CERN-TH.4563/86

SPACE-TIME VERSUS WORLD-SHEET SUPERSYMMETRY IN THE HETEROTIC STRING

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ABSTRACT

We discuss the relation between the Neveu-Schwarz-Ramond and the Green-Schwarz formulations of superstrings at higher orders in α' . On a Kähler background, renormalization of the type IIB superstring will preserve all world-sheet and space-time supersymmetries provided the conditions for the cancellation of conformal anomalies are satisfied. On the other hand, in the heterotic string theory the renormalization scheme must respect Yang-Mills covariance. The differing constraints on the renormalization schemes for the type IIB and heterotic theories account for the finite renormalization of the embedding condition needed to pass between the two theories. This accounts for the preservation of $E_6 \times E_8$ as the unbroken symmetry in the renormalized Calabi-Yau compactification, despite the internal manifold's having $U(3)$ holonomy.

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To lowest order in α' , six-dimensional Ricci-flat Kähler manifolds provide compactifying solutions to the effective field equations of the heterotic string theory. The Ricci-flat Kähler condition is also sufficient to guarantee four-dimensional space-time supersymmetry in the compactified theory to this order in α' ¹⁾. The Ricci-flat Kähler solutions in Ref. 1) were obtained by choosing an embedding of the tangent space group into the Yang-Mills group, setting the Yang-Mills connection A equal to the embedded spin connection ω and setting H_{ijk} to zero. Corrections to the effective field equations beginning at order α'^3 require that the condition of Ricci-flatness be modified²⁾⁻⁴⁾ to set the Ricci form equal to a cohomologically trivial form constructed from curvatures and their covariant derivatives⁵⁾. At the same order in α' , Yang-Mills covariance of the effective field equations requires that the Yang-Mills ansatz be modified as well if space-time supersymmetry is to be maintained⁶⁾. We shall be assuming throughout that the purely bosonic parts of the corrections to the supersymmetry transformations are those proposed in Refs. 4)-6).

The modifications to the $A = \omega$ "embedding condition" seem paradoxical at first glance, for if one had quantized the type IIB string, four-dimensional space-time supersymmetry would have been preserved on a Kähler background satisfying the condition for conformal invariance⁷⁾. The paradox arises from the fact that the (1,1) σ -model describing the type IIB string in the conformal gauge is obtained from the (1,0) σ -model describing the heterotic string in the conformal gauge by imposing $A = \omega$ ^{8),9)}. If $A = \omega$ classically, then the renormalized spin and gauge connections should remain equal if the quantization preserves (1,1) world-sheet supersymmetry. On the other hand, if Yang-Mills invariance is to be maintained in the heterotic theory while preserving space-time supersymmetry in the vacuum, the renormalized connections cannot be set equal. It is to this question that the present paper is addressed.

Let us first review the relation between world-sheet and space-time supersymmetry in the σ -models describing the type IIB theory; see Refs. 10) and 11) for details. We work in the light-cone gauge and consider curved eight-dimensional transverse spaces K_8 . In the Neveu-Schwarz-Ramond (NSR) formalism, the theory is described by a (1,1) supersymmetric σ -model with manifest world-sheet supersymmetry in which the world-sheet fermions carry a space-time vector index. On the other hand, space-time supersymmetry is most easily discussed in the Green-Schwarz (GS) formalism, using the σ -model of the general form discussed in Ref. 12), in which the world-sheet fermions are space-time spinors. For general transverse space-times, it is not clear how the two formalisms are related.

However, for space-times admitting covariantly constant spinors, there is a relation, employing the triality amongst the 8_v , 8_s and 8_c representations of $SO(8)$, the transverse Lorentz group¹⁰⁾.

Specifically, the 8_v NSR world-sheet Majorana spinor λ^a can be expressed in terms of the 8_s GS world-sheet spinor S using a real commuting 8_c spinor, world-sheet scalar ξ by

$$\lambda^a = \bar{\xi} \Gamma^a S, \quad (1)$$

where Γ^a is a Dirac matrix for $SO(8)$. Owing to triality, the relation (1) is invertible:

$$S = -\Gamma^a \xi \lambda^a, \quad (2)$$

provided ξ is normalized such that

$$\bar{\xi} \xi = 1. \quad (3)$$

If ξ is covariantly constant, then substituting (1) into the NSR σ -model action gives precisely the GS σ -model action of Ref. 12). For the Kähler-Ricci-flat spaces of Ref. 1), there are two such ξ and the NSR and GS formalisms are classically equivalent. However, the $O(\alpha'^3)$ solutions to the field equations are Kähler but not Ricci-flat, so there are no covariantly constant spinors. Nevertheless, there is a gauge-covariantly-constant spinor, η ⁶⁾. The gauge-covariant constancy condition for η is

$$\nabla_i \eta - \frac{i}{2} V_i \eta = 0 \quad (4)$$

in which V_i is a vector potential for the Ricci form P_{ij} , i.e., $P_{ij} \equiv \frac{1}{2} R_{ijkl} J^{kl} = \partial_i V_j - \partial_j V_i$, where J^{kl} is the complex structure on the Kähler manifold. We shall consider an arbitrary Kähler K_8 and examine the consequences of taking ξ in Eqs. (1)-(3) to be the real part of η . In particular, substituting (1) into the NSR action will give a GS action involving explicit complex structures.

In the NSR formation, for Kähler K_8 , there is a (2,2) world-sheet supersymmetry parametrized by the world-sheet Majorana spinors - $SO(8)$ scalars α^1, α^2 . There is also a shift symmetry¹³⁾

$$\delta \lambda_L^a = \beta_L^a, \quad \delta X^i = 0, \quad (5)$$

provided β^a is covariantly constant. Each covariantly constant vector on K_6 gives two such symmetries, since the left- and right-handed parts of λ (λ_L, λ_R) can be shifted independently. For compactifications to four-dimensional Minkowski space, $K_8 = K_6 \times R^2$, there are two covariantly constant vectors corresponding to the flat directions, and so four β -symmetries.

On substituting (1) into the NSR action, the resulting GS action is invariant under the δ - and ε -supersymmetries

$$\begin{aligned} \delta_\delta S &= \delta, & \delta_\delta X^i &= 0 \\ \delta_\varepsilon X^i &= \bar{\varepsilon}_L \Gamma^i S_R + \bar{\varepsilon}_R \Gamma^i S_L \\ \delta_\varepsilon S_L &= \frac{1}{4} (\delta_\varepsilon X^i) \tilde{\omega}_i^{ab} \Gamma_{ab} S_L + i \bar{\varepsilon}_R \Gamma^i \partial_- X^i \\ \delta_\varepsilon S_R &= \frac{1}{4} (\delta_\varepsilon X^i) \tilde{\omega}_i^{ab} \Gamma_{ab} S_R + i \bar{\varepsilon}_L \Gamma^i \partial_+ X^i \end{aligned} \quad (6)$$

where we have set $B_{ij} = 0$ for simplicity, $\partial_\pm = \partial_\tau \pm \partial_\sigma$, $\delta = -(\Gamma^i \xi) \beta_i$ and $\varepsilon = \alpha \xi$. We have derived the transformation rules (6) from those of the NSR action using (1) and (2); ξ is the real part of η , which satisfies (4), and $\tilde{\omega}_i^{ab} = \omega_i^{ab} - \frac{1}{2} V_i J^{ab}$. In the Ricci-flat case, the transformations (6) reduce to those of Refs. 10)-13). On $R^2 \times K_6$, there will be four ε and four δ supersymmetries - and hence $N = 2$ space-time supersymmetry in $d = 4$ - whenever there is a spinor ξ that can be used to generate a triality transformation. This spinor corresponds to the Killing spinor of the effective field theory analysis [cf. Refs. 1), 4) and 6)].

In the preceding discussion we have deliberately not specified whether the Kähler space K_6 is Ricci flat. This generalizes the discussion of Refs. 10) and 11) for the Ricci-flat case. In that case, the transition between the NSR and GS formalisms given above can be stated group theoretically¹⁾ by noting that, for manifolds $R^2 \times K_6$ of $SU(3)$ holonomy, the 8_v and 8_s representations of $SO(8)$ decompose identically into $3 + \bar{3} + 1 + 1$. In the present more general case, the

holonomy group is $U(3)$ and we need to keep track of the additional $U(1)$ charges. Now the 8_v decomposes as $3_2 + \bar{3}_{-2} + 1_0 + 1_0$, while the 8_s decomposes as $3_{-1} + \bar{3}_1 + 1_3 + 1_{-3}$. So, for a non-Ricci-flat manifold K_6 , we need to translate between the two using $SU(3)$ singlets of $U(1)$ charges ± 3 . These are provided by the gauge-covariantly constant 8_c spinors η, η^* , which are the singlets in the decomposition of 8_c into $3_0 + \bar{3}_0 + 1_3 + 1_{-3}$.

The above considerations have not taken into account the effects of renormalization of the σ -model perturbation theory in powers of α' . For the NSR formalism, $(1,0)$ world-sheet supersymmetry relates all of the renormalizations to those of the background metric $g_{ij}(X)$, the antisymmetric tensor $B_{ij}(X)$ and the Yang-Mills gauge connection $A_i(X)$. Requiring conformal invariance then gives the effective field equations for the renormalized background, and these determine that the metric is not Ricci-flat. In the GS formalism, on the other hand, one does not have the constraint of manifest world-sheet supersymmetry, and in principle one should impose conditions for conformal invariance separately for all the couplings of the theory. Comparison with the NSR formalism, where the renormalization is much more controllable, shows what the structure of the bare GS superstring action must be.

The bare type IIB NSR action (with $B_{ij} = 0$) is

$$I_{\text{NSR}} = \frac{1}{4\pi\alpha'} \int d^2\sigma \left\{ g_{ij} \partial_+ X^i \partial_- X^j + i \lambda_L^a \nabla_+ \lambda_L^a + i \lambda_R^a \nabla_- \lambda_R^a + \frac{1}{2} R_{abcd} \lambda_L^a \lambda_L^b \lambda_R^c \lambda_R^d \right\} \quad (7)$$

Using (1) and (4), one obtains the corresponding bare GS action

$$I_{\text{GS}} = \frac{1}{4\pi\alpha'} \int d^2\sigma \left\{ g_{ij} \partial_+ X^i \partial_- X^j + i \bar{S}_L \tilde{\nabla}_+ S_L + i \bar{S}_R \tilde{\nabla}_- S_R \right. \\ \left. + \frac{1}{32} (R_{ijkl} - \frac{1}{2} J_{ij} P_{kl} - \frac{1}{2} J_{kl} P_{ij} + \frac{1}{8} R J_{ij} J_{kl}) \bar{S}_L \Gamma^{ij} S_L \bar{S}_R \Gamma^{kl} S_R \right\} \quad (8)$$

which is invariant under the supersymmetry transformations (6), where

$$\tilde{\nabla}_\pm \equiv \partial_\pm - \frac{1}{4} \tilde{\omega}_k^{ab} \Gamma_{ab} \partial_\pm X^k = \nabla_\pm + \frac{1}{8} \Gamma^{ij} J_{ij} V_k \partial_\pm X^k \quad (9)$$

Note that on a Ricci-flat background the GS action (8) reduces to the standard GS action discussed in Ref. 1). For the non-Ricci-flat backgrounds that give conformal invariance to order α'^3 , the new terms in (8) involving explicit complex structures do not vanish, but are of order α'^3 .

The action (8) is invariant under the ϵ - and δ -supersymmetries (6). The complex structures appearing in (8) mean that it, unlike (7), is not universal in that it cannot be written down for arbitrary backgrounds. For the purposes of the present paper, the importance of the GS formalism is that it establishes a link between the world-sheet supersymmetries of the NSR formalism and the space-time supersymmetry of the vacuum. Thus, for now it is sufficient that we can write a supersymmetric GS action for a class of backgrounds (i.e., Kähler) that includes those of interest here. Hence, for the type IIB superstring compactified on a Calabi-Yau space with higher-order corrections²⁾⁻⁶⁾, renormalization consistent with (2,2) world-sheet supersymmetry yields ϵ - and δ -supersymmetries in the GS model, and so $N = 2$ space-time supersymmetry in the compactified four-dimensional theory.

For the heterotic string, similar considerations apply. We again derive a GS action from the NSR action by using (1) and (4). Just as our type IIB GS action (8) contains additional terms involving the complex structure tensor that are absent in the Ricci-flat case¹⁰⁾⁻¹³⁾, so here our heterotic GS action will contain extra terms. These will include, for example, quartic fermion terms involving $F_{kl}^A J^{kl} J_{ij}$.

For the heterotic σ -model, there are no right-handed α - or β -supersymmetries. For (2,0) world-sheet (α) supersymmetry, K_6 must be Kähler, and we must have⁸⁾

$$J_{[i}^k F_{j]k}^A = 0 \quad (10)$$

Note that for manifolds of vanishing first Chern class, there is no anomaly in (2,0) supersymmetry⁸⁾. For β_L supersymmetry (5), in addition to $\nabla_i \beta_L^a = 0$, one also needs

$$F_{ij}^A \beta_L^j = 0 \quad (11)$$

The (2,0) α -supersymmetry and the two β -supersymmetries become two ϵ - and two δ -supersymmetries in our heterotic GS action upon using (1) and (2) to

rewrite the transformation rules in terms of the GS variables. Note that, by construction, the GS action has these ϵ -supersymmetries for arbitrary Kähler backgrounds and Yang-Mills configurations satisfying the hermiticity condition (10). It is known from the structure of the theory at lowest order in α' that $N = 1$ space-time supersymmetry of the vacuum follows from the ϵ - and δ -supersymmetries of the GS action provided the conditions for conformal invariance are met and the gauge field satisfies an additional condition, which is to lowest order

$$F_{ij}^A \Gamma^{ij} \epsilon = 0 \quad (12)$$

This latter condition appears in the effective theory as the requirement that the gluino vanish in the vacuum, since the leading term of the gluino transformation is

$$\delta \chi^A = -\frac{1}{4} F_{IJ}^A \Gamma^{IJ} \epsilon \quad (13)$$

where ϵ now denotes the ten-dimensional space-time supersymmetry parameter.

As with the conditions for conformal invariance, the gluino transformation (13) and hence the vacuum condition (12) for the gauge field will acquire corrections at higher orders in α' . These corrections will imply corrections to the Yang-Mills effective field equations and thus could also be studied directly from the requirement of conformal invariance in the Yang-Mills sector. Either way, one finds that the corrections to the Ricci-flat Kähler lowest-order solutions of Ref. 1) require a modification at $O(\alpha'^3)$ to the original $A = \omega$ embedding condition. Our task now is to understand how these modifications to the embedding condition arise.

In order to preserve the $N = 1$ space-time supersymmetry in the four-dimensional vacuum for the heterotic theory, it is necessary to renormalize the corresponding σ -model so as to preserve the (2,0) world-sheet supersymmetry as well as the Yang-Mills covariance. Now, one could alternately have passed before renormalization to the type IIB theory by the original $A_i^{ab} = \omega_i^{ab}$ embedding of the spin connection in the Yang-Mills gauge group. After renormalization, there must be some embedding condition that converts the heterotic into the type IIB σ -model. We have seen above that such a passage to the type IIB theory appears to be consistent with $N = 1$ space-time supersymmetry of the four-dimensional vacuum; indeed, the type IIB theory yields an $N = 2$ four-dimensional supersymmetry.

To see that the embedding condition for the renormalized theory must acquire corrections with respect to the original $A = \omega$ condition, we shall now amplify the discussion of the effective field equations given in Ref. 6). Embedding changes a (1,0) α -supersymmetric NSR action into a (1,1) α -supersymmetric NSR action together with 24 free right-handed spinors. Since this process is more general than the passage from (2,0) to (2,2) NSR actions discussed above, it is clear that any corrections to the embedding condition must be consistent with a general Riemannian structure of the background. In other words, the corrections to the embedding condition must be consistent with universality, and in particular must not involve complex structures.

It is easiest to see the need for corrections to the $A = \omega$ embedding condition in the $E_8 \times E_8$ heterotic string. In order for the embedding condition to be consistent with $N = 1$ space-time supersymmetry of the four-dimensional effective field equations, the supersymmetry variation of the gluino must vanish in the vacuum. $E_8 \times E_8$ covariance of any possible corrections to the supersymmetry transformation (13) of the gluino imposes a very strong constraint. Equation (13) could receive corrections at order α'^3 of the generic forms $R^3 F \epsilon$, $R^2 F^2 \epsilon$, $R F^3 \epsilon$ and $F^4 \epsilon$, plus terms involving covariant derivatives. In these expressions, the products of Yang-Mills field strengths F_{IJ}^A will be contracted into group-invariant tensors, leaving one free adjoint group index. The relevant $E_8 \times E_8$ invariant tensors for the forms with two and three F 's are $\{C_{ABC}\}$ and $\{\delta_{AB} \delta_{CD}, \delta_{AC} \delta_{BD}, \delta_{AD} \delta_{BC}, C_{ABE} C_{CD}^E, C_{ACE} C_{BD}^E\}$. Note that for E_8 , by contrast with the orthogonal groups, there are only five rather than six independent four-index tensors, since E_8 has no independent fourth-order Casimir invariant.

To see that the original embedding condition $A_i^{ab} = \omega_i^{ab}$ on a six-dimensional Kähler space can no longer be maintained in the presence of the α'^3 corrections, consider making the original embedding and then projecting the corrected gluino transformation onto its $U(1)$ part. That is, upon embedding, $E_8 \times E_8$ is restricted to the $U(3)$ subgroup that is being identified with the holonomy group of the manifold, so the index A can be replaced by a $U(3)$ adjoint representation pair of antisymmetrized indices ab . The $U(1)$ projection is made by contracting the ab index pair with the complex structure J_{ab} . The problem with the original $A_i^{ab} = \omega_i^{ab}$ embedding can now be seen: all of the possible forms for the α'^3 corrections with 1, 2 or 3 F 's either give zero when contracted with J_{ab} in this way, or they give terms involving the Ricci form and are thus of higher order than α'^3 . On the other hand, on embedding and projection, the leading term (13) in $\delta \chi^{ab}$ gives

$-1/4 P_{ij} \Gamma^{ij} \epsilon$, which would have to be cancelled by order α'^3 corrections. The terms of type F^4 are of no help since upon embedding they become R^4 terms, while the Ricci form has the structure

$$P_{ij} = (\nabla_i \nabla_{\hat{j}} - \nabla_{\hat{i}} \nabla_j) S, \quad (14)$$

where $\nabla_{\hat{i}} = J_i^k \nabla_k$.

In the case of the $SO(32)$ heterotic superstring a similar conclusion can be reached, but the argument is complicated by the existence of an independent fourth-order Casimir invariant. It is possible in this case to construct an $SO(32)$ covariant term $\alpha'^3 D_I T_J^{ab} \Gamma^{IJ} \epsilon$ that cancels the $U(1)$ part of the leading term (13), where

$$T_I^{ab} = D^S F_{IK}^{ad} F_{ST}^{de} F^{TK eb} \quad (15)$$

However, there is more in (13) to cancel than just the contraction with J_{ab} on the group indices. Consideration of all possibilities shows that there is no $SO(32)$ covariant expression that cancels the full leading term (13) in the original $A = \omega$ embedding. Upon embedding, the leading term is proportional to $P^{ab} \epsilon$. The Ricci form has appeared because of the Kähler structure of the internal manifold and the covariant constancy of ϵ , as explained in Ref. 4), which enables us to replace $\Gamma^{ij} \epsilon$ by $J^{ij} \epsilon$ in (13). Starting from the Ricci form given in (14) and attempting to find an $SO(32)$ covariant object that could cancel it upon setting $A = \omega$, we find that there are terms arising from the commutation of covariant derivatives in which every index is connected via contraction or by pairing between curvature two-form or tangent space indices to a world index. Thus, such terms could never be cancelled by an $SO(32)$ covariant expression.

The above arguments have shown that the original $A = \omega$ embedding condition must be modified at order α'^3 . However, since we do not yet know the precise form of the order α'^4 corrections to the Yang-Mills equation, nor of the corresponding order α'^3 corrections to the supersymmetry transformation rules, we are not in a position to give the precise form of the embedding condition at this order. For example, our arguments above were insensitive to the presence of terms of the form $\alpha'^3 C^{ABE} C^{CD}{}_E D F^B D F^{CD}$ or $\alpha'^3 C^{ACE} C^{DB}{}_E D F^B D F^{CD}$ in the gluino

transformation rule. The embedding condition can only be determined precisely once such ambiguities are resolved. A related approach would be to calculate the order α'^4 Yang-Mills corrections to the effective action, and to determine the modifications to the embedding condition by requiring that the resulting Yang-Mills equation be satisfied.

We have seen that gauge covariance of the supersymmetry transformation rules for the $E_8 \times E_8$ heterotic string is a more powerful constraint than in the case of the $SO(32)$ heterotic string. However, the NSR action for the $E_8 \times E_8$ heterotic string has only the $SO(16) \times SO(16)$ subgroup as a manifest gauge symmetry. The remainder of the $E_8 \times E_8$ is implemented by operators that act non-linearly on the 32 fermionic variables, interchanging periodic fields with antiperiodic ones¹⁴⁾. Nevertheless, without using the details of how this works, one can make deductions about the allowable $SO(16) \times SO(16)$ structure of the effective field equations.

For example, we have seen that whereas $SO(16)$ has six independent four-index group-invariant tensors t^{ABCD} , E_8 has only five. Truncation of an $E_8 \times E_8$ covariant set of effective field equations to the fields of the $SO(16) \times SO(16)$ theory cannot involve the $SO(16)$ -invariant tensor that does not exist for E_8 . Conversely, renormalization of the $SO(16) \times SO(16)$ σ -model must be adjusted to avoid the appearance of the disallowed $SO(16)$ invariant in the β -functions. This will most likely involve a non-minimal subtraction, since otherwise one would expect all $SO(16)$ invariant to occur. For example, one might find that it was necessary to correct the Yang-Mills background gauge potential A_I^{ab} appearing in the NSR action by a term of the form $\alpha'^3 T_I^{ab}$ as given in (15). This term, which involves the use of the sixth $SO(16)$ -invariant tensor in its construction, would have to occur with a coefficient adjusted specifically so as to remove the unwanted invariant tensor from the Yang-Mills β -function at the α'^4 order. In other words, one may expect to need $SO(16) \times SO(16)$ -invariant finite local counterterms in the NSR action in order to achieve consistency with $E_8 \times E_8$ covariance.

We now return to the issue of space-time versus world-sheet supersymmetry and the necessary corrections to the embedding condition. This is also an issue of finite renormalization. The $SO(32)$ or $SO(16) \times SO(16)$ covariant σ -models must be renormalized to preserve their Yang-Mills symmetries. [This involves correcting the gauge transformation of B_{ij} , and adding Chern-Simons terms⁸⁾ to H_{ijk} , but these issues are not relevant in the present context.] On the other hand, had

one made the standard $A_i^{ab} = \omega_i^{ab}$ embedding at the outset to obtain (1,1) world-sheet supersymmetry, the renormalization would then have been required to preserve this (1,1) world-sheet supersymmetry. There is no reason to expect the Yang-Mills covariant and the (1,1) world-sheet-supersymmetric subtraction schemes to give identical results. The correction to the embedding condition needed in the Yang-Mills covariantly-renormalized theory can be understood as a finite renormalization of the embedding condition necessary to correct for the differences between the two subtraction schemes.

In other words, one could define a (1,1) world-sheet supersymmetric subtraction scheme for the heterotic theory by the requirement that one be able to pass to the (1,1) theory by making the original $A_i^{ab} = \omega_i^{ab}$ embedding, but this scheme would not respect the Yang-Mills covariance of the heterotic theory. Yang-Mills covariance could then be reobtained by the addition of finite local counterterms, but these would then violate the (1,1) world-sheet supersymmetry if one made the original $A_i^{ab} = \omega_i^{ab}$ embedding. In the Yang-Mills-covariantly renormalized heterotic theory, there does exist an embedding condition that allows one to pass to the (1,1) world-sheet-supersymmetric theory, but this embedding condition needs finite renormalization corrections. Thus, a (1,0) world-sheet-supersymmetric theory compactified on a six-dimensional Kähler space, together with a vacuum Yang-Mills potential satisfying the corrected embedding condition, will yield an $N = 1$ supersymmetric effective theory in four dimensions. The gauge group $E_8 \times E_8$ $[SO(32)]$ will then be broken down to $E_6 \times E_8$ $[SO(26) \times U(1)]$.

To conclude, we note from the definition $H = dB + \alpha' (\omega_{3Y} - \omega_{3L})$ that the vacuum values of H and B will be non-zero at order α'^4 by virtue of the correction to the embedding condition at order α'^3 . This indicates that there will be H -independent corrections to the effective equation for H at order α'^4 that for the first time do not vanish subject to the original $A_i^{ab} = \omega_i^{ab}$ embedding condition. In the effective action, there will be corresponding order α'^4 terms linear in H that do not vanish subject to $A_i^{ab} = \omega_i^{ab}$.

Finally, we note that similar questions have recently been discussed from a rather different viewpoint by Sen¹⁵⁾.

ACKNOWLEDGEMENTS

We would like to thank Paul Townsend for stimulating discussions.

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