MUON CAPTURE IN HYPERFINE STATES OF MUONIC ATOMS

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Among six coupling constants involved in weak vector and axial vector currents, the pseudoscalar term is less known, since it can not be studied in nuclear beta decay. This can, however, be done in muon capture in complex nuclei. We describe here the results of our extensive studies of the muon capture in ^{12}C , which has no spin. Longitudinal and average polarizations of the daughter nucleus, ^{12}B , are suitable physical observables for our purpose, because they are functions of the pseudoscalar coupling constant, but less dependent on the adopted nuclear model. Muon capture reaction is also investigated in spin-nonzero nuclei theoretically. In these nuclei, the muon capture takes place from two hyperfine states of a relevant muonic atom. A formalism of this reaction is made, and explicit formulas are given for the capture rate, longitudinal and average polarizations, and average alignment of the recoil nucleus. Analyzing these observables in the muon captures from two hyperfine states, we can obtain further information on the pseudoscalar coupling constant. In a few lightest muonic atoms, these physical quantities are numerically given.

1. INTRODUCTION

In this paper, I would like to describe the results of our recent investigations on weak nuclear processes, particularly on muon capture reactions starting from hyperfine states of muonic atoms. Our purpose to study these reactions is to reveal detailed structure of the weak interactions in complex nuclei. There are two hyperfine states, which have their spins $F = J \pm 1/2$, of the muonic atom, just before the muon capture takes place. Here J is the spin of the initial nuclear state, and the muon is in its K orbit. The energy difference of these two hyperfine states is assumed to be small compared with the energy transfer in the muon capture reactions. Populations of these two hyperfine states and these of the magnetic substates for each hyperfine state are known to be controlled by the recent advanced experimental technique. Consequently, we can measure various kinds of orientation parameters, such as polarization and alignment, of the daughter nucleus. Experimental data of these physical observables together with their theoretical expressions provide us detailed knowledge on the weak interactions in nuclei and that of the nuclear structure, such as exchange currents and are core polarization effects.

Before discussing our topics further, we have to mention about the electroweak interactions of Glashow, Weinberg, and Salam on the levels of

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quarks and leptons. These are established at extremely high energies. It is very natural to adopt these for deriving the weak interactions in nuclei. That is, the weak interactions on the level of nucleons and leptons may be written in terms of the fundamental lepton-quark interactions. For example, many groups are studying semi-leptonic decays of the baryon octet by the fundamental weak interactions with help of the dynamics of quarks and gluons. There are various theories for qualitative understanding of this problem, while a quantitative answer could be found by describing each member of the octet as a reasonable set of configurations of the quark-gluon systems. In this connection, we have been performing our theoretical investigations on the weak nuclear processes by assuming weak interactions between nucleons and leptons. Investigations on these interactions have been providing us many useful informations, like symmetry properties to construct fundamental interactions, in the past, and various useful boundaries to help future understanding of weak nuclear processes in terms of weak quark processes.

Now let us proceed with our discussions on the weak interactions in nuclei. Since the weak bosons $W^\pm$, which mediate between quarks in nuclei and leptons, have their masses of 82 GeV. This is very large compared with the energy transfer in weak nuclear processes, and it guaranties the following form of the current-current type as the matrix element of the interaction Hamiltonian for the muon capture, $\mu^- + p \rightarrow n + \nu_\mu$, where p may be a free proton or the proton in a nucleus¹:

$$H_I = (V_\alpha + A_\alpha)L_\alpha \quad (1)$$

with

$$L_\alpha = -i\bar{\psi}_\nu \gamma_\alpha (1 + \gamma_5) \psi_\mu / \sqrt{2} \quad (2)$$

$$V_\alpha = i\bar{u}(p_f)[g_V \gamma_\alpha + (g_W/2M)\sigma_{\alpha\beta}q_\beta + i(g_S/m_\mu)q_\alpha]\tau^{(-)}u(p_i), \quad (3)$$

$$A_\alpha = i\bar{u}(p_f)[g_A \gamma_\alpha + i(g_P/m_\mu)q_\alpha + (g_T/2M)\sigma_{\alpha\beta}q_\beta]\gamma_5 \tau^{(-)}u(p_i), \quad (4)$$

with

$$q_\alpha = (p_i - p_f)_\alpha, \quad \sigma_{\alpha\beta} = (1/2i)[\gamma_\alpha, \gamma_\beta]. \quad (5)$$

Here M and m_μ are the nucleon and muon masses, respectively. This form described by Eqs. (1)-(5) is given by assuming the invariance under proper Lorentz transformation. Each nucleon current of the vector and axial vector types, V_α and A_α , contains three components. For the vector, they are called the main term, weak magnetism, and induced scalar, while for the axial vector, the main term, induced pseudoscalar, and induced tensor.

By assuming universality of the muon and electron, the weak interactions have the same form as above expression in nuclear beta decays, where the coupling constants g can be studied. The results are summarized as follows¹:

$$g_V = (1.41271 \pm 0.00032) \times 10^{-49} \text{ erg cm}^3, \quad (6)$$

$$g_W/g_V = \mu_p - \mu_n = 3.706, \quad (7)$$

$$g_S \approx 0, \quad (8)$$

$$g_A/g_V = 1.262 \pm 0.005, \quad (9)$$

$$g_T/g_A = 0.08 \pm 0.15. \quad (10)$$

Here, the ratio g_A/g_V was recently given by a new experiment on the beta-ray angular distribution of polarized neutrons². The strength of the induced tensor, g_T/g_A , is also renewed by a joint work of Osaka group in theory³ and experiment⁴. The coupling constants g are generally dependent on the square of the four-momentum transfer q^2 as summarized in ref. 1). Numerical values of g 's derived from nuclear beta decays are those corresponding to $q^2 \approx 0$. In the muon capture, for example, in ^{12}C , $q^2 = 9.79 \times 10^3 \text{ (MeV/c)}^2$, and g_V and g_A are by about 2 % smaller than those for the beta decay of the neutron.

We can not obtain information on the induced pseudoscalar coupling constant g_p from nuclear beta decays. The reason for this is as follows. In Eq. (4), the induced pseudoscalar current contains the four-momentum transfer of the nucleon which we replace by that of the lepton, because of the four-momentum conservation rule. Next, we apply the Dirac equation to the lepton part. As a consequence, the induced pseudoscalar term becomes a product of the nucleon and lepton currents of the pseudoscalar type multiplied by the lepton mass, if we neglect the neutrino mass. In the case of the beta decay, the relevant lepton is the electron, whose mass is 207 times smaller than that of the muon. Because of this, effects of the induced pseudoscalar current can not be detected unless accuracy of experimental data in beta decay becomes extremely high.

On the other hand, the coupling constant g_p can be studied in the muon capture reactions in nuclei. Its magnitude is evaluated as

$$g_p/g_A = 2M_\mu / (m_\pi^2 + q^2) = 7.2, \quad (11)$$

for $q^2 = m_\mu^2$, by the PCAC hypothesis, which is subject to experimental study and theoretical analysis.

The muon capture in hydrogen seems to be the most ideal case for studying induced pseudoscalar coupling constant. There are, however, a few ambiguities remaining in the interpretation of experimental data, as described in section 2. Recent precision experiments on the polarizations of the recoil ^{12}B in the muon capture in ^{12}C and their theoretical interpretation provide us the most reliable value of g_p/g_A in section 3. Possible future experiments are discussed on the muon capture reactions starting from hyperfine states of muonic atoms to obtain further information on g_p/g_A in section 4. A short summary is also given in section 5.

2. MUON CAPTURE IN HYDROGEN

The muon capture in hydrogen is, in principle, one of the most suitable cases to study the pseudoscalar coupling constant, since there is no ambiguity due to nuclear structure. Because of this, the muon capture experiments have been repeatedly performed by several groups in different experimental conditions, and the results are summarized⁵ in Table 1. As is seen there, the

TABLE 1 Experimental data on muon capture rate in hydrogen (s^{-1}), and g_p/g_A derived from these data.

Exp. group and year	Target	Capture rate	g_p/g_A	
			I	II
Chicago, 1963	Bubble Chamber	428 ± 85	16.1 ± 6.9	14.5 ± 7.3
CERN-Bologna, 1962	Bubble Chamber	450 ± 50	14.2 ± 4.2	12.7 ± 4.2
Columbia, 1962	Liquid	515 ± 85	7.0 ± 6.5	4.9 ± 6.6
Columbia, 1963	Liquid	464 ± 42	11.0 ± 3.4	9.0 ± 3.5
Saclay-CERN-Bologna, 1981	Liquid	460 ± 20	11.3 ± 1.6	5.8 ± 2.5
CERN-Bologna, 1962	Gas	651 ± 57	8.4 ± 3.2	
Dubna, 1974	Gas	686 ± 88	6.4 ± 4.8	
Average of last five group			10.3 ± 1.2	7.1 ± 1.6

Numerical values of g_p/g_A are equal to $g_p/1.22$, where g_p are given in ref. 5), in the case I, no ortho-para transition between two-proton states in muonic hydrogen molecule, and in the case II, the ortho-para transition rate of $(4.1 \pm 1.4) \times 10^4 \text{s}^{-1}$, respectively.

experimental data on muon capture rate in hydrogen vary by different groups at different periods in a relatively wide range, though most of those coincide with each other in experimental error bars.

Interpretation of the experimental data varies also with each group of authors. Formerly, the experimental data in liquid hydrogen were thought to be the muon capture rate from the ortho state of two protons in muonic molecule. This brings us relatively larger values of g_p/g_A . Recently⁵, the Saclay-CERN-Bologna group made a new measurement of the muon capture in hydrogen. Furthermore, they measured also an ortho-para transition rate in two-proton states. This means that the muon capture takes place starting from the ortho state as well as para state of two protons. Taking into account this situation, the authors of ref. 5) derived numerical values of g_p from individual experimental data in liquid hydrogen. As is seen in the fourth and fifth columns in Table 1, g_p becomes considerably smaller if this correction is taken into account. It is interesting to note that the average value of g_p/g_A with the above correction is in a good agreement with the canonical value in Eq. (11), although individual values are quite different from this. In a later publication⁶, one of the authors of ref. 5) gave a different value of g_p from their original value.

3. POLARIZATION OF RECOIL NUCLEUS ^{12}B IN MUON CAPTURE IN ^{12}C

The muon capture rate increases with Z_{eff}^4 . Here Z_{eff} is called the effective charge of the nucleus which is slightly smaller than the atomic number in light nuclei. Therefore, measurement of the muon capture rate would be much easier in these nuclei than in hydrogen. And we could have additional information on the ratio g_p/g_A of the coupling constants from measured muon capture rates in complex nuclei, by accumulating experimental data in a shorter period. On the other hand, there is a serious disadvantage of ambiguities in nuclear wave functions. That is, the muon capture rate depends, generally, strongly on the adopted nuclear model than the ratio g_p/g_A . To see this in detail, let us consider a special case of the Gamow-Teller transition, where the nuclear spin changes by a unity, while the nuclear parity does not change. If we assume the canonical value of g_p/g_A , the induced pseudoscalar affects the partial muon capture rate by several percent. Here the partial rate is the transition rate where the final nuclear state is separately observed. This means that a few-percent change of calculated muon capture rate introduces a rather large ambiguity in g_p/g_A which is derived from analysis of experimental data. Therefore our choice of a particular nucleus is most essential, and its nuclear wave functions should be well established.

For our purpose, the most celebrated case is the $A = 12$ system in Fig. 1. The nuclear wave functions are of the general 0p-shell so as to fit various energy levels in the whole region of the 0p-shell nuclei. They are also adjusted by reproducing experimental data on beta-decay rate of ^{12}B , electron scattering in ^{12}C , muonic X-rays in muonic ^{12}C , etc. In these studies, there are two types of well known wave functions given by Cohen and Kurath⁷, and Hauge and Maripuu⁸. With these two kinds of wave functions, we calculated the partial muon capture rate $\Lambda(1^+)$ in ^{12}C leading to the ground state of ^{12}B , and obtained the results⁹,

$$\Lambda(1^+) = 6.16 \times 10^3 \text{ s}^{-1} \quad \text{and} \quad 7.61 \times 10^3 \text{ s}^{-1}, \quad (12)$$

for the Cohen-Kurath and Hauge-Maripuu models, respectively, and $g_p/g_A = 7$. Here the symbol 1^+ in the parentheses of Λ refers to the spin and parity of the daughter nucleus. Furthermore, the theoretically calculated values of the muon capture rate depends rather strongly on nuclear parameters, such as oscillator strength parameter for the radial wave functions, even in the same model. This fact clearly shows the difficulty of deriving an accurate value of g_p/g_A from an analysis of muon capture rates.

In order to avoid this difficulty we have been studying several kinds of relative transition probabilities, such as angular distribution of an emitted particle in the reaction or polarization of the daughter nucleus^{1,9-11}. These physical quantities are much less sensitive to the nuclear model, because they are expressed by combinations of ratios of the various types of nuclear matrix elements to the Gamow-Teller matrix element in the same

FIGURE 1
Energy levels of the $A=12$ system

transition, and they are insensitive to the adopted nuclear model. One exception is the matrix element of the time component of the axial vector current, because of its different behavior from the others under the time reversal operation. Fortunately, this term is a correction term, although we have to treat it very carefully.

In this connection, we shall discuss the following two kinds of nuclear polarization of the daughter nucleus ^{12}B . The longitudinal polarization P_L is defined by the nuclear polarization with respect to the momentum $\vec{q}$ of the recoil nucleus,

$$P_L = (\vec{J} \cdot \vec{q}) / |\vec{J} \cdot \vec{q}|, \quad (13)$$

$\vec{J}$ being the spin of the daughter nucleus. The average polarization P_{AV} is defined by the nuclear polarization with respect to the muon spin polarization $\vec{P}_\mu$

$$P_{AV} = (\vec{J} \cdot \vec{P}_\mu) / |\vec{J} \cdot \vec{P}_\mu|. \quad (14)$$

These polarizations satisfy the following equation:

$$(2J + 1)^2 [P_L(J)]^2 + 2[3P_{AV}(J) + 2(J + 1)]P_L(J) + 9[P_{AV}(J)]^2 = 0 \quad (15)$$

This equation holds without taking special assumption for weak interactions, except for the time reversal invariance and the nuclear spin change, $0 \rightarrow J$ ^{10,12}

In order to see dependence on the adopted nuclear models, we have our calculated values of the average polarization of ^{12}B in the muon capture of ^{12}C ,

$$P_{AV}(1^+) = 0.551 \quad \text{and} \quad 0.542 \quad (16)$$

for the Cohen-Kurath and Hauge-Maripuu models, respectively⁹. Here the canonical value of g_p/g_A is assumed. Comparing Eq. (12) with Eq. (16), we can see a very reduced nuclear model dependence of the average polarization.

Now let us introduce our results on nuclear polarizations in detail. The numerical values in Eqs. (12) and (16) are given in the impulse approximation. That is, the muon capture in ^{12}C is understood as a simple sum of the muon captures in individual protons of this nucleus. In addition to these processes, the muon is absorbed by a virtual pion which mediates between two nucleons, and this pion emits a neutrino, see Fig. 2a. As is seen in Fig. 2,

there are several other processes similar to the process described as above. We call these the exchange currents.

It is interesting to note that the exchange currents reduce the Gamow-Teller matrix by about 5 %, while they enhance the time component of the axial vector current by about 40 %. Next we will consider the effect of the detailed structure of the nuclear wave functions. We have started from the general 0p-shell wave functions. Configuration mixing of higher shells than the 0p-shell should be introduced by perturbation due to the residual interaction. We call this the core polarization effect, which depends on the adopted residual interaction. This reduces the Gamow-Teller matrix and the time-component of the axial vector by about 5 % and 13 %, respectively.

Taking into account these two effects, we have⁹

$$\Lambda(1^+) = 5.28 \times 10^3 \text{ s}^{-1}, \quad (17)$$

and

$$P_{AV}(1^+) = 0.532, \quad (18)$$

for the Hauge-Maripuu model with the Sussex interaction¹³ as the residual interaction, since in this model the effective interaction is derived from the same interaction. Our calculations have been made up to the second order of the core polarization effects and the cross term of the first-order core

FIGURE 2
Exchange currents

polarization and the exchange current effects. We do not present diagrams here, since they are too many.

Before comparing the theory with experiment, we have to make an important consideration on the experimentally observed data. The experimental value of the nuclear polarization is given by measuring asymmetry of the beta-ray angular distribution from the ground state of ^{12}B which is produced as the daughter nucleus of the muon capture in ^{12}C . There are, however, several processes of producing the ground state of ^{12}B . The direct process is the muon capture leading to the 1^+ state. As is seen in Fig. 3, there are four excited states, each of these states is also produced as a consequence of the muon capture in ^{12}C . It emits gamma rays and cascades down to the ground state which has the half life of 20 ms. Since the gamma decays are very fast compared with the beta decay, experimentally observed beta-rays are due to all of these processes. We can derive the polarization $P^{\text{exp}}(J)$, where J is 1 in our case, of the ground state J from the measured value P^{exp} by subtracting contributions of the muon captures leading to the excited states of spin J_a ,

$$P^{\text{exp}}(J) = P^{\text{exp}} + \sum_a [\Lambda(J_a)/\Lambda(J)] [P^{\text{exp}} - A(J_a)P(J_a)], \quad (19)$$

with the attenuation factor through gamma decay, $A(J_a)$, which has an explicit formula in ref. 9). The polarizations $P(J_a)$ of the excited states J_a are given theoretically, since we have no experimental data and they are less sensitive to the nuclear model. On the contrary to this, the branching ratio $\Lambda(J_a)/\Lambda(J)$ of the muon capture rates $\Lambda(J_a)$ are also dependent on the assumed nuclear model strongly. We could not compare the muon capture rate for a positive parity state with that for a negative parity state, although we have wave functions obtained in a theoretical work on the radiative pion capture¹⁴

FIGURE 3
Energy levels of ^{12}B

in ^{12}C . Taking into account these considerations, we derive the experimental value $P_{AV}^{\text{exp}}(J)$ from Eq. (19) with experimental data of P^{exp} and those of $\Lambda(J_a)/\Lambda(J)$.

There are experimental data on average polarization P_{AV} and on the ratio of the average and longitudinal polarizations,

$$R = P_{AV}/P_L, \quad (20)$$

for which the systematic error in experiment is reduced considerably.

$$P_{AV}^{\text{exp}} = 0.452 \pm 0.042^{15}, \quad (21)$$

$$= 0.456 \pm 0.047^{16}, \quad (22)$$

and

$$R^{\text{exp}} = -0.516 \pm 0.041^{17}, \quad (23)$$

Two experimental data on P_{AV} are given by the ETH¹⁵ and Tokyo¹⁶ groups and they are in a good agreement. In order to derive $P_{AV}^{\text{exp}}(1^+)$ and $R^{\text{exp}}(1^+)$ from the above data with Eq. (19), we have to use experimental values of the partial muon capture rates, for which there are two sets of data given by the ETH¹⁸ and Saclay¹⁹ groups, as summarized in Table 2.

TABLE 2 Experimental values of partial muon capture rates $\Lambda^{\text{exp}}(J^\pi)$ for four bound states of $^{12}\text{B}(10^3\text{s}^{-1})$

	1^+	2^+	2^-	1^-
Roesch et al. ^a	6.28 ± 0.29	0.27 ± 0.10	0.12 ± 0.08	0.38 ± 0.10
Giffon et al. ^b	5.974 ± 0.350	0 ^c	0.06 ± 0.20	1.080 ± 0.125

^aRef. 18).

^bRef. 19).

^cAssumed to be zero.

If we adopt the set given by the ETH group¹⁸, we have

$$P_{AV}^{\text{exp}}(1^+) = 0.458 \pm 0.047 \quad (24)$$

and

$$R^{\text{exp}}(1^+) = -0.499 \pm 0.044. \quad (25)$$

In Fig. 4, the theoretical values of the average polarization $P_{AV}(1^+)$ are given by the full line, where the effects of the core polarization and exchange

currents are taken into account up to the second order, together with the experimental data in Eq. (24). Here we find

$$g_P/g_A = 10.6^{+2.3}_{-2.7}. \quad (26)$$

For the purpose of comparison, the theoretical values are also given by the dashed line, where the effects of the core polarization and exchange currents are taken into account up to the first order only, and by the dotted line in the impulse approximation. In Fig. 5, the theoretical values of $R(1^+)$ are given similarly as in Fig. 4, together with the experimental data in Eq. (25). we find

$$g_P/g_A = 9.5 \pm 1.7 \quad (27)$$

for the second-order calculation, while we have

$$g_P/g_A = 9.1 \pm 1.8 \quad (28)$$

FIGURE 4
Average polarization $P_{AV}(1^+)$
of ^{12}B as a function
of g_P/g_A . Theoretical
lines are given in ref. 11).
For detail see text.

FIGURE 5
Ratio $R(1^+)$ of the average
polarization to the longitudinal
polarization of ^{12}B as a
function of g_P/g_A . Theoretical
lines are given in ref. 11).

for the first-order calculation. Since the difference between the results in Eqs. (27) and (28) is not appreciable, we could expect a convergence of higher order calculations.

Finally if we adopt the sets of the Saclay group¹⁹ in Table 2, we have

$$R^{\text{exp}}(1^+) = -0.524 \pm 0.048. \quad (29)$$

Drawing this value in Fig. 5, we find

$$g_p/g_A = 8.5 \pm 1.9. \quad (30)$$

This should be compared with the value in Eq. (27). The difference of the results in Eqs. (27) and (30) means that the experimental branching ratios affect our conclusion on g_p/g_A appreciably.

4. MUON CAPTURE IN HYPERFINE STATE OF MUONIC ATOMS

To find some other physical observables suitable for determining g_p/g_A , we have studied muon capture reactions in spin-nonzero nuclei, many of which are available in nature. In these cases, the muon capture takes place from two hyperfine states of a muonic atom which consists of the nucleus of spin J and the muon in its K orbit. Since there is an energy difference between these two hyperfine states, electromagnetic transitions are possible from the upper state to the lower, or vice versa. The natural decay of the muon in these two states is also observed. Experimentally, the transition rates of the above three processes can be determined separately. Furthermore, orientation parameters, such as polarization and alignment, of the hyperfine states of spin $F=J\pm 1/2$ can be controlled by the present advanced technique. Because of this, there are more physical quantities available than in the case of spin-zero nuclei. Experimental data of the same physical quantities in the muon capture reactions starting from two hyperfine states give us invaluable informations on the pseudoscalar coupling constant g_p in the weak nucleon current of axial vector type.

We have made a general formalism of the muon capture reaction in hyperfine states of muonic atom. Explicit formulas are given for partial capture rate, longitudinal and average polarizations, and average alignment of recoil nucleus^{20,21}. Here we report the results of our theoretical studies in the following three cases of the Gamow-Teller transitions:

$$^{11}\text{B}(\frac{3}{2}^-, \frac{1}{2} : \text{gr.}) \rightarrow ^{11}\text{Be}(\frac{1}{2}^-, \frac{3}{2} : 320 \text{ keV}), \quad (31)$$

$$^{13}\text{C}(\frac{1}{2}^-, \frac{1}{2} : \text{gr.}) \rightarrow ^{13}\text{B}(\frac{3}{2}^-, \frac{3}{2} : \text{gr.}), \quad (32)$$

$$^{14}\text{N}(1^+, 0 : \text{gr.}) \rightarrow ^{14}\text{C}(2^+, 1 : 7.01 \text{ MeV}). \quad (33)$$

In the parentheses, three labels for each nuclear state refer to the spin and parity, isospin, and ground state or energy for excited state, respectively.

In the case of ^{11}B , the muonic atom has the hyperfine states of $F=1$ and 2 . Experimental data are available on the total muon capture rates starting from these two hyperfine states, though they are very preliminary ones²². Assuming refined experimental data coming in near future, we have a theoretical investigation on muon capture rates²⁰. Here the total capture rate refers to the sum of the partial capture rates over possible final states. We have taken into account two bound states of ^{11}Be and all states which consist of a free neutron and a ^{10}Be nucleus. Total muon capture rates are given in Table 3 for the finite-size nucleus and an appropriate strength of the well-depth parameter in the continuum-neutron model. We have also a partial muon capture

TABLE 3 Total muon capture rates (10^3 s^{-1}) starting from hyperfine states of muonic ^{11}B .

g_p/g_A	$\Lambda_{F=1}$	$\Lambda_{F=2}$	Λ_{AV}	$\Lambda_{F=2}/\Lambda_{F=1}$
0	28.6	12.6	18.6	0.44
3.5	26.5	11.8	17.3	0.45
7	24.6	11.3	16.3	0.46
10.5	22.9	10.8	15.3	0.47
14	21.4	10.6	14.7	0.50
Exp ^a	22 ± 2	19 ± 2	20 ± 2	0.86 ± 0.09

^aRef. 22). In the fourth column, Λ_{AV} is a statistically averaged value of $\Lambda_{F=1}$ and $\Lambda_{F=2}$.

rate averaged over two hyperfine states in the process of Eq. (31) in comparison with an old data²³, though a quantitative discussion for this is not possible.

There are almost no experimental data on the muon captures in ^{13}C and ^{14}N , which are processes indicated in Eqs. (32) and (33), respectively. We have, however, performed several calculations to search these reactions for suitable

physical quantities which show strong g_p/g_A dependence²¹. In Fig. 6, we have the average muon capture rate Λ_{AV} and the ratio R of the partial muon capture rates in the muon capture in ^{13}C and ^{14}N . Here the former is the statistically averaged value of the partial muon capture rates in two hyperfine states, and the latter is defined as

$$R = \Lambda(F = J - 1/2) / \Lambda(F = J + 1/2). \quad (34)$$

The average capture rate is relatively insensitive to g_p/g_A , while the ratio is sensitive to it. As we see in Fig. 6, the magnitude of R is rather small. This is because the dominant Gamow-Teller matrix elements vanish for the hyperfine state of $F = J - 1/2$ due to the selection rule for angular momenta. Numerical value of R may vary appreciably, if we include corrections of, e. g., exchange current and core polarization.

In Fig. 7, we show longitudinal polarization P_L (with the solid line) of the daughter nucleus in the muon capture starting from the hyperfine state of $F = J + 1/2$. The statistically averaged value of longitudinal polarizations for two hyperfine states, shown with the dashed line, is not too different from the solid line. This is because the capture rate from the hyperfine state of

FIGURE 6
Average muon capture rates Λ_{AV} (10^3 s^{-1}) and ratios R (%) in Eq. (34) of muon capture rates from two hyperfine states for $A = 13$ and $A = 14$ given in ref. 21) The left (right) hand ordinate refers to Λ_{AV} (R).

$F = J - 1/2$ is very small, as is seen in Fig. 6. Average polarization of ^{12}B is also shown there, for comparison. In Fig. 8, we show the average polarization and the average alignment of the daughter nucleus. From these investigations, we see that the polarization and alignment of the recoil nucleus for $A = 13$ and 14 are relatively insensitive to the adopted nuclear model and they are useful quantities to obtain information on g_p/g_A .

5. CONCLUSION

In order to study the pseudoscalar coupling constant in the weak nucleon current of axial vector type, we have made an extensive investigation of various physical quantities, which are observable in muon capture reactions. There are, generally, a few ambiguities in molecular physics or nuclear physics to find g_p/g_A from experimental data on muon capture rates. On the other hand, the recoil polarization and alignment are relatively free from these difficulties. The g_p/g_A values derived from our work on polarization of ^{12}B are the most reliable values at present. This value should be compared with that of the nucleon, since we are taking into account various nuclear effects in our analysis. Accuracy is still limited since there is an ambiguity of the

FIGURE 7
Longitudinal polarizations of ^{12}B , ^{13}B , and ^{14}C given in ref. 21)

FIGURE 8
Average polarizations and average alignments of ^{12}B , ^{13}B , and ^{14}C given in ref. 21)

branching ratios of muon capture in ^{12}C leading to excited states of ^{12}B . New measurement of these branching ratios is strongly recommended. Future experiments on polarization and alignment in the muon capture in hyperfine states of muonic atoms are also promising to study the weak nucleon currents in detail.

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