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BEAUTIFUL AND OTHER HEAVY BARYONS REVISITED

A. Martin

CERN - Geneva

and

J.M. Richard

L.P.T.H.E., Université Pierre et Marie Curie
4, place Jussieu, 75231 Paris Cedex 05

A B S T R A C T

We discuss the mass range for the beautiful baryons Λ_b , Σ_b and Σ_b^* allowed by rigorous inequalities which are derived in the framework of potential models but have probably a broader domain of validity. We obtain at the same time lower bounds on the masses of other baryons. We also compare the numerical prediction of several explicit models. There are convergent indications towards a mass of about 5.6 GeV for the Λ_b .

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1. - INTRODUCTION

Since the discovery of the upsilon and its excited states¹⁾, we know that the b quark exists. B mesons have been directly observed in e^+e^- collisions²⁾ and in fixed target experiments³⁾. Naturally, beautiful baryons must exist too. A group has already seen a signal interpreted as a Λ_b ⁴⁾, while others disagree with this finding⁵⁾. As theoreticians, we cannot decide who is right, except perhaps on one point, which is to confront the mass of the Λ_b candidate with theoretical predictions.

Recently, a calculation using "scalar lattice QCD" has been reported⁶⁾. The authors find a value of 5900 ± 35 MeV for the Λ_b mass and state: "according to our calculation it [the candidate of Ref. 4) at about 5.5 GeV] is much too light to be the Λ_b ". We disagree completely with this conclusion. The hadron spectrum exhibits striking regularities and the naive estimate, obtained by ignoring the recoil of the heavy quark:

$$\Lambda_b \cong \Lambda_c + \frac{3}{4}(B^* - D^*) + \frac{1}{4}(B - D) = 5.62 \text{ GeV} \quad (1)$$

is likely to be reasonably accurate. Beyond simple additive rules like (1), inequalities have been derived, relating various meson masses or various baryon masses or mesons and baryons. These inequalities, which are satisfied by the already known hadrons, allow for safe extrapolations to the beauty sector, with the prediction that the Λ_b should lie around 5.6 GeV and the Σ_b and Σ_b^* near 5.8 GeV.

In this letter, we shall review the existing inequalities obtained in potential models and derive some new extensions concerning in particular spin- $\frac{1}{2}$ baryons and some three-body interaction inspired by the string model. We also present one more potential model calculation of beautiful baryons which is compared to some previous ones.

2. - UPPER BOUNDS ON BEAUTIFUL HADRON MASSES

The non-relativistic Hamiltonian describing a Qq_1q_2 baryon or a $Q\bar{q}$ meson with a heavy quark Q is of the form

$$H = \frac{H_1}{M_Q} + H_2 \quad , \quad (2)$$

where H_1 contains kinetic energy and spin-dependent pieces. H_2 is independent of M_Q if one assumes flavour independence of the central potential. H being linear in $1/M_Q$, its lowest binding energy for a given spin configuration is a concave function of $1/M_Q$ by a well-known theorem⁷⁾. This is also true if the kinetic energy of the quarks is treated relativistically, because $\sqrt{p^2 + M_Q^2} - M_Q$ is a concave function of $1/M_Q$. Extrapolating in $1/M_Q$ provides upper bounds for the masses of beautiful hadrons ($Q=b$) from the charmed ($Q=c$) and strange ($Q=s$) systems⁸⁾. For instance, one can get an excellent fit⁹⁾ of the $b\bar{b}$, $c\bar{c}$ and $s\bar{s}$ mesons with the masses $m_b = 5.174$ GeV, $m_c = 1.8$ GeV and $m_s = 0.518$ GeV. This particular choice, combined with the experimental input¹⁾

$$\begin{array}{ll} K = 0.496 \text{ GeV} & K^* = 0.892 \\ D = 1.869 & D^* = 2.009 \\ \Lambda = 1.116 & \Lambda_c = 2.282 \\ \Sigma = 1.193 & \Sigma_c = 2.450 \end{array} \quad (3)$$

leads to⁸⁾

$$\begin{array}{l} B < 5.263 \\ B^* < 5.340 \\ \Lambda_b < 5.629 \\ \Sigma_b < 5.826 \end{array} \quad (4)$$

Only the first of these inequalities is slightly violated by experiment, by less than two standard deviations.

The criticism one can make is that one has to know a set of constituent quark masses obtained from some potential model. These masses depend to a certain extent on the choice of the potential. However, the mass differences and, in particular, $m_b - m_c$, are much better constrained, and turn out, indeed, to be rather model-independent. For instance, while the above calculation corresponds to $m_b - m_c = 3.374$ GeV, Stanley and Robson¹⁰⁾ use $m_b - m_c = 3.350$ GeV with both $m_b = 5.050$ and $m_c = 1.700$ GeV systematically lower. A slightly higher value of $m_b - m_c$ could also be reasonably considered. Following Ref. 7), one has

$$(b\bar{b}) - (c\bar{c}) = 2(m_b - m_c) - \int_{m_c}^{m_b} \frac{T(m)}{m} dm, \quad (5)$$

where $T(m)$ is the kinetic energy of a quarkonium system with common mass m . $T(m)$ is well approximated by the formula⁷⁾

$$T = \frac{3}{4} (E_{1P} - E_{1S}) \left[1 + \frac{7}{9} \left(\frac{E_{2S} + E_{1S} - 2E_{1P}}{E_{2S} - E_{1S}} \right)^2 \right], \quad (6)$$

which is inspired by the rigorous inequality

$$T \geq \frac{3}{4} (E_{1P} - E_{1S}), \quad (7)$$

and which is exact in the cases of a Coulomb potential and a harmonic oscillator. This gives

$$T(c\bar{c}) \simeq T(b\bar{b}) \simeq 0.41 \text{ GeV}. \quad (8)$$

Approximating in (5) T by this constant in the mass range $m_c < m < m_b$ (T is strictly constant for a logarithmic potential) leads to

$$m_b - m_c = \frac{1}{2} \left[\frac{3}{4} \gamma + \frac{1}{4} \eta_b - \frac{3}{4} \gamma_\psi - \frac{1}{4} \eta_c + 0.41 \ln \frac{m_b}{m_c} \right] \simeq 3.405 \quad (9)$$

Correspondingly, the upper bounds (4) become

$$\begin{aligned} B &< 5.292 \text{ GeV} \\ B^* &< 5.370 \\ \Lambda_b &< 5.659 \\ \Sigma_b &< 5.856 \end{aligned} \quad (10)$$

A similar $m_b - m_c \simeq 3.4 \text{ GeV}$ is used in the fit by Basdevant and Boukaraa¹¹⁾. On the other hand, Olsson¹²⁾ extracted a lower value $m_b - m_c \simeq 3.34 \text{ GeV}$ from the B, B^* and D, D^* systems. This shows the limitations of this kind of approach or its reliability, depending on our degree of pessimism or optimism.

It is needless to say that a measurement of the Σ_c^* mass would provide an upper bound for the Σ_b^* . In Ref. 13), it is argued that naively, à la DGG¹⁴⁾, $\Sigma_Q^* - \Sigma_Q$ behaves like $1/M_Q$. This leads to $\Sigma_c^* \simeq 2.505 \text{ GeV}$ and thus $\Sigma_b^* < 5.911 \text{ GeV}$.

3. - LOWER BOUNDS ON BARYON MASSES

There exists a not very well founded rule which says that the quark-quark potential in baryons can be deduced from the quark-antiquark potential in mesons by

$$V_{Q_1 Q_2} = \frac{1}{2} V_{Q_1 \bar{Q}_2} . \quad (11)$$

This rule is exact for colour-octet exchange. It has the merit that the baryon-meson comparison is relatively insensitive to the choice of quark masses, since the change

$$\begin{aligned} m'_Q &= m_Q - \Delta \\ V'_{Q_1 \bar{Q}_2} &= V_{Q_1 \bar{Q}_2} + 2\Delta \end{aligned} \quad (12)$$

leaves all masses invariant if one neglects the change of kinetic energy.

The rule (12) leads, in model calculations, to noticeable successes¹⁰⁾, such as the parameter free calculation of the mass of the Ω^- ¹⁵⁾, from a quarkonium potential⁹⁾.

On the basis of (12), an inequality was derived¹⁶⁾ relating the spin averaged baryon masses to the spin averaged meson masses, or the spin-3/2 baryon to spin-1 meson:

$$2 M(QQQ) > 3 M(Q\bar{Q}) . \quad (13)$$

The proof is based on the fact that in a common fictitious harmonic oscillator potential stabilizing the centre-of-mass, the baryon wave function $\phi(x_1 x_2 x_3)$ can be considered, for any value of x_3 , as a trial wave function for the $Q_1 \bar{Q}_2$ meson.

This inequality was generalized to the unequal mass case in the form^{17),18)}

$$M(Q_1 Q_2 Q_3) > \frac{1}{2} [M_{Q_1 \bar{Q}_2} + M_{Q_2 \bar{Q}_3} + M_{Q_3 \bar{Q}_1}] . \quad (14)$$

Note, however, that some other generalizations that one might be tempted to do are not allowed and are easily shown to fail. For instance

$$\bar{Q}\bar{Q}\bar{Q} + qqq > 3 Q\bar{q} \quad (15)$$

is not true for very large values of the mass ratio M/m , so that, for instance, a $(\bar{t}\bar{t}\bar{t})$ antibaryon does not annihilate on ordinary matter.

We now extend the inequality (15) to spin- $\frac{1}{2}$ baryons. Neglecting, as usual, the effect of tensor forces on ground states, the Hamiltonian, besides kinetic energy and central potential, contains only spin-spin potential. In mesons, one has $\sigma_{12} = \langle \sigma_1 \cdot \sigma_2 \rangle = 1$ for spin 1, and $\sigma_{12} = -3$ for spin 0. Baryons with at least two quarks of equal masses, have also well-defined values of the σ_{ij} 's. In a Λ , $\sigma_{12} = -3$, and $\sigma_{13} = \sigma_{23} = 0$. For a Σ , $\sigma_{12} = 1$, and for $\sigma_{13} = \sigma_{23} = -2$ and for a Σ^* , $\sigma_{12} = \sigma_{13} = \sigma_{23} = 1$. One has thus to introduce fictitious mesons with $\sigma = -2$ or $\sigma = 0$, but, thanks to the above-mentioned concavity property, one has

$$\begin{aligned} M(\sigma=-2) &> \frac{3}{4} M(\sigma=-3) + \frac{1}{4} M(\sigma=1) \\ M(\sigma=0) &> \frac{1}{4} M(\sigma=-3) + \frac{3}{4} M(\sigma=1). \end{aligned} \quad (16)$$

The inequality now reads

$$M(qqQ) > \frac{1}{2} M(q\bar{q}, \sigma_{12}) + \frac{1}{2} M(Q\bar{q}, \sigma_{13}) \quad (17)$$

and, using (16):

$$M(\Lambda_Q) > \frac{1}{2} M(q\bar{q}, J=0) + \frac{3}{4} M(Q\bar{q}, J=1) + \frac{1}{4} M(Q\bar{q}, J=0) \quad (18)$$

$$M(\Sigma_Q) > \frac{1}{2} M(q\bar{q}, J=1) + \frac{1}{4} M(Q\bar{q}, J=1) + \frac{3}{4} M(Q\bar{q}, J=0) \quad (19)$$

Naturally, one can have many objections to the derivation of these inequalities, concerning in particular the validity of the " $\frac{1}{2}$ " rule, the neglect of possible three-body forces and the non-relativistic treatment. However, at least for the case of spin- $\frac{3}{2}$ baryons, Nussinov¹⁷⁾ has a derivation purely based on QCD for which all these objections disappear and we may hope that something will also survive for spin- $\frac{1}{2}$ baryons. In this line, one should remark that, at large distances, the linear potential λr binding meson is better generalized in the baryon case by a Y-shape string¹⁹⁾ having a potential energy

$$V(Q_1 Q_2 Q_3) = \lambda \inf_P (d_1 + d_2 + d_3), \quad (20)$$

where the d 's are the distances of the quarks to the point P , which is easily shown to be very close but always larger than the potential arising from the " $\frac{1}{2}$ " rule,

$$V_{Q_1 Q_2 Q_3} \geq \frac{1}{2} \sum \lambda r_{ij} \quad (21)$$

so that the inequalities a fortiori hold for this three-body potential.

We note also that using a relativistic form of kinetic energy with instantaneous interactions would not modify the proof.

In the case of spin-3/2 [already studied in Ref. 16)] the inequalities give

$$\begin{array}{ll} \Omega^- > \frac{3}{2} \phi & 1.672 > 1.530 \text{ GeV} \\ \Delta > \frac{3}{2} \rho & 1.232 > 1.155 \\ \Sigma^* > K^* + \frac{1}{2} \rho & 1.385 > 1.275 \\ \Xi^* > \rho + \frac{\phi}{2} & 1.532 > 1.280 \end{array} \quad (22)$$

and the prediction

$$\Sigma_b^* > B^* + \frac{\rho}{2} = 5.710 \text{ GeV}. \quad (23)$$

For Σ -like baryons, we obtain

$$\begin{array}{ll} N > \frac{3}{4} \rho + \frac{3}{4} \pi & 0.938 > 0.681 \\ \Sigma > \frac{1}{2} \rho + \frac{3}{4} K + \frac{1}{4} K^* & 1.190 > 0.979 \\ \Xi > \frac{1}{2} \phi + \frac{3}{4} K + \frac{1}{4} K^* & 1.319 > 1.104 \\ \Sigma_c > \frac{1}{2} \rho + \frac{3}{4} D + \frac{1}{4} D^* & 2.450 > 2.288 \end{array} \quad (24)$$

and the prediction

$$\Sigma_b > \frac{1}{2} \rho + \frac{3}{4} B + \frac{1}{4} B^* = 5.670 \text{ GeV}. \quad (25)$$

For Λ -like baryons, we have

$$\begin{array}{ll} \Lambda > \frac{1}{2} \pi + \frac{3}{4} K^* + \frac{1}{4} K & 1.116 > 0.863 \\ \Lambda_c > \frac{1}{2} \pi + \frac{3}{4} D^* + \frac{1}{4} D & 2.282 > 2.042 \end{array} \quad (26)$$

and the prediction

$$\Sigma_b > \frac{1}{2} p + \frac{3}{4} B + \frac{1}{4} B^* = 5.670 \text{ GeV}. \quad (27)$$

We see that all inequalities are satisfied and that the maximum difference between the bound and the actual value is 250 MeV. If one adds 250 MeV to the lower bound on Λ_b , one finds something close to the upper bound obtained in Section 2.

In the charmed-strange sector²⁰⁾, we obtain for the T(css), with new name Ω_c

$$\Omega_c > \frac{1}{2} \phi + \frac{3}{4} D_s + \frac{1}{4} D_s^* \quad 2.740? > 2.515. \quad (28)$$

For the A (new name Ξ_c) and the S (new name Ξ_c' ?) one is tempted to write

$$S' > \frac{1}{2} K^* + \frac{3}{8} D + \frac{1}{8} D^* + \frac{3}{8} D_s + \frac{1}{8} D_s^* \simeq 2.400 \quad (29)$$

and

$$A > \frac{1}{2} K + \frac{1}{8} D + \frac{3}{8} D^* + \frac{1}{8} D_s + \frac{3}{8} D_s^* \quad 2.460 > 2.272 \quad (30)$$

In fact, there is a mixing between the Λ -type and the Σ -type configurations. This pushes the S up, so that (29) is safe. It pushes the A down but the effect is very small^{21),22)}, so that (30) is also guaranteed.

4. - EXPLICIT MODEL CALCULATIONS. CONCLUSIONS

A very thorough calculation of hadron masses has been done some years ago by Stanley and Robson¹⁰⁹⁾, who used relativistic kinematics and the " $\frac{1}{2}$ " rule. Although it could be criticized for having many parameters²³⁾, their model has had an undeniable explanatory and predictive power. Prior to the discovery of the naked beauty, they predicted the masses $B = 5.308 \text{ GeV}$ and $B^* = 5.344 \text{ GeV}$, only about 30 MeV too high. They also calculated the D_s and D_s^* ($c\bar{s}$) states in better agreement with today's experiments¹⁾ than with the numbers they quote in their paper. This is also the case for the Σ_c . Their predictions for the beautiful baryons are listed in the Table.

More recently, Isgur and co-workers²⁴⁾ have done a "relativistic" calculation of baryon masses and properties. For comparison, the results of Basdevant and Boukraa¹¹⁾ are also displayed in the Table. Finally, we have extended a non-relativistic calculation of ground-state baryons by Taxil and one of us²²⁾, using the " $\frac{1}{2}$ " rule and the $b\bar{b}$ ground state to fix the b-quark mass at $m_b = 5.250$ GeV. In contrast, we present also the bag model calculation of Izatt et al.²⁵⁾, where the light quarks are treated relativistically. These results are summarized in the Table, together with the upper and lower bounds obtained in Sections 2 and 3.

The convergence of the results is impressive. On the other hand, the Λ_b prediction of Ref. 6) disagrees violently with this overall picture. Furthermore, it predicts a very unlikely $\Sigma_b - \Lambda_b$ mass difference of 70 MeV, while the trend observed, from $\Sigma - \Lambda = 77$ MeV to $\Sigma_c - \Lambda_c = 169$ MeV is an increase of the mass difference, as one would expect from an approach of DGG type^{13),14)}.

From our analysis, the mass reported by the Zichichi group⁴⁾ is compatible with the inequalities on the Λ_b and marginally compatible with the explicit calculations. Needless to say that, beyond the mass spectrum, the measurement of the lifetime and branching ratios of beautiful baryons will greatly contribute to our understanding of weak and strong interactions. A characteristic signature of the Λ_b would be a lifetime of the order of 10^{-12} seconds, which could be seen in a fixed target experiment. Finally, let us make the obvious remark that should the top meson mass be known, all these considerations could be applied to top baryons, and more generally to any other flavour if it exists.

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	Stanley Robson	Maltman Isgur	Basdevant Boukraa	Richard Taxil	Izatt et al.	Upper bound	Lower bound
Ref.	10)	21)	11)	22) extended	25)		
Λ_b	5.596	5.585	5.547	5.580	5.64	5.659	5.379
Σ_b	5.859	5.795	5.714	5.800	5.78	5.856	5.670
Σ_b^*	5.877	5.805	5.766	5.841	5.82	5.911?	5.710

- TABLE -

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