpha(r) \approx \frac{1}{\ln(k_c/k_h)} \left[\beta - \ln(k_h r) - \frac{\cos(k_c r)}{(k_c r)^2} - \frac{2 \sin(k_c r)}{(k_c r)^3} \right] \quad (34)$$

where $\beta \equiv 1 - \gamma_E = 0.423$ and $\gamma_E = 0.577$ is Euler's constant.

In Fig. 2, we show the predicted string density, normalized to the average density, as a function of r for $\phi_0/\sigma = 0, 1, 2, 3, 4, 5$. The choice $\phi_0 = 0$ corresponds to calculating $1 + \xi_s$, where ξ_s is the correlation function of string sites. On the same scale, the results for a Poisson random field would be imperceptibly different from a horizontal line. Several points should be clear from the picture. First, there is negligible large-scale structure in the standard model. Second, for our model, peaks of a few σ in $\phi(x)$ are associated with voids in the string distribution much larger than k_c^{-1} . This is a form of biasing³⁵⁾ that involves only the primordial distribution of strings. It does not depend on feedback mechanisms or timing arguments³⁶⁾ to suppress galaxy formation in voids. The "1 σ " peaks in voids are not prevented from turning into galaxies because neighbouring 3σ peaks inhibit them in some complicated way. Rather, they do not turn into galaxies simply because there are no "1 σ " peaks in the voids. We do not know if this mechanism makes it possible to reconcile our prejudice that the Universe has critical density $\Omega = 1$ with the value of $\Omega \sim 0.2$ deduced from observing the luminous matter in clusters of galaxies³⁷⁾.

It is interesting to study the voids in our model and make a comparison with the observed Universe. From Eq. (30) for $\phi_0^2/\sigma^2 \equiv v^2 \gg 1$ we get a characteristic void size, R_v , given by

$$\alpha(R_v) = \sqrt{-\frac{2}{v} \ln(f)} \quad (35)$$

where f is the depletion of bright galaxies in the void. For the void in Boötes, $f \approx 0.25$. From

$$R_v = k_h^{-1} e^\beta (k_c/k_h)^{-\sqrt{-\frac{2}{v} \ln(f)}} \quad (36)$$

it is difficult to decide what the size of a typical void is. Let us take

$$K_h = 2\pi / (6000 h^{-1} \text{ mpc})$$

and

$$K_c = 2\pi / (2 h^{-1} \text{ mpc})$$

corresponding to the typical separation between bright galaxies. The volume of the void should be weighted according to the probability of the appropriate value of v . Then the typical void is characterized by a v_{max} which is the value of v which maximizes the expression

$$e^{-\frac{v^2}{2}} \left(\frac{K_c}{K_h} \right)^{-3\sqrt{-\frac{2}{v}} \ln(f)} \quad (37)$$

For the visible Universe we have $v_{\text{max}} = 3.42$ and the corresponding $R_v = 30h^{-1}\text{ mpc}$.

It is tempting to regard the close agreement between this result and the observed size of the void in Boötes⁷⁾ as a dramatic success for our model. However, the result is better than it deserves to be. This estimate does not include any consideration of the dynamical evolution of string segments or the frequency of such "typical" voids. Also, it is clear that what one labels as voids in the observed galaxy distribution does not truly sample the empty space. To make a real comparison of this model with observations would require extensive numerical simulations looked at through the same eyes that one uses when observing the real sky.

Let us summarize the results of this paper. We find a conflict between inflation and the cosmic string theory of galaxy formation if the strings are formed in a thermal phase transition. We propose that it is possible to resolve this conflict by coupling the string and inflaton fields. In some cases this leads to a new picture for large-scale distribution of galaxies. There are a number of theoretical and observational issues raised in this model which require further work. For example, what is the global anisotropy in deep galaxy counts expected in this picture and is it consistent with observations? What is the expected distribution of void sizes and shapes? To what extent will the inclusion of dynamical effects alter the simple estimates presented above? Can we find evidence for very large-scale self-similarity in the galaxy distribution?

ACKNOWLEDGEMENTS

We would like to thank N. Turok for useful conversations. The work of KAO was supported in part by DOE grant DE-AC02-83ER-40/05 at the University of Minnesota.

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FIGURE CAPTIONS

Fig. 1 : Various constraints on the parameters in our model. We have scaled the results to $\mu_I = 10^{-4} M_{Pl}$, whereas the constraint on μ_I is $\mu_I < 5 \times 10^{-4} M_{Pl}$. For large λ and small v^2 strings form before galactic scales cross the horizon. For large λ and large v^2 , our analysis of inflation is not consistent. For $\lambda > \lambda_c$ (OLD) the large-scale network of strings is the same as would be produced in a thermal phase transition. For $\lambda < \lambda_c$ (NEW), the nature of the large-scale network is discussed in Section 4. The dashed curve illustrates how, for large v^2 , the parameter a must be adjusted to attain a string tension of $G\mu = 10^{-6}$. The "x" shows the parameters for the toy model constructed at the end of Section 3.

Fig. 2 : String density L around a point with the field value $\phi_0 = v\sigma$. The string density is normalized to the average string density. Each line is labelled by the corresponding value of v . The wave number cut-offs taken are $k_c = \pi h \text{ mpc}^{-1}$ and $k_h = \pi/3000 h \text{ mpc}^{-1}$. The radius r is given in units of $h^{-1} \text{ mpc}$.

Fig. 1

Fig. 2