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NEUTRON-ANTINEUTRON OSCILLATIONS IN ^{16}O

C.J. Batty¹⁾, M. Fidecaro²⁾ and H.B. Prosper¹⁾

ABSTRACT

We have made a careful analysis of the effect of statistical uncertainty in the form of the antineutron-nucleus potential on the value of the neutron-antineutron mixing time. Using a bias-free optical potential derived from fits to the ($\bar{p}$, ^{12}C) scattering data at 47 MeV, together with the IMB and Kamiokande results for the lifetime of ^{16}O , we set lower limits on the neutron-antineutron mixing time $\tau_{n\bar{n}}$ of $>1.1 \times 10^8$ s and 1.5×10^8 s, respectively, both at the 90% confidence level. We adopt the theoretical approach of Dover, Gal and Richard to extract these limits from the ^{16}O data.

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1) Rutherford Appleton Laboratory, Chilton, Didcot, OXON OX11 0QX, UK.

2) CERN, CH-1211 Geneva 23, Switzerland.

1. INTRODUCTION

The failure to observe proton decay at the predicted rates probably rules out grand unification theories (GUTs) based on the group $SU(5)$; however, this failure is not necessarily a catastrophe for the idea of grand unification. On the contrary, considerable progress has been made with a theory, based on the group $SO(10)$ [1], which predicts a value for the mean proton lifetime which exceeds the present experimental lower limit [2, 3]. Of particular interest also is the $SO(10)$ prediction of neutron–antineutron ($n\bar{n}$) oscillations with a measurable value for the mixing time $\tau_{n\bar{n}}$. The most direct limit on the value of $\tau_{n\bar{n}}$ is obtained using free neutrons. Indeed a reactor experiment has set a lower limit on the mixing time of 1.3×10^6 at 90% CL [4]. Alternatively, a limit can, in principle, be obtained by using the data from proton decay experiments combined with an appropriate nuclear physics calculation. This method of extracting a limit on $\tau_{n\bar{n}}$ has received some attention in recent years [5–8]. That this method generally leads to larger limits than the current free-neutron limit is of considerable interest given that the present $SO(10)$ prediction for $\tau_{n\bar{n}}$ is $\tau_{n\bar{n}} \approx (1 \rightarrow 10) \times 10^8$ s; a free-neutron experiment to explore the aforementioned range of values for the mixing time is still some years off.

In this paper we consider the effect of statistical uncertainty in the form of the anti-neutron–nucleus potential on the limit for $\tau_{n\bar{n}}$ obtained from the ^{16}O data. For our analysis the most convenient method is that of Dover, Gal and Richard (DGR) [8]; accordingly, we have followed their approach.

Using a variety of the available phenomenological optical potentials for the systems ($\bar{p}, ^{16}\text{O}$) and ($\bar{p}, ^{12}\text{C}$) obtained from fits to $\bar{p}$ -atom and $\bar{p}$ -scattering data these authors solve the coupled Schrödinger equations (SE) for the ($n, \bar{n}$) system in ^{16}O , and calculate for each optical potential a value for the *reduced lifetime* T_R (T_R^{-1} in DGR's notation). The reduced lifetime is related to $\tau_{n\bar{n}}$ through

$$\tau_{n\bar{n}}^2 = T T_R, \quad (1.1)$$

where T is the mean lifetime of the nucleus. DGR obtain a set of values for T_R in the range

$$0.63 \times 10^{-23} \text{ s} \leq T_R \leq 2.17 \times 10^{-23} \text{ s}. \quad (1.2)$$

For each value of T_R they infer a limit on $\tau_{n\bar{n}}$ by using the IMB [2] limit for the mean lifetime T of ^{16}O , namely,

$$T > 2.4 \times 10^{31} \text{ y} \quad \text{at 90\% CL},$$

from which they obtain for $\tau_{n\bar{n}}$ the following range of values,

$$7 \times 10^7 \text{ s} \leq \tau_{n\bar{n}} \leq 13 \times 10^7 \text{ s}. \quad (1.3)$$

Two comments are appropriate here: first, in going from T_R to $\tau_{n\bar{n}}$, DGR have neglected the statistical uncertainty in the value of T_R arising from statistical uncertainty in the parameters of the corresponding optical potential. Their interval for $\tau_{n\bar{n}}$ therefore reflects the magnitude of the bias introduced in the calculation of $\tau_{n\bar{n}}$ when prechosen functional forms are used for potentials whose radial form is *a priori* unknown. The second point is that the DGR interval for $\tau_{n\bar{n}}$, being essentially a systematic error interval and not one reflecting statistical error, cannot for this reason be incorporated into the IMB data in such a way as to lead to a unique statistical limit at a specified level of confidence.

In this paper we apply the formalism of DGR in a way which leads to a unique limit. The paper is organized as follows: in Section 2 we summarize the features of DGR's method; in Section 3 we describe our calculation of T_R , whilst Section 4 explains our method of correcting the value of T_R , because we have used the potential for ($\bar{p}, ^{12}\text{C}$) rather than that for ($\bar{n}, ^{15}\text{O}$). Section 5 deals with the statistical analysis leading to a lower limit on the value of $\tau_{n\bar{n}}$. Our conclusions are given in Section 6.

2. THE FORMALISM

The basic assumption of the DGR method is that a non-relativistic approach is adequate for describing $n\bar{n}$ oscillations in nuclei. A further assumption is that the nucleus ^{16}O can be described as a two-body system $(n, ^{15}\text{O})$ with reduced mass $\mu = 880.8 \text{ MeV}$, in which an effective mean field interaction exists between the neutron or antineutron and the remaining 15 nucleons. This system is described by the following coupled SE equations:

$$i\hbar \partial/\partial t \begin{pmatrix} \psi_n \\ \psi_{\bar{n}} \end{pmatrix} = \begin{pmatrix} -\hbar^2 \nabla^2/2\mu + V_n & \delta m \\ \delta m & -\hbar^2 \nabla^2/2\mu + V_{\bar{n}} + iW \end{pmatrix} \begin{pmatrix} \psi_n \\ \psi_{\bar{n}} \end{pmatrix}, \quad (2.1)$$

where V_n and $V_{\bar{n}} + iW$ are the neutron and antineutron potentials, respectively, and δm , assumed constant, is the $(\Delta B = 2)$ interaction energy.

An important additional assumption is that the nuclear medium does not greatly alter the overall strength of the $(\Delta B = 2)$ interactions; consequently, the magnitude of δm in the nucleus will be close to that of δm in free space. The nucleus becomes unstable because of the antineutron annihilation represented by the term $iW(r)$. Writing in the usual way

$$\psi(r,t) = \phi(r) \exp [(E - i\Gamma/2)t/i\hbar]$$

with

$$\phi = R(r)Y(\theta, \phi) = 1/r U(r) Y(\theta, \phi)|_{n, \bar{n}},$$

and working with the dimensionless quantities

$$x = kr \quad (k^2 = 2\mu E_c/\hbar^2, \text{ with } E_c \text{ such that } k^2 = 1 \text{ fm}^{-2}, \text{ i.e. } E_c = 22.104 \text{ MeV}),$$

$$v_n = V_n/E_c; \quad v_{\bar{n}} + iw = (V_{\bar{n}} + iW)/E_c; \quad \eta_{\ell j}^2 = |E_{\ell j}|/E_c,$$

$$u_n(x) = U_n(r),$$

$$u_{\bar{n}}(x) = U_{\bar{n}}(r)(E_c/\delta m),$$

leads to

$$\{d^2/dx^2 - [\ell(\ell+1)/x^2 + v_n(x) + \eta_{\ell j}^2]\} u_n(x) = 0, \quad (2.2)$$

$$\{d^2/dx^2 - [\ell(\ell+1)/x^2 + v_{\bar{n}}(x) + iw(x) + \eta_{\ell j}^2]\} u_{\bar{n}}(x) = u_n(x), \quad (2.3)$$

when we neglect terms of $O(\delta m^2)$ noting that the amplitude $\phi_{\bar{n}}(r)$ is expected to be of $O(\delta m)$, as may be shown by considering a model with constant potentials.

The antineutron annihilation rate $\Gamma = 1/T$ is given by

$$\Gamma = T_R/\tau_{n\bar{n}}^2$$

where

$$T_R = 2\tau_c \int_0^\infty \langle |u_{\bar{n}}|^2 \rangle w(x) dx, \quad (2.4)$$

with

$$2\tau_c = 2\hbar/E_c = 5.956 \times 10^{-23} \text{ s}.$$

The brackets $\langle \rangle$ denote the average

$$\langle |u_{\bar{n}}|^2 \rangle = (2|u_{\bar{n}}|^2 s_{1/2} + 4|u_{\bar{n}}|^2 p_{3/2} + 2|u_{\bar{n}}|^2 p_{1/2})/8, \quad (2.5)$$

which is the average single-particle density distribution for the antineutron. The DGR procedure is to solve the homogeneous SE for the neutron wave function with shell energy $E_{\ell j}$ and then to solve the corresponding inhomogeneous SE for the induced antineutron wave function. For the neutron, DGR

use a Woods–Saxon (WS) potential

$$V_n(r) = -V_{\ell j}/\{1 + \exp [(r - R)/a]\} \quad (2.6)$$

for each (ℓ, j) state. Taking $R = 2.545$ fm, $a = 0.5$ fm, they choose the coefficient $V_{\ell j}$ so as to reproduce the correct shell energy $E_{\ell j}$. Their parameters are given in Table 1. The anti-neutron–nucleus interaction is described by a phenomenological optical potential.

3. CALCULATION OF T_R

3.1 Numerical procedure

Following DGR we keep only the lowest-order terms in the coupled equations, thereby decoupling them into an homogeneous equation for the $(n, {}^{15}\text{O})$ system and an inhomogeneous equation for the system $(\bar{n}, {}^{15}\text{O})$. In order to be able to compare our calculation with theirs, we use the parameters of table 1 for ${}^{16}\text{O}$.

In this work we solve both SEs using Numerov’s method to integrate outward from $x = 0$ [9]. We have checked our numerical algorithm by comparing the numerical solutions with analytical solutions based on several test potentials.

We seek the solution $u_{\bar{n}}(x)$ of the inhomogeneous equation (2.3), which vanishes both at the origin and at infinity. This solution can of course be obtained in many ways; the following procedure is convenient. Given *any* solution $Y(x)$ of the inhomogeneous equation and the solution $C(x)$ of the associated homogeneous equation, we obtain the antineutron wavefunction from the formula

$$u_{\bar{n}}(x) = Y(x) - \alpha C(x), \quad (3.1)$$

with $\alpha = \lim_{x \rightarrow \infty} Y(x)/C(x)$ as $x \rightarrow \infty$, provided that both $Y(x)$ and $C(x)$ are zero at $x = 0$. The expression for α follows from the fact that $C(x)$ diverges as $x \rightarrow \infty$. This is so because the ${}^{16}\text{O}$ shell energies are not eigenvalues of the $(\bar{n}, {}^{15}\text{O})$ system.

3.2 Choice of the $(\bar{n}, {}^{15}\text{O})$ optical potential

An unbiased limit for $\tau_{n\bar{n}}$ can be arrived at by using a bias-free optical potential whose statistical uncertainty as a function of the radial distance is well determined. Therefore, use of the prechosen functional forms for the potential is immediately ruled out. A potential with the required properties has been obtained [10–12] from fits to the $(\bar{p}, {}^{12}\text{C})$ scattering data at 47 MeV obtained at the CERN Low-Energy Antiproton Ring (LEAR) [13]. Regrettably, no data exist for the system $(\bar{p}, {}^{16}\text{O})$ at the same energy. In the following we therefore use for $(\bar{n}, {}^{15}\text{O})$ the potential obtained for $(\bar{p}, {}^{12}\text{C})$. A correction factor for T_R arising from the difference in mass number A is estimated in the next section.

The $(\bar{p}, {}^{12}\text{C})$ potential is written as a finite Fourier–Bessel (FB) series

$$V(r) = \sum_{n=1}^N a_n j_0(n\pi r/R_c), \quad r \leq R_c, \quad (3.2a)$$

$$V(r) = 0, \quad r > R_c, \quad (3.2b)$$

with complex coefficients $\{a_n\}$. The cut-off radius $R_c \approx 7$ fm, and N is typically equal to 3. The potential, together with error bars, is shown in fig. 1. It is clear that the potential is well determined only at the extreme surface of the nucleus.

3.3 Monte Carlo calculation of the T_R distribution

If we knew the functional dependence of T_R on the coefficients $\{a_n\}$ we could in principle calculate analytically the error in T_R from the errors in the coefficients. However, we have no

analytical formula for the function $T_R(a_1, a_2, \dots)$, and even if we did, the large relative errors in the values of the coefficients would render invalid the usual propagation of error formulae; nor could we assume *a priori* that the probability density distribution of T_R would be normal. In general for complicated multiargument functions, even when the explicit functional form is known, it is more satisfactory and in fact simpler to calculate the correct distribution of the function by a Monte Carlo procedure.

We therefore adopt the following procedure to propagate the errors in the coefficients $\{a_n\}$ through to the quantity T_R . Using the covariance matrix of the $\{a_n\}$, we generate a random vector of coefficients $(a_1, a_2, \dots)$ according to a multivariate normal distribution [14]. With these coefficients we compute T_R by solving the inhomogeneous equation with the corresponding FB potential. Another random vector is generated and the corresponding value of T_R is calculated; this procedure is repeated ≈ 2000 times.

The FB potential shown in fig. 1 is the result of averaging over several fits to the 47 MeV $(\bar{p}, {}^{12}\text{C})$ scattering data with differing values of the cut-off parameter R_c . For simplicity, however, we use a single two-term FB potential with the parameters listed in table 2. This gives a good representation of the FB potential of fig. 1, as is clear from inspection of fig. 2. In fig. 3 we show for both the neutron and the antineutron the calculated average single-particle density $\langle |u(x)|^2 \rangle$. The resulting distribution of T_R has a mean and a standard deviation of, respectively,

$$T_R = 1.638 \times 10^{-23} \text{ s} \quad (3.3a)$$

and

$$\sigma = 0.464 \times 10^{-23} \text{ s} . \quad (3.3b)$$

4. CORRECTION OF T_R

In the previous section we used the potential appropriate for $(\bar{n}, {}^{12}\text{C})$; however, we actually need the potential for $(\bar{n}, {}^{15}\text{O})$. Here we correct T_R to account for the difference in mass number A .

The correction factor is estimated from two values of T_R : one is obtained using a particular representation of the $(\bar{p}, {}^{12}\text{C})$ potential, whilst the other is obtained using a potential for $(\bar{n}, {}^{15}\text{O})$ derived by extrapolation from the former potential. The extrapolation is effected by writing the optical potential in terms of a phenomenological antinucleon-nucleon interaction coupled to the nuclear matter distribution. We assume that the $\bar{n}$ -nucleus optical potential can be modelled by the following finite-range Gaussian (FRG) potential [11]:

$$V(r) = (V_g + iW_g) \int \exp[-(\mathbf{x} - \mathbf{r})^2/a_g^2] \varrho(\mathbf{x}) d^3\mathbf{x} , \quad (4.1)$$

where (V_g, W_g, a_g) are the interaction parameters assumed constant in the range $12 \leq A \leq 15$, and $\varrho(r)$ is the nuclear matter distribution normalized to A . We approximate $\varrho(r)$ by a WS form with diffusivity parameter $a = 0.5$ fm and radius parameter $R = 1.0234 A^{1/3}$ fm, i.e. $R = 2.343$ fm for ${}^{12}\text{C}$ and 2.524 fm for ${}^{15}\text{O}$. The parameters of the FRG potential are estimated by fitting the latter to the FB potential derived from the $(\bar{p}, {}^{12}\text{C})$ scattering data. The parameters obtained are

$$V_g = 7.5 \text{ MeV} , \quad (4.2a)$$

$$W_g = 20.6 \text{ MeV} , \quad (4.2b)$$

$$a_g = 1.6 \text{ fm} . \quad (4.2c)$$

Using the same parameter set (V_g, W_g, a_g) we compute the FRG potential for $(\bar{n}, {}^{15}\text{O})$. For the $(\bar{p}, {}^{12}\text{C})$ and $(\bar{n}, {}^{15}\text{O})$ FRG potentials we obtain $T_R(12) = 1.842 \times 10^{-23}$ s and $T_R(15) = 2.016 \times 10^{-23}$ s, respectively. We take as our estimate of the correction factor for T_R the ratio $T_R(15)/T_R(12) = 1.094$.

A simpler extrapolation procedure is merely to fit a WS form to the ($\bar{p}$, ^{12}C) FB potential and to obtain the corresponding WS form for ($\bar{n}$, ^{15}O) by scaling the radius parameters by the factor $(15/12)^{1/3}$. With this method we obtain $T_R(12) = 1.796 \times 10^{-23}$ s and $T_R(15) = 1.923 \times 10^{-23}$ s. These values for T_R are in good agreement with those given above, suggesting that T_R is not too sensitive to the manner in which the extrapolation is performed.

We note that the correction factor is close to unity; moreover, it is the square root of this factor which appears in the final value of $\tau_{\bar{n}\bar{n}}$. Therefore we conclude that for the problem addressed here we are in fact justified in using the potential derived from the ($\bar{p}$, ^{12}C) scattering data.

5. RESULTS

The mean lifetime T of a nucleus can be written as

$$T = \epsilon N_0 T_0 / \theta, \quad (5.1)$$

where ϵ , N_0 , T_0 , and θ are respectively the overall $n \rightarrow \bar{n}$ detection efficiency, the total number of neutrons under observation, the observation time, and the mean number of observable $n \rightarrow \bar{n}$ events. From the IMB paper [2],

$$\epsilon = 0.50, \quad (5.2a)$$

$$N_0 = 8.9 \times 10^{32}, \quad (5.2b)$$

$$T_0 = 0.36 \text{ y}, \quad (5.2c)$$

and the number of candidate events $D = 3$. With the above parameters we can write $\tau_{\bar{n}\bar{n}}$ as follows:

$$\tau_{\bar{n}\bar{n}}^2 = K(T_R/\theta), \quad (5.3)$$

where K is numerically equal to $5.05 \times 10^{16} \text{ s}^2$ with T_R in units of 10^{-23} s. The corresponding values for Kamiokande are $D = 0$ events and $K = 3.12 \times 10^{16} \text{ s}^2$.

We now proceed to set a lower limit on the quantity T_R/θ . Applying the correction factor to T_R gives

$$T_R = 1.79, \quad (5.4a)$$

$$\sigma = 0.51, \quad (5.4b)$$

as our estimate of the reduced lifetime of ^{16}O . A good fit (with $\chi^2/n_D = 44.1/34$) to the corrected T_R distribution function $P(T_R)$ is obtained with

$$P(T_R) = Q[\omega G(T_R/\mu_1, \sigma_1) + (1 - \omega)G(T_R/\mu_2, \sigma_2)], \quad (5.5)$$

where

$$G(x|\mu, \sigma) = \{\exp[-(x - \mu)^2/2\sigma^2]\}/\sqrt{2\pi}\sigma$$

and

$$Q = 192.2; \quad \omega = 0.493,$$

$$\mu_1 = 1.919; \quad \sigma_1 = 0.281,$$

$$\mu_2 = 1.672; \quad \sigma_2 = 0.595.$$

The fit is shown in fig. 4. The 68.3% central error interval

$$1.35 \leq T_R \leq 2.23 \quad (5.6)$$

for this distribution replaces the systematic error interval of DGR, i.e. eq. (1.2). If we neglect the statistical error in T_R we can obtain a 90% lower limit on the ratio T_R/θ , namely

$$T_R/\theta > 0.27, \quad (5.7)$$

by using for θ the upper limit of 6.68 corresponding to the three candidate events observed by the IMB Collaboration.

In trying to take account of the error in T_R we are confronted with an unsolved problem — that of setting classical limits on the ratio of numbers drawn from two different distributions. Here, the numerator T_R is drawn from the distribution $P(T_R)$, whilst the denominator D is a Poisson variate. Our method of incorporating the T_R error into the limit for the ratio T_R/θ is therefore approximate. It is based upon the following observation: if N and D are integers distributed according to Poisson distributions, a procedure exists whereby exact classical limits can be obtained for the ratio $F = \lambda/\theta$, where $\lambda = \langle N \rangle$ and $\theta = \langle D \rangle$ [15]. We therefore approximate the corrected distribution of T_R by a discrete distribution, namely the distribution of the quantity ηN . With the values $\eta = 0.105$ and $\langle N \rangle = 17.43$ we obtain a reasonable discrete representation of the T_R distribution. An integer N is then chosen so that

$$T_R = \eta N \quad (5.8)$$

is satisfied as closely as possible. We find $N = 17$. The distribution of the quantity ηN is shown, in fig. 5, superimposed on the corrected distribution for T_R . From the pair of integers (N, D) a lower limit on the ratio $F = \lambda/\theta$ is obtained as the solution of

$$\sum_{j=0}^{N-1} [(N+D)!/(N+D-j)!j!] p^j (1-p)^{N+D-j} = \beta, \quad (5.9)$$

where $p = \lambda/(\lambda + \theta)$, i.e. $F = p/(1-p)$, and β is the confidence level.

For $(N, D) = (17, 3)$ we obtain for $\beta = 0.90$,

$$F > 2.29,$$

and hence

$$T_R/\theta = \eta F > 0.24. \quad (5.10)$$

A comparison of the above lower limit with that obtained earlier [eq. (5.7)] shows that the effect of the statistical uncertainty in the value of T_R is equivalent to increasing the upper limit on θ by 12%, that is from 6.7 to 7.5 events, or lowering the limit on $\tau_{n\bar{n}}$ by 6%. Therefore, from the IMB results we extract the following limit for the neutron-antineutron mixing time:

$$\tau_{n\bar{n}} > 1.1 \times 10^8 \text{ s} \quad \text{at 90\% CL.} \quad (5.11)$$

The Kamiokande Collaboration, however, observe no events and therefore $(N, D) = (17, 0)$. This gives the limit

$$F > 6.89,$$

that is

$$T_R/\theta > 0.72,$$

and therefore

$$\tau_{n\bar{n}} > 1.5 \times 10^8 \text{ s} \quad \text{at 90\% CL.} \quad (5.12)$$

6. CONCLUSIONS

The currently available $\bar{p}$ -atom and $\bar{p}$ -nucleus scattering data provide reliable information essentially for the surface of nuclei. When potentials with prechosen functional forms are used to analyse these data, the results are inevitably biased by virtue of the predetermined analytical continuation of the potential from the surface to the centre of the nucleus. Consequently, the central region of the nuclear potential can appear to be well determined when in fact it is not. However, for observables which involve a radial integral over the potential, it is necessary to have realistic

estimates of the errors as a function of the radial distance in order to have realistic estimates of how well we can measure such observables. The $n\bar{n}$ mixing time is a case in point. In this paper we have correctly propagated the statistical uncertainty in the $\bar{n}$ -nucleus optical potential for $(\bar{n}, {}^{15}\text{O})$ through to $\tau_{n\bar{n}}$ and have thereby derived a meaningful statistical limit. However, despite the very large statistical uncertainty in the form of the potential within the nucleus we have obtained a limit in agreement with the range of values given by DGR [8]. The limit is just at the lower bound of the current SO(10) predictions [1] for the $n\bar{n}$ mixing time.

We have shown that the effect of statistical uncertainty in the $\bar{n}$ -nucleus potential is to reduce the limit on $\tau_{n\bar{n}}$ by $\approx 6\%$ relative to the limit obtained when this extra source of statistical uncertainty is excluded.

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REFERENCES

- [1] D. Chang, R.N. Mohapatra, J.M. Gipson, R.E. Marshak and M.K. Parida, Phys. Rev. **D31** (1985) 1718.
- [2] T.W. Jones et al. (IMB Collaboration), Phys. Rev. Lett. **52** (1984) 720.
- [3] M. Takita et al. (Kamiokande Collaboration), UT-ICEPP-86-04 (1986).
- [4] G. Fidecaro et al. (CERN-ILL-Padua-RAL-Sussex Collaboration), Phys. Lett. **156B** (1985) 122.
- [5] P.G.H. Sandars, J. Phys. **G6** (1980) L161.
- [6] W.M. Alberico, J. Bernabeu, A. Bottino and A. Molinari, Nucl. Phys. **A429** (1984) 445.
- [7] C.B. Dover, A. Gal and J.M. Richard, Phys. Rev. **D27** (1983) 1090.
- [8] C.B. Dover, A. Gal and J.M. Richard, Phys. Rev. **C31** (1985) 1423.
- [9] J.P. Killingbeck, Microcomputer quantum mechanics (A. Hilger, Bristol, 1983), p. 127.
- [10] C.J. Batty, E. Friedman and J. Lichtenstadt, Phys. Lett. **142B** (1984) 241.
- [11] C.J. Batty, E. Friedman and J. Lichtenstadt, Nucl. Phys. **A436** (1985) 621.
- [12] E. Friedman and J. Lichtenstadt, Nucl. Phys. **A455** (1986) 573.
- [13] D. Garreta et al., Phys. Lett. **135B** (1984) 266.
- [14] F. James, Rep. Prog. Phys. **43** (1980) 1145; and private communication.
- [15] F. James and M. Roos, Nucl. Phys. **B172** (1980) 575.

Table 1

Shell energies and associated potential strengths for ^{16}O
 $(R = 2.545 \text{ fm}; a = 0.5 \text{ fm})$

State	$S_{1/2}$	$P_{3/2}$	$P_{1/2}$
$E_{\ell_j} \text{ (MeV)}$	40.0	21.85	15.67
$V_{\ell_j} \text{ (MeV)}$	69.9	75.3	65.6

Table 2

FB coefficients and covariance matrix for two-term FB potential
 $(R_c = 6.410 \text{ fm})$

Coefficients $\{a_n\}$				
	Re a_1	Re a_2	Im a_1	Im a_2
	7.9210	7.2657	20.2766	18.7007
Covariance matrix of $\{a_n\}$				
	Re a_1	Re a_2	Im a_1	Im a_2
Re a_1	13.5	22.5	-4.5	-9.0
Re a_2	22.5	45.0	-18.0	-27.0
Im a_1	-4.5	-18.0	54.0	99.0
Im a_2	-9.0	-27.0	99.0	198.0

Figure captions

- Fig. 1** FB potential obtained from fits to the 47 MeV ($\bar{p}$, ^{12}C) scattering data.
- Fig. 2** a) Fit of two-term FB potential to the FB potential obtained from the 47 MeV ($\bar{p}$, ^{12}C) scattering data.
b) Two-term FB potential with error bars computed from the parameters given in table 2.
- Fig. 3** Average single-particle density $\langle |u(x)|^2 \rangle$ for n and $\bar{n}$ in ^{16}O . The error bars shown for the $\bar{n}$ density reflect the uncertainty in the $\bar{n}$ -nucleus potential.
- Fig. 4** Two-Gaussian fit to corrected T_R distribution.
- Fig. 5** ηN distribution, with $\eta = 0.105$ and $\langle N \rangle = 17.43$ (dashed curve), superimposed on corrected T_R distribution (continuous curve).

Fig. 1

Fig. 2

Fig. 3

Fig. 4

Fig. 5