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ONE-LOOP EFFECTIVE POTENTIAL IN $D = 4$ THEORIES CONSISTENT WITH
CLASSICAL SYMMETRIES OF $D = 10$ SUPERSTRINGS

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ABSTRACT

The four-dimensional effective action consistent with classical symmetries of the $E_8 \times E_8$ superstring is studied and compared with the theory obtained by performing a truncation of $D=10$ supergravity in which all heavy Kaluza-Klein modes are discarded. It is assumed that supersymmetry is broken in the hidden sector by non-perturbative effects. As in the truncation model, it is found that at tree level there are no soft supersymmetry breakings in the observable sector. However, contrary to the truncation result, one-loop corrections break supersymmetry in the observable sector providing a common mass for all matter scalars $m_0 \sim m_{3/2}(m_{3/2}/M_P)^{1/3}$. Finally, in contrast to the truncation model, there are no gauge non-singlet (N) flat directions. These effects generate an intermediate scale $\langle N \rangle \lesssim m_0^{1/4} M_P^{3/4}$.

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In the last two years, the ten-dimensional superstring theory¹ with gauge group $E_8 \times E_8$ has been shown to be a good candidate to provide a consistent description of all known interactions. It is therefore of considerable interest to extract the consequences of superstrings for low-energy physics. A first step in this direction was taken by Candelas et al.² who showed that to lowest order in the string tension the heterotic string theory admits vacuum configurations of the form $(\text{Minkowski})_4 \times K_6$ that preserve an $N=1$ supersymmetry in four dimensions and have a realistic spectrum provided K_6 is a Kähler manifold with $SU(3)$ holonomy, i.e. a Calabi-Yau manifold. Topological properties of K_6 determine the number of massless matter fields. The resulting four-dimensional theory should then correspond to $N=1$ supergravity coupled to vector and chiral supermultiplets.

To make further progress, in connection with low energy phenomenology, a field theory limit of superstrings, namely ten-dimensional supergravity³, has been assumed. Starting from this point of view, Witten⁴ has proposed a dimensional reduction of the ten-dimensional action which gives a four-dimensional theory that can be written in the standard form of $N=1$ supergravity⁵ coupled to one generation of matter fields. Some phenomenological implications of this supergravity model have recently been considered by several authors⁶.

In the case of an arbitrary number of generations, the effective action for the low-energy matter has been deduced⁷ by performing a truncation of ten-dimensional supergravity in which all heavy Kaluza-Klein modes are discarded. This approach gives an $N=1$, $D=4$ supergravity that generalizes the

result of ref. (4). However, the truncation procedure can only be expected to reproduce the correct effective action, obtained by integrating out the heavy modes, up to terms suppressed by inverse powers of the heavy mass^{7,8}. The low-energy action found in refs. (4) and (7) involves non-polynomial couplings of the light fields which, for the reasons mentioned above, could be artifacts of the truncation approximation. The correct properties of the low energy action have been determined by constructing the most general effective action consistent with the symmetries of the underlying ten-dimensional string theory^{7,9}.

In this note we analyze the physical consequences of the effective action constrained by the classical symmetries obtained in ref. (7). We assume supersymmetry is broken by non-perturbative effects in the hidden sector¹⁰. We find that at tree level there are no gauge-non-singlet flat directions and no soft supersymmetry breakings in the observable sector. This latter result is modified by one-loop corrections which generate a common non-vanishing mass for all matter scalars. A general analysis, including the breaking of the classical symmetries by string loop effects, will be considered elsewhere¹¹.

We will consider the following gauge kinetic function and Kahler potential consistent with the classical symmetries⁷ (unless explicitly mentioned, we work in units of $M_P/\sqrt{8\pi}$)

$$f_{\alpha\beta} = S \delta_{\alpha\beta} \tag{1}$$

$$\mathcal{G} = -\ln(S+S^*) - 3\ln(T+T^*) + h(u) + \ln|P|^2 \tag{2}$$

where S and T are gauge singlet fields associated with the dilaton ϕ and the

breathing mode σ . Here we have neglected other massless modes arising from the ten-dimensional metric. The real parts of S and T are related to products of powers of ϕ and e^σ and since ϕ is the string coupling constant we must have

$$S + S^* > 0 \quad ; \quad T + T^* > 0 \quad (3)$$

The function h is arbitrary and the variable u is given by

$$u = (C^*_{KK} C^K_K + B^{*XL} B_{XL}) / (T + T^*) \quad (4)$$

where C^K_K ($K=1, \dots, b_{11}$) and B_{XL} ($L=1, \dots, b_{12}$) denote scalars transforming in the $\overline{27}$ and 27 of E_6 respectively.

The superpotential P in eq. (2) is a cubic function of the charged scalars given by

$$P = W(C, B) = \xi_{LMN} d_{XYZ} C^X_L C^Y_M C^Z_N + \zeta_{IJK} d^{XYZ} B_{XI} B_{YJ} B_{ZK} \quad (5)$$

For ease of notation, we will represent all charged scalars by ϕ^a where a is a mixed gauge and family index. Then, the superpotential can simply be written as

$$P = W(\phi) = \gamma_{abc} \phi^a \phi^b \phi^c \quad (6)$$

In this notation the variable u becomes

$$u = \phi_a \phi^a / t \quad (7)$$

where $t = T + T^*$ and $\phi_a = (\phi^a)^*$. We will also use the conventions $\mathfrak{F}_i = \partial \mathfrak{F} / \partial z^i$ and $\mathfrak{F}^i = \partial \mathfrak{F} / \partial z_i$ where $z^i = (S, T, \phi^a)$.

We will assume that supersymmetry is broken by gaugino condensation

in the hidden sector and by a non-vanishing vacuum expectation value for the antisymmetric tensor field strength¹⁰. This means that the theory defined by eqs. (1), (2) and (6) is valid at scales above the condensation scale Λ_c given by

$$\Lambda_c^2 = \langle 1/(S+S^*) \rangle \exp(-\langle S+S^* \rangle/2b_0) \quad (8)$$

where b_0 is the coefficient of the one-loop β -function of the strong hidden gauge group. At scales below Λ_c a superpotential for the field S is generated; this is¹⁰

$$P = \Omega(S) + W(\Phi) \quad (9)$$

where $W(\Phi)$ is given by eq. (6) and

$$\Omega(S) = c + r \exp(-3S/2b_0) \quad (10)$$

The supersymmetric limit can be recovered at any stage by setting $\Omega(S) \equiv 0$.

By discarding the heavy Kaluza-Klein modes of the ten-dimensional theory the function h in eq. (2) is found to be⁷

$$h(u) = -3 \ln(1-2u) \quad (11)$$

This result has also been obtained⁴ by performing a dimensional reduction of the D-10 action that corresponds to a compactification on a Calabi-Yau manifold with $b_{11}=1$, $b_{12}=0$. In this particular case the heavy Kaluza-Klein modes do not couple linearly to the massless modes⁸ and therefore do not induce effective interactions at tree level. In a more general manifold (i.e. $b_{11} \geq 1$, $b_{12} \geq 0$) the effective couplings generated by the heavy modes will

translate into a function h different from that in eq.(11). We will often compare our results with those obtained by using eq.(11).

The function h is constrained by the positivity of the matrix g^i_j which determines the kinetic Lagrangian of the chiral scalars and fermions. An analysis of the eigenvalues shows that g^i_j is positive definite only in the region defined by

$$\beta = h' + uh'' > 0 ; h' > 0 \quad (12)$$

The tree-level Lagrangian is completely specified by eqs. (1), (2) and (9). The scalar potential is given by

$$V = V_1 + V_2 + V_3 + V_D \quad (13)$$

where

$$V_1 = (e^{h/(S+S^*)} t^3) |\Omega + W - (S+S^*)\Omega_S|^2 \quad (14)$$

$$V_2 = (e^{h/(S+S^*)} t^2 h') |W_a|^2 \quad (15)$$

$$V_3 = (e^{h/(S+S^*)} t^3) (\lambda/\alpha\beta h') |u(\Omega+W)h' + 3W|^2 \quad (16)$$

$$V_D = (S+S^*)^{-1} (h'/t)^2 d^\alpha d^\alpha \quad (17)$$

where

$$\lambda = (h')^2 - 3h'' \quad (18)$$

$$\alpha = 3 + uh' \quad (19)$$

$$d^\alpha = \Phi_a (T^\alpha)^a_b \Phi^b \quad (20)$$

We will now discuss the structure of eq. (13).

Included in the charged scalar sector there are two fields in each 27 ($\overline{27}$) which are standard model singlets*. We will denote them generically by N ($\overline{N}$). E_6 -invariance forces these fields to have at most linear couplings in the cubic superpotential. The rest of charged fields will be denoted by X ($\overline{X}$).

As a consequence of the constraints given by eqs. (3) and (12) V_1 , V_2 and V_D are semi-positive definite. Therefore, the choice of function h given by eq. (11), which makes $\lambda=0$ and $V_3=0$, results in a semi-positive definite potential. Its minimum is at

$$\Omega(S_0) = (S_0 + S_0^*) \Omega_S(S_0) \quad ; \quad X = \overline{X} = 0 \quad ; \quad N = \overline{N}^* \quad (21)$$

The VEV's of N and $\overline{N}$ remain undetermined, the direction $N = \overline{N}^*$ being both F-flat and D-flat. The VEV of the field T is undetermined as well. At this minimum, $V=0$ and supersymmetry is spontaneously broken. The gravitino mass is given by

$$m_{3/2}^2 = (S_0 + S_0^*)^{-1} t_0^{-3} e^{h(0)} |\Omega(S_0)|^2 \quad (22)$$

Since t_0 is undetermined, the value of $m_{3/2}$ is not fixed at tree level.

If λ is semi-positive definite in the region given in eq. (12), the global minimum is still given by eq. (21). However, V_3 now generates a potential for N whose lowest order term is

$$V = 4 [(S_0 + S_0^*)^{-1} t_0^{-5}] [h'(0) \lambda(0) / \alpha(0) \beta(0)] |\Omega(S_0)|^2 |N|^4 \quad (23)$$

* One of them is an $SO(10)$ singlet as seen from $27 = 16 + 10 + 1$; the other is an $SU(5)$ singlet included in the 16.

Therefore, at the minimum

$$N = \bar{N} = 0 \quad (24)$$

In other words, there are no (non-singlet) flat directions*. N will acquire a non-vanishing VEV if its squared-mass (arising from supersymmetry breaking at one-loop level, as explained later) is driven negative by the renormalization group equations. To compute this VEV we can write the potential (23) for properly normalized N fields as

$$V = [4 \lambda(0)/\alpha(0)\beta(0)h'(0)] m_{3/2}^2 |N|^4 \quad (25)$$

We then obtain

$$\langle N \rangle = \langle \bar{N} \rangle \sim m_0 (M_P/m_{3/2}) \quad (26)$$

where m_0 is the scale of supersymmetry breaking at low energy ($m_0 \leq 1\text{TeV}$).

In the case where λ is not semi-positive definite but $\lambda(0) > 0$, the extremum $S=S_0, u=0$ and T undetermined becomes a local minimum. However there will be other minima with $V < 0$ and broken gauge symmetries. Finally, if λ is not semi-positive definite and $\lambda(0) < 0$ the extremum mentioned above is a saddle point.

Another issue which is interesting for phenomenological applications is the breaking of supersymmetry in the observable sector. Expanding the

* Notice that in the supersymmetric limit $\Omega(S)=0$, the direction $N=\bar{N}^*$ remains flat. The existence of flat directions in the supersymmetric case has been discussed in ref. (12) from a different point of view.

Lagrangian about the vacuum (in the case $\lambda(0) > 0$) we observe that at tree-level there are no soft breaking terms like scalar or gaugino masses nor there are terms proportional to $W(\phi)$. However, at tree-level there appears a hard-breaking operator (see for instance eq. (23)) which is a good candidate to generate scalar masses through one-loop radiative corrections. For $\lambda = 0$ the observable sector is completely supersymmetric, this leads to the well known result¹³ that the choice (11) for the function h will produce no scalar masses at one-loop. We now wish to investigate to what extent such result is general.

The one-loop contribution to the effective potential is given by

$$V_{1\text{-loop}} = (1/64\pi^2) \{ \Lambda_c^2 \text{STr} M^2 + \text{STr} M^4 \ln(M^2/\Lambda_c^2) + \text{STr}(\Lambda_c^4 - M^4) \ln[1 + (M^2/\Lambda_c^2)] \} \quad (27)$$

where the condensation scale Λ_c is the natural cut-off of the theory. Expanding eq. (27) in M^2/Λ_c^2 yields

$$V_{1\text{-loop}} = (1/64\pi^2) \{ 2\Lambda_c^2 \text{STr} M^2 + \text{STr} M^4 [\ln(M^2/\Lambda_c^2) - 1/2 + \mathcal{O}(M^2/\Lambda_c^2)] \} \quad (28)$$

Since we are interested in finding the soft breaking terms, we will compute the dominant quadratic divergence in (28).

In the gauge $\gamma^\mu \psi_\mu = 0$, the supertrace of M^2 is given by

$$\text{STr} M^2 = \text{Tr} M^2_0 + 3\text{Tr} M^2_1 - 2\text{Tr} M^{\dagger}_{1/2} M_{1/2} + 4m^2_{3/2} \quad (29)$$

The contributions from the different spins are given by

$$\text{Tr} M_0^2 = 2(g^{-1})^i_j V^j_i \quad (30)$$

$$\begin{aligned} \text{Tr} M_{1/2}^\dagger M_{1/2} = e^{\mathbf{g}} \tilde{Y}_{ij} \tilde{Y}^{*}_{ji} + (2/S+S^*)(4 \mathbf{g}^a_b + \mathbf{g}^a \mathbf{g}_b) d^{\alpha b} d^{\alpha}_a \\ + \dim(\text{Adj}) e^{\mathbf{g}} |(S+S^*) \mathbf{g}^S|^2 \end{aligned} \quad (31)$$

$$\text{Tr} M_1^2 = (4/S+S^*) \mathbf{g}^a_b d^{\alpha b} d^{\alpha}_a \quad (32)$$

$$m_{3/2}^2 = (e^h/(S+S^*) t^3) |\Omega + \mathbf{W}|^2 \quad (33)$$

where we have already used that $f_{\alpha\beta} = S \delta_{\alpha\beta}$. $\tilde{Y}_{ij}$ is the normalized mass matrix for the chiral scalars

$$\tilde{Y}_{ij} = (g^{-1})^k_l Y_{kj} \quad (34)$$

and Y_{ij} is defined as

$$Y_{ij} = g_{ij} - g_i g_j - g_k (g^{-1})^m_k g^m_{ij} \quad (35)$$

A lengthy calculation shows that $\text{Tr} M_0^2$, $\text{Tr} M_{1/2}^\dagger M_{1/2}$ and $\text{Tr} M_1^2$ contain dimension-two and dimension-three operators in Φ (soft supersymmetry breaking terms) plus higher-dimension operators including renormalizations of the tree-level potential. The terms in eqs. (29), (30) and (31) that can give soft breakings are found to be

$$\begin{aligned} \text{Tr} M_0^2 = 2m_{3/2}^2 + (2e^h(S+S^*)^3/t) |\Omega_{SS}|^2 + (2e^h/(S+S^*)t(h')^2) W_{ab} W^{ab} \\ + (2e^h/(S+S^*) t^3) (\lambda/\alpha\beta) ((2n+2) |\Omega|^2 u + (3(n+3)/h')(\Omega^\dagger W + \Omega W^\dagger)) \\ + (4h'/(S+S^*)t) d^{\alpha}_a d^{\alpha a} \end{aligned} \quad (36)$$

$$\text{Tr} M_1^2 = (4h'/(S+S^*)t) d^{\alpha}_a d^{\alpha a} \quad (37)$$

$$\begin{aligned} \text{Tr} M_{1/2}^\dagger M_{1/2} &= 4m_{3/2}^2 + [e^h(S+S^*)^3/t] |\Omega_{SS}|^2 + [8h'/(S+S^*)t] d_a^\alpha d^{\alpha a} \\ &+ [e^h/(S+S^*)t(h')^2] \{W_{ab} W^{ab} + (2\lambda/t^2)(\Omega^\dagger W + \Omega W^\dagger)\} \end{aligned} \quad (38)$$

where n is the total number of charged scalars. Combining these results into $\text{STr} M^2$ yields

$$\begin{aligned} \text{STr} M^2 &= -2\{[e^h/(S+S^*)t^3] |\Omega|^2 + [e^h/(S+S^*)t^3] (\Omega^\dagger W + \Omega W^\dagger)\} \\ &+ [2e^h/(S+S^*)t^3] [\lambda/\alpha\beta] \{(2n+2) |\Omega|^2 u + [(3n+10)/h'](\Omega^\dagger W + \Omega W^\dagger)\} \\ &+ \text{higher-order terms} \end{aligned} \quad (39)$$

Let us remark that when $\lambda=0$ (i.e. h given by eq. (11)) the first term in eq. (39) seems to lead to soft breakings destabilizing the minimum and breaking all gauge symmetries at the condensation scale. However, in this particular case $V_{1\text{-loop}}$ depends on the charged scalars through the combination $(t-2\Phi_a\Phi^a)$. As argued in ref. (13) the negative mass vanishes when the full one-loop potential is considered provided $\partial V_{1\text{-loop}}/\partial t = 0$. For any other choice of the function h this argument is not valid and h has to satisfy the condition

$$2(n+1)\lambda(0) \geq 3(h'(0))^2 \quad (40)$$

which guarantees the stability of the minimum.

For properly normalized scalar fields, the supersymmetry breaking mass m_0 and the trilinear coupling A can be expressed as

$$m_0 = C_0 (m_{3/2})^{4/3} \quad (41)$$

$$A = C_1 (m_{3/2})^{5/3} \quad (42)$$

where

$$C_0 = (g_{\text{GUT}}/4\pi)(b_0 g_{\text{GUT}}/6r)^{1/3} \{[(2n+2)\lambda(0)/3(h'(0))^2] - 1\}^{1/2} \quad (43)$$

$$C_1 = (g_{\text{GUT}}^3/16\sqrt{2}\pi^2)(b_0 g_{\text{GUT}}/6r)^{2/3} \{[(3n+10)\lambda(0)/3(h'(0))^2] - 1\} [h'(0)]^{-3/2} \quad (44)$$

Here we have used the relationship between Λ_0 and $m_{3/2}$

$$\Lambda_0 = g_{\text{GUT}} (b_0 g_{\text{GUT}}/6r)^{1/3} (m_{3/2})^{1/3} \quad (45)$$

where $g_{\text{GUT}}^2 = \langle 2/(S+S^*) \rangle$.

We obtain $C_0 \sim 1/3$ by putting r , $\lambda(0)$, $h'(0)$ and g_{GUT} of $\mathcal{O}(1)$; $n \sim \mathcal{O}(10^2)$ and taking the hidden gauge group as $SU(N)$, $N \leq 8$. Therefore, the bound $m_0 \leq 1$ Tev translates into $m_{3/2} \leq 10^7$ Gev. We then find that $A/m_0 \leq 10^{-4}$ which implies that for practical purposes A can be set to zero when running the renormalization group equations from Λ_0 down to M_W .

We can now estimate the VEV of the field N . By using the value (41) for the scalar masses in eq. (26) we find

$$\langle N \rangle \leq (m_{3/2})^{1/3} \sim (m_0)^{1/4} \quad (46)$$

Numerically, this gives an intermediate scale of the order of 10^{14} Gev.

We will conclude with a few remarks. We have studied four-dimensional theories consistent with the string classical symmetries and have found that the main features of the D=4 effective theory obtained upon truncation of D=10 supergravity (existence of non-singlet flat directions at tree-level and unbroken supersymmetry in the scalar observable sector up to one loop) are no longer valid when heavy degrees of freedom are

taken into account. The one-loop effective potential is still monotonically decreasing along the T direction so that the radius of the internal manifold remains undetermined in the low-energy theory. This disease might be cured when the symmetry constraints are relaxed by considering string quantum corrections. The general structure of these theories will be analyzed in a separate publication¹¹.

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