computing of this type is that the designer can interrupt at certain stages of the process and, from the graphical display, can decide whether to continue or to modify the convergence conditions demanded. This greatly speeds the design and can result in more sophisticated design. With a little experience the work of five or six runs using a conventional offline batch process program can be performed in an hour at the display.

BETON is still being developed and a number of additional facilities are in preparation. The program has already been applied in the design of the fast ejected proton beam-line used in the neutrino experiments, for the p10 and m12 beam-lines, and in a study of an improved ejected proton beam-line for the next series of neutrino experiments. It was also used in modifying the parameters of the 70 GeV fast ejected beam at Serpukhov, taking into account the final installation details. The results of this work were transmitted to the Soviet Union only hours before they were successfully used for the first beam ejection at the beginning of February. Secondary beams for Gargamelle, with and without r.f. separation, are also being studied using the BETON interactive program.

A second interactive program with graphical output is at present under development for studying the optical system of a Cherenkov counter. The Cherenkov light emitted by different particles along their trajectory in the counter is simulated for different values of velocity, and for different directions and distances of emission of the particle from the optical axis.

The data for display consists of the point of intersection of a ray with the focal plane for about one thousand different rays from each particle. The points are transformed into a density function by summing the number of rays falling within a certain area, thus producing a function which is then output in the form of a perspective view of a three-dimensional surface.

At present the program is being run in a semi-interactive mode through the FOCUS system with output to a T4002 display and hard copy on microfilm. When more experience of the problems involved has been gained, it is planned to provide an interactive version of the program using the ARGUS display.

Mathematical computing

a) In theoretical studies

B. Lautrup

Theoretical physicists mostly use the computer facilities for numerical evaluation of theories in order to compare theory with experiment (or theory with theory). Usually this only puts moderate demands on the computer and its software. Most programs require only a library of special functions and virtually never use tapes or other 'complicated' peripheral equipment. They are reasonably satisfied with a card reader, for input and a line printer for output. About 80% of the computing jobs from the CERN Theory Division are of this scale. They would in fact be even better off in an interactive environment with an 'advanced desk calculator'. The GAMMA system described above is, among other things, a sophisticated attempt to cater for these needs.

Some theorists use computers on a grander scale. Numerical integration in many dimensions, phase shift analysis and multiparameter fits to phenomenological models lead to large central processor time consumption and require fast computers. This kind of computation is analogous to the last stage in the analysis of experimental results.

A more exotic use of the computer by theorists is in the field of algebraic manipulations. A theoretical expression may not immediately submit to numerical evaluation but require a certain amount of algebraic simplification before numbers can be extracted. This is for instance the case in the evaluation of Feynman graphs in quantum electrodynamics and related fields. A Feynman graph leads to multi-dimensional integrals, over infinite regions, of complicated products of matrices, vectors and spinors. Not only is the integration region infinite but the integrand has singularities within this region. This prevents a direct numerical attack on the integral and other methods are needed.

First of all the problem is to transform the integration region into a finite domain. This is done in a series of well-defined steps reducing the integral to a standard form. In this process matrices and tensors are multiplied out, new integration variables introduced, vectors multiplied with each other to yield physical scalar quantities, the old integration variables integrated out, etc.

Although this is quite simple to do by hand for uncomplicated graphs, it becomes extremely cumbersome for higher order graphs. The number of terms is so large that it is beyond the ability of the human mind to handle them. As the operations performed on each term are almost identical, the problem is mostly of a bookkeeping nature. This is an ideal task for the computer since it is able to do the same operation over and over again without tiring, or making errors.

At CERN a large-scale algebraic manipulation program (SCHOONSCHIP) capable of carrying out the reduction described above is available. It was written by a single person, M. Veltman from Utrecht, in the middle of the 1960's and has been continually expanded and modified since then. The program accepts input in the form of an expression containing gamma-matrices, vectors, scalars and even spinors. It allows for expansion, trace calculation, collection of identical terms, a variety of substitutions, limited factorisation, asymptotic expansion, etc. It has even been possible to perform several-dimensional analytic integration using the substitution facilities.

Another program capable of carrying out the same kind of calculations will eventually become available at CERN. This is the language oriented program REDUCE written by A. Hearn from Salt Lake City. It is more flexible than SCHOONSCHIP but requires much more software. It has been implemented interactively on a PDP-10.

Several times in the above articles we have run into the use of computers for 'numerical mathematics'. The following two articles cover some typical uses by theoreticians and by accelerator designers.