**FOUR-DIMENSIONAL SUPERGRAVITY THEORIES
CONSISTENT WITH THE SYMMETRIES OF SUPERSTRINGS**

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ABSTRACT

We analyse four-dimensional supergravity theories obtained as the low-energy limit of heterotic superstring compactification. Using the restrictions implied by the classical symmetries on the effective action for the massless modes, we derive general properties of the scalar potential. It is found that the global positivity of the scalar potential is not guaranteed: an extra condition on the Kähler potential is required. In this case, even if supersymmetry is broken by non-perturbative effects in the hidden sector, there is no supersymmetry breaking in the observable sector at tree level. However, the existence of charged flat directions at tree level and the absence of supersymmetry breaking in the scalar sector at one-loop are artefacts of the naïve truncation to the massless sector. Modifications to the classical action that might be induced by string-loop effects are considered. The possibility of fixing dynamically the radius of the internal manifold is discussed.

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1. INTRODUCTION

Superstring theories [1] are nowadays the best candidates for being fundamental theories encompassing all known interactions.

The major ingredient required by string theories in order for them to be consistent quantum theories is space-time supersymmetry, as well as the fact that they describe extended objects of a size of the order of the Planck length ($\sim 10^{-33}$ cm), identified with the inverse string tension rather than point-like objects. However, for low-energy physics, i.e. for distances much larger than the Planck length, string theories can be approximated by an effective Lagrangian for their massless modes [2]. These massless modes are usually identified with elementary particles surviving in the 'low-energy domain'.

Another aspect of the physical relevance of strings is their spontaneous compactification to $D=4$ space-time dimensions. Among possible compactifications of superstrings, the most interesting are those preserving some supersymmetry. The relevance of supersymmetry below the compactification scale is twofold: on the one hand, compactifications preserving some supersymmetry are probably quantum mechanically stable. On the other hand, low-energy supersymmetry is believed to cure the so-called hierarchy problem, which, to some extent, is also a sign of instability of the masses of scalar particles in the presence of quantum fluctuations.

Since our low-energy world is chiral, general arguments lead to the conclusion that, at most, $N=1$ supersymmetry should survive beyond the compactification scale. Calabi-Yau spaces with a surviving gauge group $G \times E_8$, where $G \subseteq E_6$, are examples of perturbative compactifications of the heterotic superstring [3] preserving $N=1$ supersymmetry with a net number of chiral families in $D=4$ given by one half the Euler number of the internal manifold [4]. To lowest order in α' these spaces are Ricci flat, with holonomy group $SU(3)$. The properties of spontaneous compactification in the supergravity limit have been studied by Candelas et al. [4].

With these considerations in mind, superstring theories at low energy and after spontaneous compactification should reduce to an effective $N=1$ supergravity theory for the massless excitations, in which higher-order string effects will appear as a modification of the Einstein-supergravity Lagrangian with higher-order curvature terms and higher-derivative interactions. Since the most important terms for low-energy physics are those with the lower number of derivatives, it is crucial to have the best possible control of them and to extract as many model-independent features as possible for this low-derivative part.

A way to extract an $N=1$ supergravity Lagrangian in $D=4$ from the $D=10$ heterotic superstring has been shown by Witten [5]. By performing a suitable dimensional reduction of the $D=10$ supergravity action, he was able to obtain a matter-coupled Lagrangian with a specific form for the Kähler potential and the superpotential terms [6]. The supergravity Lagrangian worked out by Witten is actually correct for a particular compactification on a Calabi-Yau manifold with one generation, for which the truncation of the spectrum to the massless sector is consistent. For general compactifications, the theory truncated to the massless sector is inconsistent [7], and corrections to the effective action, coming from the integration over the heavy modes, must be taken into account [8]. These considerations apply in particular to realistic Calabi-Yau spaces (with at least three generations).

Since integration over the heavy fields is not generally equivalent to a truncation of the original ten-dimensional action, the basic functions which characterize the $N=1$ supergravity effective action in $D=4$ are not simply obtained by disregarding the heavy fields. So the most general ansatz for the $N=1$ supergravity action consistent with the symmetries dictated by the string must be adopted [9–11].

Apart from the $N=1$ local supersymmetry and gauge symmetries, the superstring has the following properties that can be used to obtain information about the four-dimensional effective action: i) the invariance under the change of the antisymmetric tensor B_{MN} by an exact two-form [12], which implies that in the $D=10$ supergravity Lagrangian [13] B_{MN} has only derivative couplings; also, ii) the global scale covariance under a dilaton shift [5], and iii) the rescaling property when the ten-dimensional fields and the gravitational constant are properly rescaled, implying that the dilaton plays the role of the loop counting parameter for the heterotic superstring [14].

The symmetry properties we have been discussing have recently been exploited to restrict severely the possible form of the classical low-energy supergravity Lagrangian after compactification to $D=4$. Then the question arises: Which properties of the one-generation model will also be properties of more general models?

This paper is devoted to answering this question, and in particular to looking at the implications derived from adopting background scalar field configurations which would result in supersymmetry breaking due to non-perturbative effects in the way discussed by Dine et al. [15] in the one-generation model.

In the absence of supersymmetry-breaking effects, we find that general Calabi–Yau manifolds always have a local supersymmetric minimum with vanishing cosmological constant. Also, no anti-de Sitter supersymmetric extrema exist. This is a consequence of the analysis of supersymmetry-preserving compactifications studied in Ref. [4]. Under an additional condition on the general Kähler potential of the $N=1$ effective supergravity theory, the local minimum is actually a global minimum.

On the other hand, if the Kähler potential satisfies the positivity property mentioned above, supersymmetry may be broken with vanishing vacuum energy and, as in the one-generation case, the observable sector is supersymmetric at tree level. However, unlike the one-generation model, there are no charged flat directions; the Yang–Mills group remains unbroken at the compactification scale. In the absence of quantum effects, a feature which is independent of the truncation is the sliding gravitino mass. This amounts to saying that the coupling of the would-be Goldstone fermion (decay constant) is undetermined at tree level.

Effects of quantum corrections are taken into account using two different approaches:

- i) by analysing the string n -loop correction to the potential as suggested by scaling arguments [11, 16];
- ii) by calculating explicitly the one-loop correction to the classical effective action.

Under some assumptions regarding the loop-expansion parameter in $D=4$ dimensions, it is shown that string-loop effects may not spoil the global positivity of the supersymmetric potential. The local positivity of the potential around the supersymmetric vacuum is respected even without

these assumptions, since the superpotential remains unrenormalized. We also consider the effects of supersymmetry breaking and discuss the possibility of fixing dynamically the radius of the internal manifold; we argue that, if it is going to be determined by loop effects, it has to be large compared with the Planck length.

For the one-loop effects of the effective theory, we find that the superpartners of quarks and leptons will get a mass shift because of supersymmetry breaking. Also, trilinear soft-breaking terms are generated by these radiative corrections. This is in contrast with the truncated theory in which one-loop effects, in spite of supersymmetry breaking, do not remove the mass degeneracy in the observable sector of the tree-level Lagrangian [17].

The paper is organized as follows. In Section 2 we give the superfield form of the $N=1$, $D=4$ Lagrangian as derived from the ten-dimensional theory. The theory with an antisymmetric tensor field can be duality-transformed to a ‘standard supergravity form’ in a manifestly supersymmetric way, and the rules for the super-duality transformation are given.

In Section 3, we study the properties of the general scalar potential with the form restricted by the symmetries and scaling behaviour dictated by the string-effective action, in the absence of supersymmetry breaking. We first consider the classical potential; then we look at possible corrections coming from string loops.

The modifications to the scalar potential due to the supersymmetry-breaking effects are analysed in Section 4. We again look at the classical case first, then at the one-loop effective potential, and finally at string-loop effects. Our conclusions are presented in Section 5.

2. $N=1$ FOUR-DIMENSIONAL SUPERGRAVITY FROM SUPERSTRINGS

2.1 Superfield structure of $N=1$ supergravity

The massless modes of the $E_8 \times E_8$ [and $SO(32)$] superstring in ten dimensions correspond to the field content of $D=10$, $N=1$ supergravity coupled to matter. The fields of the supergravity multiplet are: the metric g_{MN} ($M, N = 1, \dots, 10$), the antisymmetric tensor field B_{MN} , and the dilaton, together with their fermion partners ψ_M and λ . The matter multiplet is composed of the gauge fields A_M and the gauginos χ .

After compactification [4], the gauge multiplet originates the four-dimensional gauge fields as well as the charged matter (C^X) h_{11} matter fields in the $\overline{27}$ and h_{12} in the 27 of E_6 , for the $E_8 \times E_8$ string, where h_{lm} are the Hodge numbers of the Calabi-Yau manifold.

From the supergravity multiplet, apart from the $D=4$ graviton multiplet, one obtains gauge singlet matter fields, again in correspondence with the six-dimensional harmonic forms, of which we will only be concerned with the dilaton ϕ , the antisymmetric tensor $B_{\mu\nu}$ ($\mu, \nu = 1, \dots, 4$), and the modes corresponding to the Kähler form. Together they form the four-dimensional complex scalar

fields S and T [5] and their superpartners. These are the fields present in any Calabi-Yau compactification^{*)}.

The imaginary part of the complex field S is actually not present in the ten-dimensional supergravity fields, but it is related to the four-dimensional components of the $B_{\mu\nu}$ tensor by a four-dimensional duality transformation. Since superstrings are intrinsically formulated in terms of antisymmetric tensor fields, it is of interest to identify the multiplet which contains such a tensor and to see how this multiplet and its interactions are related to the standard supergravity form.

The $N = 1$ standard supergravity action is best summarized by its superspace expression [6],

$$A = S_{\Phi} + S_W + S_f \quad (2.1)$$

where S_{Φ} , S_W , and S_f denote the Kähler part, the superpotential term, and the gauge part, respectively. They are given by

$$S_{\Phi} = \int d^8\Omega \Phi(S, T, C^* e^V) \quad (2.2)$$

$$S_W = \text{Re} \int d^6\Omega W(S, T, C) \quad (2.3)$$

$$S_f = \text{Re} \int d^6\Omega f_{AB}(S, T, C) W^A W^B \quad (2.4)$$

Here $d^8\Omega$ and $d^6\Omega$ are the appropriate superspace measures. The Φ function is related to the Kähler potential K by the relation

$$K = -3 \log \Phi \quad (2.5)$$

The Lagrangian for the antisymmetric tensor is obtained by the previous one by performing a super-Legendre transformation, adding to Eq. (2.1) a term

*) We are assuming that the spectrum on Calabi-Yau is unaltered by the recent discovery of α'^3 corrections to the Ricci-flat metric [18]. The general argument is that topological quantities are not affected by those corrections. In any case, since in the end we will use only symmetry arguments, the only information we need is that there are fields which transform in the proper way under the symmetries. These criteria can actually be applied to more general compactifications.

$$-\int d^8\Omega (S+S^*) L \quad (2.6)$$

and solving for $S + S^*$ from its equations of motion. In formula (2.6) L is a linear multiplet [19], i.e. a multiplet satisfying the local analogue of $\bar{D}\bar{D}L = DD L = 0$. It can be shown that this multiplet has the field content of a Wess–Zumino chiral multiplet but with one real scalar field replaced by an antisymmetric tensor. More precisely, the multiplet L contains the curl of the $B_{\mu\nu}$ field. If the Φ function depends only on $S + S^*$, the duality transformation in superfield language can be performed and amounts to (neglecting gauge couplings)

$$L = \Phi_S(S+S^*, T, C) \quad (2.7)$$

which can be inverted to give

$$S+S^* = \Phi_S^{-1}(L, T, C) \quad (2.8)$$

From Eq. (2.7) we see that the scalar partner of $B_{\mu\nu}$, which is the first component of the linear multiplet L , is simply given by

$$c_L = L(x, \theta=0) = \frac{\partial \Phi}{\partial S} \quad (2.9)$$

The fermionic partner is obtained by varying Eq. (2.9) under a local supersymmetry transformation. Equation (2.8) allows us to express the interactions of the S chiral field in terms of interactions of the L multiplet, occurring in superstring theories. Because of the duality transformation, we will consider in the sequel the formulation in terms of the S multiplet.

In the next subsection we will collect the restrictions on the functions K (or Φ), W , and f_{AB} coming from the superstring. Here we will not consider string effects which are of higher order in the σ -model loop expansion. These effects correspond to higher-curvature terms and higher-derivative interactions which cannot be recast in the standard supergravity form [20]. These terms would become important in a situation of a strongly coupled σ -model.

2.2 Symmetries of the superstring effective Lagrangian

The correct determination of the four-dimensional supergravity action from the ten-dimensional field theory requires a mode expansion of the fields in terms of harmonics of the internal manifold and their substitution in the expression for the original action. The low-energy theory in four dimensions is obtained after integration of the six internal variables and the subsequent ‘integration’ of the heavy ($\sim M_c$) fields. At tree level this corresponds to solving the heavy fields equations as functions of the massless fields and replacing the result back into the classical action.

In practice, the process of integrating out heavy fields is extremely difficult and two methods have been followed in order to extract information about the low-energy effective action after compactification on an arbitrary Calabi–Yau manifold [9]:

- i) truncation of the theory by setting the heavy fields to zero in the action;
- ii) use of symmetries of the underlying ten-dimensional field and string theories.

Discarding the heavy fields is a dangerous procedure. Actually it has been shown that it is inconsistent, except for the case of a Calabi-Yau manifold with $h_{11} = 1$, $h_{12} = 0$ [7]. Doing it properly, however, gives an approximation of the effective theory up to terms $O(1/M_c)$ with the expression for the Kähler potential [9]

$$K = -\log(S+S^*) - 3\log(T+T^* - 2\frac{C_X C^X}{M_P}) \quad (2.10)$$

the superpotential

$$W = \lambda_{XYZ} C^X C^Y C^Z \quad (2.11)$$

and the gauge kinetic function

$$f_{AB} = S \delta_{AB} \quad (2.12)$$

where we are using the following notation: C^X denotes all possible matter fields, and $C_X \equiv (C^X)^*$; $\Theta^\phi \equiv \partial\Theta/\partial\phi^*$, $\Theta_\phi \equiv \partial\Theta/\partial\phi$ for $\phi \equiv (S, T, C^X)$. All family indices are suppressed.

Equations (2.10) to (2.12) are the trivial generalization to any number of generations of the original result, obtained by Witten [5] performing a dimensional reduction which corresponds to compactification on the (yet to be discovered) manifold with $h_{11} = 1$, $h_{12} = 0$, i.e. a one-generation model [21].

Note that K includes terms suppressed by inverse powers of the Planck mass, and the $O(1/M_c)$ -terms cannot be neglected since $M_c \lesssim m_P$. Therefore the expression for K is not reliable and we are forced to use other methods in order to extract more information about the effective action.

After integrating out the heavy fields we are left with a four-dimensional theory which inherits the higher-dimensional symmetries. This condition imposes severe restrictions on the expression for K , W , and f_{AB} . Apart from local $N=1$ supersymmetry, the four-dimensional action has ‘axion-type’ symmetries coming from the fact that the ten-dimensional field B_{MN} has only derivative couplings. There are also two classical scaling properties under which the Lagrangian scales as [9–11]

$$\mathcal{L} \longrightarrow t^4 r^{-1/2} \mathcal{L}$$

which are used to further restrict the form of the tree-level action. This analysis was carried out in Ref. [9] with the result

$$K = -\log(S+S^*) - 3\log(T+T^*) + f\left(\frac{C_X}{\sqrt{T+T^*}}, \frac{C^X}{\sqrt{T+T^*}}\right) \quad (2.13)$$

where f is an arbitrary function; W and f_{AB} are given correctly by Eq. (2.11) and (2.12).

Note that this result is valid even without needing to use the ten-dimensional field theory as an intermediate step, since the symmetries are those of the full tree-level string theory [9].

Arguments based on the reduction of the Wess-Zumino term in ten dimensions [16] have been put forward to suggest that the n -loop quantum corrections to the classical Lagrangian should scale as [11]

$$\mathcal{L}_{n\text{-loop}} \longrightarrow t^{4(1-n)} \tau^{n-\frac{1}{2}} \mathcal{L}_{n\text{-loop}}$$

which would imply a loop parameter $\sim TS^{-1}$. This corresponds to a function K of the type

$$K = -\log(S+S^*) - 3\log(T+T^*) + g\left(\frac{T+T^*}{S+S^*}, \frac{C^x}{\sqrt{T+T^*}}, \frac{C_x}{\sqrt{T+T^*}}\right) \quad (2.14)$$

where g is an arbitrary function.

Similarly, the f_{AB} function is corrected only at one loop,

$$f_{AB} = \left[S - \epsilon (aT + bC^2) \right] \quad (2.15)$$

where ϵ is a small parameter and a and b are arbitrary constants. The superpotential W is still given by Eq. (2.11) [11], [22].

In the next sections we will analyse the physical consequences of having a Kähler potential such as the one given in Eq. (2.13) instead of the usually taken Eq. (2.10). We will also discuss some consequences of considering Eqs. (2.14) and (2.15).

3. UNBROKEN SUPERSYMMETRY

3.1 Scalar potential

In this subsection we will consider the potential determined from Eq. (2.13), given that f is a function of the single variable

$$U \equiv \frac{C_x C^x}{T+T^*} \quad (3.1)$$

The function f is restricted by the following condition. The matrix K^i_j ($i, j = S, T, C^x$) provides the kinetic Lagrangian of chiral scalars and fermions, therefore it has to be positive definite. The eigenvalues of K^i_j are given by

$$\begin{aligned} \lambda_S &= \frac{1}{(S+S^*)^2} \\ \lambda_{\hat{n}} &= \frac{f'}{T+T^*} \quad \hat{n} = 1, \dots, N-1 \end{aligned} \quad (3.2)$$

$$2\lambda_{\pm} = \alpha + (U+T+T^*)\beta \pm \left\{ [\alpha + (U+T+T^*)\beta]^2 - 4\alpha\beta(T+T^*) \right\}^{\frac{1}{2}}$$

where N is the total number of chiral non-singlet fields, and α and β are the functions defined by

$$\alpha \equiv 3 + \cup f' \quad (3.3)$$

$$\beta \equiv f' + \cup f''$$

The positivity domain for K^i_j is then given by

$$\begin{aligned} \beta &> 0 \\ f' &> 0 \\ T + T^* &> 0 \end{aligned} \quad (3.4)$$

We now turn to computing the scalar potential. For this we need the expression of the inverse matrix $(K^{-1})^i_j$. A straightforward calculation gives

$$\begin{aligned} (K^{-1})^S_S &= (S + S^*)^2 & (K^{-1})^T_T &= \frac{(T + T^*)^2}{\alpha} \\ (K^{-1})^T_X &= \frac{T + T^*}{\alpha} C_X & (K^{-1})^X_T &= \frac{T + T^*}{\alpha} C^X \\ (K^{-1})^X_Y &= \frac{T + T^*}{f'} \delta^X_Y + \frac{\Delta}{\alpha \beta f'} C^X C_Y \end{aligned} \quad (3.5)$$

where

$$\Delta \equiv f'^2 - 3f'' = \frac{9}{\Phi} \Phi'' \quad (3.6)$$

and Φ is the Kähler function defined in subsection 2.1. The scalar potential can then be written as

$$\begin{aligned} V &= \frac{e^f}{(S + S^*)(T + T^*)} \left[|W|^2 + \frac{T + T^*}{f'} |W_X|^2 + \right. \\ &\quad \left. \frac{\Delta}{\alpha \beta f'} |\cup f' W + C^X W_X|^2 \right] + \frac{1}{(S + S^*)(T + T^*)} d^\alpha d_\alpha \end{aligned} \quad (3.7)$$

where $d^\alpha = C_X (T^\alpha)^{X_Y} C^Y$ is the usual D-component of the vector superfields.

We will now discuss the structure of the potential. For $\Delta \geq 0$ it is positive definite and has a global minimum at $W = 0$, $W_X = 0$ which is supersymmetric. When $\Delta \geq 0$ is not satisfied, the potential is not positive; however, it has an extremum at $W = 0$, $W_X = 0$ which is a supersymmetric minimum with vanishing cosmological constant. In general, to minimize the D^2 -terms, the charged fields have to vanish unless there are fields with opposite quantum numbers. The equation $W_X = 0$ allows for the existence of flat directions, undetermined expectation values for some of the C fields which could break the symmetry group (E_6) .

We conclude that, in any case, there is a minimum with zero cosmological constant and unbroken supersymmetry.

Note that the critical case $\Delta = 0$ ($\Phi'' = 0$) corresponds to

$$f(u) = -3 \log(1-2u)$$

which gives the Kähler potential in Eq. (2.10).

3.2 String-loop effects

In this subsection we analyse the effects of string loops in the $D=4$ effective theory as discussed in Section 2. The general Kähler potential, superpotential, and f_{AB} function are given in Eqs. (2.11), (2.14), and (2.15). We will consider first the corrections to the type (2.10) potential, i.e. to the simple one-generation model of Ref. [5].

The Kähler potential is

$$K = -\log(s+s^*) - 3\log R + g\left(\frac{R}{s+s^*}\right) \quad (3.8)$$

where

$$R \equiv T + T^* - 2C_x C^x \quad (3.9)$$

and g is an arbitrary function of $x = R/s$, $s \equiv S + S^*$, restricted by the condition of positive kinetic energy for chiral multiplets. The positivity domain for K^i_j is given by

$$\begin{aligned} A &> 0 \\ A\eta - \rho^2 &> 0 \\ B &> 0 \end{aligned} \quad (3.10)$$

where

$$A = \frac{1}{s^2} [x^2 g'' + 2x g' + 1] \quad (3.11)$$

$$\eta = \frac{1}{R^2} [x^2 g'' + 3] \quad (3.12)$$

$$B = \frac{R}{2} [3 - x g']^{-1} \quad (3.13)$$

$$P = \frac{1}{s^2} [x g'' + g'] \quad (3.14)$$

To compute the scalar potential we need the evaluation of the inverse matrix $(K^{-1})^i_j$. A straightforward calculation shows that

$$\begin{aligned} (K^{-1})^S_S &= \frac{\eta}{A\eta - P^2} & (K^{-1})^S_T &= \frac{P}{A\eta - P^2} \\ (K^{-1})^T_T &= \frac{A + 4(A\eta - P^2)B|C'|^2}{A\eta - P^2}, & (K^{-1})^S_x &= 0 \\ (K^{-1})^T_x &= 2BC_x & (K^{-1})^x_y &= B\delta^x_y \end{aligned} \quad (3.15)$$

giving for the scalar potential the expression

$$V = \frac{e^g}{R^3 s} [B|W_x|^2 + |W|^2] + \frac{1}{2} D^\alpha D^\alpha \quad (3.16)$$

where D^α are the D-components of the properly normalized vector superfields.

The potential (3.16) is positive definite, provided the conditions for positive kinetic energy (3.10) are satisfied. Of course, this result relies upon the fact that the dependence on T of the Kähler potential (3.8) is through the variable R . We have checked by explicit calculation that this result is still valid for a more general dependence on T such as

$$K = -\log(s + s^*) - 3\log(\tau + \tau^*) + f(u) + g\left(\frac{\tau + \tau^*}{s + s^*} e^{-\frac{s}{3}}\right) \quad (3.17)$$

where f is an arbitrary function allowed by the symmetries. Even for the general Kähler potential (3.17) the condition of positive kinetic energy ensures the positiveness of the scalar potential, as long as $\Delta > 0$, and the origin is a supersymmetric minimum. This is not going to be the case for the most general Kähler potential (2.14); all we can say in this case is that the origin, where $W_x = 0$ and $W = 0$, is a local supersymmetric minimum with vanishing cosmological constant in accordance with the results of Refs. [4] and [22].

We do not expect these results to be valid if supersymmetry is broken, but instead, additional requirements on the Kähler potential should be imposed to guarantee the stability of the origin, as will be analysed in the next section.

4. SUPERSYMMETRY BREAKING

4.1 Scalar potential

Following Ref. [15], we will assume that supersymmetry is broken in the hidden sector of the theory by gaugino condensation in the hidden E_8 and/or by a non-vanishing vacuum expectation value of the field strength of the antisymmetric tensor B_{MN} in the compact directions. This has been parametrized by an S dependence of the superpotential, i.e.

$$W(S, C) = \Omega(S) + \tilde{W}(C) \quad (4.1)$$

The tree-level scalar potential gets modified to

$$V = \frac{e^f}{(S+S^*)(T+T^*)^3} \left\{ |D_S W|^2 + \frac{T+T^*}{f'} |W_X|^2 + \frac{\Delta}{2\beta f'} |U f' W + C^X W_X|^2 \right\} + D^2\text{-terms} \quad (4.2)$$

with

$$D_S W \equiv W - (S+S^*) \Omega_S$$

Again the sign of Δ determines the sign of V . For $\Delta = 0$ we reproduce the situation in Ref. [15], where we have a minimum with vanishing cosmological constant at

$$D_S W = W_X = 0 \quad (4.3)$$

with supersymmetry broken by the superpartner of the T field.

A similar situation happens in the case where $\Delta > 0$. With the extra condition, we must impose

$$U f' W + C^X W_X = 0 \quad (4.4)$$

which, together with Eq. (4.3), implies $u = 0$ or $\langle C \rangle = 0$ for every matter field C^X . This differs radically from the $\Delta = 0$ case, where $W_X = 0$ would allow for flat directions on some C^X fields. When the condition $\Delta \geq 0$ is not satisfied, we will no longer have a positive potential, and at most we could have a local minimum at zero cosmological constant. An explicit computation shows that the points in field space given by (4.3) and (4.4) still lead to an extremum. However, it can be either a minimum or a saddle point.

In the T direction the potential is flat; in the S direction it has a minimum giving a mass to the S field. In the C directions we have to consider up to the fourth derivative of the potential, since there are no mass terms for these fields, giving

$$\langle V_{Y\bar{Z}}^{Y'Z'} \rangle = \frac{\langle T+T^* \rangle}{f'(0)} W_{X\bar{Y}\bar{Z}} W^{X\bar{Y}'Z'} + \frac{2\Delta(0)|W|^2}{3\langle T+T^* \rangle^2} \delta_Y^{Y'} \delta_{\bar{Z}}^{\bar{Z}'}$$

which can be positive (minimum) or negative (saddle point) depending on the value of $\Delta(0)$ and of $\langle T+T^* \rangle$ which is completely undetermined. Note that if the first term vanishes we will have a maximum in the corresponding directions if $\Delta(0) < 0$.

We therefore conclude that many of the relevant features of the Kähler potential (2.10) are preserved for a general class of functions f , satisfying $\Delta(0) \geq 0$, i.e. we get supersymmetry breaking, with vanishing cosmological constant and soft-breaking terms absent in the observable sector (scalar masses and trilinear parameters). The only fact that is characteristic of Eq. (2.10) is the existence of flat directions (which is not allowed in any other case), and therefore the gauge group remains unbroken.

In the next section we will analyse how the above facts are affected after including one-loop corrections.

We remark, however, that supersymmetry breaking with vanishing cosmological constant at tree level and positive potentials are not necessarily a consequence of the superstring compactification since there is no known reason for restricting f by $\Delta(0) \geq 0$.

4.2 The one-loop effective potential

Knowing that there are no soft supersymmetry-breaking terms at tree level, we now turn to studying whether this continues to be the case after considering one-loop corrections to the effective potential [23]. It is already known that these effects do not induce soft-breaking terms in the truncated case ($\Delta = 0$). We will show, however, that this is an artefact of the truncation.

The one-loop contribution to the effective potential is given by

$$\begin{aligned} V_{1\text{-loop}} &= \frac{1}{32\pi^2} \text{STr} \int_0^\Lambda p^2 dp^2 \log\left(1 + \frac{M^2}{p^2}\right) \\ &= \frac{1}{64\pi^2} \left\{ \Lambda^2 \text{STr} M^2 + \text{STr} M^4 \log \frac{M^2}{\Lambda^2} + \right. \\ &\quad \left. \text{STr} (\Lambda^4 - M^4) \log\left(1 + \frac{M^2}{\Lambda^2}\right) \right\} \end{aligned} \quad (4.5)$$

where the cut-off Λ is taken to be the condensation scale. Expanding Eq. (4.5) around M^2/Λ^2 , one gets

$$V_{1\text{-loop}} = \frac{1}{32\pi^2} \left\{ \Lambda^2 \text{STr} M^2 + \frac{1}{2} \text{STr} M^4 \left[\log \frac{M^2}{\Lambda^2} - \frac{1}{2} + \mathcal{O}\left(\frac{M^2}{\Lambda^2}\right) \right] \right\} \quad (4.6)$$

Since we are interested in finding the soft-breaking terms, we will analyse the dominant term in the above expression, namely $\text{STr} M^2$:

$$S \text{Tr} M^2 = \text{Tr} M_0^2 + 3 \text{Tr} M_1^2 - 2 \text{Tr} M_{\frac{1}{2}} M_{\frac{1}{2}}^\dagger + 4 m_{\frac{3}{2}}^2 \quad (4.7)$$

Using the general form of the Kähler potential from Eqs. (2.13) and (3.1) we computed $S \text{Tr} M^2$. After a rather tedious but straightforward calculation we found that there are dimension-two and dimension-three operators which break supersymmetry softly in the observable sector. There are also higher-dimension operators, including renormalization of the tree-level potential. The soft-breaking contribution to $S \text{Tr} M^2$ is given by

$$S \text{Tr} M^2 = -2 m_{\frac{3}{2}}^2 + \frac{2 e^f}{(s+s^*)(t+t^*)^3} \frac{\Delta}{4\pi} \left\{ (2N+2) |\Omega|^2 u + \frac{1}{f'} (3N+10) (\Omega^\dagger W + \Omega W^\dagger) \right\} + 2 \frac{e^f |W|^2}{(s+s^*)(t+t^*)^3} \quad (4.8)$$

We can see that if $\Delta(0) = 0$ we reproduce the result of Refs. [17] and [24].

Using the explicit form of $\Omega(s)$ [15],

$$\Omega(s) = c + h e^{-3\frac{s}{2} b_0} \quad (4.9)$$

(where c and h are constant, and b_0 is the coefficient of the one-loop beta function of the group $Q' \subset E_8$), and the expression for the condensation scale Λ

$$\Lambda^2 = \left[\langle (s+s^*)(t+t^*) \rangle \right]^{-1} e^{-\frac{1}{2b_0} \langle (s+s^*) \rangle} \quad (4.10)$$

we find the relation between Λ and $m_{3/2}$ to be

$$\Lambda = g_{\text{GUT}} \left(\frac{g_{\text{GUT}} b_0}{6 h} \right)^{1/3} \langle m_{\frac{3}{2}}^{1/3} \rangle \quad (4.11)$$

where $g_{\text{GUT}}^2 = 2/\langle S+S^* \rangle$. We can then express the soft-breaking mass m_0 and trilinear coupling A for properly normalized fields as

$$m_0 = C_0 \langle m_{\frac{3}{2}}^{4/3} \rangle \quad (4.12)$$

$$A = C_1 \langle m_{\frac{3}{2}}^{5/3} \rangle \quad (4.13)$$

where

$$C_0 = \frac{g_{\text{GUT}}}{4\pi} \left(\frac{b_0 g_{\text{GUT}}}{6 h} \right)^{1/3} \left[\frac{2(N+1) \Delta(0)}{3 f'^2} - 1 \right]^{1/2} \quad (4.14)$$

$$C_1 = \frac{g_{\text{GUT}}^3}{16\sqrt{2}\pi^2} \left(\frac{b_0 g_{\text{GUT}}}{6h} \right)^{2/3} f'(0)^{3/2} \left[\frac{(3N+10)\Delta(0)}{3(f'(0))^2} - 1 \right] \quad (4.15)$$

Using reasonable values for the parameters entering these equations we can estimate a bound for the gravitino mass provided $m_0 \leq 1$ TeV. We obtain $m_{3/2} \lesssim 10^7$ GeV and $A/m_0 \lesssim 10^{-4}$. Therefore, the soft trilinear terms are 'weaker' than the supersymmetry-breaking scalar masses.

Note that Eq. (4.8) apparently shows that even in the truncated case ($\Delta = 0$) there are (negative) scalar masses at one-loop. However, it has been argued that they will be absent provided the minimization condition $\partial V/\partial T = 0$ is satisfied, since $\partial^2 V/\partial C \partial \bar{C} \propto \partial V/\partial(T+T^*)$ for this case. Nevertheless this relation does not hold in general, and these contributions together with the ones proportional to $\Delta(0)$ will provide a supersymmetry breaking in the observable sector. These contributions will be stable as long as

$$2(N+1)\Delta(0) \geq 3f'(0)^2$$

is satisfied.

From Eq. (4.8) we can see that at the origin the one-loop effective potential is a monotonically decreasing function of $T+T^*$, $V \approx -(T+T^*)^{-4}$, and therefore there are no stable solutions to $\partial V/\partial T = 0$. This is a disease already present when $\Delta = 0$ [24], and the generalizations considered here do not improve the situation.

4.3 String-loop effects

In this subsection we will study the effects of including symmetries from string loops in the Kähler potential (3.8), and in the gauge function (2.15), in the case of broken supersymmetry. In this case Eq. (4.1) should be modified to

$$W(S, T, C) = \Omega(S + \varepsilon(aT + bC^2)) + \tilde{W}(C) \quad (4.16)$$

Using the inverse matrix (3.15), the scalar potential can be written as

$$V = \frac{e^2}{(S+S^*)R^3} \left\{ \frac{\eta}{A\eta - \rho^2} |D_S W + \frac{\rho}{\eta} D_T W|^2 + \frac{1}{\eta} |D_T W|^2 + \right. \\ \left. 4B \left| \frac{1}{2} W_x + C_x W_T \right|^2 - 3|W|^2 \right\} + D^2\text{-terms} \quad (4.17)$$

where

$$\begin{aligned}
D_S W &= \Omega_S - \left(\frac{1}{S+S^*} + \frac{R g'}{(S+S^*)^2} \right) W \\
D_T W &= \Omega_T - \left(\frac{3}{R} - \frac{g'}{S+S^*} \right) W
\end{aligned}
\tag{4.18}$$

One can check that the scalar potential (4.17) is no longer positive definite since all the no-scale structure has been broken by string-loop effects. It still has a local minimum at $\langle C^X \rangle = 0^*$ and

$$\langle S \rangle = S_0 + \mathcal{O}(\epsilon) \tag{4.19}$$

where

$$\Omega(S_0) = \frac{1}{(S_0 + S_0^*)} \Omega_S(S_0) \tag{4.20}$$

On the other hand, at the minimum (4.19) the potential (4.16) has a non-trivial structure along the directions $\text{Re } T \equiv T_R$ (because of the breaking of the no-scaleness) and $\text{Im } T \equiv T_I$ [because of the breaking of the Peccei-Quinn (PQ) symmetry by the superpotential (4.16)].

At one-loop the g-function can be written as [16]

$$g = -c \epsilon \frac{R}{S+S^*} \tag{4.21}$$

and the potential (4.16) can be expanded as

$$V(c=0, S=\langle S \rangle, T) = 2 h^2 e^{-(S_0 + S_0^*)} \left[\frac{(c-a)}{R^2} \right] \epsilon + \mathcal{O}(\epsilon^2) \tag{4.22}$$

Therefore to order ϵ (one-loop) the potential is positive definite provided $c - a > 0$, and behaves like $1/T_R^2$. The expansion parameter is ϵT_R , as can be seen from Eq. (4.21), and the potential expansion can be written as

$$V = \frac{2(c-a)h^2}{R^2} e^{-(S_0 + S_0^*)} \left[\epsilon R + \mathcal{O}(\epsilon R)^2 \right] \tag{4.23}$$

In the region of validity of the one-loop approximation, $T_R \ll 1/\epsilon$, the potential decreases monotonically. When $T_R \sim 1/\epsilon$ the one-loop approximation breaks down and we should include

*) The degenerate minimum at $\langle C^X \rangle = \mathcal{O}(\epsilon) m_P$ coming from $W_X + C_X W_T = 0$ is energetically disfavoured by the D-terms.

higher orders in the string-loop expansion that could eventually stabilize the potential. In that case, if string effects are to stabilize T_R ^{*}, a consequence of this analysis is that $\langle T_R \rangle \sim 1/\epsilon \gg 1$ ^{**}). Explicit one-loop calculations have shown that $\epsilon \sim 10^{-5}$ so that the mass of the Kaluza-Klein excitations would be $M_{KK} \sim R^{-1} \sim T_R^{-1/2} \sim \epsilon^{1/2} m_P \sim 10^{16}$ GeV. Another feature of Eq. (22) is its flatness along T_I in spite of the fact that the PQ symmetry is broken at one-loop by Eq. (4.16). This is a consequence of the supersymmetric structure of the theory. Actually, a non-trivial potential for T_I will appear at $O(\epsilon^2)$. Finally, we will observe that supersymmetry is softly broken in the observable sector: scalar (m_0) and gaugino Majorana masses ($m^{1/2}$) as well as trilinear couplings ($A\tilde{W}$) will appear with values $O(\epsilon T_R) m_{3/2}$, so that if $T_R \sim 1/\epsilon$ the solution to the hierarchy problem would imply $m_{3/2} \lesssim 1$ TeV as in minimal supergravity models. Note that these results are valid for the most general case (2.14) as long as the origin $C_X = 0$ is a local minimum.

5. CONCLUSIONS

In this paper we have analysed a certain number of issues regarding the low-energy limit of heterotic superstrings after a compactification to $D=4$ preserving $N=1$ supersymmetry.

In particular we focused our attention on properties of the $N=1$ supergravity Lagrangian which are a result of integrating out the heavy modes rather than truncating the higher-dimensional theory. There is a compelling reason for doing so, namely that the truncation of the ten-dimensional supergravity action to its massless sector is inconsistent for general Calabi-Yau manifolds.

The main conclusions are the following: For the superstring classical low-energy effective action, the global positivity of the potential, which was a characteristic of the ‘no-scale models’ [26], is not maintained unless an additional condition on the Kähler potential is imposed. This condition is not implied by the symmetry arguments given here but it may follow from the structure of the underlying ten-dimensional theory. However, in the absence of supersymmetry breaking, the only supersymmetric extrema are those with vanishing cosmological constant, and they are the same as those of the truncated theory. This is due to the fact that effects of heavy modes modify the Kähler potential but not the superpotential. This result is even true when arbitrary string loops are taken into account, provided the ‘axion’ symmetries are not broken. Since supersymmetric extrema do not fix the v.e.v. of many scalar fields, in particular the T and S fields, the low-energy effective action has still the same properties as the no-scale models with sliding scalar fields.

This state of affairs is drastically changed in the presence of supersymmetry breaking. There, the vanishing of the cosmological constant, which was a property of the truncated theory, seems to be an artefact of that approximation, unless the Kähler potential fulfils the positivity property alluded to

*) Another possible source of non-trivial potential for T can be other non-perturbative effects such as world-sheet instantons [25]. However, they have been proved to break the no-scale structure only in the case of compactification with $(2, 0)$ supersymmetry in the world sheet.

**) In the limit $T_R \gg 1$ the two-dimensional σ -model is weakly coupled and we can neglect higher-derivative interactions in it.

before. If this additional requirement is satisfied, then supersymmetry may be broken with vanishing cosmological constant. However, even in this case some of the flat directions will be removed. In particular, the extrema with non-zero v.e.v. of the charged fields which allowed breaking of the gauge group in the truncated theory are no longer solutions.

Another artefact of the inconsistent truncation was the degeneracy of masses within multiplets of the observable sector, even when field theory one-loop effects were taken into account.

The impact of these results is obvious: some of the phenomenological implications of string-inspired no-scale models must be revised in view of our results.

Finally, some considerations about string-loop effects. Under the assumption that the loop expansion parameter is TS^{-1} , there again follows a sort of non-renormalization theorem: namely that the supersymmetric extrema with vanishing cosmological constant, present in the truncated theory, remains unshifted to all orders of perturbation theory. This rather strong result depends on the fact that perturbation theory effects modify only the Kähler potential and not the superpotential. This conclusion would hold even if the four-dimensional loop expansion parameter were not given by T/S . It is interesting to note that these effects could fix the radius of the internal manifold when supersymmetry is broken.

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