CERN-TH.4564/86
A D D E N D U M

STRING LENGTHS IN COVARIANT STRING FIELD THEORY AND $OSp(26,2/2)$

A. Neveu and P. West^{*)}

In this addendum, we discuss the quantization of the closed bosonic string along the same lines. Since open and closed strings are coupled together, one should be able to obtain an $OSp(26,2/2)$ invariant action also for the closed string. It is clear that most of the discussion of the open string still applies. All fields in the action (2.1), as well as the gauge parameters $\varepsilon^\pm$ are now periodic in σ . The invariance of the action (2.1) now implies only

$$\varepsilon^+(0, \sigma) = \varepsilon^+(T, \sigma) = 0,$$

while ε^- is unrestricted apart from its periodicity. As before, the only subtlety lies in the zero modes. So, we keep $\lambda_0^\pm(\tau)$ explicit, and go to gauge where $\lambda_n^\pm(\tau) = 0$. An infinitesimal transformation away from this gauge reads

$$\delta \lambda_n^+ = \dot{\varepsilon}_n^+ - 2n \varepsilon_n^- \lambda_0^+ - 2\lambda_0^- n \varepsilon_n^+,$$

$$\delta \lambda_n^- = \dot{\varepsilon}_n^- - 2n \lambda_0^+ \varepsilon_n^+ - 2\lambda_0^- n \varepsilon_n^-,$$

$$\delta \lambda_0^+ = \dot{\varepsilon}_0^+,$$

$$\delta \lambda_0^- = \dot{\varepsilon}_0^-.$$

We see that we cannot use the gauge $\lambda_0^- = 0$, as the Faddeev-Popov propagator would have an isolated zero eigenvalue corresponding to $\varepsilon_0^- = \text{constant}$. Hence we keep the full $\lambda_0^\pm(\tau)$ functional integral. The λ_0^+ gauge fixing is as before, and the gauge fixing Lagrangian is

$$\mathcal{L}_{GF} = b\dot{\omega} + \sum_{n \neq 0} (b_n^+ \lambda_n^+ + b_n^- \lambda_n^-)$$

The Lagrangian we obtain is as given in Eqs. (2.20) and (2.21) with the appropriate change of the range of σ integration, as well as the term

$$2 \int_0^T d\tau \lambda_0^- \left[L_0 - \tilde{L}_0 - \sum_{\substack{n=-\infty \\ n \neq 0}}^{+\infty} (n \bar{c}_n^- c_n^- + n \bar{c}_n^+ c_n^+) \right]$$

We then perform explicitly the remaining $\lambda_0^-(\tau)$ integral. It trivially enforces the algebraic constraint $L_0^{\text{tot}} - \tilde{L}_0^{\text{tot}} = 0$. The ghost zero modes are thus common to both right and left movers, just as the centre-of-mass co-ordinates. The BRST operator is a trivial extension of the one of Eq. (3.15), obtained by adding the contributions of right and left movers.

It is given by

$$Q = e^{\omega} \left[c(K + \tilde{K}) - 2 \frac{\partial}{\partial c} (M + \tilde{M}) + d + \tilde{d} + \tilde{D} + \frac{\partial}{\partial \bar{c}} \left(-c \frac{\partial}{\partial c} + N + \tilde{N} + \frac{\partial}{\partial \omega} \right) \right]$$

All our comments about the similarities with the light-cone theory apply.

We only note that while in the light cone theory the constant $(L_0 - \tilde{L}_0)_{\text{transverse}} = 0$ arises from the periodicity of $x^-(\sigma)$ in the canonical GGRT treatment, this condition now arises from integration over the Lagrange multiplier $\lambda_0^-(\tau)$.

The BRST and gauge invariant actions for the closed string are given by Eqs. (3.18) and (3.20) with the above Q and an obviously extended H . The interaction contains only a cubic term as in the light cone formalism. It is of the form of Eq. (5.3) with the obvious extension of $\mathcal{V}$ to contain tilded oscillators as well as on each leg the projector factor

$$\frac{1}{2\pi} \int_0^{2\pi} d\sigma e^{i\sigma(L_0^{\text{tot}} - \tilde{L}_0^{\text{tot}})}$$

to enforce the constraint $L_0^{\text{tot}} - \tilde{L}_0^{\text{tot}} = 0$.

One could envisage different methods of gauge fixing λ_0^- which did not require the explicit enforcement of the constraint $L_0^{\text{tot}} - \tilde{L}_0^{\text{tot}}$. Presumably this would involve more zero mode ghost fields.

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A. Neveu and P. West^{*)}

CERN - Geneva

ABSTRACT

The recent work of the authors deriving a field theory with Parisi-Sourlas symmetries from the point particle is extended to the bosonic string. Upon proper gauge fixing of the classical open string the resulting BRST field theory is $OSp(26,2/2)$ -invariant and contains the co-ordinates for the proper time and the string length, as well as the additional compensating ghost co-ordinates. The corresponding free gauge theory is also presented and shown to yield the BRST theory upon gauge fixing. The interacting gauge-invariant and BRST theories are also given.

^{*)} Permanent address: Maths. Dept., King's College, Strand, London WC2R 2LS, United Kingdom.

1. - INTRODUCTION

In a recent paper¹⁾, the authors discussed previously unsuspected features of the quantization of the free point particle. It was shown that the reparametrization invariant action

$$A = \int d\tau \{ p_\mu \dot{x}^\mu - e^\omega (p_\mu^2 + m^2) \} \quad (1.1)$$

when correctly gauge-fixed and quantized leads to a field theory whose fields depended on the $D+2$ bosonic co-ordinates x^μ , ω , τ , $\mu = 0, 1, \dots, D-1$ and two anti-commuting co-ordinates c and $\bar{c}$. The BRST theory was $OSp(D, 2/2)$ -invariant, part of this symmetry being a Parisi-Sourlas symmetry²⁾. The physical states, however, were subject to $Q\chi = 0$, where Q is the BRST charge. This condition is not $OSp(D, 2/2)$ -invariant, but with the invariance it guarantees the absence of ω and τ dependence from physical processes.

In Ref. 1), a gauge-invariant field theory was given, the local invariance corresponding to the reparametrization invariance of the classical theory. This theory was shown to give the above-mentioned BRST theory upon gauge fixing and to imply on-shell the usual Klein-Gordon equation. That is, it describes on-shell one irreducible representation of the Poincaré group with spin zero.

The purpose of this paper is to extend these results to the bosonic string. Here, as discussed in Ref. 1), one expects over and above the transverse oscillators x_n^i and centre-of-mass position x^μ to find two bosonic and two fermionic co-ordinates for each mode; that is, for x^μ we have ω , τ , c , $\bar{c}$, and for x_n^i we have $x_n^\pm$, c_n , $\bar{c}_n$. How many co-ordinates one actually has is, of course, determined by a correct gauge-fixing procedure from the classical theory. In Section 2, we demonstrate that it is indeed the above count which emerges. In particular, the difference with previous discussions of this system^{3), 4)} is that we find that one cannot set the conformally-invariant piece of the two-dimensional metric to be the Minkowski metric. As for the point particle, one degree of freedom of the metric is left. As announced in Ref. 1), this is the remnant for the point particle of the string length. To understand the string length in covariant string field theory was the motivation of Ref. 1) and we will not repeat the discussion here, but draw the reader's attention to Ref. 1).

In Section 3, we derive the BRST string field theory which follows from this analysis of the classical system, with its $OSp(26,2/2)$ invariance. We also give a gauge-invariant action; upon gauge fixing, it reproduces this BRST theory, including the on-shell count of physical states, which we examine together with the cohomology of Q .

In Section 4, we come back to the point particle, in order to show by a simple example, namely ϕ^3 , how interactions can be introduced naturally into the free theory of Ref. 1), retaining the $OSp(D,2/2)$ symmetry of the BRST action. The physical states condition becomes non-linear in the fields, corresponding to the fact that the BRST charge, which remains one of the generators of $OSp(D,2/2)$, itself becomes a cubic functional of the fields. We also give the corresponding interacting gauge-invariant theory and its non-Abelian transformation laws.

In Section 5, we treat both the BRST fixed and gauge-invariant interactions of open strings. We find that the Lagrangians and their properties are extremely simple, both conceptually and practically. It turns out that all complicated calculations [gauge invariance, $OSp(26,2/2)$ algebra] follow a similar pattern to those performed twelve years ago by Mandelstam within the context of the light-cone theory. We adapt these calculations to the case of two anticommuting co-ordinates, the BRST charges Q and $\bar{Q}$ being the immediate analogues of the troublesome Lorentz generators M^{i-} of those days. We thus find the BRST physical state condition to be up to trilinear in the string fields, corresponding to the existence of the quartic open string interaction. The Hamiltonian, which generates translations in proper time, is quartic in the fields.

In the BRST fixed theory, the OSp supergroup together with the Parisi-Sourlas mechanism guarantees the correct spectrum of intermediate states in string loop calculations, all undesirable bosonic co-ordinates (including the string length) being cancelled by their fermionic counterparts. The Feynman graphs are those of the light-cone string. Lorentz invariance is substituted for by BRST invariance; the necessity of the explicit introduction of closed strings and of open-closed string interactions at the Lagrangian level comes from a breakdown (anomaly) of BRST (i.e., gauge) invariance at the level of one loop of open strings, just as in the old light-cone approach it arose from a breakdown of Lorentz invariance.

2. - THE CLASSICAL BRST ACTION

The reparametrization-invariant bosonic string can be described by the first-order classical action

$$A = \int_0^T d\tau \int_0^\pi d\sigma \left\{ p_\mu \dot{x}^\mu - \lambda^+ (p_\mu^2 + x_\mu'^2) - 2\lambda^- x_\mu' p^\mu \right\} \quad (2.1)$$

where

$$\dot{x}^\mu = \frac{\partial x^\mu}{\partial \tau}$$

and

$$x^{\mu'} = \frac{\partial x^\mu}{\partial \sigma}$$

The action for the closed string is the same except that the limits of σ integration are $-\pi$ and π . The transformations of x^μ and p^μ which are generated by the constraints $p_\mu^2 + x_\mu'^2$ and $x_\mu' p^\mu$ are given by

$$\begin{aligned} \delta x^\mu &= 2 \varepsilon^+ p^\mu + 2 \varepsilon^- x^{\mu'} \\ \delta p^\mu &= 2 \frac{\partial}{\partial \sigma} \left\{ x^{\mu'} \varepsilon^+ + \varepsilon^- p^\mu \right\} \end{aligned} \quad (2.2)$$

The invariance of the action of Eq. (1) implies that

$$\delta \lambda^+ = \dot{\varepsilon}^+ - 2\lambda^- \varepsilon^{+'} + 2\lambda^{-'} \varepsilon^+ - 2\lambda^+ \varepsilon^{-'} + 2\lambda^{+'} \varepsilon^- \quad (2.3)$$

$$\delta \lambda^- = \dot{\varepsilon}^- - 2\lambda^- \varepsilon^{-'} + 2\lambda^{-'} \varepsilon^- - 2\lambda^+ \varepsilon^{+'} + 2\lambda^{+'} \varepsilon^+ \quad (2.4)$$

However, as for the point particle, the action is only invariant if the following boundary terms vanish:

$$\int_0^\pi d\sigma (p_\mu^2 - x_\mu'^2) \varepsilon^+ \Big|_{\tau=0}^{\tau=T} = 0$$

$$\int_0^T d\tau \left[\varepsilon^+ (2 x' \cdot \dot{x} - 2 \lambda^- (p^2 + x'^2) - 4 \lambda^+ x' \cdot p) \right. \\ \left. + 2 \varepsilon^- (p \cdot \dot{x} - \lambda^+ (p^2 - x'^2) - 2 \lambda' x' \cdot p) \right] = 0 \quad (2.5)$$

The meaning of the above terms is more transparent if one works in a formalism in which the two component world indices are explicitly displayed. In this case, if ε^α is the shift in the world-sheet co-ordinates, the variation of the action is given by³⁾

$$\delta A = \int d\sigma d\tau \partial_\alpha (\varepsilon^\alpha \mathcal{L}) \quad (2.6)$$

where $\mathcal{L}$ is the Lagrangian density. Clearly, this vanishes if

$$\varepsilon^0(0, \sigma) = \varepsilon^0(\tau, \sigma) = 0 \quad \forall \sigma \\ \varepsilon^1(\tau, 0) = \varepsilon^1(\tau, \pi) = 0 \quad \forall \tau \quad (2.7)$$

that is, we only make general co-ordinate transformations that preserve the rectangle swept out by the string from $\tau = 0$ to $\tau = T$. We therefore adopt the boundary conditions of Eq. (2.7), which are equivalent to the vanishing of the terms in Eq. (2.5). We note that for the conformal gauge $\lambda^- = 0$, the usual boundary conditions imply that x'_μ vanishes at $\sigma = 0$ and $\sigma = \pi$, and so these terms vanish if

$$\varepsilon^+(0, \sigma) = \varepsilon^+(\tau, \sigma) = 0 \\ \varepsilon^-(\tau, 0) = \varepsilon^-(\tau, \pi) = 0. \quad (2.8)$$

In a more general gauge, these conditions are more complicated.

Given these boundary conditions, we must decide what are the legitimate gauge-fixing conditions we may impose on $\lambda^\pm$. In this paper, we will only concern ourselves with local considerations, and neglect possible global obstructions and Gribov horizons. We now show that if $\lambda^+ = e$ and $\lambda^- = 0$, where e is arbitrary but σ - and τ -independent, then we can by a general co-ordinate transformation reach the most general λ^+ and λ^- . It is sufficient to demonstrate this for the infinitesimal case. That is, we take λ^+ and λ^- to be e and 0 respectively, plus small deviations. Neglecting second-order terms, we find that

$$\delta \lambda^+ = \dot{\epsilon}^+ - 2e\epsilon^{-'} , \delta \lambda^- = -2e\epsilon^{+'} + \dot{\epsilon}^- \quad (2.9)$$

By integrating $\delta \lambda^+$ over σ and using Eq. (2.8), we find the result

$$\delta \left(\int_0^\pi \lambda^+ d\sigma \right) = \frac{d}{d\tau} \left(\int_0^\pi \epsilon^+ d\sigma \right) \quad (2.10)$$

As for the point particle case¹⁾, we find that we cannot use $\epsilon = \int_0^\pi \epsilon^+ d\sigma$ to set $e^\omega \equiv \int_0^\pi \lambda^+ d\sigma$ to be one, although we may use it to set $\dot{\omega} = 0$.

We can show explicitly that indeed the system of Eqs. (2.4) has solutions ϵ^+ , ϵ^- obeying the boundary conditions (2.7) for arbitrary $\delta \lambda^+$, $\delta \lambda^-$ provided that $\delta \lambda^+$ averages to zero over σ and τ :

$$\int_0^T d\tau \int_0^\pi \delta \lambda^+ d\sigma = 0 \quad (2.11)$$

To prove this, we go Euclidean in σ and τ , and without loss of generality take $e = \frac{1}{2}$, i.e., consider the system

$$\begin{aligned} \epsilon^{-'} + \dot{\epsilon}^+ &= \delta \lambda^+ \\ \dot{\epsilon}^- - \epsilon^{+'} &= \delta \lambda^- \end{aligned} \quad (2.12)$$

together with the boundary conditions of Eq. (2.7). For a given point $z = \tau + i\sigma$ in the rectangle $0 < \tau < T$, $0 < \sigma < \pi$, we define the images z_1, z_2, z_3 as in Fig. 1 by

$$\begin{aligned} z_1 &= \bar{z} + 2i\pi \\ z_2 &= -\bar{z} + 2T \\ z_3 &= -z + 2i\pi + 2T \end{aligned} \quad (2.13)$$

Using the elliptic ζ function⁵⁾ with real period $2T$ and imaginary period $2i\pi$, we can write explicitly the solution of Eqs. (2.7) and (2.12):

$$\epsilon^+(z) = \frac{1}{4\pi} \int_0^\pi d\sigma' \int_0^T d\tau' \left[\zeta(z - z') + \zeta(\bar{z} - \bar{z}') \right]$$

$$\begin{aligned}
 & + \zeta(z-z'_1) + \zeta(\bar{z}-\bar{z}'_1) + \zeta(z-z'_2) + \zeta(\bar{z}-\bar{z}'_2) \\
 & + \zeta(z-z'_3) + \zeta(\bar{z}-\bar{z}'_3) \} \delta \lambda^+(z') \\
 & + \frac{1}{4\pi i} \int_0^\pi d\sigma' \int_0^T d\tau' [\zeta(z-z') - \zeta(\bar{z}-\bar{z}') - \zeta(z-z'_1) + \zeta(\bar{z}-\bar{z}'_1) \\
 & - \zeta(z-z'_2) + \zeta(\bar{z}-\bar{z}'_2) + \zeta(z-z'_3) - \zeta(\bar{z}-\bar{z}'_3)] \delta \lambda^-(z').
 \end{aligned}$$

$$\begin{aligned}
 \varepsilon^-(z) = & - \frac{1}{4i\pi} \int_0^\pi d\sigma' \int_0^T d\tau' [\zeta(z-z') - \zeta(\bar{z}-\bar{z}') + \zeta(z-z'_1) \\
 & - \zeta(\bar{z}-\bar{z}'_1) + \zeta(z-z'_2) - \zeta(\bar{z}-\bar{z}'_2) + \zeta(z-z'_3) - \zeta(\bar{z}-\bar{z}'_3)] \delta \lambda^+(z') \\
 & + \frac{1}{4\pi} \int_0^\pi d\sigma' \int_0^T d\tau' [\zeta(z-z') + \zeta(\bar{z}-\bar{z}') - \zeta(z-z'_1) - \zeta(\bar{z}-\bar{z}'_1) \\
 & - \zeta(z-z'_2) - \zeta(\bar{z}-\bar{z}'_2) + \zeta(z-z'_3) + \zeta(\bar{z}-\bar{z}'_3)] \\
 & \cdot \delta \lambda^-(z')
 \end{aligned}$$

(2.15)

Using the properties of the ζ function under a shift of its argument by a period⁵⁾, it is straightforward to verify that the boundary conditions (2.7) are satisfied if and only if Eq. (2.11) holds.

As a consequence of this discussion, we may fix the gauge by choosing

$$\dot{\omega} = 0, \quad \hat{\lambda}^+ = \hat{\lambda}^- = 0$$

(2.16)

where

$$\lambda^+ = e^\omega + e^\omega \hat{\lambda}^+, \quad \hat{\lambda}^- = e^\omega \lambda^-$$

(2.17)

and ω is independent of σ . We make a similar decomposition on the parameter, namely:

$$\varepsilon^+ = e^\omega \varepsilon + e^\omega \hat{\varepsilon}^+, \quad \varepsilon^- = e^\omega \hat{\varepsilon}^-$$

(2.18)

where ε is σ -independent and $\hat{\varepsilon}^+$ has no σ zero mode. The BRST variations of $\hat{\lambda}^+$, ω and $\hat{\lambda}^-$ are found by replacing ε , $\hat{\varepsilon}^+$ and $\hat{\varepsilon}^-$ by $\Delta\varepsilon$, $\Delta\varepsilon^+$ and $\Delta\varepsilon^-$ respectively. One finds the result

$$\begin{aligned}
 \delta \omega &= e^{-\omega} \frac{d}{d\tau} (e^{\omega} \wedge c) + \dots \\
 \delta \hat{\lambda}^+ &= e^{-\omega} \frac{d}{d\tau} (e^{\omega} \wedge c^+) + \dots \\
 \delta \hat{\lambda}^- &= e^{-\omega} \frac{d}{d\tau} (e^{\omega} \wedge c^-) + \dots
 \end{aligned} \tag{2.19}$$

where $+\dots$ denotes terms proportional to $\hat{\lambda}^{\pm}$.

After adding the gauge-fixing term $b\dot{\omega} + b^+\dot{\lambda}^+ + b^-\dot{\lambda}^-$ to the Lagrangian, we easily find the corresponding ghost terms. We absorb terms proportional to $\hat{\lambda}^{\pm}$ in $b^{\pm}$ and $\dot{\omega}$ in b . The resulting redefined terms are of the form $b^+\dot{\lambda}^+ + b^-\dot{\lambda}^-$ and are in effect disconnected from the rest of the action, and so we drop them. The resulting BRST action is given by

$$\begin{aligned}
 A &= \int_0^T d\tau \int_0^\pi d\sigma \left\{ p_\mu \dot{x}^\mu + b\dot{\omega} - \bar{c}\dot{p} - \bar{p}\dot{c} + \bar{c}^+\dot{c}^+ \right. \\
 &\quad \left. + \bar{c}^-\dot{c}^- - \mathcal{H} \right\}.
 \end{aligned} \tag{2.20}$$

where $\mathcal{H}$ is the Hamiltonian density and is given by

$$\mathcal{H} = e^{\omega} \left\{ (p_\mu^2 + x_\mu'^2) - \bar{p}p + 2\bar{c}^+c'^+ + 2\bar{c}^-c'^- \right\} \tag{2.21}$$

The reader may verify that the BRST charge is given by

$$\begin{aligned}
 Q &= \frac{1}{2} \int_0^\pi d\sigma e^{\omega} \left\{ c(p_\mu^2 + x_\mu'^2) - \bar{p}p + 2\bar{c}^+c'^+ + 2\bar{c}^-c'^- \right. \\
 &\quad + c^+(p_\mu^2 + x_\mu'^2) + 2c^-x_\mu'p^\mu + p b - p(\bar{c}^+c'^+ + \bar{c}^-c'^-) \\
 &\quad + 2\bar{c}^+(c^+c'^+ + c^-c'^-) + 2\bar{c}^-(c^+c'^+ + c^-c'^-) \\
 &\quad \left. + 2\bar{p}(c^+c'^+ + c^-c'^-) \right\}
 \end{aligned} \tag{2.22}$$

and that it does generate a set of field variations according to the rules $\delta \cdot = [\cdot, AQ]$ which leave the above action invariant. To give just one example, we note that

$$\delta c^- = \frac{1}{2} e^{\omega} \{ 2 \Lambda c c^+ - \Lambda \bar{\rho} c^- - 2 \Lambda (c^+ c^+ + c^- c^-) \} \quad (2.23)$$

In deriving these results, we used the Poisson bracket relations

$$\begin{aligned} \{\bar{c}, \rho\} &= 1, \quad \{c, \bar{\rho}\} = -1, \quad [\omega, b] = 1, \\ \{c^+(\sigma), \bar{c}^+(\sigma')\} &= \delta, \quad \{\bar{c}^-(\sigma), c^-(\sigma')\} = \delta \\ \{x^\mu(\sigma), p^\nu(\sigma')\} &= \gamma^{\mu\nu} \delta \end{aligned} \quad (2.24)$$

where the nature of the bracket changes in the usual way according to the statistics of the field involved and δ stands for $\delta(\sigma-\sigma')$. The fields $\bar{c}$, $\bar{c}^\pm$, $\bar{\rho}$, Λ are antiHermitian while c , $c^\pm$, π , x^μ , p^μ , ω and b are Hermitian.

Since we have used different gauge-fixing conditions to Kato and Ogawa³⁾, we find a different BRST action, and charge, Q , which has one extra ghost. As we shall see, this extra ghost plays a crucial rôle in the field theory. We note that the Hamiltonian is given by

$$H \equiv \int_0^\pi d\sigma \mathcal{H} = 2i \{\bar{\rho}, \varphi\} \quad (2.25)$$

and consequently

$$[\varphi, H] = 0. \quad (2.26)$$

The anti-BRST invariance $\bar{Q}$ of the action of Eq. (2.20) can easily be worked out by interchanging ghosts and antighosts; we shall later discuss how to derive $\bar{Q}$ from the $OSp(D, 2/2)$ point of view, as well as the commutation relation

$$\varphi \bar{\varphi} + \bar{\varphi} \varphi = \varphi^2 = \bar{\varphi}^2 = [\varphi, H] = [\bar{\varphi}, H] = 0. \quad (2.27)$$

3. - OPEN STRING FIELD THEORY

The BRST field theory follows from the above BRST classical theory in a systematic manner according to the procedure set out in Ref. 1). First we replace the Poisson brackets by (anti)commutators according to the rule

$$\{ \quad \}_{\pm} \rightarrow \frac{1}{i\hbar} [\quad , \quad]_{\pm} \quad (3.1)$$

The resulting relations are represented by the replacements

$$\begin{aligned} x^{\mu} &\rightarrow x^{\mu}, \quad p^{\mu} \rightarrow i \frac{\delta}{\delta x^{\mu}}, \quad c \rightarrow c, \quad \bar{c} \rightarrow \bar{c}, \quad \omega \rightarrow \omega, \quad \rho \rightarrow i \frac{\partial}{\partial \bar{c}} \\ \bar{\rho} &\rightarrow -i \frac{\partial}{\partial c}, \quad b \rightarrow -i \frac{\partial}{\partial \omega}, \quad c^{+} \rightarrow c(\sigma), \quad \bar{c}^{+} \rightarrow i \frac{\delta}{\delta c(\sigma)}, \\ \bar{c}^{-} &\rightarrow \bar{c}(\sigma), \quad c^{-} \rightarrow i \frac{\delta}{\delta \bar{c}(\sigma)} \end{aligned} \quad (3.2)$$

It will prove convenient to extend the range of σ from $-\pi$ to π by insisting that x^{μ} and c are even while $\bar{c}$ is odd under $\sigma \rightarrow -\sigma$. The expansions of these objects in terms of their Fourier modes may then be written as

$$\begin{aligned} x^{\mu} &= \sum_{n=-\infty}^{\infty} x_n^{\mu} e^{in\sigma}, \quad c(\sigma) = \sum_{\substack{n=-\infty \\ n \neq 6}}^{\infty} c_n e^{in\sigma} \\ \bar{c}(\sigma) &= \sum_{n=-\infty}^{\infty} \bar{c}_n e^{in\sigma} \end{aligned} \quad (3.3)$$

where

$$c_n = c_{-n}, \quad c_n^{*} = c_{-n}, \quad \bar{c}_n = -\bar{c}_{-n}, \quad (\bar{c}_n)^{*} = -\bar{c}_{-n}$$

We define the annihilation and creation operators by

$$\begin{aligned} \bar{\beta}(\sigma) &= \frac{\delta}{\delta c(\sigma)} + i \bar{c}(\sigma) = \sum_{\substack{n=-\infty \\ n \neq 0}}^{\infty} \bar{\beta}_n e^{in\sigma} \\ \beta(\sigma) &= -i \frac{\delta}{\delta \bar{c}(\sigma)} + c(\sigma) = \sum_{\substack{n=-\infty \\ n \neq 0}}^{\infty} \beta_n e^{in\sigma}. \end{aligned}$$

$$p^\mu = p^\mu - \frac{\partial x^\mu}{\partial \sigma} = \sum_{n=-\infty}^{\infty} \alpha_n^\mu e^{in\sigma}$$

(3.4)

They obey the relations

$$\{\bar{\beta}_n, \beta_m\} = \delta_{n+m,0}, [\alpha_n^\mu, \alpha_m^\nu] = n \eta^{\mu\nu} \delta_{n+m,0}$$

(3.5)

with all other brackets vanishing.

With these definitions, we find that the Hamiltonian density takes the form

$$\mathcal{H} = e^\omega \left\{ \frac{1}{2} (\alpha_0^\mu)^2 - \frac{1}{2} \frac{\partial}{\partial c} \frac{\partial}{\partial \bar{c}} + \sum_{n=1}^{\infty} (\alpha_n^{\mu\dagger} \alpha_n^\mu + n \beta_n^\dagger \bar{\beta}_n + n \bar{\beta}_n^\dagger \beta_n) \right\}$$

(3.6)

It is clearly an $OSp(D-1,1/2)$ -invariant operator. As for the centre-of-mass momentum, we could define $\delta/\delta c$ and $\delta/\delta \bar{c}$ to be the zero modes of β_n and $\bar{\beta}_n$. We consider wave functions $\chi[z, \tau]$ where the co-ordinates z include

$$z = (x^\mu(\sigma), \omega, c, \bar{c}, c(\sigma), \bar{c}(\sigma))$$

(3.7)

The BRST theory is then given by

$$\int dz d\tau e^{-\omega} \left\{ \chi^* \left(i \frac{d}{d\tau} \chi - \mathcal{H} \chi \right) \right\}$$

(3.8)

Identifying $e^{-\omega} = \alpha$ where α is the + momentum component in the two extra dimensions, we can use the Fourier transform

$$\chi[\dots, \alpha] = \frac{\sqrt{\alpha'}}{2\pi} \int_{-\infty}^{\infty} d\alpha e^{i y^- \alpha} \hat{\chi}[\dots, y^-] \quad (3.9)$$

In the notation of Ref. 1), the BRST action becomes

$$\int d^{D+2} y d\bar{c} dc \omega(\bar{c}(0)) \omega(c(0)) \hat{\chi} \Delta \hat{\chi} \quad (3.10)$$

where

$$\begin{aligned} \Delta = & \frac{\partial}{\partial y^{\hat{u}}} \frac{\partial}{\partial y^{\hat{v}}} \gamma^{\hat{u}\hat{v}} - \frac{\partial}{\partial c} \frac{\partial}{\partial \bar{c}} + \sum_{n=1}^{\infty} (\alpha_n^{\mu\dagger} \alpha_n^{\mu} + n \beta_n^{\dagger} \bar{\beta}_n \\ & + n \bar{\beta}_n^{\dagger} \beta_n) \end{aligned} \quad (3.11)$$

Written in this way, the BRST action is manifestly $OSp(D, 2/2)$ -invariant in the zero mode sector.

In the form of Eq. (3.8), it is clearly invariant under

$$\delta \chi = \Lambda Q \chi + \bar{\Lambda} \bar{Q} \chi \quad (3.12)$$

since Q and $\bar{Q}$ commute with H and $id/d\tau$. As will be explained below, these are the actions of two of the $OSp(D, 2/2)$ generators.

In oscillator basis, the BRST charge Q , after appropriate normal ordering, takes the form

$$\begin{aligned} Q = & e^{\omega} K - 2 \frac{\partial}{\partial c} M e^{\omega} + e^{\omega} (d + D) + \frac{e^{\omega}}{2} \frac{\partial}{\partial \bar{c}} \left(-c \frac{\partial}{\partial c} + N + \frac{\partial}{\partial \omega} \right) \\ \equiv & e^{\omega} Q^{old} + \frac{e^{\omega}}{2} \frac{\partial}{\partial \bar{c}} \left(-c \frac{\partial}{\partial c} + N + \frac{\partial}{\partial \omega} \right) \end{aligned} \quad (3.13)$$

where

$$\begin{aligned} K = & \frac{1}{2} (\alpha_0^{\mu})^2 + \sum_{n=1}^{\infty} \alpha_n^{\mu\dagger} \alpha_n^{\mu} + n (\beta_n^{\dagger} \bar{\beta}_n + \bar{\beta}_n^{\dagger} \beta_n) - 1 \\ M = & n \sum_{n=1}^{\infty} \beta_n^{\dagger} \beta_n \end{aligned}$$

$$\begin{aligned}
 d &= \sum_{n=1}^{\infty} \beta_n^+ (L_n + (n+1) \bar{\beta}_{-n}^+ \beta_n + \frac{1}{2} \beta_n^+ (n-1) \bar{\beta}_{n+1}) \\
 D &= \sum_{n=1}^{\infty} \beta_n (L_{-n} + (n+1) \beta_m^+ \bar{\beta}_{n-n} + \frac{1}{2} \beta_{m+n}^+ (n-1) \beta_m) \\
 N &= \sum_{n=1}^{\infty} (\bar{\beta}_n^+ \beta_n - \beta_n^+ \bar{\beta}_n) + \frac{1}{2}.
 \end{aligned}
 \tag{3.14}$$

and

$$L_n = \frac{1}{4} \int_{-\pi}^{\pi} (P^\mu)^2 e^{-in\sigma} d\sigma.$$

For future use we will rescale $\bar{c} \rightarrow \bar{c}/2$ in Eq. (3.13) so as to obtain simpler-looking results.

The relation $Q^2 = 0$ is easily verified if one uses the results

$$\begin{aligned}
 (Q^{\text{old}})^2 &= 0, \quad [N, d] = -d, \quad [N, D] = -D \\
 [N, M] &= -2M.
 \end{aligned}$$

Equation (3.13) for the BRST charge Q can be rewritten

$$\begin{aligned}
 Q &= \frac{1}{2} e^\omega [c (L_0^{\text{tot}} - 1) + (L_0^{\text{tot}} - 1) c] + e^\omega \sum_{n=1}^{\infty} (L_n^{\text{tot}} \beta_n \\
 &+ \beta_n^+ L_n^{\text{tot}}) + \frac{1}{2} \frac{\partial}{\partial \bar{c}} (e^\omega \frac{\partial}{\partial \omega} + \frac{\partial}{\partial \omega} e^\omega)
 \end{aligned}
 \tag{3.15}$$

where

$$\begin{aligned}
 L_n^{\text{tot}} &= \sum_{m=-\infty}^{\infty} \left(\frac{1}{2} \alpha_{m+n}^\mu \alpha_{-m}^\mu - m \bar{\beta}_{m+n} \beta_{-m} \right) + \bar{\beta}_n \frac{\partial}{\partial \bar{c}} \\
 \text{with } \bar{\beta}_0 &= \frac{\partial}{\partial c}
 \end{aligned}
 \tag{3.16}$$

L_n^{tot} obeys the Virasoro algebra with central charge corresponding to $D = 24^*$, the $\beta, \bar{\beta}$ contribution cancelling the contribution of two of the α modes

$$[L_n^{\text{tot}}, L_m^{\text{tot}}] = (n-m) L_{n+m}^{\text{tot}} + 2(n^3-n) \delta_{n,-m}$$

*) The $\beta\bar{\beta}$ part of L_n^{tot} corresponds to giving conformal spin 1 to the ghost fields $\bar{c}(\sigma)$ and $c(\sigma)$ rather than 2 and 0 respectively.

Q as defined by Eq. (3.15) differs from Q^{old} only by zero-mode contributions.

On the form (3.15) we recognize the formal identity of Q with one of the light-cone generators M^{i-} of Goddard, Goldstone, Rebbi and Thorn: it is the generator of $OSp(26,2/2)$ which mixes the τ and c co-ordinates. The generator $\bar{Q}$ mixes τ and $\bar{c}$. It can of course be obtained by commuting Q with the $Sp(2)$ generator which exchanges ghosts and antighosts, and thus trivially squares to zero, anti-commutes with Q and commutes with H.

It is instructive to write the BRST action in terms of component fields. Writing the zero modes in χ explicitly in the form

$$\chi = \bar{\Psi} + \phi c + \bar{\Phi} \bar{c} + \psi c \bar{c} \quad (3.17)$$

one finds

$$\prod_{n=1}^{\infty} \int dx_n^\mu d\bar{c}_n dc_n \int dx^\mu d\tau d\omega \left\{ \bar{\Psi}^* i \frac{d}{d\tau} \Psi - e^\omega \bar{\Psi}^* \kappa \Psi - e^\omega \bar{\psi}^* \psi + (\bar{\Phi}^* i \frac{d}{d\tau} \phi - e^\omega \bar{\Phi}^* \kappa \phi + h.c.) \right\} e^{-\omega} \quad (3.18)$$

This has the generic form of a Parisi-Sourlas action.

As explained in Ref. 1), although the BRST action is $OSp(D,2/2)$ -invariant, the physical state condition is not and is given by

$$\bar{Q} \chi = 0 \quad (3.19)$$

We will examine the cohomology of this constraint later in this section.

The gauge-invariant field theory takes an identical form to that given¹⁾ for the point particle, namely

$$\int dz d\tau e^{-\omega} \left\{ \chi_1^* i \frac{d}{d\tau} \chi_1 - \chi_2^* \varphi \chi_1 - \chi_1^* \varphi \chi_2 \right\} \quad (3.20)$$

and it is invariant under

$$\delta \chi_1 = \varphi \Lambda, \quad \delta \chi_2 = i \frac{d}{d\tau} \Lambda - \varphi \Lambda' \quad (3.21)$$

and the corresponding complex conjugated relations. We must also impose the following mode number counting conditions

$$\begin{aligned} N' \chi_1 &= 0 \\ (N' - 1) \chi_2 &= 0 \end{aligned} \quad (3.22)$$

where

$$N' = N - c \frac{\partial}{\partial c} + \bar{c} \frac{\partial}{\partial \bar{c}} - \frac{1}{2} \quad (3.23)$$

Let us now demonstrate that the above gauge-invariant action does indeed describe the spectrum of the open bosonic string. The field equations are given by

$$i \frac{d\chi_1}{d\tau} - \varphi \chi_2 = 0 \quad ; \quad \varphi \chi_1 = 0 \quad (3.24)$$

We may use Λ to gauge $\chi_2 = 0$, leaving only χ_1 subject to $\delta \chi_1 = \varphi \Lambda$, both χ_1 and Λ being τ -independent. To gauge-fix further, we decompose χ_1 in terms of component fields using the same decomposition as given for χ in Eq. (3.17). The action of Q on $S = a_1 + a_2 c + a_3 \bar{c} + a_4 c \bar{c}$ is given by

$$\begin{aligned} \varphi S = e^{\omega} \{ & (\pm 2 m a_2 \mp (N + \frac{\partial}{\partial \omega}) a_3 + (d+D) a_1) \\ & + (\pm \kappa a_1 \mp (N + \frac{\partial}{\partial \omega} - 1) a_2 + (d+D) a_4) c + (\mp 2 m a_4 \\ & + (d+D) a_3) \bar{c} + (\mp \kappa a_3 + (d+D) a_4) c \bar{c} \} \end{aligned} \quad (3.25)$$

where the upper and lower signs are if S is even and odd respectively. If we denote the components of Λ by $(\lambda_1, \lambda_2, \lambda_3 \text{ and } \lambda_4)$, we find that

$$\delta \bar{\Phi} = 2M\lambda_4 + (d+D)\lambda_3. \quad (3.26)$$

Examining Eq. (3.23), we find that $\bar{\Phi}$ and λ_4 have the index structure $\bar{\Phi}_{n+1}^n$ and λ_{4n}^{n+1} respectively. At lowest order, we find that

$$\delta \bar{\Phi}_p = 2p\lambda_4^p. \quad (3.27)$$

and we can use λ_4^p to set $\bar{\Phi}_p$ to zero. In fact, in general there are as many λ_4 's as $\bar{\Phi}$'s, and we can use them to set $\bar{\Phi} = 0$. One can check that one may also use λ_3 to set $\phi = 0$, since $Q\chi_1 = 0$ now implies $(d+D)\phi = 2M\phi = 0$ and

$$\delta \psi = \kappa \lambda_3 + (d+D)\lambda_4. \quad (3.28)$$

As a result, χ_1 now takes the form

$$\bar{\Psi} + c\phi \quad (3.29)$$

and the equation of motion is

$$\partial^{\alpha d} (\bar{\Psi} + c\phi) = 0 \quad (3.30)$$

However, we have not completely fixed the λ_3 and λ_4 gauge invariances which are now subject to

$$\kappa \lambda_3 + (d+D)\lambda_4 = 0 = 2M\lambda_4 + (d+D)\lambda_3 \quad (3.31)$$

The transformation of $\bar{\Psi}$ and ϕ are given by

$$\begin{aligned} \delta \phi &= -\kappa \lambda_1 + \left(\frac{\partial}{\partial \omega} + \frac{1}{2}\right) \lambda_4 + (d+D) \lambda_2 \\ \delta \bar{\Psi} &= -2M\lambda_2 + \left(\frac{\partial}{\partial \omega} + \frac{1}{2}\right) \lambda_3 + (d+D) \lambda_1. \end{aligned} \quad (3.32)$$

Since ϕ and Ψ obey the same equations as λ_3 and λ_4 , we may use the latter to set the ω dependence of ϕ and Ψ to be prescribed, for example, not to depend on ω at all. We cannot gauge ϕ and Ψ to zero as λ_4 and λ_3 must vanish at $\omega = \pm \infty$ ($\alpha = 0$ and $\alpha = \infty$); see Ref. 1) in this context.

At this point we are left with $\Psi + c\phi$ subject to $Q^{\text{old}}(\Psi + c\phi) = 0$ and the fields Ψ and ϕ are functions of c_n , $\bar{c}_n$ and $x^\mu(\sigma)$; in other words, we have arrived at the formulation of the open string found in Refs. 6) and 7). However, let us continue the discussion to arrive at the physical states. We may use λ_2 to set $\phi = 0$ leaving only Ψ , which satisfies $K\Psi = 0 = (d+D)\Psi = 0$.

We now borrow with a slight modification an argument given in Ref. 8), where it was shown that once the states are on-shell, then the projector E onto physical states, that is DDF states, can be written as $E = \{S, Q^{\text{old}}\}$ for some operator S . Since Ψ is on-shell ($K\Psi = 0$) and $Q\Psi = 0$, we may write

$$E \Psi = \varphi(S \Psi) \quad (3.33)$$

As such, we may use a gauge transformation to set any state for which E is non-zero to zero. Thus we have demonstrated that we are left with only physical states. Clearly, we could gauge away less and in particular leave only ϕ_0^0 , in which case the equations of motion imply that $(L_0 - 1)\phi$, $L_n \phi = 0$, $n > 1$. In Ref. 8) it was noted that DDF operators acting on the other zero mode vacuum were also candidates for physical states, but as a consequence of χ_1 being even, these states are not included in the gauge-invariant theory.

The analysis of the cohomology of Q contains many of the features of the above discussion, except that the gauge choice is now replaced by using the equivalence class $\chi \sim \chi + QN$. A first step is to reduce χ to be of the form of Eq. (3.29), where Ψ and ϕ are ω -independent. We may then make a further choice to remove ϕ , leaving only Ψ , and finally to leave only the DDF physical states according to the same argument as given above.

We now show that we can gauge-fix the theory of Eq. (3.20) so as to obtain the BRST theory of Eq. (3.8). Let us denote by λ_i , λ_i' and ϕ_i , $i = 1, 2, 3, 4$ the components of Λ , Λ' and χ_2 respectively. Using λ_3 , λ_1 and λ_1' and λ_3' , we may choose the gauge

$$\psi - c_3 = 0 = \phi + c_1 = c_2 = c_4. \quad (3.34)$$

We find the action becomes

$$A + A_{gf} + A_{gh} \quad (3.35)$$

where A is the action of Eq. (3.8)

$$\begin{aligned} A_{gf} &= \int dx^\mu d\omega d\tau \prod_{n=1}^{\infty} dx_n^\mu d\bar{c}_n dc_n e^{-\omega} (b_4 (\psi - c_3) \\ &\quad + b_2 (\phi + c_1) + b_2' c_2 + b_4' c_4) \quad (3.36) \\ A_{gh} &= \int dx^\mu d\omega d\tau \prod_{n=1}^{\infty} dx_n^\mu d\bar{c}_n dc_n \left(\bar{\lambda}_3 (\kappa \lambda_3 + (d+D) \lambda_4 - i \frac{d}{d\tau} \lambda_3 e^{-\omega} \right. \\ &\quad - (-2m \lambda_4' + (d+D) \lambda_3')) + \bar{\lambda}_1 (-\kappa \lambda_1 + (\frac{\partial}{\partial \omega} + \frac{1}{2}) \lambda_4 + (d+D) \lambda_2 + i e^{-\omega} \frac{d}{d\tau} \lambda_1 \\ &\quad + (2m \lambda_2' - (\frac{\partial}{\partial \omega} + \frac{3}{2}) \lambda_3' + (d+D) \lambda_1')) + \bar{\lambda}_1' (i e^{-\omega} \frac{d}{d\tau} \lambda_2 - (\kappa \lambda_1' - (\frac{\partial}{\partial \omega} + \frac{3}{2}) \lambda_4' \\ &\quad + (d+D) \lambda_2')) + \bar{\lambda}_3' (i e^{-\omega} \frac{d}{d\tau} \lambda_4 - (-\kappa \lambda_3' + (d+D) \lambda_4')) \end{aligned} \quad (3.37)$$

In the above we use the same symbol for the ghost as for the parameter. At first sight, it would appear that the gauge had not been completely fixed since we have not discussed $\lambda_2, \lambda_4, \lambda_2'$ and λ_4' . One may verify that placing conditions on the other remaining fields does not further fix the gauge. For example, one finds that

$$(e^\omega \kappa - i \frac{d}{d\tau}) \delta \Phi = (d+D) \delta (\psi - c_3) - 2m \delta c_4. \quad (3.38)$$

However, it is clear from Eq. (3.21) that there is a hidden invariance $\bar{\Lambda}, \bar{\Lambda}'$ given by

$$\Lambda = \Phi \bar{\Lambda}, \quad \Lambda' = i \frac{d}{d\tau} \bar{\Lambda} - \Phi \bar{\Lambda}' \quad (3.39)$$

and these are the gauge degrees of freedom which have not so far been fixed, and now manifest themselves in the usual way by being an invariance of the ghost action.

The situation is rather like the transition from the master set $(\chi Q^{\text{old}} \chi, \delta \chi = Q^{\text{old}} \chi)$ to the BRST theory. This transition was first carried out in Ref. 6). The invariance in the ghost action is now fixed by choosing

$$\lambda_4 = \lambda'_3, \quad \lambda_2 = -\lambda'_1, \quad \lambda'_2 = \lambda'_4 = 0 \quad (3.40)$$

We then find a further hidden invariance in the ghost-for-ghost action which is fixed in a similar way ad infinitum.

The resulting BRST action can be simplified by integrating over the Lautrup fields b_2, b_4, b'_2, b'_4 , etc. Let us consider this first for the classical action. The gauge-fixing condition can be written

$$\frac{\partial}{\partial c} \chi_1 = \chi_2 \quad \text{or} \quad i \bar{\sigma} \chi_1 = \chi_2. \quad (3.41)$$

Eliminating χ_2 in Eq. (3.20) by this equation, we find that

$$\begin{aligned} & \int d\tau d\epsilon e^{-\omega} \left\{ \chi_1^* i \frac{d}{d\tau} \chi_1 - \chi_1^* \{ i \bar{\sigma}, \varphi \} \chi_1 \right\} \\ &= \int d\tau d\epsilon e^{-\omega} \left\{ \chi_1^* \left(i \frac{d}{d\tau} - \partial \right) \chi_1 \right\}. \end{aligned} \quad (3.42)$$

This is the first part of the BRST action since χ_1 is subject to $N' \chi_1 = 0$. The first set of ghosts involves $\Lambda, \bar{\Lambda}$ and Λ^1 , and after implementation of $i \bar{\rho} \Lambda = \Lambda^1$, we find an action of the same form as above, involving Λ and $\bar{\Lambda}$, which have

$$(N' - 1) \Lambda = 0 \quad (N' + 1) \bar{\Lambda} = 0 \quad (3.43)$$

Repeating this process for each set of ghosts, we recover a χ_1 (χ_1^*) not subject to any mode number constraint and having the BRST action of Eq. (3.8), which has the same form as Eq. (3.42).

This action is generically identical to the light-cone action stating that the time evolution of χ is generated by $\mathcal{H}$. The light-cone time x^+ is replaced by the proper time τ , the light-cone momentum by the string length, and the ghosts compensate for the additional bosonic dimensions.

4. - INTERACTING POINT PARTICLE

In this section we make a slight digression to extend the work of Ref. 1) to the interacting case. This will prove a useful guide for the corresponding construction of the case of the string. In this section and Section 5, it will be useful to regard χ as being defined on the range $-\infty < \alpha < +\infty$, and not to explicitly denote $\chi(-\alpha, \dots) = \chi^*(\alpha, \dots)$ for $\alpha > 0$. Having adopted this convention, it is preferable to scale χ by a $|\alpha|^{-\frac{1}{2}}$ factor. Upon making these changes, the free action of Ref. 1) becomes

$$\frac{1}{2} \int d\hat{z} \int_{-\infty}^{\infty} \frac{d\alpha}{\alpha} d\tau \left\{ \chi(-\alpha, \hat{z}, \tau) \left(i \frac{d}{d\tau} - \mathcal{H} \right) \chi(\alpha, \hat{z}, \tau) \right. \quad (4.1)$$

where

$$\mathcal{H} = \frac{1}{\alpha} \left(-\partial^2 + m^2 - \frac{\partial}{\partial c} \frac{\partial}{\partial \bar{c}} \right) \quad (4.2)$$

$$\hat{z} = x^\mu, c, \bar{c}$$

and it is invariant under $\delta\chi = \Lambda Q\chi + \bar{\Lambda} \bar{Q}\chi$ where

$$Q = \frac{1}{2} \{ c, \mathcal{H} \} - \frac{\partial}{\partial \alpha} \frac{\partial}{\partial \bar{c}} + \frac{1}{2\alpha} \frac{\partial}{\partial \bar{c}} \quad (4.3)$$

The Q given here differs slightly from the preprint of Ref. 1) by a normal ordering term that also affects the choice of measure as well as the above rescaling of χ which results in the addition of the last term.

In the following manoeuvres, the quantity α which appears in $\mathcal{H}$ should be regarded as an operator; as a result, $\mathcal{H}$ is Hermitian with the scalar product given in this paper. The operator $\mathcal{H}$ also integrates by parts as follows

$$\int d\hat{z} \int_{-\infty}^{\infty} \frac{d\alpha}{\alpha} d\tau \chi(\dots, -\alpha) \mathcal{H} Y(\dots, \alpha) = - \int d\hat{z} \int_{-\infty}^{\infty} \frac{d\alpha}{\alpha} d\tau \mathcal{H} \chi(\dots, -\alpha) Y(\dots, \alpha)$$

Let us assume that the interaction vertex in the BRST theory is a simple overlap, that is, the vertex in the representation involving the co-ordinates $z = \{x^\mu, c, \bar{c}\}$ and α takes the form

$$V(\hat{z}, \alpha) = \delta(c_1 - c_2) \delta(c_2 - c_3) \delta(\bar{c}_1 - \bar{c}_2) \delta(\bar{c}_2 - \bar{c}_3) \delta(x_1^\mu - x_2^\mu) \\ \delta(x_2^\mu - x_3^\mu) \delta(\alpha_1 + \alpha_2 + \alpha_3) \equiv U$$

(4.5)

We note that this is manifestly $OSp(D-1,1/2)$ -invariant. We therefore add to the free action the term

$$A^I = g/3 \prod_{r=1}^3 \int d\hat{x}_r d\bar{c}_r d\alpha_r \int_{-\infty}^{\infty} \frac{d\alpha_r}{\alpha_r} d\tau \cup \chi(1) \chi(2) \chi(3). \quad (4.5)$$

where $\chi(i) = \chi(z_i, \alpha_i, \tau)$.

Varying A^I under $\delta\chi = \Delta Q\chi$, we find that

$$\delta A^I = g/3 \prod_{r=1}^3 \int d\hat{x}_r \int_{-\infty}^{\infty} \frac{d\alpha_r}{\alpha_r} d\tau \cup \left\{ \left[\wedge \frac{c_1}{\alpha_1} \left(\frac{d\chi(1)}{d\tau} - i \frac{d}{d\tau} \right) \chi(1) \right] \chi(2) \chi(3) \right. \\ \left. + \left\{ \wedge \left[\frac{1}{\alpha_1} \frac{\partial}{\partial c_1} - \frac{\partial}{\partial \alpha_1} \frac{\partial}{\partial \bar{c}_1} \right] \chi(1) \right\} \chi(2) \chi(3) + \text{cyclic} \right\}. \quad (4.6)$$

The τ derivative becomes a total derivative which we can discard when the cyclic terms are included. The second term can be seen to contain a piece

$$\dots \int_{-\infty}^{\infty} \frac{d\alpha_1}{\alpha_1} \delta'(\alpha_1 + \alpha_2 + \alpha_3) \left(\sum_{s=1}^3 \frac{\partial}{\partial \bar{c}_s} \right) (\bar{c}_1 - \bar{c}_2)(\bar{c}_2 - \bar{c}_3) \quad (4.7)$$

which vanishes due to p momentum conservation. The first term is proportional to an equation of motion and may be cancelled by an addition to $\delta\chi$, namely

$$\delta\chi(z, -\alpha) = \wedge Q(1) \chi(\hat{z}_1, \tau, -\alpha) + g \int \frac{d\alpha_2}{\alpha_2} \frac{d\alpha_3}{\alpha_3} d\hat{z}_2 d\hat{z}_3 \cup \wedge c_1 \chi(2) \chi(3) \\ = \wedge Q(1) \chi(\hat{z}_1, \tau, -\alpha) + g \int \frac{d\alpha_2}{\alpha_2} \frac{d\alpha_3}{\alpha_3} \delta(\alpha_1 + \alpha_2 + \alpha_3) \wedge c_1 \\ \cdot \chi(\hat{z}_1, \tau, \alpha_2) \chi(\hat{z}_1, \tau, \alpha_3). \quad (4.8)$$

Using the δ functions we may write A^I in the form

$$A^I = g/3 \int d\hat{x}^\mu d\bar{c} d\alpha \prod_{r=1}^3 \int \frac{d\alpha_r}{\alpha_r} d\tau \delta(\alpha_1 + \alpha_2 + \alpha_3).$$

$$\chi(x^\mu, c, \bar{c}, \tau, \alpha_1) \chi(x^\mu, c, \bar{c}, \tau, \alpha_2) \chi(x^\mu, c, \bar{c}, \tau, \alpha_3) \quad (4.9)$$

Using the component expansion of Ref. 1), we find that

$$A^I = g \int dx^\mu d\tau \prod_{r=1}^3 \frac{d\alpha_r}{\alpha_r} \left\{ \bar{\Psi}(x^\mu, \tau, \alpha_1) \bar{\Psi}(x^\mu, \tau, \alpha_2) \right. \\ \left. \Psi(x^\mu, \tau, \alpha_3) + \bar{\Theta}(x^\mu, \tau, \alpha_1) \Psi(x^\mu, \tau, \alpha_2) \Theta(x^\mu, \tau, \alpha_3) \right\} \\ \delta(\alpha_1 + \alpha_2 + \alpha_3) \quad (4.10)$$

This is, of course, the interaction term for the Parisi-Sourlas action in light-cone co-ordinates. One easily verifies that the action is invariant under the above transformation to all orders in g and so no further addition to the action and transformation law is required.

The gauge-invariant interacting theory corresponding to the BRST theory discussed above is

$$A = \frac{1}{2} \int d\hat{z}_1 d\tau \int_{-\infty}^{\infty} \frac{d\alpha}{\alpha} \left\{ \chi(\hat{z}_1, \tau, -\alpha) i \frac{d}{d\tau} \chi(\hat{z}_1, \tau, \alpha) - \chi(\hat{z}_1, \tau, -\alpha) \varphi \tilde{\chi}(\hat{z}_1, \tau, \alpha) \right. \\ \left. + \tilde{\chi}(\hat{z}_1, \tau, -\alpha) \varphi \chi(\hat{z}_1, \tau, \alpha) \right\} + g \prod_{r=1}^3 \int d\hat{z}_r \int_{-\infty}^{\infty} \frac{d\alpha_r}{\alpha_r} d\tau U_C \chi(1) \chi(2) \chi(3) \quad (4.11)$$

where, in terms of the previous notation of Ref. 1),

$$\chi = \chi_1, \quad \hat{\chi} = \chi_2. \quad (4.12)$$

Varying the interaction term under the Abelian symmetries, we find the result

$$g \prod_{r=1}^3 \int d\hat{z}_r \frac{d\alpha_r}{\alpha_r} d\tau \left[+ \left\{ - \left(\sum_{s=1}^3 \varphi_s \right) U_C \right\} \left\{ \Lambda(1) \hat{\chi}(2) \chi(3) \right. \right. \\ \left. \left. - \hat{\chi}(1) \Lambda(2) \chi(3) \right\} + \left[(U_C) \left\{ -i \frac{d}{d\tau} \chi(2) + \varphi(2) \hat{\chi}(2) \right\} \chi(3) \right] \right. \\ \left. + (2 \leftrightarrow 3) \right] \Lambda(1)$$

$$- (U_C) \{ \varphi(z) \chi(z) \} (\lambda(1) \hat{\chi}(2) - \hat{\chi}(1) \lambda(2)).] \quad (4.13)$$

The first term vanishes due to the identity

$$\left(\sum_{s=1}^3 \varphi_s \right) (U_C) = 0 \quad (4.14)$$

while the latter two terms are cancelled by adding new terms to the field variations as follows

$$\begin{aligned} \delta \chi(\hat{z}_1, \tau, -\alpha_1) &= \varphi(1) \chi(\hat{z}_1, \tau, -\alpha_1) + g \int d\hat{z}_2 d\hat{z}_3 \frac{d\alpha_2}{\alpha_2} \frac{d\alpha_3}{\alpha_3} U_C [\lambda(2) \chi(3) \\ &\quad + \lambda(3) \chi(2)] \\ \delta \hat{\chi}(\hat{z}_1, \tau, -\alpha_1) &= i \frac{d}{d\tau} \hat{\chi}(\hat{z}_1, \tau, -\alpha_1) + g \int d\hat{z}_2 d\hat{z}_3 \frac{d\alpha_2}{\alpha_2} \frac{d\alpha_3}{\alpha_3} U_C [\lambda(2) \hat{\chi}(3) \\ &\quad - \hat{\chi}(2) \lambda(3)] \end{aligned} \quad (4.15)$$

We observe that $\tilde{\chi}$ acts as a Lagrange multiplier for the BRST condition on physical states and that the commutation of two transformations closes to give an Abelian algebra. Any other polynomial interaction can be treated in a similar manner.

5. - THE INTERACTING OPEN STRING

Just as the free BRST theory is $OSp(D, 2/2)$ -invariant, we must add an interacting term which preserves this symmetry in order that the non-physical co-ordinates α, τ do not occur in physical processes. In the light-cone formalism in which most of the results of this paper are expressed, we expect the vertex to have a manifest $OSp(D-1, 1/2)$ invariant, and the extra symmetries are now realized by generators which are non-linear in the fields.

The vertex is given by

$$\begin{aligned}
 V = & \langle 1 | \frac{1}{2} | \frac{1}{3} | \exp \left\{ \sum_{n,m=1}^{\infty} \sum_{r,s=1}^3 \left(N_{nm}^{rs} \left(\frac{1}{2} \alpha_n^{\mu r} \alpha_m^{\mu s} \right. \right. \right. \\
 & \left. \left. - n \beta_n^r \bar{\beta}_m^s \right) + N_n^{rs} \left(\alpha_n^{\mu r} \alpha_0^{\mu s} - n \beta_n^r \bar{\beta}_0^s - \beta_0^r \bar{\beta}_n^s \right) \right\} \\
 & \exp \left[\tau_b \sum_{s=1}^3 \left(\frac{L_0^s}{\alpha_s} - 1 \right) \right] \cdot \delta \left(\sum_{r=1}^3 \alpha_0^{\mu r} \right) \cdot \delta \left(\sum_{r=1}^3 \alpha_r \right) \\
 & \cdot \delta \left(\sum_{r=1}^3 \beta_0^r \right) \cdot \delta \left(\sum_{r=1}^3 \bar{\beta}_0^r \right)
 \end{aligned}$$

(5.1)

where

$$N_{nm}^{rs} \equiv - \frac{\alpha_1 \alpha_2 \alpha_3}{m \alpha_r + n \alpha_s} \frac{mn}{\alpha_r \alpha_s} f_m(\gamma_r) f_n(\gamma_s); \alpha_0^\mu = p^\mu \sqrt{2\alpha'}$$

$$N_n^{rs} \equiv \frac{1}{\alpha_r} f_m(\gamma_r) (\alpha_r \delta_{r+1,s} - \alpha_{r+1} \delta_{r,s}), \gamma_r \equiv - \frac{\alpha_{r+1}}{\alpha_r}$$

$$f_n(\gamma) \equiv \frac{1}{n\gamma} \binom{n\gamma}{n} = \frac{1}{n!} (n\gamma-1)(n\gamma-2) \dots (n\gamma-n+1)$$

and

$$\tau_b = \alpha_1 \ln \left(-\frac{\alpha_1}{\alpha_3} \right) + \alpha_2 \ln \left(-\frac{\alpha_2}{\alpha_3} \right)$$

Except for the ghost zero mode, the above vertex is the same as that given in Refs. 9)-11), which itself was an extension of the light-cone vertex of Refs. 12)-14) to include $\alpha_n^\pm$, β_n and $\bar{\beta}_n$. The vertex of Refs. 9)-11), although it involved string lengths, was not $OSp(D-1,1/2)$. However, for the free open bosonic string, we now have an additional zero mode which enables us easily to realize a manifest $OSp(D-1,1/2)$. The vacuum in the vertex satisfies the equations

$$\alpha_n^\mu | \rangle = \beta_n | \rangle = 0 = \bar{\beta}_n | \rangle \quad n \geq 1 \quad (5.2)$$

as far as the non-zero modes are concerned. The $\alpha_0^{\mu r}$ read off the momenta of the states, and the $\delta(\sum_{r=1}^3 \alpha_0^{\mu r})$ factor corresponds to momentum conservation. The β_0 and $\bar{\beta}_0$ act similarly and there is a corresponding momentum conservation law. We note that the addition to the action is given by

$$g_3 \prod_{r=1}^3 \int d\rho_r d\bar{\rho}_r d\bar{g}_r \frac{d\alpha_r d\tau}{\alpha_r} U |x\rangle_1 |x\rangle_2 |x\rangle_3 \quad (5.3)$$

where $\rho = \beta_0$ and $i\bar{\rho} = \bar{\beta}_0$, and includes an explicit integration over the zero mode momentum co-ordinates. In what follows below we will often take the integration over the zero modes to be understood and may not write it explicitly. We could transform this vertex to position space, the transition for the bosonic and fermionic variables being much like that of Refs. 13), 14) and 10) respectively. One finds that it is simply an overlap, namely if

$$z^r(\eta_r) = (x^{\mu r}(\eta_r), c^r(\eta_r), \bar{c}^r(\eta_r), c^r, \bar{c}^r) \quad (5.4)$$

then the vertex is given by

$$\delta(z^3(\eta_3) - \theta(-\eta_3 + \beta_2 \pi) z^2(\eta_2) - \theta(\eta_3 - \beta_1 \pi) z^1(\eta_1)) \cdot \delta(y_1^- - y_2^-) \delta(y_2^- - y_3^-) \quad (5.5)$$

We note that, unlike the vertex in Refs. 9) and 11), this vertex does not require any limiting process when written in terms of $Z^r(\eta)$.

We now prove that the theory is also invariant under the full $OSp(D, 2/2)$ group. As noted in Section 3, the extra generators $J^{1-} = Q$ and $J^{2-} = \bar{Q}$ are the generators of Ref. 15) which occur in the light-cone version of string theory, except that the number of co-ordinates has been changed and some of them are anticommuting, namely $c(\sigma)$ and $\bar{c}(\sigma)$. We may expect therefore that the proof of BRST invariance for the action considered here will be rather similar to the proof of Lorentz invariance of the light-cone theory given in Refs. 12) and 16). In fact, this is the case; inspecting the proof of Ref. 12), we see that it involves oscillator algebra, plus the fact that $[M^{i-}, M^{j-}] = 0$ in $D = 26$ if the intercept is one. In our case, varying the interacting term of Eq. (5.3) under a BRST variation, we find the result

$$\begin{aligned} & \delta_3 U \wedge Q_1 |x\rangle_1 |x\rangle_2 |x\rangle_3 + \text{cyclic} \\ & = -\delta_3 U \wedge \left(\sum_{s=1}^3 U \bar{Q}_s \right) |x\rangle_1 |x\rangle_2 |x\rangle_3 \end{aligned} \quad (5.6)$$

Hence we must evaluate the expression

$$U \sum_{s=1}^3 \overleftarrow{Q}^s \quad (5.7)$$

which is similar to Eq. (4.18) of Ref. 12). The oscillator algebra goes through as in Ref. 12), since the oscillators obey the similar equations, namely,

$$\begin{aligned} [\alpha_n^\mu, \alpha_m^\nu] &= n \eta^{\mu\nu} \delta_{n+m,0} \\ \{n \beta_n, \beta_m\} &= n \delta_{n+m,0} \end{aligned} \quad (5.8)$$

The condition $[M^{i-}, M^{j-}] = 0$ is now replaced by $Q^2 = 0$, which also occurs if $D = 26$ and the intercept is one, the contribution of the anticommuting modes just compensating for the extra bosonic modes over and above the light-cone treatment. A similar comparison with Ref. 16) can also be made. Apart from important normal ordering effects, this paper utilizes the fact that the vertex is a simple δ function in position space and so also translates straightforwardly.

As a consequence, we may carry over from Refs. 12) and 16) the result

$$U \sum_{s=1}^3 \overleftarrow{Q}^s = U c(\sigma^{int}) \sum_{s=1}^3 p_s^- \quad (5.9)$$

where, as before,

$$p_s^- = \left(\frac{K}{\alpha_s} \right) = \frac{(L_0^{tot} - 1)}{\alpha_s} \quad (5.10)$$

Substituting Eq. (5.9) in the variation of the interacting term of Eq. (5.6), we find that

$$\begin{aligned} \delta A^I &= g_{1/3} U \wedge c(\sigma^{int}) \left[\left(p_1^- - i \frac{d}{dz} \right) |x\rangle_1 \right] |x\rangle_2 |x\rangle_3 \\ &\quad + \text{cyclic} \end{aligned} \quad (5.11)$$

We may cancel this term by adding a field variation to $\delta\chi$, namely:

$$\delta \langle x | = \langle x | \wedge \varphi(1) + g \int \wedge c(\sigma^{\text{int}}) |x\rangle_2 |x\rangle_3 \quad (5.12)$$

This completes the invariance to order g , which is in complete analogy with the point particle discussed in Section 4.

In the light-cone theory, however, we know that a four-point function is necessary for Lorentz invariance. In our case this means that the order g^2 terms do not cancel by themselves, but require the existence of a four-point function in the action and a term cubic in χ in $\delta\chi$. Although the proof of Lorentz invariance of the action is implicit in the work of Ref. 12), the exact form of the four-point function was found in Refs. 13) and 14) by demanding duality. This result may be immediately carried over to the BRST theory by simply demanding a manifest $\text{OSp}(D-1,1/2)$ covariance. Consequently, we add to the action the vertex

$$\tilde{U} |x\rangle_1 |x\rangle_2 |x\rangle_3 |x\rangle_4. \quad (5.13)$$

where

$$\tilde{U} = \langle 1 | \leq 1 \leq 1 \leq 1 \leq 1 \exp \left\{ \sum_{r,s=1}^3 \sum_{n,m=0}^{\infty} \left(-\frac{1}{2} \mathcal{H}_{mn}^{rs} (\alpha_m^{rm} \alpha_n^{sm} - m \beta_m^r \bar{\beta}_n^s) \right) \right\} \quad (5.14)$$

We draw the reader's attention to Refs. 13) and 14) for the definitions of $\mathcal{H}_{mn}^{rs}$, $\mathcal{H}_{m0}^{rs}$ and $\mathcal{H}_{00}^{rs}$, and for $n\beta_n^s$ with $n=0$ we take β_0^s . As for the light-cone case, this reduces to a simple overlap condition in co-ordinate space. In fact, adding the χ^4 term to the action is not sufficient to cancel the $\delta\chi = g\Lambda\chi^2$ variation of the cubic term, and one must add a $g^2\chi^3\Lambda$ term to $\delta\chi$. These variations imply that the second-quantized Q will contain χ^3 and χ^4 terms which will be proportional to $c(\sigma^{\text{int}})$. These additions are required by the $\text{OSp}(D,2/2)$ commutation rule of

Eq. (2.25) $[H = 2i\{Q, \bar{\rho}\}]$.

The discussion of the gauge-covariant open string theory parallels that given in Section 4 for the point particle. We add to the free action of Eq. (3.20) the term

$$A^I = g \prod_{r=1}^3 \int d x_r^{\mu}(\sigma) \int_{-\infty}^{\infty} \frac{d \alpha_r}{\alpha_r} d \tau \int d \bar{c}_r(\sigma) d c_r(\sigma) d \bar{c} d c \, U(c(\sigma^{\text{int}}))$$

$$\hat{\chi}(1) \chi(2) \chi(3) = g \, U(c(\sigma^{\text{int}})) |\hat{\chi}\rangle_1 |\chi\rangle_2 |\chi\rangle_3$$

(5.15)

where $\chi_2 = \tilde{\chi}$ and $\chi_1 = \chi$. The invariance at order g goes the same way as for the point particle since $[c(\sigma^{\text{int}})]^2 = 0$. We add to $\delta\chi$, $\delta\tilde{\chi}$ terms similar to Eq. (4.15) with $Vc \rightarrow Vc(\sigma^{\text{int}})$ and other obvious replacements. For the string, however, we must add a four-point function and further terms to $\delta\chi$ in order to cancel the variation of the action at order g^2 . The quartic term in the action is of the

form

$$\prod_{r=1}^4 \int d x_r^{\mu}(\sigma) \int_{-\infty}^{\infty} \frac{d \alpha_r}{\alpha_r} \int d \bar{c}_r(\sigma) d c_r(\sigma) d \bar{c} d c d \tau \, \tilde{V}(c(\sigma^{\text{int}}))$$

$$\hat{\chi}(1) \chi(2) \chi(3) \chi(4).$$

(5.16)

where $\tilde{V}$ is the vertex of Eq. (5.14) in position space.

One may choose a gauge in which χ has the form $\Psi + c\phi$ and so the field content considered in Refs. 9)-11) where string lengths were introduced by hand. In Refs. 9) and 10), many calculations such as invariance at order g and independence of the physical three-point function on the string lengths were shown to hold. Although the formulation of this paper uses similar vertices, a gauge choice is generally only permissible in the context of an equation of motion. In fact, the calculations of Refs. 9) and 10) reflect properties that concern tree-level effects and one would not expect them to generalize to loop effects, as do the considerations of this paper.

In a future publication we will give a more detailed discussion of the invariance and closure of the interacting theory.

6. - DISCUSSION

We have quantized the bosonic string paying attention to hitherto neglected world-sheet boundary conditions in the reparametrization transformations. These conditions generalize those found for the point particle¹⁷⁾, for which a correct classical discussion of BRST invariance was first given in Ref. 18). We have shown that the correct gauge fixing leads inevitably to the introduction of the string length α and of the proper time τ , as well as the corresponding compensating anticommuting co-ordinates. The string theory is a particularly natural application of the general framework of Fradkin and Vilkovisky¹⁹⁾.

The BRST fixed string field theory resembles the old light-cone action, with the group $SO(25,1)$ replaced by the supergroup $OSp(26,2/2)$, of which the BRST charges Q and $\bar{Q}$ are two of the generators which contain interaction-dependent terms more than bilinear in the fields. In the interacting theory, the string lengths must be conserved as they are an extra momentum component. Only the physical state condition, involving Q , breaks $OSp(26,2/2)$.

Were one to write the equations for the open string in terms of component fields, one would find an $OSp(26,2/2)$ formulation of each particle generalizing that given for the spin-0 particle in Ref. 1). Presumably, this $OSp(26,2/2)$ formulation for a particle of a given spin can be viewed as a variation of the method of induced representation where, when extending the irreducible representation of the Poincaré group to a reducible representation with a projector condition (i.e., a field equation), one also extends the group to $OSp(26,2/2)$.

The structure of perturbation theory Feynman graphs is identical to that of the light-cone approach and calculations are very similar. We recall that string lengths were introduced to solve the duality problem in the first place. In the formalism of this paper, the duality problem is still solved, while avoiding the infinite degeneracy of the spectrum which such an extra unphysical parameter might have meant. We note that the epic works of the Kyoto group²⁰⁾, running to 486 pages, have demonstrated that there are insuperable problems in the incorporation of string lengths solely by hand. Further investigations will reveal more details about the connection between duality, moduli space in string theory and the τ and (via α) σ parameters which our covariant string field theory contains. We also note the added bonus that the ghost zero mode vacuum does not need the special treatment which it did in previous attempts, neither for the free action nor for the interaction.

In this covariant approach, closed strings do not arise as open string bound states any more than they did in the light-cone case. For BRST invariance at the one-loop level, they must be added in the Lagrangian together with their interactions with the open strings, as well as trilinear interactions among themselves. The treatment of closed strings along the lines of this paper requires only some extra care about zero modes, in order to see how the algebraic constraint $L_0^{\text{tot}} - \tilde{L}_0^{\text{tot}} = 0$ arises. This will be reported elsewhere²¹⁾.

REFERENCES

- 1) A. Neveu and P. West, "Unification of the Poincaré Group with the BRST and Parisi-Sourlas Supersymmetries", CERN preprint TH.45476/86 (1986), to be published in Phys. Lett. B.
- 2) G. Parisi and N. Surlas, Phys. Rev. Lett. 43 (1979) 744.
- 3) K. Kato and K. Ogawa, Nucl. Phys. B212 (1983) 443;
S. Huang, Phys. Rev. D28 (1983) 2614.
- 4) W. Siegel, Phys. Lett. 148B (1984) 556; 149B (1984) 157.
- 5) "Higher Transcendental Functions", Bateman manuscript project, A. Erdélyi ed. (McGraw Hill, New York, 1953).
- 6) A. Neveu, H. Nicolai and P. West, Phys. Lett. 167B (1986) 307.
- 7) E. Witten, Nucl. Phys. B268 (1986) 253.
- 8) M.D. Freeman and D.I. Olive, Phys. Lett. 175B (1986) 151.
- 9) A. Neveu and P.C. West, Phys. Lett. 168B (1986) 192.
- 10) A. Neveu and P.C. West, Nucl. Phys. B278 (1986) 601.
- 11) H. Hata, K. Itoh, T. Kugo, H. Kunitomo and K. Ogawa, Phys. Lett. 172B (1986) 186.
- 12) S. Mandelstam, Nucl. Phys. B64 (1973) 205.
- 13) E. Cremmer and J.-L. Gervais, Nucl. Phys. B76 (1974) 209; B90 (1975) 410.
- 14) M. Kaku and K. Kilschawa, Phys. Rev. D10 (1974) 1110, 1823.
- 15) P. Goddard, J. Goldstone, C. Rebbi and C.B. Thorn, Nucl. Phys. B56 (1973) 109.
- 16) S. Mandelstam, Nucl. Phys. B83 (1974) 413.
- 17) C. Teitelboim, Phys. Rev. D25 (1982) 3159.
- 18) S. Monaghan, Phys. Lett. 178B (1986) 231.
- 19) E.S. Fradkin and G.A. Vilkovisky, Phys. Lett. 55B (1975) 224;
I.A. Batalin and G.A. Vilkovisky, Phys. Lett. 69B (1977) 309;
E.S. Fradkin and T.E. Fradkina, Phys. Lett. 72B (1978) 343;
M. Henneaux, Phys. Reports 126C (1985) 1.
- 20) H. Hata, K. Itoh, T. Kugo, H. Kunitomo and K. Ogawa, Kyoto preprints KUNS 829, RIFP 660, 6743, 674.
- 21) A. Neveu and P. West, in preparation.