Reparametrisation - Invariant Closed String Field Theory:Foliation Structure and Physical Spectrum

by

M. Nouri-Moghadam*
Research Institute for Fundamental Physics
Kyoto University, Kyoto 606

and

C.R.R. Smith†
Department of Mathematics, Kings College, London

and

J.G. Taylor‡
CERN, Geneva

Abstract

The foliation structure and physical spectrum of a reparametrisation-invariant field theory of closed strings are analyzed to show (a) independence of the action of the foliation (b) the spectrum is that of the usual light cone gauge formulation.

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 (†) Supported by SERC of Gt. Britain.
 (‡) Permanent address: Dept. of Mathematics, Kings College, London.

1. Introduction

It is still an open question as to how to construct a field theory of interacting closed strings. One approach, which has been followed by many [1], is that of using the BRST approach to first quantisation and developing a quantum field theory from that. An alternative approach is to attempt to preserve the underlying differential geometry more explicitly at all stages instead of having it emerge rather miraculously at the end of the construction as in the BRST approach. This latter method was initially formulated by Marshall and Ramond [2]. Their analysis produced a reparametrisation-invariant field theory which included interaction string fields.

It was possibly not appreciated at that time that such a theory was in general automatically marred by anomalies due to the need for the usual normal-ordering definition of the kinetic operator in the field theory action. To avoid this anomaly it proved necessary [3] to adjoin to the string variables $X^\mu(\sigma)$ ($1 \leq \mu \leq d$, where d is the dimension of the target space, and $0 \leq \sigma \leq \pi$) a further scalar mode $\phi(\sigma)$ corresponding to the conformal mode of the world-sheet metric. It then proved possible to extend the Marshall and Ramond [2] field theory so that the reparametrisation invariance was not spoiled by the conformal anomaly; this was achieved by cancellations between the anomaly contributions for X^μ and ϕ . A similar extension, though only at the first quantised level, was proposed independently by Münster [4], and by Awada [5] and had also been considered earlier by Teitelboim [6], although we must emphasize that only in ref[3] was a second-quantised field theory constructed as a direct extension of ref[2].

The construction of Marshall and Ramond^[2] used a foliation of the embedding space in order to define the function measure $\mathcal{D}X$ arising by integrating over all string configurations. This was achieved in terms of a family of space-like hypersurfaces

$$\Sigma_t = \{X^\mu; \psi(X^\mu) = t\} \quad (1.1)$$

The interpretation is then over strings living in a Σ_t for some t .

In order to define such a foliation it is necessary to assume some pseudo-Riemannian metric on the embedding space. This could be taken as arbitrary, and our whole discussion of reparametrisation-invariant actions be given at a very general level. Indeed it is very interesting to determine what, if any, constraints can arise on such a metric in order for the construction of ref[3] to succeed and there be no anomaly. Any such constraints may be expected to be the full string analogue of the β -function constraint approach^[7]. We will not do that here but hope to return to it elsewhere.

In the previous paper^[3] it appeared necessary to add a further time-like foliation ψ to the foliation χ of (1.1) in order to achieve the correct physical spectrum for the free string. It is the purpose of this paper to analyze the nature of these foliations in more detail. In particular we wish to consider the dependence of the theory on the exact choice of the foliations χ and ψ and the exact form of the spectrum. The foliations are considered in the next section, and the physical spectrum in the remaining one. The final section contains conclusions.

2. Foliation Structure

To reproduce the correct physical spectrum and to ensure a consistent time-interpretation (with a single time) a double foliation (ψ, χ) is necessary. Here ψ and χ are smooth maps from $\mathbb{R}^d$ into $\mathbb{R}$ (Minkowski space-time). The need for a single time for each string, as indicated in the introduction, requires that $\Sigma_t = \psi^{-1}(t)$ is a space-like hypersurface Σ_t for each t (w.r.t the Minkowski metric), with

$$\Sigma_t \subset \mathbb{R}^d, \quad \bigcup_{t \in \mathbb{R}} \Sigma_t = \mathbb{R}^d \quad (2.1)$$

Then the measure $d\mu_\psi(x)$, on integration over strings with the foliation ψ , may be defined as

$$d\mu_\psi = dt \int_{\sigma, \mu} dx^\mu(\sigma) \prod \delta[t - \psi(X^\mu(\sigma))] \quad (2.2)$$

A further time-like foliation χ is also introduced, so that if $\Sigma'_t = \chi^{-1}(t)$ then (2.1) holds with Σ_t replaced by Σ'_t . The time-like foliation χ is used to restrict strings further in the integration measure over all string configurations. This measure $d\mu_{\psi, \chi}$ is therefore

$$d\mu_{\psi, \chi} = dt \int_{\sigma, \mu} dx^\mu(\sigma) d\chi^\mu(\sigma) \prod \delta[t - \psi(X^\mu(\sigma))] \prod \delta[\chi - \chi(X^\mu(\sigma))] \quad (2.3)$$

The reparametrisation invariant action of ref[3] may therefore be written as

$$S_{\psi, \chi} = \int d\mu_{\psi, \chi} (X) \mathcal{L}(\psi, \chi) \cdot \int \phi^* \wedge e(\sigma) \otimes d\mu(\sigma) \quad (2.4)$$

In (2.4) $d\mu(\sigma) = \exp[-\frac{1}{2}\phi(\sigma)]d\sigma$ is the Diff S^1 -invariant integral around the string and $\Lambda^e(\sigma)$ is the kinetic term $X'^2(\sigma) + p_\mu^2(\sigma)$, with $p_\mu = \frac{1}{2}\delta/\delta x^\mu(\sigma)$, with the addition of the term proportional to $\phi'^2 + p_\phi^2(\sigma) - 4\phi''''$, $p_\phi = \frac{1}{2}\delta/\delta\phi(\sigma)$:

$$\Lambda^e(\sigma) = [X_\mu'^2(\sigma) + p_\mu^2(\sigma)] - \kappa[\phi'^2 + p_\phi^2 - 4\phi'''] \quad (2.5)$$

and $\kappa = d[1+48\eta]^{-1}$ is chosen to cancel the conformal anomaly.

The foliation ψ and χ are determined by their unit d-vectors m and n , with

$$m_\mu \propto \partial\psi/\partial x^\mu, \quad n_\mu \propto \partial\chi/\partial x^\mu \quad (2.6)$$

An important example of a foliation is the light-cone foliation (LCF) for which

$$\begin{aligned} \psi(X) &= X^+ = \frac{1}{\sqrt{2}}(X^d + X^{d-1}), \\ \chi(x) &= X^- = \frac{1}{\sqrt{2}}(X^d - X^{d-1}). \end{aligned} \quad (2.7)$$

We note that in this case $m = (0, \dots, 0, 1, 1)$, $n = (0, \dots, 0, -1, 1)$ are both null vectors $m^2 = n^2 = 0$ with $m \cdot n = 2$. Thus the LCF is singular. However it is the most natural choice of foliations in which to derive the physical spectrum, as is shown in the next section. Effectively one has identified the time with X^+ and similarly eliminated all but the 0-mode of X^- , which is then integrated over. Then only the transverse oscillation $\underline{X}(\sigma)$ are left, along with the conformal mode $\phi(\sigma)$. Moreover the variable y plays the

role of the conjugate of the string length variable, α , so that integration over y in (2.4) corresponds to integration over all string lengths.

For non-singular foliations one can have

$$m \cdot n = 0, \quad m^2 = 1, \quad n^2 = -1 \quad (2.8)$$

Corresponding to space-like and time-like foliations for ψ and χ respectively. As an example the coordinate foliation $X^\circ = \tau$ is commonly used in developing a Hamiltonian formalism. For this one has

$$\begin{aligned} \psi &= X^d \\ \chi &= X^{d-1} \end{aligned} \quad (2.9)$$

Any other non-singular foliation has a similar form.

It is expected that physical results arising from $S_{\psi, \chi}$ of (2.4) are independent of the specific choice of ψ or χ . This might be thought to correspond to the fact that the foliations are actually a gauge choice, fixing the parametrisation on the world sheets of the strings being considered. However this is not the case since the measure $d\mu_{\psi, \chi}(X)$ of (2.3) is invariant under a reparametrisation of the strings, by the same argument as given originally for the invariance of $d\mu_\psi(X)$ of (2.2) by Marshall and Ramond [2]. This is essentially because a reparametrisation does not change the restriction imposed on the string by the δ -functions. In detail if the reparametrisation is $\sigma \rightarrow \tilde{\sigma} = \tilde{\sigma}(\sigma)$, with

$\bar{X}^\mu(\vec{\sigma}) = X^\mu(\sigma)$ then

$$\begin{aligned} d\mu_{\psi, \chi}(\bar{X}) &= d\tau d\vec{y} \prod_{\vec{\sigma}} \delta(\tau - \psi(\bar{X}^\mu(\vec{\sigma}))) \delta(y - \chi(\bar{X}^\mu(\vec{\sigma}))) \\ &= d\tau d\vec{y} \prod_{\vec{\sigma}} \delta(\tau - \psi(X^\mu(\sigma))) \delta(y - \chi(X^\mu(\sigma))) = d\mu_{\psi, \chi}(X) \end{aligned} \quad (2.10)$$

The foliation is not, therefore a complete gauge choice. It may be necessary to analyze more directly the dependence of physical results on ψ, χ in $S_{\psi, \chi}$ of (2.4). That such independence arises can be shown for a variation $\delta\psi = \epsilon g(X)$ when g is independent of X^+ by direct analysis of the variation of (2.4)

$$\frac{\delta S_\psi}{\delta g} = \left\{ \int d\vec{x} d\vec{y} d\sigma_1 d\sigma_1' g(X) \delta'(\tau - X^+(\sigma_1)) \left[d\mu \phi^* \Lambda^e(\sigma) \phi \right] \right\} \quad (2.11)$$

Since g is independent of X^+ the integration over $X^+(\sigma_1)$ may be performed to give zero. A similar result holds if g is orthogonal to the foliation ψ and χ . If g is not as orthogonal to ψ or χ a direct analysis for, say $g(X) = \epsilon X^+$ shows that changing $\tau \rightarrow \tau(1+\epsilon)$ leads to no change in $S_{\psi, \chi}$; a similar result holds for X^+ substituted by X^- .

3. Physical Spectrum

The σ -integral in the action $S_{\psi, \chi}$ of (2.4) plays the role of a Lagrangian density

$$\mathcal{L} = \int d\mu(\sigma) \phi^* \Lambda^e(\sigma) \phi \quad (3.1)$$

The ϕ -dependence of this expression is a non-trivial problem which will be addressed elsewhere, but for now we assume that the ϕ -dependence is not relevant on-shell. Arguments for this had been given in ref[3], where it was noted that, after an integration by parts, the ϕ contribution arose from an anti-harmonic oscillator-like term proportional to $(p_\phi^2 - \phi'^2)$. There are no normalisable states for such a term, so it was concluded that the ϕ dependence of ϕ must be trivial on-shell.

Replacing Λ^e by the unextended operators $p^2 + X'^2$, the Lagrangian (3.1) reduces in the LCF to

$$\mathcal{L} = \int d\mu(\sigma) \phi^* (\delta_-^2 - 2\delta_+ \delta_- - X'^2) \phi \quad (3.2)$$

In (3.2), $\delta_\pm = \delta / \delta X^\mu(\sigma)$ and δ_- denotes the transverse components of δ_μ . The LCF; $\psi(X) = X^+$ implies that

$$\delta_+ = \delta / \delta X^+(\sigma) = \partial / \partial \tau = \partial_\tau \quad (3.3)$$

so that inserting (3.3) in (3.2) leads to

$$\mathcal{L} = -2 \int d\mu(\sigma) \phi^* \partial_\tau \delta_- \phi - \mathcal{L}^T \quad (3.4)$$

where $\mathcal{L}^T$ denotes the transverse part of the Lagrangian density. We now use the LCF; $X(X) = X^-$ to obtain

$$\delta_- = \frac{\partial}{\partial y} = \partial_y \quad (3.5)$$

and that (3.4) becomes

$$\mathcal{L} = 2\psi\partial_y\phi^*\partial_\tau\phi - \mathcal{L}^T$$

(3.6)

where $\psi = \int d\sigma e^{-\frac{1}{2}\phi(\sigma)}$. The spectrum of (3.5) follows from the usual L.C. analysis, as does the Hamiltonian analysis. The standard Virasaro-Shapiro factor thus results in an evident manner.

4. Conclusions

It has been shown here that the reparametrisation-invariant approach of ref[3] has the properties (a) the action is independent of the foliations ψ or χ (b) the spectrum is that of the usual LC gauge. The latter result was obtained under the assumption that dependence on the conformal mode ϕ is trivial. However that needs more careful analysis in the manner in which integration over ϕ occurs. Since ϕ is, in fact, the bosonised ghost, as was pointed out in ref [3], it needs more careful analysis. It is hoped to give that elsewhere.

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