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STRING LENGTH AND COVARIANT STRING FIELD THEORY

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ABSTRACT

String length differentials are shown to arise naturally in a BRST-Hamiltonian approach to the closed bosonic string. A gauge-covariant quadratic action is proposed which is shown to lead to the correct equations of motion. The gauge covariant action is extended to including a three-string interaction vertex, with exact invariance under an extended transformation on the string fields.

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1. - INTRODUCTION

An attractive approach to formulating a general theory of string and superstring theory is by way of string field theory¹⁾. Such a formulation would hopefully allow explicit realizations of the full symmetries of the theories, at the same time giving access to non-perturbative aspects absent from the multiloop analyses performed up to now. This latter possibility is particularly important if a realistic understanding is to be gained of the process of compactification and localization which must occur at an early stage in the "stringy" Universe in order to transform it into the local field theoretic four-dimensional Universe observed at energies below 100 GeV or so.

The above advantages that string field theory has over the present multiloop perturbation expansion has led to a lot of effort being put into the construction of string and superstring field theory¹⁾. Yet in spite of the large number of research papers, there is no fully satisfactory interacting string field theory for open strings nor even a non-interacting one for closed strings. In the former case, this may be seen as due to the "string length" problem; in the latter from the problem of ghost counting. (We do not consider here the non-BRST approaches to string field theory along the lines of explicit reparametrization invariant constructions.) Thus if Φ is the string field, K the kinetic term and V the cubic vertex, the generic interacting string field action is schematically

$$\int [\Phi K \Phi + V \Phi^3] \quad (1)$$

In (1) the integration is over all variables on which Φ depends: the embedding variables $X^\mu(\sigma)$, ghosts $C(\sigma)$, fermions $\psi^\mu(\sigma)$ or $S(\sigma)$ and their ghosts, etc. Two of the main approaches to open string field theory^{2),3)} require the ad hoc introduction of a further variable α , termed the string length, and playing a similar role to that of πp^+ in light-cone gauge string field theory⁴⁾. From the claimed α -invariance of the theory follows²⁾ the presence of infinite factors $(\int_{-\infty}^{+\infty} d\alpha)^g$ in the g -loop amplitude. By analogy with LC string field theory, the loop α 's should play the role of moduli on the associated Riemann surface of genus g , so such infinite factors should be absent, and non-trivial integration over intermediate α 's must be present. However, it is difficult to analyze the role played by these length parameters until their origin is discovered in the structure, and especially the symmetries of the string theory itself. An ad hoc introduction of string lengths is clearly unsatisfactory for what is supposedly a fundamental

theory of nature. These lengths do not enter into the approach of Witten⁵⁾, though this approach may have difficulties at the multiloop level⁶⁾.

All of the above approaches^{2),3),5)} have difficulties for the closed string quadratic term in (1). This is due to the fact that Φ should have zero-mode ghost number -1, whilst the natural choice for K as the BRST charge Q, valid in the open string case^{3),5),7)}, clearly cannot succeed. Alternate choices so far appear to have been unsuccessful, so that the only progress has been to constrain the field Φ so that it has the mean ghost content (L+R) (L, R denoting the left- and right-going modes on the string). Besides the difficulties of the constraint, there is a degree of unnaturalness in this approach which again seems to preclude it from being regarded as a fundamental theory of strings. It is the purpose of this letter to attempt to resolve both of these questions, on string length and ghost counting, by a more fundamental analysis of the BRST approach to strings. This has already been begun by⁸⁾ [a first quantized approach along similar lines was discussed in Ref. 8a)] and is based on the earlier analyses of gauge fixing of reparametrization invariant theories by Teitelboim⁹⁾ and the application of this to the single particle by Monaghan¹⁰⁾ and Neveu and West¹¹⁾. The main theme of this work is that the Lagrange multipliers for constraints quadratic in momenta cannot be gauged to arbitrary constants. This will be reviewed briefly in the next section where it is shown how the Lagrange multipliers λ play the role of string length differentials. The construction of a gauge-invariant quadratic action for unconstrained string fields is given in the following section. This action is shown to lead to the correct equation of motion for the string field. This action is then extended to a cubic term, and the total action is shown to be invariant under an extension of the original gauge symmetry.

Most of our discussion will be for closed strings. Adaptation to the open string case and further elaboration of the content of this letter will be given elsewhere¹²⁾.

2. - BRST FOR CLOSED STRINGS

The classical constrained action for bosonic strings is well known to be

$$S_{cl} = \int_{\tau_1}^{\tau_2} d\tau \int_0^\pi d\sigma [P \dot{X} - \lambda_+ L_+ - \lambda_- L_-] \quad (2)$$

where $\lambda_\pm$ are the Lagrange multipliers for the constraints

$$L_{\pm} = \frac{1}{4} (P + X')^2 = 0 \quad (3)$$

with P_{μ} , X^{μ} the string momenta and co-ordinates for $\mu = 1, 2, \dots, d$ and the embedding space-time is taken to have Minkowski metric. Following the results of Teitelboim⁹⁾, S_{cl} is only invariant under variations of the variables X , P and $\lambda_{\pm}$

$$\begin{aligned} \delta X &= \frac{1}{2} [\varepsilon_+ (P + X') + \varepsilon_- (P - X')] \\ \delta P &= \frac{1}{2} \partial_{\sigma} [\varepsilon_+ (P + X') - \varepsilon_- (P - X')] \\ \delta \lambda_+ &= \dot{\varepsilon}_+ + \lambda'_+ \varepsilon_+ - \lambda_+ \varepsilon'_+ \\ \delta \lambda_- &= \dot{\varepsilon}_- - \lambda'_- \varepsilon_- + \lambda_- \varepsilon'_- \end{aligned} \quad (4)$$

provided

$$(\varepsilon_+ + \varepsilon_-)(\sigma, \tau_i) = 0 \quad (i=1, 2, 0 \leq \sigma \leq \pi) \quad (5)$$

The condition (5) does not allow the choice of canonical gauge-fixing and the natural proper time gauge (PTG) becomes

$$\dot{\lambda}_+ = \dot{\lambda}_- = 0 \quad (6)$$

as discussed in detail in Ref. 8).

A powerful way to develop the theory further is by means of the BRST symmetry, using suitable ghosts and their conjugates. Following the gauge-choice-independent procedure of Batalin, Fradkin and Vilkovisky¹³⁾, these additional variables will be

- (a) conjugate momenta $\pi_{\pm}(\sigma)$ to the Lagrange multipliers $\lambda_{\pm}(\sigma)$;
- (b) ghosts $C_{\pm}$ associated to the constraints $\lambda_{\pm}$;
- (c) antighosts $\bar{C}_{\pm}$ associated with $\pi_{\pm}(\sigma)$;
- (d) conjugate ghosts $B_{\pm}$ to $C_{\pm}$;
- (e) conjugate ghosts $\bar{B}_{\pm}$ to $\bar{C}_{\pm}$.

The resulting nilpotent BRST charge Q is then independent of gauge choice, and is equal to

$$Q = \int_0^\pi d\sigma [-i\bar{B}_+ \pi_+ - i\bar{B}_- \pi_- + C_+ L_+ + C_- L_- + B_+ C'_+ C_+ - B_- C'_- C_-] \quad (7)$$

The canonical Hamiltonian for PTG (6) is

$$H = \{Q, \psi\} \quad (8)$$

with

$$\psi = \int_0^\pi d\sigma [B_+ \lambda_+ + B_- \lambda_-] \quad (9)$$

so that

$$H = \int_0^\pi d\sigma [iB_+ \bar{B}_+ + \lambda_+ (L_+ + 2B_+ C'_+) + (\lambda_+ B_+) C'_+ + iB_- \bar{B}_- + \lambda_- (L_- + 2B_- C'_-) + (\lambda_- B_-) C'_-] \quad (10)$$

(the nilpotency of Q , $\{Q, Q\} = 0$, is only at the classical level; anomalies in the quantum case restrict us to take $d = 26$, as usual). The dynamical development of the string, as determined by the Hamiltonian H of (10) has an interesting feature. The Lagrange multipliers $\lambda_\pm$ may be regarded as length differentials, since they can be removed by change of variable from H (separately in L and R) as

$$d\sigma = \lambda d\bar{\sigma} \quad (10)$$

The total length of the string in the $\bar{\sigma}$ variables will now be

$$\ell = \int_0^\pi d\bar{\sigma} = \int_0^\pi d\sigma [\lambda(\sigma)]^{-1} \quad (11)$$

Since λ is positive⁹⁾, such an interpretation of ℓ is satisfactory; moreover, if a vertex has conservation of $\lambda(\sigma)$, for each σ , this guarantees conservation for ℓ . Thus at a three-string vertex, λ -conservation ensures $\ell_1 + \ell_2 = \ell_3$ (where 1, 2, 3 denote the three interacting strings). In terms of the partial length

$$\ell(\sigma) = \int_0^\sigma d\bar{\sigma} = \int_0^\sigma d\sigma' [\lambda(\sigma')]^{-1} \quad (12)$$

it is clear that $\lambda^{-1} = \ell'$, so that λ^{-1} is the length differential. It is the theme of this letter that the non-trivial presence of $\lambda_\pm$ is the origin of the lengths being introduced in an ad hoc manner in Refs. 2) and 3). It is to be noted that $\lambda_\pm$ cannot be removed from H and Q simultaneously.

The presence of $\lambda_{\pm}$ as fundamental variables is thus guaranteed by the above remarks. The field variables $Z(\sigma)$ for the closed bosonic string will therefore be the set

$$Z = \{X^M, c_{\pm}, \bar{c}_{\pm}, \lambda_{\pm}\} \quad (13)$$

with $\Phi = \Phi(Z)$ the field itself. The conjugate variables

$$\Pi = \{p_M, B_{\pm}, \bar{B}_{\pm}, \pi_{\pm}\} \quad (14)$$

will act on Φ as $-i\delta/\delta Z(\sigma)$. The proper-time action of Ref. 8) is therefore

$$\int \mathcal{D}Z d\tau \Phi^* [i\partial/\partial\tau - H(Z, \Pi)] \Phi \quad (15)$$

where functional integration of $\lambda_{\pm}$ is over positive values only.

3. - GAUGE COVARIANT QUADRATIC ACTION

It would be possible to analyze (15) to show that it has the correct equations of motion. Bypassing that straightforward step, a gauge covariant extension of this is of greatest concern. The most natural choice is the string version of that for the particle¹¹⁾

$$\int \mathcal{D}Z d\tau [\Phi_1^* i\partial/\partial\tau \Phi_1 - \Phi_1^* Q \Phi_2 - \Phi_2^* Q \Phi_1] \quad (16)$$

In (16) and (15), $\Phi^*(X, C, \bar{C}, \lambda) = \Phi^+(X, C, -\bar{C}, \lambda)$ where + denotes Hermitian conjugation; the field Φ is only defined for positive λ . The reality condition $\Phi^+ = \Phi(X, -C, -\bar{C}, \lambda)$ is assumed from now on. Expression (16) is real and invariant under the local gauge transformations

$$\begin{aligned} \delta \Phi_1 &= Q\Lambda \\ \delta \Phi_2 &= i\dot{\Lambda} \end{aligned} \quad (17)$$

This gauge invariance may be used to show that (16) has the correct physical content of (15) on-shell. This is best done by expanding Φ_1 , Φ_2 and Λ about the zero modes $c_{\pm}$, $\bar{c}_{\pm}$. Using the notation of Ref. 6), after transforming $c_{\pm}$ to $(c_+ \pm c_-)$, with Q in the new notation having zero mode (denoted by small letters) expansion⁶⁾

$$Q = (2c_+ L + c_- \Delta + b_+ M_+ + b_- M_- + \bar{b}_+ \pi_+ + \bar{b}_- \pi_- + Q'_B) \quad (18a)$$

$$\begin{aligned} \Phi_1 = & [c_+ c_- \bar{c}_+ \bar{c}_- \phi_{++--} + c_+ c_- \bar{c}_\pm \phi_{+-,\pm} + c_\pm \bar{c}_+ \bar{c}_- \phi_{\pm,+} + c_+ c_- \phi_{+-} \\ & + \bar{c}_+ \bar{c}_- \bar{\phi}_{+-} + c_\pm \bar{c}_\pm \bar{\phi}_{\pm\pm} + c_\pm \phi_\pm + \bar{c}_\pm \bar{\phi}_\pm + \phi_0] |0_\pm, \bar{0}_\pm\rangle \end{aligned} \quad (18b)$$

and similar expansions for Φ_2 (which we will not use explicitly) and Λ [which has ϕ replaced by Λ in every term in (18b)], the gauge transformations (17) are

$$\delta \Phi_2 = i \dot{\Lambda} \quad (19a)$$

$$\begin{aligned} \delta \phi_{++--} &= Q'_B \Lambda_{++--} + 2L \Lambda_{-,+-} - \Delta \Lambda_{+,+-} \\ \delta \phi_{+-,+} &= -\pi_- \Lambda_{++--} - Q'_B \Lambda_{+-,+} + 2L \bar{\Lambda}_{-+} - \Delta \bar{\Lambda}_{++} \\ \delta \phi_{+-,-} &= \pi_+ \Lambda_{++--} - Q'_B \Lambda_{+-,-} + 2L \bar{\Lambda}_{--} - \Delta \bar{\Lambda}_{+-} \\ \delta \phi_{+,+} &= -M_- \Lambda_{++--} - Q'_B \Lambda_{+,+} + 2L \bar{\Lambda}_{+-} \\ \delta \phi_{-,-} &= M_+ \Lambda_{++--} - Q'_B \Lambda_{-,-} + \Delta \bar{\Lambda}_{+-} \\ \delta \phi_{+-} &= \pi_\pm \Lambda_{+-,\pm} + Q'_B \Lambda_{+-} - \Delta \Lambda_{+} + 2L \Lambda_- \\ \delta \bar{\phi}_{+-} &= M_\pm \Lambda_{\pm,+} + Q'_B \bar{\Lambda}_{+-} \\ \delta \bar{\phi}_{\pm,\pm} &= \mp M_\mp \Lambda_{+-,\pm} \pm \pi_\mp \Lambda_{\pm,\pm} + Q'_B \bar{\Lambda}_{\pm\pm} + \left(\frac{2L}{\Delta}\right) \bar{\Lambda}_\pm \\ \delta \phi_\pm &= \mp M_\mp \Lambda_{+-} - \pi_\pm \bar{\Lambda}_{\pm,\pm} - Q'_B \Lambda_\pm + \left(\frac{2L}{\Delta}\right) \Lambda_0 \\ \delta \bar{\phi}_\pm &= \mp \pi_\mp \bar{\Lambda}_{+-} + M_+ \bar{\Lambda}_{+, \pm} + M_- \bar{\Lambda}_{-, \pm} - Q'_B \bar{\Lambda}_\pm \\ \delta \phi_0 &= M_\pm \Lambda_\pm + \pi_\pm \bar{\Lambda}_\pm + Q'_B \Lambda_0 \end{aligned} \quad (19b)$$

where $Q'_B = (\bar{b}_+ \pi_+ + \bar{b}_- \pi_-)_{\text{non-zero modes}} + Q_B$, and repeated suffixes $\pm$ are summed over in (19). After using (19a) to gauge away Φ_2 the equations of motion for the components (18b) of Φ_1 are

$$\begin{aligned} Q'_B \psi_{++--} + 2L \psi_{-,+-} - \Delta \psi_{+,+-} &= 0 \\ -\pi_- \psi_{++--} - Q'_B \psi_{+-,+} + 2L \bar{\psi}_{-+} - \Delta \bar{\psi}_{++} &= 0 \end{aligned}$$

$$\begin{aligned}
& \pi_+ \phi_{++--} - Q'_B \phi_{+-,-} + 2L \bar{\phi}_{--} - \Delta \bar{\phi}_{+-} = 0 \\
& -M_- \phi_{++--} - Q'_B \phi_{+,+} + 2L \bar{\phi}_{+-} = 0 \\
& M_+ \phi_{++--} - Q'_B \phi_{-,+} + \Delta \bar{\phi}_{+-} = 0 \\
& \pi_{\pm} \phi_{+-,\pm} + Q'_B \phi_{\pm} - \Delta \phi_{\pm} + 2L \phi_{\pm} = 0 \\
& M_{\pm} \phi_{\pm,+} + Q'_B \bar{\phi}_{+-} = 0 \\
& \mp M_{\mp} \phi_{+-,\pm} \pm \pi_{\mp} \phi_{\pm,+} + Q'_B \bar{\phi}_{\pm,\pm} + \left(\frac{2L}{\Delta}\right) \bar{\phi}_{\pm} = 0 \\
& \mp M_{\mp} \phi_{+-} - \pi_{\pm} \bar{\phi}_{\pm,\pm} - Q'_B \phi_{\pm} + \left(\frac{2L}{\Delta}\right) \phi_0 = 0 \\
& \mp \pi_{\mp} \bar{\phi}_{+-} + M_{\mp} \bar{\phi}_{+,+} + M_{\mp} \bar{\phi}_{-,+} - Q'_B \bar{\phi}_{\pm} = 0 \\
& M_{\pm} \phi_{\pm} + \pi_{\pm} \bar{\phi}_{\pm} + Q'_B \phi_0 = 0.
\end{aligned} \tag{20}$$

Then the residual gauge invariance $\delta\Phi_1 = Q\Lambda$, with $\Lambda = 0$, may be used:

$$\begin{array}{ll}
\text{with } \Lambda_{-,-} & \text{to set } \phi_{++--} \rightarrow 0 \\
\bar{\Lambda}_{-+} & \phi_{+-,+} \rightarrow 0 \\
\bar{\Lambda}_{--} & \phi_{+,--} \rightarrow 0 \\
\bar{\Lambda}_{+-} & \phi_{+,+} \rightarrow 0 \\
& \phi_{-,+} \rightarrow 0 \\
\Lambda_{-} & \phi_{+-} \rightarrow 0 \\
\bar{\Lambda}_{\pm} & \bar{\phi}_{\pm\pm} \rightarrow 0
\end{array}$$

The component field $\bar{\phi}_{+-}$ satisfies

$$L \bar{\phi}_{+-} = \Delta \bar{\phi}_{+-} = 0, \delta \bar{\phi}_{+-} = M_{\pm} \Lambda_{\pm,+} + Q'_B \bar{\Lambda}_{+-}$$

with $L\Lambda_{\pm,-} = \Delta\Lambda_{\pm,-} = 0$. This guarantees the vanishing of $\bar{\phi}_{+-}$, since it has non-vanishing ghost number, by the Kato-Ogawa theorem¹⁴⁾, along similar lines to that used in Ref. 6). Λ_0 may then be used to gauge $\phi_{\pm} \rightarrow 0$. The equations of motion $2L\bar{\phi}_{\pm} = \Delta\bar{\phi}_{\pm} = 0$ and the associated gauge invariance $\delta\bar{\phi}_{\pm} = Q'_B \bar{\Lambda}_{\pm} + \dots$ set $\bar{\phi}_{\pm} = 0$ since they have non-zero ghost number.

Thus only the component ϕ_0 is left, satisfying

$$L\phi_0 = \Delta\phi_0 = 0 \tag{21a}$$

with gauge invariance

$$\delta \phi_0 = \pi_+ \bar{\Lambda}_+ + \pi_- \bar{\Lambda}_- + M_+ \Lambda_+ + M_- \Lambda_- + Q'_B \Lambda_0 \quad (21b)$$

where $\Lambda_\pm, \bar{\Lambda}_\pm, \Lambda_0$ satisfy

$$0 = L \Lambda_- = L \bar{\Lambda}_\pm = \Delta \Lambda_+ = \Delta \bar{\Lambda}_\pm = L \Lambda_0 = \Delta \Lambda_0 = Q'_B \bar{\Lambda}_\pm = Q'_B \Lambda_\pm \quad (21c)$$

As remarked in the particle case¹¹⁾, it is not possible to use the gauge transformations (21c) to remove ϕ_0 , due to the boundary conditions.

It may be of value to describe the particle situation in detail; the extension to the string case above is immediate. The field ϕ_0 has variation

$$\delta \phi_0 = \partial \Lambda / \partial \lambda \quad (22)$$

(dropping the distinction between $\pm$). One cannot choose

$$\Lambda(\lambda, \dots) = - \int_0^\lambda \phi_0(\lambda', \dots) d\lambda' \quad (23)$$

to remove ϕ_0 since the right-hand side of (23) does not in general vanish as $\lambda \rightarrow \infty$, even though $\phi_0 \rightarrow 0$ as $\lambda \rightarrow 0, \infty$, this being the class of functions for which the variational approach is chosen to apply. Thus Λ given by (23) is not an allowed gauge transformation in the class of functions vanishing at $\lambda = 0, \infty$. To see what can be gauged away, it is always possible to choose a fixed Λ_0 with $\int_0^{+\infty} \Lambda_0(\lambda) d\lambda = 1$. Then if any ϕ_0 is expressed as

$$\phi_0 = \phi_1 + \left[\int_0^\infty d\lambda' \phi_0(\lambda') \right] \cdot \Lambda_0 \quad (24)$$

then $\int_0^{+\infty} \phi_1(\lambda) d\lambda = 0$, so that ϕ_1 can be gauged away by the choice of (23) with ϕ_0 replaced by ϕ_1 . Thus ϕ_0 can be reduced to have λ dependence given by Λ_0 . This is arbitrary; the simplest dependence on λ may be taken as $\Lambda_0 = \delta(\lambda-1)$. A similar situation occurs for the string, where λ for the particle is replaced by $\lambda_\pm(\sigma)$ at each σ ; the residual dependence of ϕ_0 is in a fixed functional Λ_0 of $\lambda_\pm(\sigma)$ with $\int \mathfrak{D} \lambda_+ \mathfrak{D} \lambda_- \Lambda_0 = 1$. Following the remarks of Teitelboim⁹⁾, it might be expected that the measure for $\lambda_\pm$ is $\lambda n \lambda_\pm$ and not $\lambda_\pm$; this will be considered in more detail when the perturbation expansion for multiloop amplitudes is explored¹²⁾.

4. - THREE-STRING VERTEX

The natural extension of the action of (16) is by addition to (16) of a three-string vertex. By trial and error the most natural choice seems to be

$$g V [\Phi_1^* \Phi_2 \Phi_3 + \Phi_1 \Phi_2^* \Phi_3^*] \quad (25)$$

where V contains δ -functions for all of the associated variables Z and is assumed to be distributive with respect to the BRST charge Q of (7). Then (16) and (25) are invariant under the extension of (17) to

$$\begin{aligned} \delta \Phi_1 &= Q \Lambda \\ \delta \Phi_2 &= i \dot{\Lambda} + 2g V (\Phi_1 \Lambda + \Phi_1^* \Lambda + \Phi_1 \Lambda^*) \end{aligned} \quad (26)$$

We note the useful feature that the invariance need only be checked to $O(g)$; there are no $O(g^2)$ or higher terms generated in (16) and (25) by (26).

The vertex V with the correct property

$$\left(\sum_{r=1}^3 Q^{(r)} \right) V = 0 \quad (27)$$

and with total ghost number zero may be constructed from the usual δ -function vertex V_0 by multiplication by ghost factors at the interaction point P :

$$V = C_+(P) C_-(P) \bar{C}_+(P) \bar{C}_-(P) V_0 \quad (28)$$

V_0 itself is constructed from the usual δ -function of all variables Z on each string, with the associated length $\ell_r = \int_0^\pi [\lambda_+^{(r)}(\sigma) + \lambda_-^{(r)}(\sigma)]^{-1} d\sigma$ (the positive sign being chosen between λ_+ and λ_- to ensure positivity of ℓ_r).

The string integration thus splits up into the various regions $\ell_1, \ell_2 < \ell_3$, etc. In each of these, the vertex factors in V_0 are chosen as having the usual parametrization, so that, for example, the δ -function for the X -variables is

$$\prod_{0 \leq \sigma \leq \pi \ell_3} \delta \left(\oplus_1 X^{(1)}(\sigma_1) + \oplus_2 X^{(2)}(\sigma_2) - X^{(3)}(\sigma_3) \right) \quad (29)$$

where $\oplus_1 = \partial(\pi \ell_1 - \sigma)$, $\oplus_2 = \partial(\sigma - \pi \ell_1)$, $\sigma_1 = \sigma / \ell_1$, $\sigma_2 = (\sigma - \pi \ell_1) / \ell_2$,
 $\sigma_3 = (\pi \ell_3 - \sigma) / \ell_3$.

The results of Ref. 2) (in particular the third paper of that reference) prove (27) for the original part of Q of (7), i.e., that other than depending on $\bar{B}_\pm \pi_\pm$. The latter term is zero by continuity of the momenta across the strings from 1 to 3 or 2 to 3, as in the proof of Ref. 2) from the usual δ -function mode analysis. Further zero mode factors are needed in both $C_\pm$ and $\bar{C}_\pm$ to give the correct perturbation expansion in physical modes for multiloop amplitudes, as is more fully detailed in Ref. 12). Thus V given by (28) is satisfactory.

5. - DISCUSSION

It has been shown in the above that the origin of string lengths for the closed bosonic string is the presence of Lagrange multipliers $\lambda_\pm$ for the constraints $L_\pm$ depending quadratically on the momenta. The use of both $\lambda_\pm$ leads to a pair of antighosts $\bar{C}_\pm$. It is the presence of both of these which allows for ghost balancing between $C_\pm$ and $\bar{C}_\pm$. This may be the origin of the extreme simplicity of the total action

$$\int \mathcal{L} d\tau \left[\Phi_1^* i \partial / \partial \tau \Phi_1 - \Phi_1^* Q \Phi_2 - \Phi_2^* Q \Phi_1 + \right. \\ \left. + g V(\Phi_1^* \Phi_1 \Phi_1 + \Phi_1 \Phi_1^* \Phi_1^*) \right] \quad (30)$$

and the variations (26) under which it is invariant. A similar construction for the open string seems more difficult since there is no longer ghost balancing; only the sole antighost $\bar{C}$ as compared to the pair of ghosts $C_\pm$. This may make for simplicity as far as having a unique length differential $\lambda(\sigma)^{-1}$, but on the other hand requires a more delicate handling of the construction of the vertex V in this case. The details of this construction and its relation to the vertex of Ref. 2) will be discussed elsewhere.

The construction presented in this paper has yet to be tested by deriving multiloop amplitudes from it, and showing that only integration over a fundamental domain of the modular group occurs in those amplitudes. That such an important feature arises is shown in Ref. 12).

As an important feature of this we note finally that the gauge choice

$$\bar{\Phi}_2 = \psi \bar{\Phi}_1 \quad (31)$$

reduces (30) to the interacting extension of (15) given by

$$\int d\tau d\tau' \{ \bar{\Phi}^* [i\partial/\partial\tau - H] \Phi + g V(\bar{\Phi}^* \Phi \Phi + \bar{\Phi} \Phi^* \Phi^*) \} \quad (32)$$

which still has the global BRST invariance $\delta\Phi = i\Lambda Q\Phi$ with Λ an anticommuting antiHermitian parameter. It is (32) which must be used to obtain multiloop amplitudes; this will also be discussed in detail elsewhere, as will the extension to the superstring case.

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