CERN-TH.4587/86

HIERARCHICAL CHIRAL SYMMETRIES AND THE QUARK MASS MATRIX

Harald Fritzsch^{*)},⁺⁾

CERN - Geneva
and
Max-Planck-Institut für Physik und Astrophysik
Werner Heisenberg Institut für Physik
München, Germany

A B S T R A C T

A hierarchical pattern of chiral symmetries is introduced, whose breaking is supposed to lead to the observed hierarchical pattern of the quark mass spectrum. Specific consequences of the interplay between the quark masses and the weak interaction mixing parameters are derived, in accordance with the observational constraints. They can be regarded as stringent constraints for any realistic dynamical theory of the quark mass spectrum. CP violation is absent in the limit $m_u = m_d = 0$. A specific way to describe the weak interaction mixing emerges.

^{*)} Supported in part by DFG-contract Fr 412/6-2.

⁺⁾ On leave from Sektion Physik, Universität München.

CERN-TH.4587/86

November 1986

It is a well-known, however yet unexplained fact that the quark states for which the charged weak currents are diagonal are not eigenstates of the quark mass matrix. In QCD the latter is identified with the operator breaking the chiral flavor symmetry. If all quark masses were zero or had a common non-zero value, the phenomenon of weak interaction mixing would not exist since in this case the eigenstates, defined by the weak interactions, and the ones given by the strong interactions would coincide. Thus the phenomenon of weak interaction mixing and the breaking of the flavor symmetry in the internal space spanned by the various quark families are expected to be related to each other. Any attempt to gain a deeper insight into the structure of the quark mass matrix is at the same time an attempt to understand the phenomenon of weak interaction mixing.

We shall adopt the usual convention and suppose that the charge 2/3 quarks are unmixed, and the weak mixing is expressed in terms of a 3 x 3 matrix V relating the weak eigenstates q' to the mass eigenstates q ($q = d, s, b$):¹⁾

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix} \quad (1)$$

Thus far it is unknown whether the elements of the mixing matrix V depend on the quark masses or might even be specific functions of quark mass ratios (for various attempts in this direction see e.g., refs. (2-5)).

In this note we shall discuss a general approach to the problem using a set of chiral symmetries which provides strong arguments for such a dependence. Under rather general circumstances we derive stringent constraints for the weak mixing angles.

The spirit of our approach is similar to the "soft pion limit" considered about 25 years ago in the physics of the strong interactions. The real world is not far from this limit, in which the mass of the pion vanishes. It was later realized that this limit coincides with the "chiral limit" of QCD, the limiting case, in which the masses of both the u and d quarks vanish.

The "chiral limit" plays a dual rôle in strong interaction physics. A number of parameters of the strong interactions (e.g. the nucleon mass or the pion decay constant) depend only very weakly on the "light" quark masses m_u and m_d , i.e. they can be calculated safely in the chiral limit $m_u = m_d = 0$. However, those parameters, which vanish in the symmetry limit (e.g. the pion mass),

depend strongly on the quark masses. The chiral symmetry of the strong interactions provides a possibility to estimate the latter in terms of the quark masses.

In this note we shall describe an analogous situation in the physics of the weak interactions. Starting from the observed world in which a particular set of quark masses and weak angles is realized, we shall move on to various fictitious worlds in which some of the quark masses vanish.

We shall suppose that there are three doublets of quarks:

$$\left\{ \begin{array}{ccc} \text{I} & \text{II} & \text{III} \\ \begin{array}{c} u \ [5.1] \\ d \ [8.9] \end{array} & \begin{array}{c} c \ [1 \ 350.] \\ s \ [175] \end{array} & \begin{array}{c} t \ [?] \\ b \ [5300] \end{array} \end{array} \right\} \quad (2)$$

(the numbers in the brackets denote the quark mass values, normalized at an energy of 1 GeV; for a review see ref. (6)).

With one possible exception, the t-quark, whose mass is not yet known, all quark masses are relatively small compared to the typical weak interaction energy scale of the order of 0.3 TeV.

Furthermore a clear hierarchical pattern has emerged: the masses of the first generation (u, d) are small compared to the corresponding masses of the second generation (c, s), and an analogous hierarchy exists if we compare the masses of the third generation (t, b) to the masses of the second generation. Thus it seems useful to consider the various stages of mass generation given in the following table.

Stage	Masses different from zero
I	none.
↓ (t, b massive)	
II	t, b.
↓ (c, s massive)	
III	t, b; c, s.
↓ (u, d massive)	
IV	t, b; c, s; u, d.

If all six quark masses vanish, the Lagrangian of the standard model is invariant under the chiral symmetry $U(3)_L \times U(3)_R$, where $U(3)$ is the symmetry group connecting the three generations ($U(3)$ stands as usual for the direct product $SU(3) \times U(1)$, and the index L(R) refers to the left-handed or right-handed quark fields q_L (q_R)). In the standard electroweak gauge theory (for recent reviews see e.g. ref. (7)) or in supersymmetric extensions of the standard model this symmetry is explicitly violated by the "Yukawa" type couplings of the quark fields to the scalar "Higgs" field.

In many composite models the chiral generation symmetry is regarded as an approximate symmetry, which is supposed to be softly broken and to provide the reason why the quark masses are small compared to the 100 GeV scale of the weak interactions.

At the present time it is completely unknown what dynamical mechanism is responsible for generating the masses of the quarks (and leptons). Nevertheless it is a fact that the chiral symmetry $U(3)_L \times U(3)_R$ in the space of the generations is exact only if all quark masses vanish, and vice versa. Furthermore in this limit all weak mixing angles vanish. Of course, in the real world this symmetry is broken. Nevertheless the chiral generation symmetry does play a relevant rôle. It guarantees that the symmetry limit $m_q = 0$ ($q = u, d, \dots$) is a natural limit, i.e. it can be realized without posing artificial constraints on the underlying yet unknown dynamical framework (whatever this may be), for example the unnatural cancellation of unrelated terms in the fermion self energies.

Stage II: $m_u = m_d = 0$, $m_c = m_s = 0$, $m_t, m_b \neq 0$.

The (u,d)-quarks as well as the (c,s)-quarks are massless. The (t,b)-quarks are massive (their masses assume the values realized in Nature).

Obviously this stage is not very far from the case given in Nature, since the masses of the (u,d; c,s)-quarks are indeed very small compared to the t and b masses and might be regarded as perturbations. In terms of mass eigenstates, the mass matrices for the (2/3) and (-1/3) charged quarks are proportional to the matrix

$$\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}. \quad (3)$$

This matrix transforms under the generation symmetry $SU(3) \times U(1)$ as a superposition of λ_8 and λ_0 (we use the conventional λ -matrices), i.e. as a

superposition of an octet and a singlet. In a group-theoretical sense the mass matrix in the stage II limit does not exhibit a particularly simple structure. Instead the breaking of the $SU(3) \times U(1)$ generation symmetry involving a λ_8 and λ_0 term is rather similar to the breaking of the $SU(3)$ flavor symmetry of the strong interactions⁸⁾ which is described by a similar superposition of λ_0 and λ_8 and is essentially provided by the mass term of the s-quark.

This situation can be obtained in a natural way only if it follows from a specific breaking of the original chiral generation symmetry. Since only the t and b quarks are massive, the original chiral symmetry $U(3)_L \times U(3)_R$ is not completely violated, but it is reduced to the subsymmetry $U(2)_L \times U(2)_R$ acting on the (u,d; c,s)-system.

What is the weak interaction mixing in the stage II ? In principle the stage II mass pattern allows a non-trivial weak mixing described by just one angle, as described by the following weak doublets:

$$\begin{pmatrix} \begin{array}{c|c} u & c \\ \hline d & s \cos\alpha + b \sin\alpha \end{array} & \begin{array}{c|c} t \\ \hline -s \sin\alpha + b \cos\alpha \end{array} \end{pmatrix} \quad (4)$$

It can easily be seen that any parametrization of the weak interaction mixing in the stage II limit can be reduced to the one given in eq. (4). Note that here the (u,d)-pair remains unmixed; this can always be arranged by a suitable unitary transformation among the two massless doublets.

The weak interaction mixing given in eq. (4) must be the consequence of a particular structure of the quark mass matrix. Let us consider this structure and in particular its properties with respect to the chiral symmetry. In eq. (4) the mixing involves the massive b-quark and the massless s-quark. The quark mass term describing such a mixing is as follows:

$$\overline{(d_0, s_0, b_0)}_R \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & B & A \end{pmatrix} \begin{pmatrix} d_0 \\ s_0 \\ b_0 \end{pmatrix}_L + \text{h.c.} \quad (5)$$

(d_0, s_0, b_0 : quark states in which the weak currents are diagonal). The parameters A and B which can be arranged to be real and positive are related to the mixing angle α and the b-mass as follows:

$$\begin{aligned} \tan\alpha &= B/A \\ m_b &= \sqrt{B^2 + A^2} . \end{aligned} \quad (6)$$

It is obvious from the matrix eq. (5) that the mixing parameter does not influence the mass of the s-quark; the determinant of the mass matrix in eq. (5) is zero, independent of the magnitude of the mixing. The mass term can be written explicitly as

$$B \cdot \bar{b}_{0R} s_{0L} + A \cdot \bar{b}_{0R} b_{0L} + \text{h.c.} \quad (7)$$

We should like to interpret the mass pattern at stage II (two heavy quarks, four massless quarks) as a result of a specific breaking of the chiral $U(3)_L \times U(3)_R$ symmetry which is broken down to $U(2)_L \times U(2)_R$. This reduction of the symmetry establishes a hierarchical pattern of the fermion masses. For this reason we refer to it as a hierarchical chiral symmetry breaking.

The masslessness of the quarks of the first two generations in the stage II can only be guaranteed if the subgroup $U(2)_L \times U(2)_R$ of the original group $U(3)_L \times U(3)_R$ is preserved. However the mass term given in eq. (7) does not respect this symmetry if the mixing term $A \cdot \bar{b}_{0R} s_{0L}$ is present; this term transforms under $U(2)_L \times U(2)_R$ as $(1/2, 0)$. For example, as a result of the chiral transformation $s_{0L} \rightarrow e^{i\alpha} s_{0L}$, $s_{0R} \rightarrow e^{-i\alpha} s_{0R}$ the first term in eq. (7) is multiplied by $e^{i\alpha}$. Thus the mixing term in eq. (7) violates the chiral $U(2)_L \times U(2)_R$ symmetry although it does not generate a mass directly. The weak interaction mixing and the chiral symmetry cannot both be implemented. Thus we conclude : there is no weak interaction mixing in the limit of stage II, i.e. in the limit $m_u = m_d = 0$, $m_c = m_s = 0$. The weak interaction doublets are simply given by the mass eigenstates:

$$\begin{pmatrix} u & | & c & | & t \\ d & | & s & | & b \end{pmatrix} \quad (8)$$

This result can be understood in the following way. At stage II only the (t,b)-quarks are massive. If these quarks mix via the weak interaction with the other (massless) quarks, violating the underlying chiral symmetry, no more guarantee can be given that these quarks stay massless. The same mechanism which generates the t and b mass, whatever this may be, will in general also generate a mass for those quarks mixed with t and b. This observation implies that the weak mixing angles are directly linked to the mechanism which generates the other masses, and it may be possible to calculate the weak mixing angles in terms of the quark masses m_q ($q = c, s; u, d$).

In reality it is observed that the (t,b) system mixes only very slightly with the other quarks, which in our view is related to the fact that the

(c,s)-masses are small compared to the (t,b)-masses and the hierarchical symmetry limit of stage II is not far away.

Stage III: $m_u = m_d = 0$, $m_c, m_s; m_t, m_b \neq 0$.

In order to arrive at stage III, the chiral symmetry is broken down to $U(1)_L \times U(1)_R$. In terms of weak eigenstates the general mass term for the charge $(-1/3)$ -quarks respecting this symmetry has the structure

$$\overline{(d_o, s_o, b_o)}_R \begin{pmatrix} 0 & 0 & 0 \\ 0 & C & B' \\ 0 & B & A \end{pmatrix} \begin{pmatrix} d_o \\ s_o \\ b_o \end{pmatrix}_L \quad (9)$$

The parameters A, B, B', C can be made real by suitable phase rotations of the quark fields. The mass matrix is Hermitian ($B = B'$) after a rotation of the right-handed quark fields (s_{oR}, b_{oR}) (angle $\arctan (B-B')/(A+C)$).

The mass term (eq. (9)) can be diagonalized by a rotation in the (s_o, b_o)-plane. Only one mixing angle α enters, and we arrive at the same weak doublets as given in eq. (2). The weak mixing matrix denoted here by $V^{(0)}$ is given by:

$$V^{(0)} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\alpha & \sin\alpha \\ 0 & -\sin\alpha & \cos\alpha \end{pmatrix}, \quad (10)$$

and the mixing angle α is given (for $B = B'$) by

$$\cos\alpha \sin\alpha = \frac{-B}{m_b - m_s} \quad (11)$$

(m_b, m_s : mass eigenvalues).

The weak mixing matrix at stage III is orthogonal - no complex phases are present. As a result the quark mass matrix does not induce CP violation. We thus conclude that in the limit $m_u = m_d = 0$ there is no CP violation. All CP violating effects must be proportional to positive powers of the u or d quark masses. Furthermore the u and d quarks remain unmixed in the limit $m_u = m_d = 0$. The Cabibbo angle vanishes in this limit, as well as the transitions $u \leftrightarrow b$ or $t \leftrightarrow d$.

This result is, of course, not valid in general, but follows in our special approach from the underlying hierarchical symmetry structure.

Stage IV: All quark masses different from zero.

At the last stage IV the (small) masses of the u and d quarks are generated. The mixing matrix V takes its final form. Since the u and d masses are small compared to the other masses, the weak mixing matrix V can be regarded as a perturbation of the matrix $V^{(0)}$ denoted in eq. (10). This implies that the elements V_{us} , V_{ub} , V_{cd} and V_{td} which vanish in the limit $m_u = m_d = 0$ can be expanded in positive powers of the ratios m_d/m_s , m_d/m_b , m_u/m_c and m_u/m_t . Of course, the corresponding expansion coefficients cannot be predicted in the general approach discussed here. This can only be done if one has a specific algebraic structure for the mass matrix. For example, in the ansatz of ref. (2) the chiral symmetry constraints discussed here are fulfilled. One finds:

$$\begin{aligned}
 V_{us} &\approx -\sqrt{m_d/m_s} + e^{i\alpha} \sqrt{m_u/m_c} \\
 V_{ub} &\approx \sqrt{m_d/m_b} \frac{m_s}{m_b} + \sqrt{m_u/m_c} (e^{i\alpha} \sqrt{m_s/m_b} - e^{i\beta} \sqrt{m_c/m_t}) \\
 V_{cd} &\approx -\sqrt{m_u/m_c} + e^{i\alpha} \sqrt{m_d/m_s} \\
 V_{td} &\approx \sqrt{m_u/m_t} \frac{m_c}{m_t} + \sqrt{m_d/m_s} (e^{i\alpha} \sqrt{m_c/m_t} - e^{i\beta} \sqrt{m_s/m_b})
 \end{aligned} \tag{12}$$

(α, β phase parameters).

All these matrix elements vanish in the limit $m_u, m_d \rightarrow 0$, in accordance with the general conclusions drawn above.

As the u and d quarks acquire their masses, the b-s mixing coefficients (V_{cb} , V_{ts}) are essentially not changed (apart from very small corrections due to unitarity).

The considerations made above can easily be extended to include further quark doublets, provided the masses of the new quarks fit into the hierarchical mass pattern as observed for the first three doublets. Suppose there exists a fourth doublet denoted by

$$\begin{pmatrix} h \\ l \end{pmatrix}$$

(h: "high", l: "low"). If the masses of these quarks are large enough, e.g. $m_l > 80$ GeV, $m_h > 300$ GeV, the set of chiral symmetries discussed above can be extended by another stage at which all masses except the h and l masses are zero. At this stage the (h,l)-doublet remains unmixed. The mixing of this doublet with the (t,b)-doublet would arise once the t and b masses are introduced.

CP-violation and the parametrization of the mixing; in the case of three quark doublets CP violation can arise once the u and d masses are introduced, since the off-diagonal matrix elements V_{ub} , V_{td} , V_{cd} and V_{us} which were previously zero may involve complex phases. However the hierarchical pattern of chiral symmetries discussed here requires that the CP violation disappears in the limit $m_u, m_d \rightarrow 0$. The phenomenological description of the mixing should account for this feature in a manifest way. Both parametrizations used in the particle data table introduced by Kobayashi and Maskawa⁹⁾ and by Maiani¹⁰⁾ do not fulfil those constraints since in the limit $m_u, m_d \rightarrow 0$ the mixing matrix is still complex. The parametrization introduced by Chau and Keung¹¹⁾ and the author¹²⁾, as well as the one given by Wolfenstein¹³⁾, fulfils the required constraints.

According to our view the weak interaction mixing starts with the heaviest quark doublet and proceeds step by step until one arrives at the mixing involving the lightest doublet (u, d). Thus one starts at the top and proceeds downward. The parametrization given by Kobayashi and Maskawa⁹⁾ is based on the opposite point of view. First the Cabibbo mixing is introduced, followed by the mixing involving the heavier quarks. Thus the mixing is built up from below and proceeds upward. Of course, from a historical point of view, this is the way the physics has developed. However this approach cannot be maintained if the weak interaction mixing is indeed, as discussed in this paper, due to an evolution of the chiral symmetry breaking.

Conclusions: In this paper we have argued that the quark mixing matrix V (eq. (1)) can be viewed as a result of an evolutionary process. It starts out from the unit matrix which is realized in the chiral limits of stages I and II. The next step is the generation of a non-zero s - b mixing by the same symmetry breaking which generates the masses of the c and s quark, while CP violation is still absent. Finally the u and d masses are introduced. At this stage the full 3×3 mixing matrix including a CP-violating phase is generated.

The algebraic framework discussed here is probably simple and general enough to be realized in a number of possible approaches, while we have not based ourselves on any particular dynamical framework of the mass generation.

In the history of particle physics the concept of the chiral symmetry and its breaking has played a crucial rôle in the development of the dynamical theory of the strong interactions. Similarly the idea of a hierarchy of chiral symmetries being important for the generation of the hierarchical pattern of the quark mass spectrum and of the weak interaction mixing may be an important constraint for the development of a dynamical theory of the quarks and leptons in the future.

ACKNOWLEDGEMENTS

This paper was partly written while the author was visiting the Theory Division of the Los Alamos Scientific Laboratories in August 1986. I would like to acknowledge the kind hospitality received there. Furthermore I am indebted to the particle theorists in Los Alamos, especially to R. Slansky, S. Raby and P. Rosen, for useful discussions.

REFERENCES

1. Review of Particle Properties, Phys.Lett. 170B (1986)
2. H. Fritzsch, Nucl.Phys. B155 (1979) 189;
See also: H. Fritzsch, Phys.Lett. 166B (1986) 423.
3. B. Stech, Phys.Lett. 130B (1983) 189.
4. G. Ecker, Z.Phys. C24 (1984) 353.
5. H. Georgi, A. Nelson and M. Shin, Phys.Lett. 150B (1985) 306.
6. J. Gasser and H. Leutwyler, Physics Reports 87 (1982) 27.
7. C. Quigg, Gauge Theories of the Strong, Weak and Electromagnetic Interactions, Benjamin/Cummings (1983);
T.P. Cheng and L.F. Li, Gauge Theory of Elementary Particle Physics, Clarendon Press, Oxford (1984).
8. M. Gell-Mann, R. Oakes and B. Renner, Phys.Rev. 175 (1968) 2195;
S. Glashow and S. Weinberg, Phys.Rev. D15 (1977) 1958.
9. M. Kobayashi and T. Maskawa, Progr.Theor.Phys. 49 (1973) 652.
10. L. Maiani, in Proc. of 1977 Int. Symposium on Lepton and Photon Interactions at High Energies, DESY, Hamburg, 1977, p. 867.
11. L.L. Chau and W.Y. Keung, Phys.Rev.Lett. 53 (1984) 1802.
12. H. Fritzsch, Phys.Rev. D32 (1985) 3058.
13. L. Wolfenstein, Phys.Rev.Lett. 51 (1984) 1945.