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POLARISATION IN THE (d,2p) REACTION AT INTERMEDIATE ENERGIES

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Abstract

Deuteron tensor polarisations t_{2i} are considered in $A(d,2p)A'$ reactions on hydrogen and nuclei for deuteron kinetic energies 100 to 2000 MeV. In those parts of phase space where the di-proton singlet or triplet final state dominates, both t_{20} and t_{22} should in general be quite large. Hence the reaction may be used as an efficient and convenient analyser of deuteron tensor polarisations.

An interesting specific case is when the di-proton is enhanced by the virtual $d^* \ ^1S_0$ state near threshold for then the polarisation of the deuteron governs the quantum numbers exchanged with the nucleus. Measurements of σ , t_{20} and t_{22} in inclusive and exclusive (d,d^*) reactions on $J^P = 0^+$ target nuclei may be used to find the spin dependence of nuclear cross sections as functions of momentum transfer. For inclusive processes, t_{20} and t_{22} should be large as for hydrogen, but with two qualitative differences: (a) pion exchange should be enhanced because of the shadowing of short-range exchanges, (b) the peak of t_{22} moves to a smaller angle because of the shadowing.

Excitation of specific final nuclear states by $\Delta S = 1$ transitions is

considered. Tensor polarisations are distinctively different for $J = L+1$, L and $L-1$ final states, and are useful tools for separating and identifying 0^- , 1^- and 2^- giant excitations.

A connection is drawn with the spin-transfer process $d(\vec{p}, \vec{n})2p$ which is used at meson factories to make polarised neutron beams. The neutron spectrum is estimated and a large variation of the polarisation transfer is found over it. A convenient formalism is given for handling spin effects involving the deuteron D-state in all the above reactions.

1. INTRODUCTION

The broad features of the reaction $A(d,2p)A'$ at intermediate energies and small momentum transfers can be described by impulse approximation where the neutron in the deuteron undergoes charge exchange, as shown in Figure 1.⁽¹⁾ The proton from the deuteron acts merely as a spectator. The amplitudes for n-p charge exchange are known from nucleon-nucleon phase shift analysis up to 800 MeV and can be used to predict the properties of the (d,p) reaction with some confidence. There is a strong peak in n-p charge exchange at small momentum transfers and this leads the (d,p) reaction to produce two protons with a small opening angle, moving close to the direction of the incident deuteron, and sharing its energy roughly equally between them. The reaction is experimentally distinctive and has a convenient geometry.

There are parts of phase space where singlet di-proton final states dominate, for example at low excitation energy where the 1S_0 virtual state d^* dominates. Triplet states are antisymmetric in $\vec{q} \cdot \vec{k}$, where $\vec{q}$ is the momentum transfer to the deuteron and $2\vec{k}$ is the relative momentum of the two protons (see Figure 1), so that singlet states also dominate near $\vec{q} \cdot \vec{k} = 0$ for all values of k . In other parts of phase space, triplet final states prevail. We demonstrate that in either region there are large deuteron tensor polarisation effects (equal and opposite for singlet and triplet) which can be used as the basis of an efficient analyser of both t_{20} and t_{22} .

There are two essential points. Firstly n-p charge exchange involves the strong spin-dependent amplitudes β , δ and ϵ of equation (2.3). Secondly the spin-independent amplitude α does not contribute to the deuteron (3S_1) $\rightarrow$ singlet di-proton transition. We show in section 2 that for an S-wave deuteron the tensor polarisation for the production of singlet final states

in $dp \rightarrow (pp)n$ is given by

$$t'_{20} = \frac{1}{\sqrt{2}} \frac{|\beta|^2 + |\gamma|^2 + |\delta|^2 - 2|\epsilon|^2}{|\beta|^2 + |\gamma|^2 + |\delta|^2 + |\epsilon|^2} \quad (1.1a)$$

$$t'_{22} = \frac{1}{\sqrt{3}} \frac{|\beta|^2 + |\gamma|^2 - |\delta|^2}{|\beta|^2 + |\gamma|^2 + |\delta|^2 + |\epsilon|^2} \quad (1.1b)$$

with analogous formulae for more complex targets. These formulae must be subjected to a small Wigner rotation (equation 2.21) in order to have the quantisation axis along the beam direction.

So long as β , δ and ϵ are large and different from one another one will in general get large cross sections and sizeable t'_{20} and t'_{22} . In contrast, elastic scattering of deuterons is dominated by the spin-independent diffraction amplitude α and therefore generally gives small analysing powers. The only exceptions are near diffraction minima where the deuteron D-state effects become important.

By suitable choices of phase space and converter thickness one should be able to achieve mean analysing powers $|t_{20}| \geq 0.37$ for all deuteron momenta above about 600 MeV/c (100 MeV) with an efficiency of the order of 2% at low momenta, falling to 0.2% at 3 GeV/c (1600 MeV). This should be useful for analysing deuteron tensor polarisations in reactions such as elastic π -d and p-d scattering as well as $pp \rightarrow d\pi^+$. Of special interest would be a measurement of t_{20} in elastic e-d scattering at $q^2 \approx 1 \text{ (GeV/c)}^2$, for the separation of the spherical and quadrupole form factors of the deuteron.⁽²⁾ In experiments at lower momentum transfers the ${}^3\text{He}(d,p){}^4\text{He}$ reaction has been used as the basis of the polarimeter.⁽³⁾ For deuterons below 40 MeV, analysing powers greater

than 0.7 are possible but efficiencies of only 0.7×10^{-4} are achieved in practical situations, due to the small deuteron range. Between 75 and 200 MeV, d-p scattering around the first diffraction minimum has a reasonable analysing power (up to 0.4), leading to an efficiency of about 0.1% in a polarimeter using the full azimuthal acceptance.⁽⁴⁾ A practical polarimeter could readily combine detection of elastic p-d scattering at large angles together with measuring the (d,2p) reaction at small angles.

Secondly, we consider nuclear reactions where the diproton has very low excitation energy, and is therefore essentially purely d*. The form of equation (1.1) is governed by the $d \rightarrow d^*$ vertex so that it goes over largely unchanged to the nuclear case, except for the replacement of β by an amplitude $\tilde{\beta}$ for the nucleon-induced nuclear transition $A \rightarrow A'$, and likewise for γ , δ and ϵ . Equation (1.1) is modified slightly by the inclusion of deuteron D-state effects but these are only important at large momentum transfers. It can then be inverted to determine $|\tilde{\beta}|^2 + |\tilde{\gamma}|^2$, $|\tilde{\delta}|^2$ and $|\tilde{\epsilon}|^2$ from experimental determinations of cross section, t_{20} and t_{22} : deuteron vector polarisation gives no further information for (d,d*) reactions. This should provide a simple and convenient technique for isolating the spin-dependence of nuclear cross sections.⁽⁵⁾ It is an extremely promising field of physics at Indiana, where tensor polarised beams are available up to about 100 MeV, and at Saclay, where the energy range 200 to 2200 MeV is accessible.

A particular case that we study is the excitation of specific nuclear states via $\Delta S = \Delta T = 1$ transitions. The condition of spin-flip ensures that the γ amplitude drops out and the β , δ and ϵ are multiplied by different Clebsch-Gordan coefficients. We therefore find that t_{20} and t_{22} should be distinctively different for final nuclear states with $J = L+1$, L and $L-1$, where L is the orbital angular momentum transfer. For example, in all heavy nuclei

a peak is evident in (p,n) reactions at about 20 MeV excitation, associated with $L = 1$ transfer.⁽⁶⁾ It is believed that this peak is due to $\Delta S = 1$ transitions but that it is a mixture of 0^- , 1^- and 2^- giant excitations. It would be of great interest to separate them.

There is a close correspondence between $(\vec{d}, 2p)$ and $(\vec{n}, \vec{p})$ reactions. Moss⁽⁷⁾ has pointed out that a measurement of spin-transfer (Wolfenstein) parameters in the $(\vec{n}, \vec{p})$ or $(\vec{p}, \vec{n})$ case may also be used to distinguish between final J-values. Bleszynski et al⁽⁸⁾ have likewise shown that Wolfenstein parameters can be used to isolate the spin dependence of nuclear cross sections and have initiated a program at LAMPF on $(\vec{p}, \vec{p}')$ reactions. Experimentally the $(\vec{p}, \vec{n})$ reaction has the disadvantage of requiring long time-of-flight paths for energy resolution on the neutron, together with a double scattering of this particle to measure its polarisation. The $(\vec{d}, 2p)$ reaction⁽⁹⁾ has in its favour the fact that no polarisation measurement is required on the final particles and the two protons are readily measured simultaneously in a large solid angle spectrometer. The energy resolution to be used is governed by that required to isolate nuclear levels: 10^{-3} of the deuteron beam energy is readily achieved and better is possible. For the same incident beam intensity, the efficiency of the $(\vec{d}, 2p)$ reaction is higher than that of the $(\vec{p}, \vec{n})$ by a factor of order 10^5 . However deuteron beam rates have to be limited in order to avoid random coincidences due to the large stripping cross sections. Another disadvantage is that nuclear distortions are larger, making the evaluation of strength functions more difficult. The physics of the two reactions is also a little different for heavy targets⁽⁹⁾ where the large neutron excess leads to Pauli blocking of some Gamow-Teller transitions in the (d,2p) case.

In the $p(d, 2p)n$ reaction, double scattering attenuates the cross section

slightly but is less important for the polarisation observables. We estimate such effects within the Glauber approximation. The fractional changes in polarisation are somewhat larger for the coherent reaction where events fall within the 1S_0 final state interaction peak, as is the case in the current Saclay experiment. For nuclear targets, shadowing is large and we are able to make only crude estimates. Nonetheless, two important qualitative effects arise which are almost certain to survive in any calculation. Firstly the amplitude due to long range π exchange is attenuated less by the shadowing than the short range (e.g. ρ) exchange. The Glauber approximation displays this effect in a qualitatively obvious way, driving t_{20} and t_{22} in the direction of values which would arise from damped pion exchange. Secondly, the peak in t_{22} , which occurs at the value of the momentum transfer q where $\text{Re } \tilde{\delta}$ goes through zero, moves to significantly smaller q . For carbon the shift is from 0.75 fm^{-1} to 0.50 fm^{-1} . The reason is that the shadowing correction has the opposite sign to $\text{Re } \delta$ for small q , but the same sign for q beyond the zero.

A third type of experiment we consider is that of the Wolfenstein parameters in the $d(\vec{p}, \vec{n})2p$ reaction. This process is used to make polarised neutron beams at the TRIUMF⁽¹⁰⁾ and LAMPF⁽¹¹⁾ meson factories. It is important to know the energy spectrum of the neutron beam, how the Wolfenstein parameters differ from those of free n - p charge exchange and how they vary over the neutron spectrum. This is necessary in assessing the absolute magnitude of the neutron beam polarisation. We stress the utility of the linear relation⁽¹²⁾ between the Wolfenstein parameters $K_{LL} + 2K_{NN} = -1$, which is valid at 0° for the hard part of the neutron spectrum, and show its connection to the deuteron tensor polarisation.

We have reported briefly on the first two topics in earlier publications^(1,5): here we give fuller details.

We introduce the basic formalism involving tensor polarisation by discussing, in section 2, matrix elements for an S-wave deuteron. This gives essentially correct results for low di-proton excitations. Section 3 develops the formalism required to handle spin effects for the deuteron D-state. It reduces the treatment of the D-state to a straightforward systematic evaluation of density matrices and the approach may be useful in other applications. The results of our calculations for a hydrogen target are given in section 4, where we discuss the polarimeter possibilities of the $p(\vec{d}, 2p)$ reaction. If we sum over all p-p excitation energies then we can get results which are essentially independent of the details of the p-p interaction. The resulting closure sum rules are developed in section 5 and numerical examples are given which might be particularly useful for the case of $\vec{d}(p, n)pp$. The coherent scattering to the d^* state is considered, for both hydrogen and nuclear targets, in section 6, where results are given for the polarisations expected for the transitions to distinct nuclear levels.

Up to this point, all the results have been derived in the single scattering approximation. It is necessary to get some feeling for the effect of double scattering, which may occur in either or both of the deuteron and the nuclear target. Section 7 treats double scattering in the deuteron in the Glauber approximation and section 8 extends this, in a simplified formalism, to multiple scattering in the nucleus. The question of Pauli blocking in nuclei is also addressed here. Finally, expressions for the Wolfenstein parameters are derived in section 9, where typical neutron beam spectra and polarisations are calculated. Our conclusions are summarised in section 10. Relations amongst tensor polarisation operators are given in an appendix.

2. FORMALISM FOR THE DEUTERON S-STATE

In the impulse approximation⁽¹³⁾ of Figure 1, the amplitude for $dp \rightarrow (pp)n$ involves the charge exchange of the neutron (1) in the deuteron on a proton (3),

$$M = \langle \vec{k}, m'_1, m'_2, m'_3 | f_{13}^{ce} e^{i\vec{q} \cdot \vec{r}} | d, m, m_3 \rangle . \quad (2.1)$$

Here $\vec{r} = \vec{r}_1 - \vec{r}_2$ is the relative coordinate in the two-nucleon system and $2\vec{k}$ the relative momentum of the two final protons in their centre-of-mass frame. In terms of the initial and final nucleon momenta $\vec{k}_i$ and $\vec{k}_f$, the momentum transfer $\vec{q} = \vec{k}_f - \vec{k}_i$.

It is convenient to define a set of orthogonal basis vectors,

$$\begin{aligned} \vec{p} &= \frac{1}{2}(\vec{k}_f + \vec{k}_i) \\ \vec{n} &= \vec{k}_i \times \vec{k}_f \\ \vec{q}' &= \vec{n} \times \vec{p} = \frac{1}{4}(\vec{k}_i + \vec{k}_f)^2 \vec{q} + \frac{1}{2}(k_i^2 - k_f^2) \vec{p} . \end{aligned} \quad (2.2)$$

The formula of equation (2.1) is written in the deuteron Breit frame where for low p-p excitation energies, $k_i^2 \approx k_f^2$ and the directions of $\vec{q}$ and $\vec{q}'$ essentially coincide. For the low momentum transfers in which we are interested, there is little difference between that and the deuteron rest frame except that the magnetic quantum numbers m refer to quantisation along the direction of $\vec{p}$. This means that away from the forward direction we must perform a Wigner rotation before comparing our predictions with experimental data (see equation 2.21).

The n-p charge exchange amplitude is decomposed in conventional form,

using the axes of equation (2.2), as

$$f_{13}^{ce} = \alpha - i\gamma(\vec{\sigma}^1 + \vec{\sigma}^3) \cdot \hat{n} + \beta(\vec{\sigma}^1 \cdot \hat{n})(\vec{\sigma}^3 \cdot \hat{n}) + \delta(\vec{\sigma}^1 \cdot \hat{q}')(\vec{\sigma}^3 \cdot \hat{q}') + \epsilon(\vec{\sigma}^1 \cdot \hat{p})(\vec{\sigma}^3 \cdot \hat{p}). \quad (2.3)$$

This is normalised such that the unpolarised n-p charge exchange differential cross section with respect to four-momentum transfer t is given by

$$\frac{d\sigma}{dt} = |\alpha|^2 + |\beta|^2 + 2|\gamma|^2 + |\delta|^2 + |\epsilon|^2. \quad (2.4)$$

Likewise for small p-p excitation energies the unpolarised deuteron charge exchange cross section is given in terms of the matrix elements of equation (2.1) by

$$d\sigma = \frac{1}{6} \frac{p_f}{p_i} \sum_{\text{spins}} |M|^2 d^3k dt \quad (2.5)$$

where p_f/p_i is the ratio of the outgoing neutron momentum to that of the incident proton in the centre-of-mass frame.

As was mentioned in the introduction, triplet final p-p states give contributions to deuteron tensor polarisations of opposite signs to singlet. We shall have to consider the two cases separately and this is conveniently done using the triplet and singlet projection operators, respectively

$$\begin{aligned} T^{12} &= \frac{1}{4}(3 + \vec{\sigma}^1 \cdot \vec{\sigma}^2), \\ S^{12} &= \frac{1}{4}(1 - \vec{\sigma}^1 \cdot \vec{\sigma}^2). \end{aligned} \quad (2.6)$$

For an S-wave deuteron we can factorise simply the spin and spatial degrees of freedom;

$$M_S = A_S(\vec{k}, \vec{q}) Z. \quad (2.7)$$

Z is an operator in the nucleon spin spaces;

$$Z = X^{12} f_{13} T^{12} \quad (2.8)$$

where X is either S or T depending upon the final p - p spin state.

(a) The Form Factor

The function

$$A_S(\vec{k}, \vec{q}) = \int \psi_{\vec{k}}^{X(-)*}(\vec{r}) e^{i\vec{q} \cdot \vec{r}/2} \psi_S(\vec{r}) d^3 r \quad (2.9)$$

depends upon the spatial part of the deuteron (S -wave) and p - p scattering wave functions, $^{(13)} \psi_S$ and $\psi_{\vec{k}}$ respectively.

Low energy p - p scattering is dominated by the virtual d^* state in the 1S_0 channel and this is clearly seen in charge exchange reactions involving the deuteron. With the exception of this, we shall not be concerned with the fine details of the p - p spectrum so that for the triplet final states we shall just take antisymmetric plane waves,

$$\psi_{\vec{k}}^{t(-)}(\vec{r}) = \frac{1}{\sqrt{2}} \frac{1}{(2\pi)^{3/2}} (e^{i\vec{k} \cdot \vec{r}} - e^{-i\vec{k} \cdot \vec{r}}) \quad (2.10)$$

This leads to a triplet form factor

$$A_S^t(\vec{k}, \vec{q}) = \frac{1}{\sqrt{2}} \left[\phi(\frac{1}{2}\vec{q} - \vec{k}) - \phi(\frac{1}{2}\vec{q} + \vec{k}) \right] \quad (2.11)$$

which depends upon the deuteron wave function in momentum space $\phi(\vec{K})$.

The 1S_0 interaction in the singlet final states may be most easily introduced with the help of a separable potential $^{(14)}$

$$m_p < \vec{k}' | v_{pp} | \vec{k} > = - \frac{\lambda}{(k^2 + \beta^2)(k'^2 + \beta^2)} \quad (2.12)$$

where m_p is twice the proton reduced mass.

The resulting scattering wave function then corresponds to plane waves for all except the S-state:

$$\psi_k^{s(-)}(\vec{r}) = \frac{1}{\sqrt{2}} \frac{1}{(2\pi)^{3/2}} \left[e^{i\vec{k} \cdot \vec{r}} + e^{-i\vec{k} \cdot \vec{r}} + 2e^{-i\delta} \sin \delta (e^{-ikr} - e^{-\beta r})/kr \right] \quad (2.13)$$

with a phase shift δ given by

$$k \cot \delta = (k^2 + \beta^2)^2 / 2\pi^2 \lambda + (k^2 - \beta^2) / 2\beta . \quad (2.14)$$

We use $\lambda = 0.153 \text{ fm}^{-3}$ and $\beta = 1.22 \text{ fm}^{-1}$. These are derived⁽¹⁵⁾ from the Coulomb-corrected scattering length and effective range but we shall neglect the Coulomb repulsion in the wave functions of equations (2.10), (2.13).

The singlet matrix element becomes

$$A_S^s(\vec{k}, \vec{q}) = \frac{1}{\sqrt{2}} \left[\phi(\frac{1}{2}\vec{q} + \vec{k}) + \phi(\frac{1}{2}\vec{q} - \vec{k}) + 2 \frac{e^{i\delta} \sin \delta}{k} \left(\frac{2}{\pi} \right)^{\frac{1}{2}} Y_0^0(\hat{q}) \times \right. \\ \left. \int_0^\infty (e^{ikr} - e^{-\beta r}) j_0(\frac{1}{2}qr) \psi_S(r) r dr \right]. \quad (2.15)$$

This may be evaluated analytically with the parameterisation of the Paris deuteron wave function for which the radial parts of the S and D components are

$$\psi_S(r) = \sum_i c_i \frac{e^{-\alpha_i r}}{r} , \quad (2.16a)$$

$$\psi_D(r) = \sum_j d_j \frac{e^{-\gamma_j r}}{r} (1 + 3/\gamma_j r + 3/\gamma_j^2 r^2) . \quad (2.16b)$$

In momentum space these become

$$\phi_S(p) = \left(\frac{2}{\pi} \right)^{\frac{1}{2}} \int_0^\infty r^2 dr j_0(pr) \psi_S(r) = \frac{1}{\pi\sqrt{2}} \sum_i \frac{c_i}{(p^2 + \gamma_i^2)} , \quad (2.16c)$$

$$\psi_D(p) = - \left(\frac{2}{\pi} \right)^{\frac{1}{2}} \int_0^\infty r^2 dr j_2(pr) \psi_D(r) = - \frac{1}{\pi\sqrt{2}} \sum_j \frac{d_j}{(p^2 + \gamma_j^2)} . \quad (2.16d)$$

In order that these functions be well behaved at the origin the coefficients c_i and d_j must satisfy linear constraints and great care must be taken to ensure that the cancellation amongst the large number of terms is sufficiently precise numerically.

With this parameterisation the singlet form factor becomes⁽¹⁴⁾

$$A_S^s(\vec{k}, \vec{q}) = \frac{1}{\pi^{1/2}} \sum_i c_i \left[\frac{1}{(\vec{k} + \frac{1}{2}\vec{q})^2 + \alpha_i^2} + \frac{1}{(\vec{k} - \frac{1}{2}\vec{q})^2 + \alpha_i^2} + \frac{2}{q(k \cot \delta - ik)} \times \right. \\ \left. \left\{ \tan^{-1} \left(\frac{\alpha_i q}{\alpha_i^2 + k^2 - \frac{1}{4}q^2} \right) - 2 \tan^{-1} \left(\frac{\frac{1}{2}q}{\alpha_i + \beta} \right) + \frac{1}{2}i \log \left(\frac{\alpha_i^2 + (k + \frac{1}{2}q)^2}{\alpha_i^2 + (k - \frac{1}{2}q)^2} \right) \right\} \right] \quad (2.17)$$

where the values of the arctangents lie between 0 and π .

(b) Spin Algebra

The formalism to describe deuteron tensor polarisation observables has been well summarised by Ohlsen⁽¹⁷⁾. In order to calculate these in our model we must project out linear combinations of various m values in the amplitudes of equation (2.5). Following Hamilton,⁽¹⁸⁾ we define vector and tensor operators in terms of the spin operations of the individual nucleons $\vec{\sigma}^1$ and $\vec{\sigma}^2$ and total spin $S = \frac{1}{2}(\vec{\sigma}^1 + \vec{\sigma}^2)$ by

$$\Omega_{20} = \frac{1}{\sqrt{2}} (3S_z^2 - 2) = \frac{1}{2\sqrt{2}} (3\sigma_z^1 \sigma_z^2 - \vec{\sigma}^1 \cdot \vec{\sigma}^2), \quad (2.18a)$$

$$\Omega_{2\pm 1} = \mp \frac{1}{\sqrt{3}} (S_{\pm} S_z + S_z S_{\pm}) = \mp \frac{1}{\sqrt{3}} (S_x S_z + S_z S_x \pm i S_y S_z \pm i S_z S_y) \\ = \mp \frac{1}{\sqrt{3}} (\sigma_x^1 \sigma_z^2 + \sigma_z^1 \sigma_x^2 \pm i \sigma_y^1 \sigma_z^2 \pm i \sigma_z^1 \sigma_y^2), \quad (2.18b)$$

$$\Omega_{2\pm 2} = \frac{1}{\sqrt{3}} (S_{\pm})^2 = \frac{1}{\sqrt{3}} (S_x \pm i S_y)^2 = \frac{1}{\sqrt{3}} (\sigma_x^1 \sigma_x^2 - \sigma_y^1 \sigma_y^2 \pm i \sigma_x^1 \sigma_y^2 \pm i \sigma_y^1 \sigma_x^2), \quad (2.18c)$$

$$\Omega_{10} = \sqrt{\frac{3}{2}} S_z, \quad (2.18d) \quad \Omega_{1\pm 1} = -\frac{1}{\sqrt{3}} (S_x \pm i S_y). \quad (2.18e)$$

Then, to find the polarisation, we must evaluate the traces of bi-linear

combinations of the spin operators Z of equation (2.8) with or without the above tensor operators. This must be done separately for the singlet and triplet final states. Explicitly

$$\langle Z Z^+ \rangle_s = |\gamma|^2 + |\beta|^2 + |\delta|^2 + |\epsilon|^2, \quad (2.19a)$$

$$\langle Z Z^+ \rangle_t = 3|\alpha|^2 + 5|\gamma|^2 + 2|\beta|^2 + 2|\delta|^2 + 2|\epsilon|^2, \quad (2.19b)$$

$$\langle Z \Omega_{20} Z^+ \rangle_s = - \langle Z \Omega_{20} Z^+ \rangle_t = \frac{1}{\sqrt{2}} \left[|\gamma|^2 + |\beta|^2 + |\delta|^2 - 2|\epsilon|^2 \right], \quad (2.19c)$$

$$\langle Z \Omega_{2\pm 2} Z^+ \rangle_s = - \langle Z \Omega_{2\pm 2} Z^+ \rangle_t = \frac{1}{2} \sqrt{3} \left[|\gamma|^2 + |\beta|^2 - |\delta|^2 \right], \quad (2.19d)$$

$$\langle Z \Omega_{2\pm 1} Z^+ \rangle = \langle Z \Omega_{10} Z^+ \rangle = \langle Z \Omega_{1\pm 1} Z \rangle_s = 0 \quad (2.19e)$$

$$\langle Z \Omega_{1\pm 1} Z^+ \rangle = \mp 2\sqrt{3} i \operatorname{Im} ((\alpha + \beta)^* \gamma) \quad (2.19f)$$

These trace relations provide the key to understanding the essential features of (d,2p) reactions. Unpolarised cross sections and polarisations t'_{Lm} (in the Madison convention) are given for singlet and triplet final states x by

$$\frac{d^4\sigma}{dtd^3k} = \frac{1}{3} |A_S^x|^2 \frac{p_f}{p_i} \langle Z Z^+ \rangle_x, \quad (2.20a)$$

$$t'_{Lm} \frac{d^4\sigma}{dtd^3k} = (-1)^{L-m} \frac{1}{3} |A_S^x|^2 \frac{p_f}{p_i} \langle Z \Omega_{Lm} Z^+ \rangle_x \quad (2.20b)$$

Note that if $|A_S^t| = |A_S^s|$, the net tensor polarisation is zero because of the opposite signs in equation (2.19) for singlet and triplet states. Also, in this limit, it follows from equations (2.19a) and (2.19b) that the cross section to the sum of all final p-p states is proportional to the free n-p cross section of equation (2.4).

The results of equation (2.19) and (2.20) have been obtained by quantising along the $\hat{p}$ axis of equation (2.2). These "primed" variables can be converted into ones referring to quantisation along the beam direction by using the standard transformations for tensor polarisations (17,18).

$$\begin{aligned}
 t_{20} &= \frac{1}{2}(3\cos^2\chi - 1)t'_{20} - \sqrt{6} \sin\chi \cos\chi \operatorname{Re} t'_{21} + (3/2)^{\frac{1}{2}} \sin^2\chi \operatorname{Re} t'_{22} , \\
 t_{21} &= (2\cos^2\chi - 1)\operatorname{Re} t'_{21} - \sin\chi \cos\chi (\operatorname{Re} t'_{22} - (3/2)^{\frac{1}{2}} t'_{20}) \\
 &\quad + i \cos\chi \operatorname{Im} t'_{21} - i \sin\chi \operatorname{Im} t'_{22} , \\
 t_{22} &= \frac{1}{2}(1 + \cos^2\chi) \operatorname{Re} t'_{22} + \sin\chi \cos\chi \operatorname{Re} t'_{21} + \frac{1}{2}\sqrt{6} \sin^2\chi t'_{20} \\
 &\quad + i \cos\chi \operatorname{Im} t'_{22} + i \sin\chi \operatorname{Im} t'_{21} .
 \end{aligned} \tag{2.21}$$

where χ is the (positive) angle of rotation about $\hat{n}$ between the two laboratory frames.

Discussion of the numerical values of t_{20} , t_{21} and t_{22} in the different parts of the p-p phase space will be deferred until after the evaluation of the D-state effects in the next section. We remark however here on one special case which gives very simple but revealing results, namely pure pion exchange. In the poor person's absorption model for the n-p charge exchange amplitude, where the contact term in the s-wave has been removed, the only surviving contributions are⁽¹⁹⁾

$$\begin{aligned}
 \beta(q) &= C, \\
 \delta(q) &= C(\mu^2 - q^2)/(\mu^2 + q^2)
 \end{aligned} \tag{2.22}$$

where C is a constant and μ is the mass of the pion. For pure singlet final states, in this model t'_{20} takes its maximum positive value $(1/\sqrt{2})$ and t'_{22} increases initially like q^2 to reach its maximum allowed value $(\frac{1}{2}\sqrt{3})$ for $q = \mu$. This describes quite well the qualitative features of our full predictions at small q for the highest energy studied in section 4.

3. INCLUSION OF THE DEUTERON D-STATE

The angular part of the deuteron wave function may be written compactly in terms of the tensor operators Ω_{2m} of equation (2.18). Eigenstates of J_z are related to those of S_z by

$$|J_z = m\rangle = \frac{1}{\sqrt{5}} \sum_m Y_2^{m*}(\hat{r}) \Omega_{2m} |S_z = m\rangle. \quad (3.1)$$

The tensor operators were essentially designed so as to satisfy this relation. Use of equation (3.1) reduces the spin algebra needed to evaluate the matrix element of equation (2.1) to straightforward manipulation of tensor operators Ω_{2m} . However their presence in the deuteron D-state wave function complicates the formulae for the t_{2i} .

The impulse approximation of equation (2.1) is linear in the S and D-state wave functions. It is given as an operator in nucleon spin space by

$$M^X = \int \psi_{\vec{k}}^{X(-)*}(\vec{r}) X^{12} f_{13}^{ce} e^{i\vec{q}\cdot\vec{r}/2} \left[\psi_S(r) Y_0^0(\hat{r}) + \frac{1}{\sqrt{5}} \psi_D(r) \sum_m Y_2^{m*}(\hat{r}) \Omega_{2m} \right] T^{12}. \quad (3.2)$$

The S-wave part of this has already been evaluated in the previous section. For the D-wave contribution we must decompose the matrix element as a series of terms

$$M_D^X = \sum_m A_D^{X,m}(\vec{k}, \vec{q}) Z_D^m \quad (3.3a)$$

where the form factor

$$A_D^{X,m}(\vec{k}, \vec{q}) = \frac{1}{\sqrt{5}} \int \psi_{\vec{k}}^{X(-)*}(\vec{r}) e^{i\vec{q}\cdot\vec{r}/2} Y_2^{m*}(\hat{r}) \psi_D(r) d^3r \quad (3.3b)$$

is split off from the spin dependence contained in

$$Z_D^m = X^{12} f_{13}^{ce} \Omega_{2m} T^{12}. \quad (3.3c)$$

For the plane waves used for the triplet state in equation (2.10) the

form factor is easily evaluated using the identity

$$e^{i\vec{p} \cdot \vec{r}} = 4\pi \sum_{\ell, m} i^m j_{\ell}(pr) Y_{\ell}^{m*}(\hat{p}) Y_{\ell}^m(\hat{r}) . \quad (3.4)$$

This leads to

$$A_D^{t,m}(\vec{k}, \vec{q}) = - \frac{1}{\sqrt{10}} \left[\phi_D(q_+) Y_2^{m*}(\hat{q}_+) - \phi_D(q_-) Y_2^{m*}(\hat{q}_-) \right] \quad (3.5)$$

where we have defined the vectors $\vec{q}_{\pm} = \frac{1}{2}\vec{q} \pm \vec{k}$.

The singlet form factor has the additional final state interaction:

$$A_D^{s,m}(\vec{k}, \vec{q}) = - \frac{1}{\sqrt{10}} \left[\phi_D(q_+) Y_2^{m*}(\hat{q}_+) + \phi_D(q_-) Y_2^{m*}(\hat{q}_-) \right] \\ + 2 \frac{e^{i\delta} \sin\delta}{k} \left(\frac{2}{\pi} \right)^{\frac{1}{2}} Y_2^{m*}(\hat{q}) \int_0^{\infty} j_2(\frac{1}{2}qr) (e^{ikr} - e^{-\beta r}) \psi_D(r) r dr. \quad (3.6)$$

In deriving the expressions for the polarisation observables, we must take traces of expressions of the form $z' O z^+$, where z, z' are either the S-wave Z of equation (2.8) or any of the Z_D^m of equation (3.3c), and the operator O may be 1 or any of the Ω_{Lm} . We found it convenient first to simplify the part of the expression containing $\Omega T O T^+ \Omega^+$ and some results on tensor operators, which are helpful for this purpose, are collected together in the appendix. It is only necessary to preserve terms which are proportional to $I, \sigma_y^1 + \sigma_y^2, \sigma_x^1 \sigma_x^2, \sigma_y^1 \sigma_y^2, \sigma_z^1 \sigma_z^2$ since anything else, when sandwiched between Xf_{13} and its Hermitean conjugate, yields zero trace. Thus we only have to keep terms proportional to $I, \Omega_{11}, \Omega_{20},$ and $\Omega_{22} + \Omega_{2-2}$. As a consequence the final results may be expressed in terms of the same traces as the S-wave case of equation (2.19).

$$3 \frac{d\sigma}{dtd^3k} = V_1 \langle Z Z^+ \rangle + V_2 \langle Z \Omega_{20} Z^+ \rangle + V_3 \langle Z \Omega_{22} Z^+ \rangle \quad (3.7a)$$

$$+ i \left[3\sqrt{2} \operatorname{Re} A_D^1 \operatorname{Im} A_D^0 + \sqrt{3} \operatorname{Im} (A_D^1 (A_D^{2*} - A_D^{-2})) \right] \langle Z \Omega_{11} Z^+ \rangle,$$

$$\begin{aligned}
3 \, t'_{20} \frac{d\sigma}{dt d^3 k} &= v_2 < Z \, Z^+ > \\
&+ \left[|A_S|^2 - \sqrt{2} \operatorname{Re}(A_S^* A_D^0) + \frac{3}{2} |A_D^0|^2 - 2 |A_D^1|^2 + \frac{1}{2} (|A_D^2|^2 + |A_D^{-2}|^2) \right] < Z \, \Omega_{20} \, Z^+ > \\
&+ \operatorname{Re} \left[(\sqrt{2} A_S^* + A_D^{0*}) (A_D^2 + A_D^{-2}) - \sqrt{6} A_D^{-1} A_D^{1*} \right] < Z \, \Omega_{22} \, Z^+ > \\
&+ i \left[3\sqrt{2} \operatorname{Re} A_D^1 \operatorname{Im} (A_S - A_D^0/\sqrt{2}) + (3/2)^{\frac{1}{2}} \operatorname{Im} (A_D^1 (A_D^{2*} - A_D^{-2})) \right] < Z \, \Omega_{11} \, Z^+ >, \quad (3.7b) \\
3 \, t'_{22} \frac{d\sigma}{dt d^3 k} &= \left[\operatorname{Re} (A_S + A_D^0/\sqrt{2}) (A_D^2 + A_D^{-2}) + \frac{1}{2} \sqrt{3} A_D^{-1} A_D^{1*} \right] < Z \, Z^+ > \\
&+ \left[\operatorname{Re} (A_S^* + A_D^{0*}/\sqrt{2}) (A_D^2 + A_D^{-2})/\sqrt{2} - (3/2)^{\frac{1}{2}} A_D^{-1} A_D^{1*} \right] < Z \, \Omega_{20} \, Z^+ > \\
&+ \left[|A_S + A_D^0/\sqrt{2}|^2 + 3 A_D^{-2} A_D^{2*} \right] < Z \, \Omega_{22} \, Z^+ > \\
&+ i \left[\frac{3}{2} \operatorname{Im} (A_D^1 (A_D^2 - A_D^{-2*})) - \sqrt{3} \operatorname{Re} A_D^1 \operatorname{Im} (A_S + A_D^0/\sqrt{2}) \right] < Z \, \Omega_{11} \, Z^+ >, \quad (3.7c) \\
3 \, t'_{21} \frac{d\sigma}{dt d^3 k} &= - \left[\operatorname{Re} (2 A_S - A_D^0/\sqrt{2}) A_D^1 - \frac{1}{2} \sqrt{3} A_D^1 (A_D^{-2} + A_D^{2*}) \right] < Z \, Z^+ > \\
&+ \left[\operatorname{Re}(A_S) A_D^1/\sqrt{2} - \frac{5}{2} \operatorname{Re}(A_D^0) A_D^1 + \frac{1}{2} (3/2)^{\frac{1}{2}} (A_D^1 (A_D^{2*} + A_D^{-2})) \right] < Z \, \Omega_{20} \, Z^+ > \\
&+ \left[\sqrt{3} \operatorname{Re} (A_S + A_D^0/\sqrt{2}) A_D^1 - \frac{3}{2} A_D^1 (A_D^2 + A_D^{-2*}) \right] < Z \, \Omega_{22} \, Z^+ > \\
&+ i \left[\frac{3}{\sqrt{2}} \operatorname{Im} (A_S^* A_D^0) - \frac{1}{2} \sqrt{3} \operatorname{Im} ((A_S^* - A_D^0/\sqrt{2}) (A_D^2 + A_D^{-2})) \right] < Z \, \Omega_{11} \, Z^+ >, \quad (3.7d)
\end{aligned}$$

where the following combination of form factors will be of use later.

$$v_1 = |A_S|^2 + |A_D^2|^2 + |A_D^{-2}|^2 + 2 |A_D^1|^2 + |A_D^0|^2 \quad (3.8a)$$

$$v_2 = 2 \operatorname{Re}(A_S^* A_D^0) + (|A_D^2|^2 + |A_D^{-2}|^2 - |A_D^0|^2 - |A_D^1|^2)/\sqrt{2} \quad (3.8b)$$

$$V_3 = \text{Re}((2A_S^* + \sqrt{2}A_D^{O*})(A_D^2 + A_D^{-2}) + \sqrt{3} A_D^{-1} A_D^{1*}). \quad (3.8c)$$

Equations (3.7) generalise equations (2.19) and (2.20) to include effects of the deuteron D state.

In writing these formulae we have used the fact that the final state interaction does not contribute to $A_D^{\pm 1}$ because of the $Y_2^m(\hat{q})$ in equation (3.6). There is therefore a symmetry relation, valid for both singlet and triplet final states,

$$A_D^{-1} = - A_D^{1*}. \quad (3.9)$$

For completeness we give the corresponding formulae for the vector polarisations:

$$\begin{aligned} 3 t'_{10} \frac{d\sigma}{dtd^3k} &= \frac{1}{2} \sqrt{3} \left[|A_D^{-2}|^2 - |A_D^2|^2 \right] \left[\langle Z \Omega_{20} Z^+ \rangle + \sqrt{2} \langle Z Z^+ \rangle \right] \\ &+ \sqrt{3} \text{Re} \left[(\sqrt{2} A_S^* + A_D^{O*})(A_D^{-2} - A_D^2) \right] \langle Z \Omega_{22} Z^+ \rangle \\ &+ i \left[\sqrt{6} \text{Im} A_D^1 \text{Re}(A_S + A_D^O/\sqrt{2}) + \frac{3}{\sqrt{2}} \text{Im} (A_D^{1*} (A_D^2 + A_D^{-2*})) \right] \langle Z \Omega_{11} Z^+ \rangle, \end{aligned} \quad (3.10a)$$

$$\begin{aligned} 3 i t'_{11} \frac{d\sigma}{dtd^3k} &= \left[-\frac{3}{\sqrt{2}} A_D^{1*} \text{Im} A_D^O \right. \\ &\quad \left. + \frac{1}{2} \sqrt{3} i A_D^1 (A_D^{2*} - A_D^{-2}) \right] \langle Z Z^+ \rangle \\ &+ \left[-\frac{3}{\sqrt{2}} A_D^{1*} \text{Im} (A_S - A_D^O/\sqrt{2}) + \frac{1}{2} \left(\frac{3}{2} \right)^{\frac{1}{2}} i A_D^1 (A_D^{2*} - A_D^{-2}) \right] \langle Z \Omega_{20} Z^+ \rangle \\ &+ \left[-\frac{3}{2} i A_D^{1*} (A_D^{2*} - A_D^{-2}) + \sqrt{3} A_D^1 \text{Im} (A_S + \frac{A_D^O}{\sqrt{2}}) \right] \langle Z \Omega_{22} Z^+ \rangle \\ &+ \left[|A_S|^2 - \frac{1}{\sqrt{2}} \text{Re} (A_S^* A_D^O) - |A_D^O|^2 + \frac{3}{2} A_D^{1*} A_D^{-1} \right. \\ &\quad \left. - \frac{1}{2} \sqrt{3} \{ (A_S - A_D^O/\sqrt{2}) A_D^{2*} + (A_S^* - A_D^{O*}/\sqrt{2}) A_D^{-2} \} \right] \langle Z \Omega_{11} Z^+ \rangle. \end{aligned} \quad (3.10b)$$

It is important to note that both t'_{22} and $i t'_{11}$ are almost purely real and in all the applications in this paper it is a good approximation to discard their imaginary parts.

4. NUMERICAL RESULTS AND POLARIMETER STUDY

In the coordinate system of equation (2.2), we take the relative momentum $\vec{k}$ of the two final protons to be related to $\vec{q}'$ by polar angle θ and azimuthal angle ϕ such that

$$(\vec{k} \cdot \hat{q}', \vec{k} \cdot \hat{n}, \vec{k} \cdot \hat{p}) = k(\cos\theta, \sin\theta \sin\phi, \sin\theta \cos\phi) . \quad (4.1)$$

Thus $\phi = 0$ when $\vec{k}$ lies in the plane of scattering of the deuteron and di-proton.

The tensor polarisations are symmetric in ϕ ,

$$t'_{2m}(\phi) = t'_{2m}(-\phi) . \quad (4.2)$$

This follows because under the transformation $\phi \rightarrow -\phi$, the spherical harmonics of equation (3.6) change as $Y_2^m(\hat{q}_\pm) \rightarrow (-1)^m Y_2^{-m}(\hat{q}_\pm)$ and the resulting form factors satisfy $A_S \rightarrow A_S$, $A_D^m \rightarrow (-1)^m A_D^{-m}$. On the other hand, under the transformation $\phi \rightarrow \pi - \phi$, $Y_2^m(\hat{q}_\pm) \rightarrow (-1)^m Y_2^m(\hat{q}_\pm)$, $A_S \rightarrow A_S$, $A_D^m \rightarrow (-1)^m A_D^m$, so that

$$t'_{2m}(\pi - \phi) = (-1)^m t'_{2m}(\phi) . \quad (4.3)$$

Because of the identity of the two final state protons, all observables are symmetric under $\vec{k} \rightarrow -\vec{k}$, i.e. under $\phi \rightarrow \phi + \pi$, $\theta \rightarrow \pi - \theta$. It then follows from equation (4.3) that

$$t'_{2m}(\cos\theta) = (-1)^m t'_{2m}(-\cos\theta) . \quad (4.4)$$

These symmetry relations cut down considerably the amount of phase space to be investigated. The antisymmetry of t'_{21} in equations (4.3, 4.4) ensures that it vanishes when integrated over all values of $\vec{k}$, and this result we shall find again in the closure sum rules of section 5.

We take the nucleon-nucleon amplitudes, required as input for our calculation, from the updated phase shift analysis of Dubois et al.,⁽²⁰⁾

which goes up to a proton laboratory kinetic energy T_p of 800 MeV. Illustrative examples of charge exchange amplitudes are shown at 142 MeV in Figure 2a and 425 MeV in Figure 4 of Ref. 21. The centre-of-mass amplitudes change little with energy with the exception of one feature which is crucial to the present analysis. At low energies $|\epsilon(0)| > |\beta(0)| = |\delta(0)|$ in the forward direction but at high energies the inequality is reversed, as can be seen in Figure 2b. The smoothness in the energy variation supports our neglect of Fermi motion when evaluating the impulse approximation matrix element.

The broad trend for singlet final states is shown in Figure 3 as a function of q and T_p . They have been integrated over all θ and ϕ and in k from 0 to 280 MeV/c.

It can be seen from equation (2.19c) that for a purely S-wave deuteron the relative magnitudes of β , δ , and ϵ have a strong influence on t'_{20} . In the forward direction the energy dependence evinced by Figure 2b is reflected by t'_{20} being negative at low energies but strongly positive at high energies; the cross-over takes place at about $T_p = 280$ MeV. This is not changed by the deuteron D-state modifications after integrating over the angles of $\vec{k}$.

From equations (2.19c,d), the predictions for singlet and triplet have opposite signs,

$$\left[t'_{20} \frac{d\sigma}{dtd^3k} \right]_t = - \frac{|A_S^t|^2}{|A_S^s|^2} \left[t'_{20} \frac{d\sigma}{dtd^3k} \right]_s \quad (4.5)$$

Away from $q = 0$ there are D-state contributions proportional to $\langle Z Z^+ \rangle$ in equation (3.7), which disturb this simple relationship. However equation (4.5) suggests that significant net values of t_{20} and t_{22} will only be seen in regions of phase space where either the singlet or triplet form factor

is dominant. These are the parts of phase space that one must be able to select if one wants to use the reaction in the construction of a polarimeter.

Figure 4 shows cross sections for the production of singlet and triplet final states, integrated over all (θ, ϕ) and bins of width 17.5 MeV/c in both k and q . Singlet final states dominate for $k < 50$ MeV/c, especially for small q . This is because the triplet contribution must vanish at $k = 0$, whereas the singlet is enhanced there by the 1S_0 interaction.

Singlet final states are also pre-eminent near $\cos\theta = 0$ for all values of k due to the antisymmetry of the triplet wave function. This is illustrated on a logarithmic scale in Figure 5 where the cross section is now also binned in $\cos\theta$ in units of 0.1. Because of the closure sum rules guaranteeing conservation of triplet and singlet cross sections after integrating over all $\vec{k}$ (see section 5), the triplet strength must be found at larger k where it dominates over singlet near $\cos\theta = \pm 1$. A useful rule-of-thumb is that singlet cross sections dominate by at least a factor of 10 : 1 providing either $2k|\cos\theta| < 100$ MeV/c or $k|\cos\theta| + \frac{1}{2}q < 100$ MeV/c. The former constraint corresponds to small opening angles between the two protons, while the latter restricts the momentum of both protons transverse to the beam direction.

To predict the detailed variation of tensor polarisation with k , θ and ϕ , one needs a computer program to evaluate the equations given in sections 2 and 3; ours is available upon request. Tensor polarisations are shown in Figure 6, summed over singlet and triplet states and integrated over ϕ . For large k , and values of $\cos\theta$ close to ± 1 , triplet states dilute considerably t_{20} and t_{22} . At large q , where the triplet cross sections exceed those of the singlet final states, the signs of the polarisations can even be changed.

For large q , the contributions from the deuteron D-state become of similar size to those of the S-state and the large S-D interference in

equation (3.7) gives rise to strong ϕ dependence. This is illustrated in Figure 7 for representative values of k and $\cos\theta$. For $k > 2q$ and $\cos\theta$ close to 0.5, t_{20} becomes very large near $\phi = 0$ and also near $\phi = \pi/2$, but with the opposite sign. Because of the symmetry requirement of equation (4.3), t'_{21} goes to zero at $\phi = \pi/2$. However for $k > 2q$ and $\cos\theta$ between 0.5 and 1.0, t_{21} becomes large in the vicinity of $\phi = 0$. These large values of t_{20} and t_{21} , because they depend upon interference between S and D waves, are always associated with small values of the cross sections. Nevertheless they offer useful analysing powers.

The observables directly measured experimentally t_{2m} are of course slightly different from the t'_{2m} because of the Wigner rotation described by equation (2.21). After this has been done the figure of merit for any polarimeter is $\sigma(t_{2m})^2$, and the optimum analysing power is obtained by weighting different parts of phase space in proportion to $\sigma(t_{2m})^2$. This enhances regions of large analysing power. The regions of positive and negative analysing power must both be used, in the latter case subtracting the contribution from the total. We are thus led to study rather $|t_{2m}|$ which has an average value, with our weighting procedure, of

$$\overline{|t_{2m}|} = \int dt d^3k \frac{d\sigma}{dtd^3k} |t_{2m}| t_{2m}^2 / \int dt d^3k \frac{d\sigma}{dtd^3k} t_{2m}^2. \quad (4.6)$$

The corresponding effective cross section is

$$\sigma_{\text{eff}}^m = \int dt d^3k \frac{d\sigma}{dtd^3k} t_{2m}^2 / \overline{|t_{2m}|}^2 \quad (4.7)$$

though for some applications it is convenient to quote rather values of $\sigma_{\text{eff}}^m \overline{|t_{2m}|}^2$.

Our predictions of $\overline{|t_{2m}|}$ and σ_{eff}^m , obtained by integrating in both k and

q up to 280 MeV/c, are shown in Table 1 both with and without the D-state contributions. In the 250 - 300 MeV domain, where t_{20} is small in the forward direction because of the cancellation between β , δ and ϵ , $\overline{t_{20}}$ is very small for the purely S-wave deuteron. The S-D interference induces large analysing powers for some regions of ϕ and if those are used a respectable value of about 0.4 is achieved. The corresponding σ_{eff}^0 is reduced slightly because we are then also employing regions of small cross sections, but of course $\sigma_{\text{eff}} \overline{t_{20}}^2$ is enhanced.

The uncertainties of our predictions originate to a large part from those of the nucleon-nucleon phase shifts. We estimate errors of ± 0.03 in the tensor polarisations for $T_p = 210$ to 500 MeV, ± 0.05 for 140 and 800 MeV, but ± 0.08 below 100 MeV due to this cause. The double scattering corrections of section 7 should be much less than this at high energies but at the lowest energies the charge exchange impulse approximation diagram may not describe the amplitude sufficiently accurately, especially away from the peak of the quasi-free kinematics. Even if this were the case, our calculations indicate in which areas of phase space to look to find a sizeable polarisation signal for polarimeter purposes.

It is realistic to use a liquid hydrogen converter 1 m long, except for deuteron momenta below 1100 MeV/c, when the deuteron range imposes a limitation. For $p_d = 600$ MeV/c a target thickness of only 20 cm is more appropriate but this is compensated by the larger effective cross sections of Table 1 at lower energies. For such thicknesses, efficiencies of 0.75% at 600 MeV/c, 1.7% at 1100 MeV/c falling to 0.24% at 3000 MeV/c, could be achieved.

It would be convenient to use carbon rather than hydrogen as converter. It is then mechanically straightforward to achieve a large cross sectional

area and hence large solid angle. We expect analysing powers for inclusive reactions on carbon to be broadly similar to those on hydrogen. A quantitative discussion is deferred to section 8, where it is shown that the number of useful neutrons is about 0.7 per carbon nucleus.

5. CLOSURE SUM RULES

We have so far made estimates in impulse approximation of the observables in the fully constrained $\vec{d}p \rightarrow (pp)n$ reaction within a simple model for the p-p final state interaction. If the only kinematic information available were the momentum transfer to the di-proton pair then, summing over all p-p excitation energies, these observables satisfy sum rules analogous to the ones successfully used to describe the inclusive break-up cross section⁽²²⁾. These sum rules, which are independent of the choice of the p-p final state interaction, may be especially useful for a high energy proton charge exchange from a polarised deuterium target where the produced neutron's direction is well determined but its energy is not. They also provide valuable checks on the more detailed calculations presented in the previous section.

The inclusive charge exchange cross section $\vec{d}p \rightarrow (pp)n$ on an unpolarised proton may be obtained by summing the contributions of equation (2.1) over all antisymmetric final di-proton states $|F\rangle$, as well as summing and averaging over the unobserved nucleon polarisations:

$$\left(\frac{d\sigma}{dt}\right)_{\vec{d}p \rightarrow ppn} = \frac{1}{4} \sum'_{F, m'_3, m_3} \left| \langle F, m'_3 | f_{13}^{ce} e^{i\vec{q} \cdot \vec{r}/2} | d, M, m_3 \rangle \right|^2. \quad (5.1)$$

The restraint on the summation may be lifted providing that we antisymmetrise the operator with respect to the interchange 1 $\leftrightarrow$ 2.

$$\left(\frac{d\sigma}{dt}\right)_{\vec{d}p \rightarrow ppn} = \frac{1}{4} \sum_{F, m'_3, m_3} \left| \langle F, m'_3 | f_{13}^{ce} e^{i\vec{q} \cdot \vec{r}/2} - f_{23}^{ce} e^{-i\vec{q} \cdot \vec{r}/2} | d, M, m_3 \rangle \right|^2. \quad (5.2)$$

We now invoke closure over the final di-proton states. This is a valid approximation providing the operators do not have a significant energy dependence and the reaction takes place at sufficiently high energies that the

restrictions on the (p-p) phase space are unimportant. There are two identical sets of terms and we are left with

$$\left(\frac{d\sigma}{dt}\right)_{dp \rightarrow ppn} = O_{M,M} \quad (5.3a)$$

where

$$O_{M',M} = \frac{1}{2} \sum_{m_3} \langle d, M', m_3 | f_{13}^{ce} + f_{13}^{ce} - f_{23}^{ce} + f_{13}^{ce} e^{i\vec{q} \cdot \vec{r}} | d, M, m_3 \rangle \quad (5.3b)$$

and other spin observables are determined by the off-diagonal elements of the density matrix O .

After taking the trace over the spin of nucleon 3 we find

$$\begin{aligned} O_{M',M} = & \langle d, M' | (|\alpha|^2 + |\gamma|^2)(1 - e^{i\vec{q} \cdot \vec{r}}) + (|\beta|^2 + |\gamma|^2)(1 - \vec{\sigma}_1 \cdot \hat{n} \vec{\sigma}_2 \cdot \hat{n} e^{i\vec{q} \cdot \vec{r}}) \\ & + |\delta|^2 (1 - \vec{\sigma}_1 \cdot \hat{m} \vec{\sigma}_2 \cdot \hat{m} e^{i\vec{q} \cdot \vec{r}}) + |\epsilon|^2 (1 - \vec{\sigma}_1 \cdot \hat{k} \vec{\sigma}_2 \cdot \hat{k} e^{i\vec{q} \cdot \vec{r}}) \\ & + \text{Im}(\alpha\gamma^* + \beta\gamma^*)(\vec{\sigma}_1 \cdot \hat{n} + \vec{\sigma}_2 \cdot \hat{n}) (1 - e^{i\vec{q} \cdot \vec{r}}) | d, M \rangle \end{aligned} \quad (5.4)$$

This only involves deuteron ground state information which may be expressed in terms of the form factors of the S and D states

$$\begin{aligned} S_a(q) &= \langle S | j_0(qr) | S \rangle; \quad S_b(q) = \langle D | j_0(qr) | D \rangle; \quad S_c(q) = \langle S | j_2(qr) | D \rangle; \\ S_d(q) &= \langle D | j_2(qr) | D \rangle; \quad S_e(q) = \langle D | j_4(qr) | D \rangle \end{aligned} \quad (5.5a)$$

and the spherical, quadrupole and magnetic combinations

$$\begin{aligned} S_s(q) &= S_a(q) + S_b(q) \\ S_Q(q) &= 2S_c(q) - S_d(q)/\sqrt{2} \\ S_M(q) &= S_a(q) - \frac{1}{2}S_b(q) + S_c(q)/\sqrt{2} + \frac{1}{2}S_d(q) \end{aligned} \quad (5.5b)$$

The evaluation of equation (5.4) necessitates extensive spin algebra and leads to the following expressions for the observables

$$\begin{aligned} \frac{d\sigma}{dt} = & (|\alpha|^2 + |\gamma|^2) [1 - s_s(q)] + (|\beta|^2 + |\gamma|^2 + |\epsilon|^2) [1 - \frac{1}{3}(s_s(q) + \sqrt{2}s_Q(q))] \\ & + |\delta|^2 [1 - \frac{1}{3}(s_s(q) - 2\sqrt{2}s_Q(q))], \end{aligned} \quad (5.6a)$$

$$\begin{aligned} t'_{20} \frac{d\sigma}{dt} = & -\frac{1}{2}(|\alpha|^2 + |\gamma|^2) s_Q(q) + \frac{\sqrt{2}}{3}(|\beta|^2 + |\gamma|^2) [s_a(q) + \frac{1}{10} s_b(q) - \frac{5}{\sqrt{2}} s_c(q) \\ & - \frac{1}{28} s_d(q) - \frac{27}{70} s_e(q)] + \frac{\sqrt{2}}{3} |\delta|^2 [s_a(q) + \frac{1}{10} s_b(q) + \frac{1}{\sqrt{2}} s_c(q) + \frac{11}{28} s_d(q) \\ & + \frac{54}{35} s_e(q)] - \frac{\sqrt{2}}{3} |\epsilon|^2 [2s_a(q) + \frac{1}{5} s_b(q) - \frac{1}{\sqrt{2}} s_c(q) - \frac{11}{28} s_d(q) + \frac{81}{70} s_e(q)], \end{aligned} \quad (5.6b)$$

$$\begin{aligned} t'_{22} \frac{d\sigma}{dt} = & -\frac{1}{2}(3/2)^{\frac{1}{2}}(|\alpha|^2 + |\gamma|^2) s_Q(q) + \frac{1}{\sqrt{3}}(|\beta|^2 + |\gamma|^2) [s_a(q) + \frac{1}{10} s_b(q) \\ & + \frac{1}{\sqrt{2}} s_c(q) - \frac{1}{4} s_d(q) + \frac{9}{10} s_e(q)] - \frac{1}{\sqrt{3}} |\delta|^2 [s_a(q) + \frac{1}{10} s_b(q) + \frac{1}{\sqrt{2}} s_c(q) \\ & + \frac{11}{28} s_d(q) + \frac{54}{35} s_e(q)] + \sqrt{3} |\epsilon|^2 [\frac{1}{\sqrt{2}} s_c(q) - \frac{1}{28} s_d(q) + \frac{3}{14} s_e(q)], \end{aligned} \quad (5.6c)$$

$$it'_{11} \frac{d\sigma}{dt} = -\frac{2}{\sqrt{3}} \text{Im}(\alpha\gamma^* + \beta\gamma^*) [s_M(0) - s_M(q)] = \frac{\sqrt{3}}{2} P_y \frac{d\sigma}{dt}, \quad (5.6d)$$

$$t'_{10} = t'_{21} = 0. \quad (5.6e)$$

The expression for the unpolarised cross section agrees with that derived by Dean⁽²³⁾. At large momentum transfers, where interference and final state interaction effects are negligible, the results of equation (5.6) go over into the free nucleon-nucleon ones. The tensor observables, being proportional to the deuteron form factors, vanish in this limit and we are left solely with the vector analysing power P_y .

For zero momentum transfer only t_{20} is non-vanishing and equal to

$$t_{20} = \sqrt{2} \left(1 - \frac{9}{10} P_D\right) \frac{(|\beta|^2 - |\epsilon|^2)}{(2|\beta|^2 + |\epsilon|^2)} \quad (5.7)$$

where P_D is the deuteron D-state probability. For a purely S-wave deuteron this coincides with the result of equation (2.19) because at $q = 0$ the impulse approximation only permits transitions to the spin-singlet 1S_0 state. The dilution due to the deuteron D-state, even though the final p-p system is then in the singlet 1D_2 configuration, is also consistent with the results of section 3 after integration over the unobserved variables.

In Figure 8 we show the predictions of the sum rules for the cross section, t_{20} and t_{22} at $T_p = 800$ MeV: the vector analysing power remains small over this domain of momentum transfer. Away from $q = 0$ the polarisation signals fall away fast as compared to the pure singlet results shown in Figure 3. Nevertheless the maximum value of t_{22} at around $4 - 5^\circ$ may be useful because of the associated large cross section.

The numerical values of the sum rules should be independent of the specific form of the p-p final state interaction but this does influence greatly the value of the p-p excitation energy E_x to which one must integrate in order to saturate the sum rules. Care should be taken in using the sum rules at low incident energies where not all parts of the p-p phase space are accessible and the use of closure is dubious. Furthermore, at low energies there can be effects due to the longitudinal momentum transfer, which depends upon E_x , and has been neglected in the derivation.

6. COHERENT CHARGE EXCHANGE REACTIONS

(a) On Hydrogen

In the SPES IV experiments carried out at Saclay^(9,24) the two protons from the A(d,2p) reaction are detected in the same high resolution spectrometer. The experimental cuts on opening angle and relative momentum ensure that very few events survive with a di-proton excitation energy E_x above 1 MeV. This system therefore provides a very clean sample of coherent charge exchange data to the 1S_0 (d*) final state.

The cross section and polarisation, for a hydrogen target, of $dp \rightarrow d^*n$ may be deduced from the more general result of equation (3.7) which allows for triplet final states and singlet states other than d*:

$$\frac{d\sigma}{dtd^3k} = \frac{1}{3} \{ (|\beta|^2 + |\gamma|^2 + |\epsilon|^2) |S^-(k, q/2)|^2 + |\delta|^2 |S^+(k, q/2)|^2 \} , \quad (6.1a)$$

$$t'_{20} \frac{d\sigma}{dtd^3k} = \frac{1}{3\sqrt{2}} \{ (|\beta|^2 + |\gamma|^2 - 2|\epsilon|^2) |S^-(k, q/2)|^2 + |\delta|^2 |S^+(k, q/2)|^2 \} , \quad (6.1b)$$

$$t'_{22} \frac{d\sigma}{dtd^3k} = \frac{1}{2\sqrt{3}} \{ (|\beta|^2 + |\gamma|^2) |S^-(k, q/2)|^2 - |\delta|^2 |S^+(k, q/2)|^2 \} , \quad (6.1c)$$

$$t'_{21} = 0 . \quad (6.1d)$$

These forms generalise the basic equations (2.19), (2.20) to include the dependence of σ , t_{20} and t_{22} on the deuteron D state. They give all the essential physics of the ($\vec{d}, d^*$) reaction and we shall refer to them frequently.

In the above

$$S^+(k, q) = S_S(k, q) + \sqrt{2} S_D(k, q) ,$$

$$S^-(k, q) = S_S(k, q) - S_D(k, q)/\sqrt{2} \quad (6.2)$$

are linear combinations of the transition form factors from the S and D states

of the deuteron to the d^* state;

$$\begin{aligned} S_S(k,q) &= \langle d^* | j_0(qr) | S \rangle, \\ S_D(k,q) &= \langle d^* | j_2(qr) | D \rangle. \end{aligned} \quad (6.3)$$

These form factors may be estimated by using the S-wave part of the p-p final state wave function of equation (2.13),

$$\phi_k^{(-)}(r) = \frac{1}{2\pi^{3/2}} e^{-i\delta} \{ \cos\delta \sin(kr) + \sin\delta (\cos(kr) - e^{-\beta r}) \} / kr. \quad (6.4)$$

The q -dependence of the S varies significantly with excitation energy as may be seen from the examples given in Figure 9 calculated with the Paris deuteron wave function⁽¹⁶⁾. Thus S_S has a maximum in q for $q \approx k$ which gets broader as k increases; the D-state form factor becomes more important as q increases.

We have neglected completely Coulomb effects and, whereas this is reasonable for the low resolution polarimeter studies of sections 2 and 3, they lead to major cross section reductions in the spectrometer experiments. Within a crude nucleon-nucleon model, Phillips⁽²⁵⁾ suggests that with an experimental cut at $E_x \leq 1$ MeV the Coulomb repulsion should lower the measured intensity by about 30% at low q . This must be included before a detailed interpretation of the Saclay counting rates can be carried out. It should however be noted that the Coulomb distortions affect but weakly the tensor polarisation predictions of equation (6.1). This is confirmed through numerical evaluation of the p-p wave function for the Paris potential with the Coulomb interaction⁽²⁶⁾.

The Coulomb repulsion pushes events to higher values of k where they are discarded on account of the experimental cuts. This can be understood through the closure sum rules

$$\begin{aligned}
 2\pi \int_0^\infty k^2 dk |S_S(k, q)|^2 &= \langle S | (j_0(qr))^2 | S \rangle \\
 2\pi \int_0^\infty k^2 dk |S_D(k, q)|^2 &= \langle D | (j_2(qr))^2 | D \rangle
 \end{aligned} \tag{6.5}$$

where the integration is over only one hemisphere in $\vec{k}$ due to the identity of the two protons. These sum rules are valid independent of the particular p-p final state interaction, with or without Coulomb forces.

(b) On Complex Nuclei

In impulse approximation, the matrix element corresponding to Figure 1 is

$$\begin{aligned}
 M &= \langle d^* | e^{i\vec{q} \cdot \vec{r}} \sigma_y^1 | d \rangle \langle A' | (\beta \sigma_y^3 - i\gamma) e^{i\vec{q} \cdot \vec{r}} | A \rangle \\
 &+ \langle d^* | e^{i\vec{q} \cdot \vec{r}} \sigma_x^1 | d \rangle \langle A' | \delta \sigma_x^3 e^{i\vec{q} \cdot \vec{r}} | A \rangle \\
 &+ \langle d^* | e^{i\vec{q} \cdot \vec{r}} \sigma_z^1 | d \rangle \langle A' | \epsilon \sigma_z^3 e^{i\vec{q} \cdot \vec{r}} | A \rangle .
 \end{aligned} \tag{6.6}$$

If we abbreviate the nuclear part of the matrix element as $\tilde{\beta} - i\tilde{\gamma}$, $\tilde{\delta}$ and $\tilde{\epsilon}$, then equations (6.1) carry over to this case with $\tilde{\beta}$, $\tilde{\gamma}$, $\tilde{\delta}$ and $\tilde{\epsilon}$ replacing β , γ , δ and ϵ . The form of these relations is a property of the $d \rightarrow 2p$ vertex, specifically of equation (2.8). They thus apply to inclusive reactions as well as exclusive processes. The complexity of the nuclear physics is buried in the specific forms of these modified amplitudes and how they depend upon q .

Equations (6.1) can be inverted to derive $|\tilde{\beta}|^2 + |\tilde{\gamma}|^2$, $|\tilde{\delta}|^2$ and $|\tilde{\epsilon}|^2$ from experimental determinations of σ , t_{20} and t_{22} . In this inversion, one has to correct for the (small) effects of double scattering in the deuteron. This is considered further in sections 7 and 8.

Here we consider in detail $\Delta S = 1$ transitions to specific nuclear levels.

The (p,n) reaction at 0^0 has been a valuable source of information on $L = 0$ Gamow-Teller transitions⁽²⁷⁾. If, for the sake of discussion, we neglect configuration mixing, one studies collective transitions of neutrons in the outermost shell of the nucleus into unfilled proton levels of the same shell, i.e. particle-hole states built on the valence shell. Because of the neutron excess in heavy nuclei, the Pauli principle inhibits the inverse charge exchange reactions (n,p) or (d,2p). However for transitions with $L = 1$ the (d,2p) reaction becomes a useful tool to investigate the promotion of one of the outermost protons to a neutron level which is vacant in the next shell. As an analogue of this, a peak at 20 MeV excitation energy has been seen in $^{90}\text{Zr}(p,n)^{90}\text{Nb}$ and this is believed to be due to such an $L = 1$ transition with $\Delta S = 1$ ⁽²⁸⁾.

In a previous publication⁽⁵⁾ we showed that the measurement of the tensor polarisations t_{20} and t_{22} in $(\vec{d},2p)$ reactions contains essentially the same nuclear structure information as the determination of spin transfer parameters in the $(\vec{n},\vec{p})$ reaction. The selection rules for this were studied by Moss⁽⁷⁾ who looked in detail at transitions between nuclear states with $\Delta S = 1$. Thus, starting from a spin zero nucleus, states with $J = L \pm 1$ and L could be accessed. Because of the spin-flip requirement the amplitude $\alpha(q) + i\gamma(q) \sigma_N^1$ does not contribute in PWIA and he showed that in this approximation the terms in $|\gamma|^2$, $|\beta|^2$ and $|\epsilon|^2$ are multiplied by a factor $\bar{x}_1^{J,L} |S_{J,L}(q)|^2$ and $|\delta|^2$ by $\bar{x}_2^{J,L} |S_{J,L}(q)|^2$. The nuclear form factor $S_{J,L}$ is sensitive to the details of the radial dependence of the transition density,

$$S_{J,L}(q) = \langle A^* | |j_L(qr)| | A \rangle \quad (6.7)$$

but the values of the coefficients $\bar{x}$ follow from angular momentum considerations *. For $L > 0$,

* Our present normalisation differs from that of Moss by a factor of 2.

$$\bar{\ell}_1^{L+1,L} = \frac{1}{2}(L+2); \quad \bar{\ell}_1^{L,L} = \frac{1}{2}(2L+1); \quad \bar{\ell}_1^{L-1,L} = \frac{1}{2}(L-1)$$

$$\bar{\ell}_2^{L+1,L} = L+1; \quad \bar{\ell}_2^{L,L} = 0; \quad \bar{\ell}_2^{L-1,L} = L. \quad (6.8a)$$

They satisfy the constraints

$$\sum_J \bar{\ell}_i^{J,L} = (2L+1) \quad (6.8b)$$

$$L \bar{\ell}_i^{L+1,L} = (L+1) \bar{\ell}_i^{L-1,L} + \bar{\ell}_i^{L,L}. \quad (6.8c)$$

In the case of $L=0$, $\bar{\ell}_i^{1,0} = 1$.

As an example, the transverse spin-transfer coefficient is predicted in Moss's formalism to be

$$D_{NN} = \frac{\bar{\ell}_1^{J,L} (|\gamma|^2 + |\beta|^2 - |\epsilon|^2) - \bar{\ell}_2^{J,L} |\delta|^2}{\bar{\ell}_1^{J,L} (|\gamma|^2 + |\beta|^2 + |\epsilon|^2) + \bar{\ell}_2^{J,L} |\delta|^2}. \quad (6.9)$$

The $(\vec{p}, \vec{n})$ experiments at Indiana have confirmed the gross dependence of $D_{NN}(0^0)$ on the nuclear transition⁽²⁹⁾. At their energy (160 MeV) the effective β , δ and ϵ amplitudes are of very similar magnitude in the forward direction so that for a Gamow-Teller transition $D_{NN}(0^0) \approx -\frac{1}{3}$, as was found experimentally.

Because the (d, d^*) charge exchange vertex and the nuclear excitation are decoupled by the trace over spins (providing one does not use information about the nuclear decay), Moss's formulae translate almost unchanged to the deuteron case and we find for the cross section and polarisation

$$\frac{3}{2} \frac{d\sigma}{dtd^3k} = \frac{P_f}{P_i} S_{J,L}(q)^2 \{ \bar{\ell}_1^{J,L} (|\beta|^2 + |\epsilon|^2) |S^-(k, \frac{1}{2}q)|^2 + \bar{\ell}_2^{J,L} |\delta|^2 |S^+(k, \frac{1}{2}q)|^2 \}, \quad (6.10e)$$

$$\frac{3}{\sqrt{2}} t'_{20} \frac{d\sigma}{dt d^3 k} = \frac{P_f}{P_i} S_{J,L}(q)^2 \{ \bar{\ell}_1^{J,L} (|\beta|^2 - 2|\epsilon|^2) |S^-(k, \frac{1}{2}q)|^2 + \bar{\ell}_2^{J,L} |\delta|^2 |S^+(k, \frac{1}{2}q)|^2 \}, \quad (6.10b)$$

$$\sqrt{3} t'_{22} \frac{d\sigma}{dt d^3 k} = \frac{P_f}{P_i} S_{J,L}(q)^2 \{ \bar{\ell}_1^{J,L} |\beta|^2 |S^-(k, \frac{1}{2}q)|^2 - \bar{\ell}_2^{J,L} |\delta|^2 |S^+(k, \frac{1}{2}q)|^2 \}, \quad (6.10c)$$

$$t'_{21} = 0. \quad (6.10d)$$

These forms follow the fundamental equations (6.1) in their dependence on β , γ , δ and ϵ , but project out in addition the dependence of σ , t'_{20} and t'_{22} on the quantum numbers of the final nuclear state. It should be noted that they can be expressed as linear combinations of cross sections and spin-transfer parameters of the $(\vec{n}, \vec{p})$ charge exchange reaction.

For completeness we give the expressions for $\Delta S = 0$ Fermi transitions which, for the (d, d^*) reaction, proceed through the spin-orbit $i\gamma(q)\sigma_N^1$ amplitude in PWIA.

$$\frac{3}{2} \frac{d\sigma}{dt d^3 k} = (2L + 1) \frac{P_f}{P_i} S'_{L,L}(q)^2 |S^-(k, \frac{1}{2}q)|^2 |\gamma|^2, \quad (6.11a)$$

$$t'_{20} = 1/\sqrt{2}, \quad (6.11b)$$

$$t'_{22} = \frac{1}{2}\sqrt{3}, \quad (6.11c)$$

$$t'_{21} = 0, \quad (6.11d)$$

where S'_{LL} is the nuclear transition form factor corresponding to $\Delta S = 0$. The values of t'_{20} and t'_{22} are maximal in this case but it must be borne in mind that $\gamma(q)$ vanishes for $q = 0$. Transitions may of course be mixtures of $\Delta S = 0$ and $\Delta S = 1$.

We see from equations (6.8a), (6.10) and (6.11) that, if all the form factors for a given L were equal, then summing the $\Delta S = 0$ and $\Delta S = 1$

observables over all J would lead to a result similar to equation (6.1).

For $L = 0$ the formulae for t_{20} and t_{22} are the same as for hydrogen, equations (6.1), except for the absence of $|\gamma|^2$ terms. In practice, $|\gamma|^2$ is negligibly small, so t_{20} and t_{22} are given, except for nuclear distortion, by the curves shown in Figure 3. The $(\vec{d}, 2p)$ reaction may be useful for locating Gamow-Teller strength at high excitation, amidst a background of natural parity.

For $L = 1$, excitations with $J^P = 0^-$ have $t'_{20} = 1/\sqrt{2}$ and $t'_{22} = -1/\sqrt{3}$. (Note that the corresponding $\sigma \rightarrow 0$ as $q \rightarrow 0$). The polarisation predictions shown in Figure 10 are very different depending upon the J and L. Thus for isolated resonances a clear identification of J should be possible once the L value has been deduced from the shape of the angular distribution. On the other hand, for overlapping states neither the $(\vec{d}, d^*)$ nor $(\vec{n}, \vec{p})$ reaction can distinguish for example between a 2^- and an appropriate mixture of 0^- and 1^- levels. This is because all the observables in the two cases are linear in the $\bar{x}_i^{J,L}$ which are in turn subject to the linear constraint of equation (6.8b). We can however form linear superpositions of observables which can enhance or suppress nuclear structure of reaction features. For example, allowing for the possibility of $\Delta S = 0$ transitions,

$$\frac{1}{2}(1 - \sqrt{2} t'_{20}) \frac{d\sigma}{dt} = \frac{P_f}{P_i} \bar{x}_1^{J,L} S_{J,L}(q)^2 |S^-(k, \frac{1}{2}q)|^2 |\epsilon|^2 d^3k, \quad (6.12a)$$

$$\frac{1}{2}(1 + t'_{20}/\sqrt{2} - \sqrt{3}t'_{22}) \frac{d\sigma}{dt} = \frac{P_f}{P_i} \bar{x}_2^{J,L} S_{J,L}(q)^2 |S^+(k, \frac{1}{2}q)|^2 |\delta|^2 d^3k, \quad (6.12b)$$

$$\begin{aligned} \frac{1}{2}(1 - A_{yy}) \frac{d\sigma}{dt} &= \frac{1}{2}(1 + t'_{20}/\sqrt{2} + \sqrt{3} t'_{22}) \frac{d\sigma}{dt} \\ &= \frac{P_f}{P_i} |S^-(k, \frac{1}{2}q)|^2 \{ \bar{x}_1^{J,L} S_{J,L}(q)^2 |\beta|^2 + (2L+1) S'_{L,L}(q)^2 \delta_{J,L} |\gamma|^2 \} d^3k \end{aligned} \quad (6.12c)$$

decouple most of the exchange amplitudes. This may also be used profitably for the elementary $dp \rightarrow d^*n$ reaction.

On the structure side, any $J = L = 1$ state is eliminated in equation (6.12b) and any $J = 0, L = 1$ by equation (6.12a) or (6.12c). Unfortunately away from the vicinity of 0° the Saclay detection system is mainly sensitive to the Cartesian combination A_{yy} of equation (6.12). This is because the principal axis of the polarised beam is perpendicular to the mutual plane of the accelerator and spectrometer^(8,22). Thus at present the β may be isolated but the δ and ϵ cannot be separated. To exploit the full potential of the $(\vec{d}, 2p)$ reaction therefore necessitates a solenoid to precess the deuteron spin. This must be done in order to look for such crucial features as the pion exchange amplitude $\delta(q)$ for large q to investigate possible precursor phenomena.

The formalism described here is based upon the assumption that the state of a given J corresponds to a unique value of L . A prime example to the contrary is of course the $d \rightarrow d^*$ transition itself where the S and D waves interfere coherently in the form factor combinations S^+ and S^- of equation (6.2). The $\bar{x}_i^{J,L}$ only give correctly the incoherent superposition but this can be remedied and offers the possibility of disentangling the different orbital contributions to the excitation of a state of known J .

Distortions will play an even greater role in the $(d, 2p)$ reaction than in (p, n) where, in the case of the ground state Gamow-Teller transition $p^{12}C \rightarrow n^{12}N$ at 800 MeV, it reduces the cross section by a factor of about 2.5 in the forward direction. However from the comparison of the spectra produced by (p, n) , (d, d^*) and $(^3He, t)$ reactions on a number of targets it seems that in general the excitation of levels by all these probes involves primarily single-step processes⁽²⁴⁾. We therefore expect the main features

of the selection rules contained in equation 6.7 to persist when the effects of nuclear damping are included, though there will be quantitative changes due for example to the multiple scatterings being alignment dependent. This is confirmed for Gamow-Teller transitions in ^{12}C by calculations presented in section 9.

It is well known that n-p charge exchange is dominated by spin-dependent amplitudes from $T_p = 200$ to 400 MeV, which makes the Saturne accelerator ideal for studying $\Delta S = 1$ transitions. From Figures 3a and 3b, we see that t_{20} and t_{22} become more pion-like at the highest energies; this implies that the cross section for exciting 0^- states is cleanest there. In the energy range below 100 MeV, which is accessible at the Indiana cyclotron, the spin-independent charge-exchange amplitude α becomes large. Though this cannot excite the d^* state, it can contribute to strong Fermi transitions to final triplet p-p states. With an apparatus with a large acceptance in both angle and momentum it might be possible to study both Gamow-Teller and Fermi transitions by varying the part of phase space in which the p-p final state is detected, e.g. flipping between $\cos\theta \approx 1$ (wide angle p-p pairs) and 0 (p-p pairs close in laboratory angle), as was discussed in section 4 for the deuteron polarimeter. Solid state telescopes, used so far⁽³⁰⁾, detect only a very small part of the whole solid angles.

7. DOUBLE SCATTERING IN THE DEUTERON

For energies of a few hundred MeV and above, the Glauber model provides a good estimate of double scattering effects in proton-deuteron collisions⁽³¹⁾. In the cases of total and inclusive cross sections the effects are typically 5%; for elastic scattering they increase quickly with momentum transfer due to the rapid decline of the single scattering contribution. Analogous results should hold for the deuteron charge exchange reaction where, if we sum over many singlet (triplet) final states, then the double scattering changes in t_{20} and t_{22} are expected to be less than 0.02. On the other hand the effects can be much larger for the coherent (d,d*) transition at large q where the single scattering is much reduced.

The single and double scattering amplitudes with charge exchange are

$$F(\vec{q}) = \langle pp | f_{np}^{ce}(\vec{q}) e^{i\vec{q} \cdot \vec{r}/2} + \frac{i}{2\pi^{3/2}} \int e^{i(\vec{q}' - \frac{1}{2}\vec{q}) \cdot \vec{r}} f_{np}^{ce}(\vec{q}') \bar{f}_{NN}(\vec{q} - \vec{q}') d^2 q' | d \rangle \quad (7.1)$$

where the intermediate momentum transfer $\vec{q}'$ lies in an arbitrary direction in the transverse plane,

$$\vec{q}' = q'(\cos\phi, \sin\phi, 0). \quad (7.2)$$

The formula is derived in the deuteron rest frame but it is essentially frame-independent for small momentum transfers. It looks different from the standard Glauber result because of the invariant normalisation used in equation (2.4).

The evaluation of equation (7.1) involves multidimensional integration: two dimensions for q' and three more for the form factor of the deuteron D state, equation (3.3b). For a given experimental configuration, i.e. $\vec{q}$ and $\vec{k}$, this presents no problem, but it uses too much computer time to be worth pursuing in the present general exploration. We therefore make two approximations.

Firstly, we keep only the dominant spin-isospin independent NN amplitude $\bar{\alpha}_{NN} = \frac{1}{2}(\alpha_{pp} + \alpha_{np})$ of equation (2.3) in the shadowing. Above $T_p \approx 100$ MeV, other amplitudes are smaller by at least a factor of 5 in the important region near $\theta = 0^\circ$. $\text{Im } \bar{\gamma}$ gets as large as $\text{Im } \bar{\alpha}$ for centre-of-mass angles greater than about 55° , but its effect is damped by the phasefactor $\exp(i(\vec{q}' - \frac{1}{2}\vec{q}) \cdot \vec{r})$ in equation (7.1). We have convinced ourselves by alternative approximations that the other spin-dependent contributions to $\bar{f}_{NN}$ affect tensor polarisation predictions by less than 0.01. Below 100 MeV the rapid rise in the 1S_0 and 3S_1 amplitudes makes our approximation unreliable, but so then is the Glauber model.

Secondly, equation (7.1) may be simplified considerably if we restrict the study to the purely s-wave d^* final state which is appropriate for the SPES IV experiments⁽²⁴⁾. It is then straightforward but tedious to show that the effect of double scattering is to make the following replacements in the observables estimated in single scattering approximation in equation (6.1)

$$\epsilon(q)S^-(k, q/2) \rightarrow \epsilon(q)S^-(k, q/2) + \frac{i}{2\pi^{3/2}} \int d^2q' \bar{\alpha}(\vec{q}-\vec{q}') \epsilon(q')S^-(k, Q), \quad (7.3a)$$

$$\begin{aligned} \gamma(q)S^-(k, q/2) \rightarrow \gamma(q)S^-(k, q/2) + \frac{i}{2\pi^{3/2}} \int d^2q' \bar{\alpha}(\vec{q}-\vec{q}') \gamma(q') \times \\ \{S^-(k, Q) \cos \phi + \frac{3}{2\sqrt{2}} \frac{qq'}{Q^2} \sin^2 \phi S_D(k, Q)\}, \end{aligned} \quad (7.3b)$$

$$\begin{aligned} \delta(q)S^+(k, q/2) \rightarrow \delta(q)S^+(k, q/2) + \frac{i}{2\pi^{3/2}} \int d^2q' \bar{\alpha}(\vec{q}-\vec{q}') [\delta(q')\{S^+(k, Q) \cos^2 \phi \\ - \frac{3}{2\sqrt{2}} \frac{qq'}{Q^2} \cos \phi \sin^2 \phi S_D(k, Q)\} + \beta(q') \{S^-(k, Q) \sin^2 \phi - \frac{3}{2\sqrt{2}} (qq' \cos \phi - \frac{1}{2}q^2) \times \\ S_D(k, Q) \sin^2 \phi / Q^2\}], \end{aligned} \quad (7.3c)$$

$$\begin{aligned} \beta(q) S^-(k, q/2) \rightarrow \beta(q) S^-(k, q/2) + \frac{i}{2\pi^{3/2}} \int d^2q' \bar{\alpha}(\vec{q}-\vec{q}') [\beta(q') \{S^-(k, Q) \cos^2 \phi \\ + \frac{3}{2\sqrt{2}} \frac{qq'}{Q^2} \cos \phi \sin^2 \phi S_D(k, Q)\} + \delta(q') \{S^+(k, Q) \sin^2 \phi + \frac{3}{2\sqrt{2}} (qq' \cos \phi - \frac{1}{2}q^2) \times \\ S_D(k, Q) \sin^2 \phi / Q^2\}] , \end{aligned} \quad (7.3d)$$

where

$$\vec{Q} = \vec{q}' - \frac{1}{2}\vec{q} . \quad (7.4)$$

Note that as $q \rightarrow 0$ the double scattering corrections to γ and $(\beta - \delta)$ tend to zero, as they must do on invariance grounds.

The equations do not mix up different values of k so that the double scattering effects may be estimated separately for each excitation energy. At high incident energies, $\bar{\alpha}$ is almost purely imaginary and, since the charge exchange amplitudes are dominantly real, then so are the double scattering contributions shown in equation (7.3).

In Figure 11 we show the results for the moduli of the single scattering terms in equation (7.3) as functions of q for $T_p = 800$ MeV at the peak of the d^* spectrum ($E_x = 600$ keV, $k = 0.12$ fm $^{-1}$). In addition we show those parts of the double scattering terms which interfere with the single, viz.

$$\tilde{A} = \text{Re}(A_{DS} A_{SS}^*) / |A_{SS}| . \quad (7.5)$$

The remainder of the double scattering is of little practical consequence. In general the interference is destructive (shadowing) and for $q = 0$ the ratio of the double to single scattering contributions, $\Delta = \tilde{A} / |A_{SS}|$, is $\Delta_\beta = \Delta_\delta = -3\%$ but $\Delta_\epsilon = -5\%$. This difference arises primarily because the average $\beta + \delta$ amplitude has a steeper slope in q than the ϵ and this reduces the value of the shadowing integral. Equivalently in configuration space, the ϵ amplitude has a shorter range and hence is damped more. At $q = 1$ fm $^{-1}$, $\Delta_\epsilon \approx -15\%$ but $\Delta_\beta \approx -10\%$ despite the ϵ and β terms having similar slopes.

This is due to the coupling of the δ and β amplitudes in equation (7.3d). The D-state modifications of the shadowing are of only minor importance and the explicit $S_D(k,Q)$ terms occurring in equations (7.3b,c,d) are negligible until very large values of q . Apart from these terms, the shadowing integrals for δS^+ and βS^- look very similar with the interchange $\sin\phi \rightarrow \cos\phi$. This only makes a big difference at high q and so the large δ slope explains the comparatively small damping in β .

The real part of $\delta(q)$ vanishes for $q \approx 0.75 \text{ fm}^{-1}$ and the dip in the double scattering curve here is an artefact of the definition in equation (7.5), where the interference changes from destructive to constructive. If the form factor $S^+(k,Q)$ were peaked sharply for small values of Q , then we would expect the double scattering to vanish for $q \approx 1.5 \text{ fm}^{-1}$; the zero actually comes a little after 2 fm^{-1} .

Since for $q > 0.75 \text{ fm}^{-1}$ the δ amplitude is enhanced, while the β amplitude is shadowed, the double scattering decreases the size of t'_{22} here and this is shown in Figure 12b. On the other hand, the larger damping of the ϵ , combined with the antishadowing of the δ , increases the t'_{20} to be found in Figure 12a. The situation is qualitatively the same at higher d^* excitation energies.

We show also in Figure 12 our results for $T_p = 425 \text{ MeV}$. Double scattering modifications of the polarisation are of similar appearance at this lower energy, though it must be stressed that the Glauber approach must be less valid as the energy is decreased.

Double scattering effects for the $d \rightarrow d^*$ charge exchange or $d \rightarrow d$ elastic scattering reactions become more important with increasing q due to the rapid decrease of the single scattering contributions to these coherent processes. This is to be contrasted to the relative smoothness of the double

scattering terms shown in Figure 11. The q dependence of the single scattering will be far less drastic for an inclusive deuteron break-up reaction, with or without charge exchange. It is however tedious to evaluate the double scattering contribution as accurately as for the coherent processes due to the increased number of relevant variables and this will not be attempted here.

8. NUCLEAR DISTORTIONS

At small momentum transfers, charge exchange on nuclear targets is inhibited by the Pauli principle and this reduces the efficiency of a nuclear converter in a deuteron polarimeter. For example, in the simplest closed $p_{3/2}$ harmonic oscillator shell model for ^{12}C , the impulse approximation to the inclusive proton charge exchange cross section is

$$\left(\frac{d\sigma}{dt}\right)_{p^{12}\text{C} \rightarrow nX} = \left[6 - \frac{2}{9} \{19 + (3 + D_{qq})q^2/\alpha^2 + q^4/\alpha^4\} e^{-q^2/2\alpha^2} \right] \left(\frac{d\sigma}{dt}\right)_{pn \rightarrow np} \quad (8.1)$$

where D_{qq} is the charge-exchange Wolfenstein parameter defined by equation (9.2) and we have only kept the spin-spin terms in the amplitude. Thus at $q^2 = 0$ only 16/9 neutrons contribute to the charge exchange and these correspond to the transitions $p_{3/2}$ neutron $\rightarrow$ $p_{1/2}$ proton. For a harmonic oscillator parameter $\alpha^2 = 0.4 \text{ fm}^{-2}$, the effective neutron number increases to 3.5 for $q = 100 \text{ MeV/c}$. It is however found experimentally⁽³²⁾ that in the forward direction at 800 MeV the ratio is only $\frac{16}{9} \times (0.39 \pm 0.05)$, where we have used the n - p amplitudes of ref(20). Under these conditions there are only 0.7 useful neutrons per nucleus and this extra reduction is mainly a consequence of nuclear distortions which will be even stronger for an incoming deuteron. In order to estimate what differences will appear in t_{20} and t_{22} between hydrogen and nuclei, we need a model of the nuclear amplitudes β , γ , δ and ϵ .

It is known that at $q^2 \approx 0$ the excitation of the ground state of ^{12}N takes a large part of the Gamow-Teller transition strength⁽³³⁾ and it is much easier theoretically to study the effects of distortions in such coherent processes. We shall therefore calculate the amplitudes for the reaction $p^{12}\text{C} \rightarrow n^{12}\text{N}_{\text{gs}}$ at 800 MeV using Glauber theory and compare the resulting cross section with the available data. We shall then feed these

amplitudes into the single and double scattering deuteron amplitudes developed in section 7. This provides us with crude estimates of the cross section and asymmetries for $d^{12}\text{C} \rightarrow d^{*12}\text{B}_{\text{gs}}$, though the procedure probably underestimates the deuteron distortion.

In impulse approximation the scattering amplitude for the Gamow-Teller transition $p^{12}\text{C} \rightarrow n^{12}\text{N}_{\text{gs}}$ may be written as

$$F_{12\text{C}}^{\text{I}} = B_{\text{GT}}^{\frac{1}{2}} \left[(\beta\sigma_y - i\gamma) S_y + \delta\sigma_x S_x + \epsilon\sigma_z S_z \right] S_{1+}(q) \quad (8.2)$$

where $\vec{\sigma}$ refers to the incident nucleon and the $\vec{S}$ to the polarisation of the ^{12}N ground state. The transition form factor to the 15.1 MeV isobaric analogue state has been measured in magnetic electron scattering and, after corrections for meson exchange effects, the results may be parameterised as⁽³⁴⁾

$$S_{1+}(q) = (1 - 0.587q^2) \exp(-0.822q^2) \exp(q^2/48\alpha^2). \quad (8.3)$$

A Tassie-Barker factor has been included to account approximately for the centre-of-mass constraint in the harmonic oscillator limit. The Gamow-Teller strength is found to be

$$B_{\text{GT}} = | \langle \vec{\sigma} \rangle |^2 = 0.97. \quad (8.4)$$

As discussed in section 6, the unpolarised cross section is the incoherent sum of four terms

$$\left(\frac{d\sigma}{dt} \right)^{\text{I}} = \frac{P_f}{P_i} \frac{1}{3} B_{\text{GT}} \left[|\beta|^2 + |\gamma|^2 + |\delta|^2 + |\epsilon|^2 \right] S_{1+}(q)^2. \quad (8.5)$$

The eikonal DWIA is an approximation to Glauber theory where only those terms proportional to a single-step process are retained. The scattering amplitude takes the form⁽³⁵⁾

$$F_{12\text{C}}^{\text{I}}(\vec{q}) = (i/2\pi)^{3/2} \exp(q^2/48\alpha^2) \int e^{i\vec{q} \cdot \vec{b}} (1 - \Gamma_{\text{el}}(b))^{11/12} \Gamma_{1+}(\vec{b}) d^2b, \quad (8.6)$$

If the elastic $p\text{-}^{12}\text{C}$ profile function $\Gamma_{el}(b)$ is calculated from Glauber theory, using only the free spin-independent N-N amplitude, it leads to a reasonable description of the small angle elastic $p\text{-}^{12}\text{C}$ differential cross section.

The transition profile function $\Gamma_{1+}(\vec{b})$ must be such that when the distortion factor is neglected the amplitude of equation (8.6) reduces to the impulse approximation of equation (8.2). As in the deuteron double scattering problem of section 7, the multiple scatterings mix the β and δ amplitudes because of the azimuthal integration over the impact parameter vector $\vec{b}$. The resulting amplitude has the same structure as that of equation (8.2).

$$F_{12C}(\vec{q}) = B_{GT}^{\frac{1}{2}} \left[(\tilde{\beta}\sigma_y - i\tilde{\gamma}) S_y + \tilde{\beta}\sigma_x S_x + \tilde{\epsilon}\sigma_z S_z \right] \quad (8.7)$$

with modified components

$$\tilde{\beta}(q) = \langle \Gamma^{\delta+\beta}(b) \rangle_0 - \langle \Gamma^{\delta-\beta}(b) \rangle_2 \quad (8.8a)$$

$$\tilde{\delta}(q) = \langle \Gamma^{\delta+\beta}(b) \rangle_0 + \langle \Gamma^{\delta-\beta}(b) \rangle_2 \quad (8.8b)$$

$$\tilde{\gamma}(q) = \langle \Gamma^{\gamma}(b) \rangle_1 \quad (8.8c)$$

$$\tilde{\epsilon}(b) = \langle \Gamma^{\epsilon}(b) \rangle_0 . \quad (8.8d)$$

We here use the notation

$$\langle \Gamma^{\alpha}(b) \rangle_n \equiv \exp(q^2/48\alpha^2) \int_0^{\infty} b \, db \, J_n(qb) \, \Gamma^{\alpha}(b) \, (1 - \Gamma_{el}(b))^{11/12} . \quad (8.9)$$

The transition profile functions $\Gamma^{\alpha}(b)$ now depend only upon the magnitude of $\vec{b}$ and their values are easily obtained by demanding that equation (8.9) reduces to the impulse approximation in the absence of distortion.

In Figure 13 we show the unpolarised cross section for $p\text{-}^{12}\text{C} \rightarrow p\text{-}^{12}\text{C}_{15.1}^*$

at 800 MeV compared with the predictions of equation (8.6) divided by an isospin factor of 2. The individual contributions of the β , δ and ϵ amplitudes to the cross section are separately illustrated, the spin-orbit γ term being negligible in this experiment. The forward value of the amplitude is damped by the multiple scatterings by a factor of 0.57 whereas for the β and δ terms it is only 0.7. As discussed in section 7, this can be understood in terms of the long range of pion exchange and the coupling of the β and δ terms. It has the effect of making the amplitudes more pion-like in the forward direction. The effect would be more pronounced for $T < 300$ MeV, where $|\epsilon(0)| \leq |\beta(0)|$.

In impulse approximation, all three contributions have a common zero at $q^2 = 1.7 \text{ fm}^{-2}$ arising from the transition form factor. Nuclear distortions push all three minima to smaller values of q^2 but the effect is much larger in the δ term due to the coupling in equation (8.6). As a consequence, although all three minima are predicted to be very deep, the unpolarised cross section is expected to show little more than a change of slope. The shape of the cross section agrees quite well with the LAMPF experimental data^(36,37) though the normalisation is about a factor of 1.4 too low. The LAMPF analysis also required a slight upward adjustment but it also predicted a broad minimum at $q^2 \approx 1.2 \text{ fm}^{-2}$ which is not found in the data. The origin of this discrepancy is probably the Love-Franey nucleon-nucleon charge exchange amplitudes⁽³⁸⁾ that they used. These assume that $\beta(q) = \epsilon(q)$ for all q . This may be tolerable at low energies but at 800 MeV our amplitudes suggest that $|\epsilon(0)|/|\beta(0)| \sim 0.4$.

If we neglect double scattering in the deuteron, then the tensor observables for $d^{12}\text{C} \rightarrow d^{12}\text{B}_{\text{gs}}$ are given by equation (6.1) but with $\gamma = 0$ and the β , δ and ϵ as calculated above. The predictions for t_{20} and t_{22}

in this model are displayed in Figure 14. In the forward direction $t_{20} = 0.58$ to be compared with an impulse approximation value of 0.53. This increase in analysing power is probably typical of all Gamow-Teller transitions. Since these dominate the forward cross section this is an indication that the use of a nuclear converter would not destroy the polarimeter signal.

The $p^{12}\text{C} \rightarrow n^{12}\text{N}_{\text{gs}}$ amplitude may then be fed into the deuteron double scattering integrals of equation (7.3) to estimate the effects of deuteron distortion. Taking $\bar{a}$ to be the $N^{12}\text{C}$ elastic amplitude calculated in Glauber theory, this still does not account for all the multiple scatterings in the $d^{12}\text{C} \rightarrow d^{*12}\text{B}$ amplitudes. It neglects contributions where the target nucleus is excited by a collision with one of the nucleons in the deuteron, the final state being reached through an interaction with the other.

In this approximation the deuteron double scattering reduces the zero degree cross section by a further 12% and increases the value of t_{20} by only 1%. The predicted angular dependence of t_{20} and t_{22} for $d^{12}\text{C} \rightarrow d^{*12}\text{B}_{\text{gs}}$, with and without the deuteron double scattering but including D-state effects, is shown in Figure 14 for a d^* excitation energy $E_x = 600$ keV. The strong oscillations in the asymmetries arise from the minima in the β , δ and ϵ cross sections of Figure 14 and are typical of coherent nuclear reactions. For the purposes of a polarimeter, where we exploit the inclusive cross section to measure t_{20} , the zero degree results are more indicative. We can therefore expect at this energy about 0.65 effective nucleons per carbon atom but with an analysing power marginally higher than for free protons.

The zero of the δ amplitude has moved from $q \approx 0.75 \text{ fm}^{-1}$ for free np charge exchange to $q \approx 0.5 \text{ fm}^{-1}$, and with it the first peak of t_{22} . The origin of this effect is the destructive interference between single and multiple scattering at small q and constructive interference for $q > 0.75 \text{ fm}^{-1}$.

We anticipate that this will be a general feature of both inclusive and exclusive (d,2p) reactions on carbon and neighbouring nuclei.

9. WOLFENSTEIN PARAMETERS FOR THE $d(p,n)2p$ REACTION

There is an intimate link between the tensor polarisation observables in the $p(\vec{d},2p)n$ reaction and the spin transfer parameters K_{ij} when the reaction is looked at as $d(\vec{p},\vec{n})2p$.

In the deuteron Breit frame, the Wolfenstein parameters⁽³⁹⁾ are given in terms of our impulse approximation matrix element M by

$$K_{SS}^B = R^B = D_{qq} \cos(\tfrac{1}{2}\theta_B) + D_{pq} \sin(\tfrac{1}{2}\theta_B), \quad (9.1a)$$

$$K_{LS}^B = A^B = -D_{qq} \sin(\tfrac{1}{2}\theta_B) + D_{pq} \cos(\tfrac{1}{2}\theta_B), \quad (9.1b)$$

$$K_{SL}^B = R'^B = -D_{pq} \cos(\tfrac{1}{2}\theta_B) + D_{pp} \sin(\tfrac{1}{2}\theta_B), \quad (9.1c)$$

$$K_{LL}^B = A'^B = D_{pq} \sin(\tfrac{1}{2}\theta_B) + D_{pp} \cos(\tfrac{1}{2}\theta_B), \quad (9.1d)$$

where θ_B is the scattering angle in the Breit frame. The normal component involves no spin rotation and is simply

$$K_{nn}^B = D^B = D_{nn}. \quad (9.1e)$$

In these expressions we have used the notation

$$3 D_{ij} \frac{d\sigma}{dtd^3k} = \text{Tr}(M \sigma_1^3 M^\dagger \sigma_j^3) \quad (9.2)$$

where the unpolarised cross section is given by equation (3.7a) and the i, j refer to the axes defined by equation (2.2). The S and L components are then modified by the standard Wigner rotation⁽³⁹⁾ from the Breit to the laboratory frame, though this has only a small effect under the experimental conditions studied here.

The traces in equation (9.2) are easily evaluated in terms of the same

grouping of matrix elements defined in equation (3.8) for the tensor polarisation case.

$$3 D_{ij} \frac{d}{dtd^3k} = v_1 v_1 + v_2 v_2 + v_3 v_3 . \quad (9.3)$$

The v_i are similar combinations of n-p amplitudes to those occurring in equation (2.19) and are different for singlet and triplet final states:

$$v_1^s = |\gamma|^2 + a_1 |\beta|^2 + a_2 |\delta|^2 + a_3 |\epsilon|^2 , \quad (9.4a)$$

$$v_1^t = 3|\alpha|^2 + (2 + 3a_4) |\gamma|^2 + 2a_1 |\beta|^2 + 2a_2 |\delta|^2 + 2a_3 |\epsilon|^2 , \quad (9.4b)$$

$$v_2^s = -v_2^t = (|\gamma|^2 + a_1 |\beta|^2 + a_2 |\delta|^2 - 2a_3 |\epsilon|^2) / \sqrt{2} , \quad (9.4c)$$

$$v_3^s = -v_3^t = \frac{1}{2}\sqrt{3}(|\gamma|^2 + a_1 |\beta|^2 - a_2 |\delta|^2) \quad (9.4d)$$

with

$$(a_1, a_2, a_3, a_4) = (+1, -1, -1, +1) \text{ for } D_{nn} , \quad (9.5a)$$

$$= (-1, +1, -1, -1) \text{ for } D_{qq} , \quad (9.5b)$$

$$= (-1, -1, +1, -1) \text{ for } D_{pp} . \quad (9.5c)$$

On the other hand D_{pq} depends upon the interference between the spin-orbit amplitude γ and the other terms:

$$3 D_{pq} \frac{d\sigma}{dtd^3k} = (2v_1 + v_2 \sqrt{2} + v_3 \sqrt{3}) \text{Re}(\beta^* \gamma) \text{ for singlet final states, } (9.6a)$$

$$= v_1 (4\text{Re}(\beta^* \gamma) - 6\text{Re}(\alpha^* \gamma)) - (v_2 \sqrt{2} + v_3 \sqrt{3}) \text{Re}(\beta^* \gamma)$$

for triplet final states. (9.6b)

These formulae represent a generalisation of those of Moss⁽⁷⁾ to include

S - D interference and arbitrary final p-p angular momentum states but the

singlet D_{ij} reduce to his for a purely S-wave deuteron.

The results simplify significantly at $q = 0$ when only singlet p-p final states are allowed. If then we integrate over the directions of the relative momentum $\vec{k}$ of the undetected protons we find that $V_2 = V_3 = 0$ and we are left with

$$D_{ij} = \frac{(a_1 + a_2)|\beta|^2 + a_3|\epsilon|^2}{2|\beta|^2 + |\epsilon|^2} . \quad (9.7)$$

This yields the well-known $q = 0$ constraint^(12,17)

$$K_{LL} + 2 K_{SS} = -1 . \quad (9.8)$$

The tensor polarisation also simplifies at this point;

$$t_{20} = \frac{(\phi_S(k))^2 + \frac{1}{10}(\phi_D(k))^2}{(\phi_S(k))^2 + (\phi_D(k))^2} \frac{1}{\sqrt{2}} (K_{SS} - K_{LL}) . \quad (9.9)$$

Integration over all values of k leads to the closure sum rule result of equation (5.7). If however we restrict attention to the vicinity of $k = 0$ which corresponds to the $pd \rightarrow nd^*$ reaction, we obtain

$$T_{20} = \frac{1}{\sqrt{2}} (K_{SS} - K_{LL}) \text{ for } q = k = 0. \quad (9.10)$$

The vital relations equations (9.8), (9.10) follow directly from spin-parity considerations for $pd \rightarrow nd^*$ at $q = 0$ and are thus independent of the impulse approximation that we have used. They are unaffected by the deuteron D wave and one can show explicitly that the double-scattering corrections of section 7 do not affect the relations at $q = 0$.

For free n-p exchange above 100 MeV, $R = K_{SS}$ has a large negative peak at a momentum transfer $q \approx 140$ MeV/c, arising from the zero of $\text{Re}(\delta(q))$ which is associated with the rapidly varying pion-exchange amplitude (see Figure 2). This large value of R is used at TRIUMF to make a polarised neutron beam via

the $d(\vec{p}, \vec{n})2p$ reaction at a laboratory angle of 9° . Alternatively the 0° value of the parameter $A' = K_{LL}$ increases in magnitude with energy and, above 650 MeV, becomes large enough to make a polarised neutron beam in the forward direction. This is the technique used at LAMPF⁽¹¹⁾. In both cases it is important to know the neutron spectrum and how the spin-transfer parameters vary across the spectrum.

We first have to relate the neutron energy to q and k . Providing the recoil of the target is non-relativistic, the neutron kinetic energy for free $pn \rightarrow np$ scattering is related to the beam energy by

$$T_n(H) = T_B - q^2/2m \quad (9.11a)$$

whereas for the reaction in deuterium

$$\begin{aligned} T_n(D) &= T_B - (\vec{k} + \frac{1}{2}\vec{q})^2/2m - (\vec{k} - \frac{1}{2}\vec{q})^2/2m - B \\ &= T_B - k^2/2m - q^2/4m - B \end{aligned} \quad (9.11b)$$

with m the nucleon mass and B the deuteron binding energy. The difference in neutron energy between reactions on hydrogen and deuterium targets is

$$\Delta T = T_n(H) - T_n(D) = B + k^2/m - q^2/4m. \quad (9.11c)$$

We define the difference in Wolfenstein parameters for free n - p scattering and charge exchange in deuterium by

$$\Delta K = K(H) - K(D). \quad (9.12)$$

Estimates of ΔK have been made by Dass and Queen⁽⁴⁰⁾, Cromer⁽⁴¹⁾ and Reay et al.⁽⁴²⁾, but all ignoring the deuteron D-state. We have evaluated predictions from 140 to 800 MeV. Illustrative examples, obtained by integrating over the unobserved proton angles (θ, ϕ) , are to be found in Figure 15 for 515 MeV.

The neutron energy spectrum is shown for laboratory angles of 3, 6 and 9°. At the smallest angle it has a half-width of 2 MeV, dropping roughly like $1/(\Delta T)^2$. The spectrum gets much broader with increasing angle, but the mean value of ΔT , which is 3.4 MeV at 3°, drops to 2.0 MeV at 9°.

The Wolfenstein parameters, shown in Figure 15, vary rapidly over the neutron spectrum because singlet final states are enhanced at small k and this must be compensated by triplet dominance at large k . The neutron beams have normally a wide energy acceptance and it is of interest to know how the mean values of the Wolfenstein parameters vary with the upper limit set on ΔT . This is also shown in Figure 15 for R and A' .

Table 2 displays our predicted values of ΔK for all five Wolfenstein parameters for a variety of energies and angles, assuming a cut on the neutron spectrum at $\Delta T = 50$ MeV. From their measurements on the neutron beam at TRIUMF, Axen et al.⁽⁴³⁾ deduced values of K_{SS} and K_{SL} for hydrogen at 9° in the laboratory as a function of energy. On the basis of the present calculation we are able to improve their evaluation of deuteron effects and, in Table 3, we give revised values of these two parameters. The changes are small, but in several instances improve agreement with phase shift analysis.

We show additionally in Table 2 the input hydrogen values of K_{LL} and K_{SS} as well as the deuterium value of the combination $K_{LL} + 2K_{SS}$ as a function of angle at 515 MeV. It can be seen from this that equation (9.8) breaks down extremely quickly as one moves away from $q = 0$ and this is true either in transverse or longitudinal momentum transfer (associated with the excitation energy of the p - p pair). The reasons are two-fold:

- (a) the free K_{SS} value varies rapidly with q due to pion exchange, and
- (b) the admixture of triplet final states increases fast with q .

For $A' = K_{LL}$ we predict a significant 0° difference ΔK_{LL} between the

free p-n values and those for deuterium, which should increase with energy. However at 790 MeV Riley et al.⁽¹¹⁾ measured $K_{LL}(D) = -0.65 \pm 0.044$ for deuterium, whereas Ransome et al.⁽⁴⁴⁾ obtained a value very close to that for hydrogen, namely -0.65 ± 0.11 (at 2.9° c.m.). The error bars on the latter are large so that the disagreement with the K_{LL} calculated here or by Riley⁽¹¹⁾, who neglected the D-state effect, is not too serious.

Chalmers et al.⁽¹²⁾ have recently remeasured K_{SS} and K_{LL} at 0° in charge exchange in deuterium at 500, 650 and 800 MeV. They quote a mass resolution of $\pm 10 \text{ MeV}/c^2$ together with an angular acceptance of less than 0.2° . Averaging our predictions over this excellent resolution we estimate that the influence of triplet states should dilute the right hand side of equation (9.8) by less than 0.02. However their results fail to satisfy this relation by much more than this, finding the sum to lie in the range -0.76 to -0.82 with statistical errors of ± 0.03 but a systematic error of ± 0.10 . The systematic uncertainty arises almost entirely from the analysing power for neutrons, which is derived from the analysing power of n-p elastic scattering, A_{np} . There are however major (20%) discrepancies in the latter between LAMPF and TRIUMF at energies near 500 MeV and it is clear that an alternative means of calibration is needed for neutron polarisation. We consider that equation (9.8) might fulfil this role. The one requirement is that q should be small, less than say $20 \text{ MeV}/c$, although corrections for triplet final states are readily calculable providing the neutron resolution is known accurately. Since the condition appears to be met by the data of Chalmers et al.⁽¹²⁾, we suggest that the polarisation of the LAMPF neutron beam^(12,46) could be a factor of about 1.25 higher than their results indicate, such that equation (9.8) would be satisfied at all three energies within statistical errors. As a consequence, the values of A_{np} they use would be a factor of 1.25 too

high. If these are scaled down by this amount they agree with the TRIUMF value of Clough et al. at 495 MeV, which claims a $\pm 3\%$ absolute normalisation⁽⁴⁵⁾.

After scaling the data of Chalmers et al⁽¹²⁾ in this way, we can deduce values of t_{20} from equation (9.10). This procedure gives $t_{20} = 0.30, 0.49$ and 0.46 , all with statistical errors of ± 0.02 , at proton energies of 500, 650 and 800 MeV respectively. These are in general accord with the predictions of Figure 3 and provide the first published experimental check on the large values of t_{20} expected for the $\vec{dp} \rightarrow d^*n$ reaction.

10. CONCLUSIONS

We have presented calculations which show that tensor polarisation effects in charge exchange reactions involving the deuteron have an important bearing on various problems in nuclear and particle physics. The fundamental results for σ , t'_{20} and t'_{22} in the (d,d^*) reaction on hydrogen are given by equations (6.1) and (6.2). For nuclei, the free np charge exchange amplitudes β , γ , δ and ϵ are replaced by distorted amplitudes $\tilde{\beta}$, $\tilde{\gamma}$, $\tilde{\delta}$ and $\tilde{\epsilon}$ of equation (8.8). For other singlet and triplet final pp states, similar but more complicated formulae are given by equations (3.7).

It is vital for the isolation of the deuteron monopole form factor that we have an efficient analyser of the polarisation of the recoil deuteron in elastic e-d scattering⁽²⁾. Away from the low energy domain, where the ${}^3\text{He}(d,p){}^4\text{He}$ serves as basis⁽³⁾, elastic d-p scattering looks promising⁽⁴⁾. Now even for deuterons of 200 MeV the theoretical figure of merit of a polarimeter based upon $dp \rightarrow (pp)n$ looks much higher. The situation becomes even more favourable for charge exchange vis-a-vis elastic scattering as the energy is raised. It should however be feasible to devise an analysing system which detects and uses both reactions in the same apparatus. This would provide consistency checks which would be especially valuable for t_{20} , whose measurement depends upon an absolute rate.

The theoretical estimates of $dp \rightarrow (pp)n$ presented here should be quite reliable since the reaction involves only small momentum transfers to the deuteron. Our predictions are indeed in accord with the first very preliminary SPES IV data⁽⁴⁷⁾. The results then may be turned around and used to give information on n-p charge exchange amplitudes for which only models exist above 800 MeV⁽⁴⁸⁾.

The use of carbon as a converter in a polarimeter is hindered by the

Pauli blocking and nuclear distortion. There may be less than one useful proton per carbon nucleus but this may be still helpful for deuterons around 500 MeV because of the enhanced t_{20} we expect to be generated by multiple scattering.

Experimental tests of our calculations for the deuteron tensor polarisation in $(\vec{d}, 2p)$ reactions will be useful for confirming our estimates for the nucleon spin-transfer parameters in $d(\vec{p}, \vec{n})pp$ which also involves a tensor combination of spin variables. This is very important for neutron beam studies. The relation $K_{LL} + 2K_{SS} = -1$ at $q = 0$ can serve to calibrate neutron beam polarisations.

The excitation of specific nuclear levels through the (d, d^*) reaction or the separation of longitudinal and transverse response functions of a nucleus could prove to be as fruitful as for the (p, n) case. We emphasise that from experimental determinations of σ , t_{20} and t_{22} one can invert equations (6.1) to determine $|\tilde{\gamma}|^2 + |\tilde{\beta}|^2$, $|\tilde{\epsilon}|^2$ and $|\tilde{\delta}|^2$. This should be a valuable way of isolating experimentally the various spin-dependent contributions to nuclear transitions. In this procedure, one has to factor out the deuteron form factors $S^\pm(k, \frac{1}{2}q)$ and unfold the double scattering effects in the deuteron along the lines outlined in section 8.

An even more efficient probe of Gamow-Teller transitions would be through the polarised $(\vec{^6\text{Li}}, ^6\text{He})$ reaction where only a single unpolarised stable particle needs to be detected in the final state. This has all the same spin-parity selection rules as the (d, d^*) case but is even more confined to the nuclear periphery by the damping. It is hoped that polarised lithium isotopes will be accelerated in Saturne in the near future. It is possible to calibrate ^6Li beam polarisations in terms of deuteron ones by comparing the forward analysing powers in $d\vec{^6\text{Li}} \rightarrow d^*{}^6\text{He}$ and $\vec{^6\text{Li}}d \rightarrow ^6\text{He}d^*$.

An independent absolute calibration of the polarisation of the deuteron beam is very desirable and this might be achieved through (d,d*) for nuclear transitions $0^+ \rightarrow 0^-$. An alternative possibility⁽⁴⁹⁾ is to study two-particle transfer reactions such as $d^4\text{He} \rightarrow \pi^+{}^6\text{He}$ ($1^+0^+ \rightarrow 0^-0^+$) which have a unique t_{20} but they have very small cross sections at high energies.

We have only considered nucleonic degrees of freedom in this paper but the Saclay experiments are also capable of measuring very inelastic (d,d*) reactions which excite the Δ resonance from either hydrogen or complex nuclei⁽²⁴⁾. This will be the subject of later work but several points can be made here. The extension of the present impulse approximation formalism to the Δ region is complicated because of the large number of $NN \rightarrow N\Delta$ amplitudes⁽⁵⁰⁾ and non-resonant π -N states will also have to be included. The longitudinal momentum transfer is large and care has to be taken with the effect on this of Lorentz transformations. On the other hand, measuring deuteron tensor observables through (d,d*) in deuterium and carbon should tell us whether the mass of a longitudinally excited Δ is shifted more in a nucleus than a transverse one⁽⁵¹⁾. It may give us further clues on the quenching of Gamow-Teller sum rules⁽²⁷⁾. Such questions are central to intermediate energy nuclear physics.

We are grateful to the members of the SPES IV (d,d*) team, in particular C. Gaarde, T. Hennino, T. Jørgensen and P. Radvanyi for discussions on their experiment. We also acknowledge talks and correspondence with L. Antonuk, J.M. Cameron, R.C. Johnson and F. Lehar.

APPENDIX

We give here some useful relations which help in the manipulation of tensor operators⁽¹⁸⁾. It is readily seen from their definition in equation (2.18) that the Ω_{2m} commute with the triplet operator and that

$$\Omega_{2m}^T = T \Omega_{2m} = \Omega_{2m} \quad (A1)$$

The Ω_{2m} may therefore be used in the spin-one space and are conveniently represented by 3×3 matrices:

$$\Omega_{20} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad \Omega_{21} = \left(\frac{3}{2}\right)^{\frac{1}{2}} \begin{bmatrix} 0 & -1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}, \quad \Omega_{22} = \sqrt{3} \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad (A2)$$

with

$$\Omega_{2-m} = (-1)^m \Omega_{2m}^+ \quad (A3)$$

From the operator or matrix forms, one readily derives the following relations which are used in the evaluations of the traces of $Z \Omega Z^+$ in sections 2 and 3:

$$\Omega_{20} \Omega_{20} = 1 - \Omega_{20}/\sqrt{2},$$

$$\Omega_{20} \Omega_{22} = \Omega_{22} \Omega_{20} = \Omega_{22}/\sqrt{2},$$

$$\Omega_{22} \Omega_{2-2} + \Omega_{2-2} \Omega_{22} = 2 + \sqrt{2} \Omega_{20},$$

$$\Omega_{21} \Omega_{21} = -\frac{1}{2}\sqrt{3} \Omega_{22},$$

$$\Omega_{21} \Omega_{2-1} + \Omega_{2-1} \Omega_{21} = -2 + \Omega_{20}/\sqrt{2},$$

$$\Omega_{20} \Omega_{21} + \Omega_{21} \Omega_{20} = -\Omega_{21}/\sqrt{2},$$

$$\Omega_{21} \Omega_{2-2} + \Omega_{2-2} \Omega_{21} = \sqrt{3} \Omega_{2-1},$$

$$\Omega_{21} \Omega_{20} \Omega_{21} = (3/2)^{\frac{1}{2}} \Omega_{22},$$

$$\Omega_{21} \Omega_{20} \Omega_{2-1} + \Omega_{2-1} \Omega_{20} \Omega_{21} = 1/\sqrt{2} + 2 \Omega_{20},$$

$$\Omega_{2-2} \Omega_{22} \Omega_{2-2} = 3 \Omega_{2-2} ,$$

$$\Omega_{2-1} \Omega_{22} \Omega_{2-1} = \frac{1}{2}\sqrt{3} (\sqrt{2} \Omega_{20} - 1) ,$$

$$\Omega_{20} \Omega_{21} \Omega_{2-1} + \Omega_{2-1} \Omega_{21} \Omega_{20} = \frac{1}{2}(\sqrt{2} - 5 \Omega_{20}) ,$$

$$\Omega_{2-2} \Omega_{21} \Omega_{2-1} + \Omega_{2-1} \Omega_{21} \Omega_{2-2} = - 3 \Omega_{2-2} , \quad (A4)$$

with other relations following from the hermiticity shown by equation (A3)

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P_d (MeV/c)	T_p (MeV)	Without Deuteron D-state				Including Deuteron D-state					
		$ \overline{t}_{20} $	σ_{eff}^0 (mb)	$ \overline{t}_{22} $	σ_{eff}^2 (mb)	$ \overline{t}_{20} $	σ_{eff}^0 (mb)	$ \overline{t}_{21} $	σ_{eff}^1 (mb)	$ \overline{t}_{22} $	σ_{eff}^2 (mb)
436	25	0.17	7.16	0.04	8.25	0.37	4.32	0.29	1.03	0.29	1.66
621	50	0.41	5.63	0.07	6.90	0.45	8.92	0.28	0.81	0.22	3.84
889	100	0.41	3.32	0.20	3.44	0.46	5.92	0.25	0.91	0.24	5.98
1071	142	0.34	2.09	0.24	2.06	0.43	4.12	0.24	0.86	0.26	4.33
1324	210	0.27	1.42	0.29	1.43	0.37	2.42	0.23	0.72	0.30	3.12
1692	325	0.13	0.57	0.37	0.80	0.42	0.53	0.22	0.45	0.36	1.71
1978	425	0.17	0.38	0.39	0.54	0.41	0.41	0.22	0.32	0.38	1.15
2220	515	0.23	0.43	0.43	0.39	0.38	0.51	0.22	0.23	0.41	0.82
2927	800	0.43	0.33	0.50	0.21	0.45	0.57	0.23	0.13	0.48	0.45

Table 1: Mean tensor analysing powers and effective cross section for $p(d,2p)n$ estimated with and without the deuteron D-state. Note that t'_{21} vanishes for a purely S-wave deuteron so that $|\overline{t}_{21}|$ will be small in that case.

T_p (MeV)	θ_{lab} (deg)	$K_{LL}(H)$	$K_{SS}(H)$	ΔK_{LL}	ΔK_{NN}	ΔK_{SS}	ΔK_{LS}	ΔK_{SL}	$K_{LL}(D) + 2K_{SS}(D)$
142	0	0.0086	-0.3667	0.0709	0.1015	0.1015	0	0	-0.999
	9	0.2286	-0.2356	0.0355	0.0610	0.0741	-0.0172	0.0111	
210	0	-0.1624	-0.3209	0.0581	0.0680	0.0680	0	0	-0.998
	9	0.0872	-0.8462	0.0203	0.0281	0.0378	-0.0099	0.0071	
325	0	-0.3649	-0.1499	0.1218	0.1066	0.1066	0	0	-0.997
	9	-0.1389	-0.8038	0.0298	0.0244	0.0473	-0.0193	0.0145	
425	0	-0.4015	-0.1220	0.1239	0.1128	0.1128	0	0	-0.994
	9	-0.1840	-0.7958	0.0250	0.0186	0.0362	-0.0146	0.0129	
515	0	-0.4303	-0.0430	0.1897	0.1425	0.1425	0	0	-0.991
	0.5	-0.4225	-0.0594	0.1930	0.1407	0.1406	-0.0033	0.0037	-1.017
	3	-0.3259	-0.2636	0.1557	0.1081	0.1296	-0.0166	0.0180	-1.268
	6	-0.1818	-0.6351	0.0788	0.0412	0.1049	-0.0217	0.0190	-1.741
	9	-0.2374	-0.6750	0.0336	0.0211	0.0384	-0.0131	0.0123	-1.698
	12	-0.3702	-0.5238	0.0202	0.0267	0.0098	-0.0026	0.0089	-1.418
	15	-0.4768	-0.3397	0.0158	0.0306	-0.0284	0.0046	0.0073	-1.115
	18	-0.5341	-0.1544	0.0145	0.0316	-0.0319	0.0088	0.0062	-0.794
800	0	-0.5943	0.0550	0.2234	0.1366	0.1366	0	0	-0.981

Table 2: Predictions of the differences $\Delta K = K(H) - K(D)$ in Wolfenstein parameters between hydrogen and deuterium for selected energies and angles. Note that the combination $K_{LL}(D) + 2K_{SS}(D)$ varies very rapidly with angle and is not equal to -1 at $\theta = 0^\circ$ due to the longitudinal momentum transfer.

Mean Neutron Energy (MeV)	Scattering Angle in c.m. (deg)	R'_t	R_t
225.0	160.97	-0.028 ± 0.010	-0.801 ± 0.0025
332.5	160.50	0.010 ± 0.010	-0.804 ± 0.026
434.7	160.06	0.032 ± 0.010	-0.771 ± 0.024
506.4	159.76	0.040 ± 0.010	-0.684 ± 0.021

Table 3: A revision of the values of Axen et al.⁽⁴³⁾ for $R'_t = K_{SL}$ and $R_t = K_{SS}$ for free n-p scattering, taking into account the more refined deuteron corrections presented in section 9.

FIGURE CAPTIONS

- Figure 1. Impulse approximation diagram for the small angle charge exchange $dp \rightarrow (pp)n$ or $dA \rightarrow (pp)A'$ reactions, defining the kinematics in the deuteron rest frame and the particle labels.
- Figure 2. (a) Real and (b) imaginary parts of np charge exchange amplitudes⁽²⁰⁾ at 142 MeV normalised to $d\sigma/d\Omega_{cm}$: β (long dashes), $-i\gamma$ (dash-dot), δ (full line) and ϵ (short dashes); (c) $|\beta(0)| = |\delta(0)|$ (full line) and $|\epsilon(0)|$ (dashed line) versus deuteron momentum p_d and nucleon energy T_p .
- Figure 3. Values of (a) t_{20} and (b) t_{22} for the (d,2p) reaction on hydrogen leading to singlet di-proton states. The predictions are integrated over $\cos\theta$ and ϕ and over k from 0 to 280 MeV/c. Numbers indicate nucleon laboratory energies in MeV.
- Figure 4. Cross sections for p-p singlet (solid curves) and triplet (broken) final states for the $dp \rightarrow (pp)n$ reaction at $T_d = 200$ MeV. The results are integrated over $\cos\theta$ and ϕ and binned in intervals of $\Delta q = \Delta k = 17.5$ MeV/c. (a) - (f) are centred at $q = 35, 70, 105, 140, 175$ and 210 MeV/c respectively.
- Figure 5. Cross sections for p-p singlet (solid curves) and triplet (broken) final states for the $dp \rightarrow (pp)n$ reaction at $T_d = 200$ MeV. The results are integrated over ϕ and binned in intervals of $\Delta q = \Delta k = 17.5$ MeV/c and $\Delta(\cos\theta) = 0.1$. (a) - (c) are centred at $q = 70$ MeV/c with $k = 35, 70$ and 105 MeV/c, while (d) - (f) are at $q = 140$ MeV/c and $k = 35, 70$ and 105 MeV/c respectively.
- Figure 6. Values of t_{20} and t_{22} for the $dp \rightarrow (pp)n$ reaction at $T_d = 200$ MeV summed over p-p singlet and triplet final states and integrated over ϕ . (a) - (c) refer to t_{20} and (d) - (f) to t_{22} at $q = 35, 70$ and 105 MeV/c respectively. Full, dashed and chain curves correspond to $k = 35, 70$ and 105 MeV/c respectively.
- Figure 7. Values of (a) t_{20} , (b) t_{21} and (c) t_{22} for the $dp \rightarrow (pp)n$ reaction at $T_d = 200$ MeV and $q = 70$ MeV/c, summed over singlet and triplet final states. Full curve corresponds to $k = 70$ MeV/c, $\cos\theta = 0.9$, long dashes $k = 70$ MeV/c, $\cos\theta = 0.1$, short dashes $k = 140$ MeV/c, $\cos\theta = 0.9$, dash-dot $k = 140$ MeV/c, $\cos\theta = 0.5$, dash-dot-dot $k = 140$ MeV/c, $\cos\theta = 0.1$ and dotted $k = 210$ MeV/c, $\cos\theta = 0.5$.

- Figure 8. Impulse approximation sum rules of equation (5.6) for the cross section (chain curve), t_{20} (solid) and t_{22} (broken) for $\vec{p}d \rightarrow npp$ at 800 MeV.
- Figure 9. Transition form factors of equation (6.3) for the excitation of the d^* system from the S and D-state components of the deuteron. The solid curves correspond to $k = 0.12 \text{ fm}^{-1}$, the broken to $k = 0.6 \text{ fm}^{-1}$.
- Figure 10. Mean values of (a) t_{20} and (b) t_{22} averaged over $0 < k < 70 \text{ MeV/c}$, $-0.5 < \cos\theta < 0.5$ for nuclear spin-dipole excitations induced by the (d,d^*) reaction estimated neglecting nuclear distortion. The predictions are for a 1^- level (full curve $T_d = 650 \text{ MeV}$, dash-dot 100 MeV, dash-dot-dot 1600 MeV) and 2^- level (short dashes 650 MeV, long dashes 100 MeV and dotted 1600 MeV).
- Figure 11. Moduli of the single scattering amplitudes for $dp \rightarrow d^*n$ at $T_d = 1600 \text{ MeV}$ and $E_{d^*} = 600 \text{ keV}$. The solid curve corresponds to the δ term, long dash to the β , short dash to the ϵ and chain curve to the γ . Illustrated below these are those parts of the Glauber double scattering amplitudes which interfere with the single, as defined by equation (7.5), multiplied by -1 .
- Figure 12. Predictions of (a) t'_{20} and (b) t'_{22} for $dp \rightarrow d^*n$ with $E_{d^*} = 600 \text{ keV}$ at 1600 and 850 MeV. The broken curve is in single scattering approximation whereas the solid curve includes the Glauber double scattering term.
- Figure 13. Differential cross section for $p^{12}\text{C} \rightarrow p^{12}\text{C}_{15.1}^*$ at 800 MeV, the experimental data being taken from ref (34) (circles) and (35) (crosses). The individual contributions of the δ (solid curve), β (short dashes) and ϵ (long dashes) terms of equation (8.8) are shown as well as their sum (solid curve).
- Figure 14. Tensor asymmetries t'_{20} and t'_{22} for $d^{12}\text{C} \rightarrow d^*^{12}\text{B}_{\text{gs}}$ at $T_d = 1600 \text{ MeV}$ and $E_{d^*} = 600 \text{ keV}$ evaluated with the nuclear amplitudes of equation (8.8). The dashed curves assume that the proton of the deuteron acts as a spectator particle; the solid curves include some deuteron multiple scattering through the replacements of equation (7.3).

Figure 15. Predictions for the charge exchange $\vec{p}d \rightarrow \vec{n}pp$ at 515 MeV and neutron laboratory angles of 3° , 6° and 9° . The solid curves show, on arbitrary scales, the spectra of neutron energy loss ΔT , defined by equation (9.11c), integrated over the unobserved proton variables. Values of Wolfenstein parameters $A' \equiv K_{LL}$ and $R \equiv K_{SS}$ are shown by the dash-dot-dot curve (scale to the left) and dash-dot (scale to the right) respectively. Mean values integrated up to ΔT are shown by short and long dashes respectively.

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