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THERMAL GREEN'S FUNCTIONS OF THE ENERGY-MOMENTUM TENSOR AND
TRANSPORT COEFFICIENTS OF THE SU(3) YANG-MILLS GAS

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A B S T R A C T

The calculation of transport coefficients for the SU(3) Yang-Mills gas on a Euclidean lattice is discussed. We present first results for the shear viscosity η obtained from measurements of thermal correlation functions of the energy-momentum tensor on a $8^3 \times 4$ lattice.

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Forthcoming heavy ion experiments¹ will probe the existence of a quark-gluon plasma at high temperatures. The equilibrium properties of the plasma phase have been studied in lattice Monte Carlo simulations². However, in a heavy ion experiment, a hot QCD plasma will not be produced in equilibrium and moreover will cool down rapidly during the subsequent expansion. To judge, for instance, the temperature decrease during the expansion phase of the plasma, an understanding of dissipative effects will be essential.

Transport coefficients are important ingredients in the description of dissipative effects in an expanding quark-gluon plasma using hydrodynamic equations of motion³⁻⁶. For instance, the change in energy density ϵ is determined by

$$\frac{d\epsilon}{d\tau} + \frac{1}{\tau} (\epsilon + p) = \left(\frac{4}{3} \eta + \xi\right) / \tau^2 \quad (1)$$

with p denoting the pressure and $\eta(\xi)$ the shear (bulk) viscosity. Transport coefficients like η or ξ can be calculated by means of Monte Carlo simulations. This, however, requires the analytic continuation of thermal correlation functions of the energy-momentum tensor to real time^{4,7}, which in general is difficult as the knowledge of the spectral decomposition of the thermal Green's functions⁸⁻¹⁰ is needed. We will avoid this problem by using an ansatz for the spectral density function involving a few parameters. These are determined from a fit of the Monte Carlo data for the thermal Green's functions.

In the following, we will concentrate on a discussion of the calculation of the shear viscosity η and present first results from a Monte Carlo simulation on a $8^3 \times 4$ lattice for pure SU(3) gauge theory¹¹.

From a Kubo type formula, we find for the shear viscosity⁹

$$\eta = - \int d^3x' \int_{-\infty}^t dt_1 e^{\varepsilon(t_1-t)} \int_{-\infty}^{t_1} dt' \langle T_{12}(x,t), T_{12}(x',t') \rangle_{\text{ret}} \quad (2)$$

Here the limit $\varepsilon \rightarrow +0$ has to be taken, and $T_{\mu\nu}$ denotes the energy-momentum tensor, which in terms of the field strength tensor $F_{\mu\nu}$, is given by

$$T_{\mu\nu} = 2 \text{Tr} (F_{\mu\alpha} F_{\nu\alpha} - \frac{1}{4} \delta_{\mu\nu} F_{\alpha\beta} F_{\alpha\beta}) \quad (3)$$

The retarded Green's function $\langle A(x,t) B(x',t') \rangle_{\text{ret}}$ is defined as

$$\langle A(x,t), B(x',t') \rangle_{\text{ret}} = -i \theta(t-t') \langle [A(x,t), B(x',t')] \rangle_0 \quad (4)$$

with $\langle \dots \rangle_0$ denoting the usual thermal expectation value of the correlation function with respect to the thermal equilibrium distribution. The correlation functions appearing in Eq. (2) represent correlations in real time. In a non-perturbative lattice

calculation on a Euclidean lattice, we obtain instead thermal Green's functions. The determination of the retarded Green's functions in Eq. (2) thus requires an analytic continuation of the thermal Green's functions

$$G_{\beta}(x, \tau) = -i \langle R(x, \tau) B(0, 0) \rangle_0, \quad 0 < \tau < \beta = 1/T \quad (5)$$

The formalism to do this has recently been discussed by Hosoya et al.⁸⁾ [see also Ref. 10)]. Knowing the thermal Green's function $G_{\beta}(x, \tau)$, we can determine the Fourier transform $G^{\beta}(p, \omega_n)$ at the discrete set of frequencies $\omega_n = 2\pi n/\beta$ with $n = 0, \pm 1, \dots$. In terms of the spectral function $\rho(p, \omega)$, G^{β} is given as

$$G^{\beta}(p, \omega) = \int d\omega \frac{\rho(p, \omega)}{i\omega_n - \omega} \quad (6)$$

Once the spectral function ρ is known, the analytic continuation is immediate as the retarded Green's function has the same spectral representation as G^{β} with $i\omega_n$ replaced by $p_0 + i\varepsilon$.

Thus the aim of a non-perturbative Monte Carlo calculation of transport coefficients should be to extract from thermal correlation functions of suitable components of the energy-momentum tensor the spectral function $\rho(p, \omega)$, and use it as an input for the analytic continuation. In practice, however, we are far away from this ideal situation. On lattices with a rather small number of sites in the temporal direction, the thermal correlation functions can only be determined for a few points. A more pragmatic way to proceed is to start with a reasonable ansatz for the spectral density which contains a few parameters. Using the Fourier transform of Eq. (6) gives then an ansatz for the thermal correlation function $G_{\beta}(x, \tau)$ which can be used to fit the Monte Carlo data.

As an ansatz for the spectral density $\rho(\omega)$ [only the zero momentum part contributes in Eq. (2)], we use¹¹

$$\rho(\omega) = \frac{A(1 - e^{-\beta m})}{\pi} \left[\frac{\gamma}{(m - \omega)^2 + \gamma^2} - \frac{\gamma}{(m + \omega)^2 + \gamma^2} \right] \quad (7)$$

which is motivated by the spectral density for a free bosonic field and attempts to include the effect of interactions through the parameter γ . The above ansatz for $\rho(\omega)$ has three free parameters A , m and γ which have to be determined from fits of Monte Carlo data with the analytic form obtained from Eq. (7), i.e.,

$$G_\beta(\tau) = -i \int d\omega \frac{e^{\beta\omega}}{e^{\beta\omega} - 1} e^{-\omega\tau} \rho(\omega) \quad (8)$$

Once these parameters are determined we get the shear viscosity η as

$$\eta = 2A(1 - e^{-\beta m}) \frac{2\gamma m}{(\gamma^2 + m^2)^2} \quad (9)$$

or in terms of dimensionless lattice parameters $A_\ell = Aa^5$, $m_\ell = ma$ and $\gamma_\ell = \gamma a$

$$\eta/T^3 = N_\tau^3 \frac{2A_\ell}{m_\ell} (1 - e^{-N_\tau m_\ell}) \frac{2\gamma_\ell/m_\ell}{(1 + (\gamma_\ell/m_\ell)^2)^2} \quad (10)$$

where we have used $\beta \equiv 1/T = N_\tau a$, a being the lattice spacing and N_τ the number of lattice sites in temporal direction of the lattice.

To test the applicability of this approach, we have performed a Monte Carlo simulation on a $8^3 \times 4$ lattice. To evaluate η we have to measure thermal correlations of off-diagonal spacelike components of the energy-momentum tensor $T_{\mu\nu}$. Here we take advantage of the cubic symmetry of space components of $T_{\mu\nu}^{11}$ which allows us to express these correlation functions in terms of diagonal elements,

$$\langle T_{12}(x)T_{12}(y) \rangle = \frac{1}{2} [\langle T_{11}(x)T_{11}(y) \rangle - \langle T_{11}(x)T_{22}(y) \rangle] \quad (11)$$

The diagonal components of $T_{\mu\nu}$ represent the energy (T_{00}) and pressure (T_{ii}) in thermal equilibrium. Thus we can obtain $T_{\mu\mu}$ as a suitable derivative of the partition function of the SU(3) Yang-Mills system¹². This way we also obtain the correct trace anomaly for the energy-momentum tensor^{13,14}. As is usual in thermodynamic calculations on the lattice we have neglected the $O(g^2)$ terms in the definition of $T_{\mu\mu}$ in our present calculation. As in calculations of T_{00} we expect the influence of the neglected terms to be small. Thus we use for the diagonal components of the energy-momentum tensor

$$T_{\mu\mu}(x) = \frac{2}{g^2} \left[-\sum_{\nu \neq \mu} \text{Tr} U_{x,\mu\nu} + \sum_{\substack{\nu \neq \mu \\ \nu > \mu}} \text{Tr} U_{x,\nu\mu} \right] \quad (12)$$

On a $8^3 \times 4$ lattice, we can measure thermal correlation functions at distance $\tau = 0, 1, 2$. As we have to determine three parameters to extract η from the Monte Carlo data, calculations on such a small lattice can only serve as a test of the procedure used. We have performed simulations at various values of the bare coupling β in the confined and deconfined region. It turned out that the correlation functions drop very fast, making a measurement at distance $\tau = 2$ difficult even with runs of 180 000 iterations¹¹. In Fig. 1 we show results at $6/g^2 = 5.6$, which is close to the deconfinement transition ($6/g_c^2 = 5.68$) in the confined region. Figure 1a shows fits of $G(\tau)$ for fixed $m\beta = 11.6$ and various values of m/γ whereas in Fig. 1b we show fits for fixed $m/\gamma = 0.02$ and various values of $m\beta$. The third parameter, A , has been fixed to be identical to $G(0)$. Clearly, larger lattices will be necessary to do a better fit.

Fig. 1a Thermal Green's function $G_\beta(\tau)$ versus τ for $m\beta = 11.6$ and various values of the interaction parameter $\gamma/m = 0.0, 0.02$ and 0.04 . Also shown are Monte Carlo data from simulations at $6/g^2 = 5.6$ for the shear viscosity correlation function.

Fig. 1b Same as Fig. 1a, however, for fixed $\gamma/m = 0.02$ and $m\beta = 11.2, 11.6$ and 12.0 .

We see that while the mass parameter seems to be well determined ($m_a = 2.5 \pm 0.5$) the value of the interaction parameter γ/m is difficult to determine and we can only give an upper bound for it

$$\gamma/m < 0.05 \quad (13)$$

Similar results hold slightly above the deconfinement transition¹¹. From our present calculations, we thus can give only quite loose bounds for η/T^3 for temperatures close to T_c

$$0 \leq \eta/T^3 \leq 9.5 \quad (14)$$

It would be of considerable interest to get better bounds on η/T^3 as this would give indications for the applicability of hydrodynamical models for the expansion of a quark-gluon plasma⁶. To achieve this, it seems that other methods have to be tested which allow us to improve the signal to noise ratio in our data. Like in glueball calculations where similar correlation functions are studied, it may be useful to develop source methods for the energy momentum tensor.

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