223-87/28

UPDATE ON ANOMALOUS THEORIES*

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Based on Lectures delivered at the
Summer Meeting on Quantum Mechanics of Fundamental Systems
Centro de Estudios Científicos de Santiago (CECS)
Santiago, Chile
December 1985

To be published by Plenum Press
as part of the CECS Scientific Series
C. Teitelboim, ed.

Typeset in $\mathcal{T}_E\mathcal{X}$ by Roger L. Gilson

CTP# 1436

December 1986

* This work was supported in part by funds provided by the U. S. Department of Energy (D.O.E.) under contract #DE-AC02-76ER03069.

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UPDATE ON ANOMALOUS THEORIES

R. Jackiw

I. WHAT IS AN ANOMALOUS THEORY?

It is known that the quantization procedure can spoil classical symmetries. The problem afflicts continuous chiral symmetries and gravitational symmetries of massless [Weyl] fermions, the former in any even dimensional space-time, the latter in space-times with dimensionality $4k+2$, $k=0,1,\dots$. [Similar quantum breaking afflicts discrete symmetries (P,T) in odd dimensions, and scale/conformal symmetries in any dimension; we shall not be concerned with these.] As a consequence, the symmetry current, whose classical conservation is assured by Noether's theorem, ceases to be conserved after quantization. We call such a current *anomalous*; it possesses an *anomalous divergence*, and the coupling of gauge fields to this current becomes problematical.¹

The difficulty with gauge field couplings is seen from the equations of motion for the gauge field A_μ [taken to be an anti-Hermitian matrix in the Lie algebra of the internal symmetry group].

$$F_{\mu\nu} \equiv \partial_\mu A_\nu - \partial_\nu A_\mu + [A_\mu, A_\nu] \quad (1)$$

$$D_\mu F^{\mu\nu} \equiv \partial_\mu F^{\mu\nu} + [A_\mu, F^{\mu\nu}] = J^\nu \quad (2)$$

Since $D_\mu D_\nu F^{\mu\nu} = 0$, Eq. (2) requires $D_\mu J^\mu = 0$. If on the other hand, the current J^μ acquires an anomalous divergence,

$$D_\mu J^\mu = \text{anomaly} \neq 0 \quad (3)$$

it appears that the theory can be consistent only on the subspace where the anomaly vanishes. Moreover, aside from issues of dynamical consistency, questions about gauge invariance and unitarity arise. Finally, renormalizability of the quantum theory must be re-examined, since that desirable property is frequently linked with gauge invariance.

Another viewpoint is gotten from a functional integral formulation. Upon integrating over the chiral fermions that couple to the gauge field, one is left with an effective action.

$$W(A) = -i \ln \mathcal{D}(A) \quad (4)$$

$$\mathcal{D}(A) \equiv \det(\not{D} + \not{A}) \quad (5)$$

Under a gauge transformation by the group element g ,

$$A_\mu \rightarrow A_\mu^g \equiv g^{-1} A_\mu g + g^{-1} \partial_\mu g \quad (6)$$

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which can be taken infinitesimally,

$$g = I + \theta + \dots \quad (7)$$

$$\delta A_\mu = D_\mu \theta \quad (8)$$

$\mathcal{D}(A)$ changes as

$$\delta \mathcal{D} \equiv \mathcal{D}(A + D\theta) - \mathcal{D}(A) = \int \text{tr} (D_\mu \theta) \frac{\delta}{\delta A_\mu} \mathcal{D}(A) \quad (9)$$

The variation is the vacuum matrix element of the current in the presence of an external gauge field.

$$\langle J^\mu \rangle_A = - \frac{\delta}{\delta A_\mu} \mathcal{D}(A) \quad (10)$$

Hence

$$\delta \mathcal{D} = \int \text{tr} \theta D_\mu \langle J^\mu \rangle_A \quad (11)$$

and the effective action for chiral fermions is not gauge invariant when the fermion current is not conserved [in its matrix elements]. Thus the total action, gauge field action $I(A)$ plus effective fermion action $\mathcal{D}(A)$, loses gauge invariance and it is unclear whether the gauge invariant $I(A)$ can be consistently combined with the gauge non-invariant $\mathcal{D}(A)$.

Models which possess the above described gauge pathology are called *anomalous theories*.²

An approach to anomalous theories that has been advocated in the past, is to eliminate the anomaly, i.e., the [chiral] fermion content is adjusted so that the anomalous divergence vanishes.¹ More specifically, in four-dimensions, the anomalous divergence reads¹

$$(D_\mu J^\mu)_a \propto \frac{1}{24\pi^2} \text{tr} T^a \partial_\mu \epsilon^{\mu\alpha\beta\gamma} \left(A_\alpha \partial_\beta A_\gamma + \frac{1}{2} A_\alpha A_\beta A_\gamma \right) \quad (12)$$

Here, the T^a 's comprise anti-Hermitian matrices which provide a basis for the Lie algebra, in the representation of the fermions.

$$A_\alpha = A_\alpha^a T^a \quad (13)$$

The proportionality constant is fixed by the number and chirality of fermions that interact with A_μ . The most important term in the divergence is the first. Because of cyclicity of the trace, it involves

$$\frac{i}{2} d_{abc} \equiv \text{tr} T^a \{T^b, T^c\} \quad (14)$$

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Fig. 1: Triangle graph which spoils gauge invariance in a 4-dimensional chiral gauge theory.

where the "b" and "c" indices refer to the gauge fields that couple to the current "a", as represented by the triangle diagram of Fig. 1.

The trace is unaffected by group transformations, hence an invariant statement for the absence of anomalies in four dimensions is that fermions belong to representations for which d_{abc} vanishes. [One can show that the rest of the anomaly also vanishes once d_{abc} does.]

This requirement of anomaly cancellation has been widely accepted and has two notable successes. First, when applied to the electroweak unified $SU(2) \times U(1)$ theory, which is potentially anomalous, it predicts that the number of quarks balances against the number of leptons, thus providing us with the only theoretical explanation of this apparently true experimental fact. Second, when applied to the construction of superstring models, the rank of the internal symmetry group is fixed uniquely at sixteen and the group is essentially predicted. All the other imponderables in the string program make this one fixed fact very important indeed.

Precisely because of the central role that anomaly cancellation plays in modern theoretical particle physics, it is worthwhile to look deeper into the matter, and to inquire whether a gauge theory which does possess anomalies is in fact meaningless, or whether some sense can be made of it. The question is especially appropriate at the present time, when as it happens a consistent mathematical framework for discussing anomalies has emerged.

The conclusion of several investigations is that unitarity and positivity need not be destroyed by the anomaly. Renormalizability of an anomalous theory remains unsettled.

DISCUSSION

C. Teitelboim (Santiago and Austin):

I would like to ask you something very naive: Is it possible to have anomalies in quantum mechanics, as opposed to quantum field theory?

R.J.:

Yes, several examples are given in the exercises to my Les Houches lectures, reprinted in Ref. [1]. The same examples, and others, are also found in a paper by Elitzur *et al.* [*Nucl. Phys.* B273, 93 (1986)].

Teitelboim:

So there can be a case where something is anomalously broken, without infinities.

R.J.:

Quantum mechanics does not suffer from field-theoretic ultraviolet infinities. However, there are infinities associated with "infinite dimensional matrices". *etc.* Indeed, in the examples mentioned by me, one is evaluating determinants of differential operators.

II. MATHEMATICALLY COHERENT FRAME FOR ANOMALIES

When anomalous non-conservation of currents was first encountered, this unexpected phenomenon was associated with the ultraviolet divergences of perturbation theory.¹ Later, a Hamiltonian analysis of the effect related it to symmetry breaking aspects of the filled "negative energy sea" — again a quantity which cannot be described in finite terms.¹

However, more recent discussions of the anomaly make use of various well-defined mathematical characteristic classes: Chern-Pontryagin, Chern-Simons, *etc.*, and attention is drawn to the fact that various quantities of interest like the anomalous divergence of a chiral current, the anomalous response to a gauge transformation of a fermion determinant, *etc.*, are expressed in terms of these.¹ The mathematical connection has come to a sharper focus in the characterization of an anomalous gauge theory by the fact that commutators of gauge transformation generators are anomalous, and do not follow the Lie algebra of the gauge group.³

It was appreciated already in the earliest days of research on anomalies that fermion charge densities $\rho_a \equiv J_a^0$ satisfy anomalous commutators.¹ Since ρ_a generates gauge transformations only on the fermion degrees of freedom, it is natural, in a mathematical/algebraic framework for the gauge theory, to consider the complete generator, G_a , given by ρ_a supplemented by $-(DE)_a$. [E_a is the non-Abelian electric field F_a^{i0} — it is the negative of the momentum conjugate to the canonical coordinate A_a^i .] G_a generates the complete gauge transformation, which modifies the gauge field degrees of freedom as well. It was conjectured³ and then verified⁴ that the anomaly in the commutator of the G_a 's is also given by a mathematically determined quantity.

$$[G_a(r), G_b(r')] = if_{abc} G_c(r) \delta(r-r') \pm \frac{i}{24\pi^2} \epsilon^{ijkl} \text{tr} \{T^a, T^b\} \partial_i A_j \partial_k \delta(r-r') \quad (15)$$

The sign is determined by the fermion chirality, and we observe that d_{abc} controls the anomaly in the commutator as well. [The third representation matrix making up d_{abc} resides in the gauge field: $A_j \equiv A_j^c T^c$.]

A finite gauge transformation with $g = e^{g^a T^a}$ is implemented in the quantum theory by the unitary operator

$$U(g) = \exp i \int d^3x G_a \quad (16)$$

The fact (15) that the Lie algebra of the generators does not follow that of the group, but possesses an extension, means that the composition law for the operators U differs from the group composition law.

$$g_1 g_2 = g_{12} \quad (17)$$

$$U(g_1)U(g_2) = e^{-i2\pi\omega_2(A;g_1,g_2)}U(g_{12}) \quad (18)$$

The additional phase, called a *2-cocycle*, when expanded for g_1 and g_2 near the identity reproduces the extension in (15).

Thus, the anomaly phenomenon is just the statement that in the quantized field theory, the gauge group is represented projectively, as in (18).

Before proceeding with the discussion of anomalous theories, let us pause for a mathematical message about cocycles.

III. MATHEMATICAL ASIDE ON COCYCLES

The mathematical theory for representing transformation groups by unitary operators on a linear vector or Hilbert space meets its objective correlative — its specific application — in quantum mechanics, which naturally is concerned with operators acting on physical states. When one is dealing with an abstract group of transformations, g , which obey the composition law (17) and which transform variables q according to a definite rule,

$$q \xrightarrow{g} q^g \quad (19)$$

then the simplest representation of this on functions of q , $\Psi(q)$, is obtained by associating with g an operator $U(g)$, which implements (19) as,

$$U(g)U^{-1}(g) = q^g \quad (20)$$

acts on $\Psi(q)$ according to,

$$U(g)\Psi(q) = \Psi(q^g) \quad (21)$$

and satisfies a composition law which parallels (17).

$$U(g_1)U(g_2) = U(g_{12}) \quad (22)$$

However, it is possible to elaborate this simplest realization of the transformation group by introducing phases into formulas (21) and (22). Such phases are called *cocycles* and the various conditions that they must satisfy have been elaborated in mathematics.

It has been known for some time that cocycles are present in the quantum mechanics of point particles. As mentioned above, modern gauge field theory makes use of them as well, to give a coherent mathematical framework for the anomaly phenomena.

A. 1-COCYCLE

In the simplest generalization of (21) and (22), a phase factor is inserted in the action of the representation,

$$U(g)\Psi(q) = e^{-i2\pi\omega_1(g;g^g)}\Psi(g^g) \quad (23)$$

Consistency with (22) requires that ω_1 satisfy

$$\omega_1(g^g; g_2) - \omega_1(g; g_{12}) + \omega_1(g; g_1) = 0 \quad (\text{mod integer}) \quad (24)$$

A quantity $\omega_1(g; g)$ depending on one member of the transformation group, g , and possibly on the variable acted upon, g , is called a *1-cocycle* if it obeys (24).

Representing Galilean boosts in quantum mechanics makes use of a 1-cocycle. The Abelian group of Galileo transformations of the position vector $\mathbf{r}$ [in the general discussion] is defined to act as $\mathbf{r} \rightarrow \mathbf{r} + \mathbf{v}t$. The transformation is labelled by $\mathbf{v}$ [corresponding to g in the general discussion] and is implemented by the operator

$$U(\mathbf{v}) = e^{i\mathbf{v} \cdot (\mathbf{p}t - m\mathbf{r})} \quad (25)$$

[The second term in the exponential involving $m\mathbf{r}$ is present so that in the Galilean invariant theory of a non-relativistic free particle with mass m , the exponent is the appropriate constant of motion: $(d/dt)(\mathbf{p}t - m\mathbf{r}) = \mathbf{p}t + \mathbf{p} - m\dot{\mathbf{r}} = 0$ when $\mathbf{p} = m\dot{\mathbf{r}}$.] One verifies with the help of the Heisenberg algebra

$$\begin{aligned} [\mathbf{r}^i, \mathbf{p}^j] &= 0 = [\mathbf{p}^i, \mathbf{p}^j] \\ [\mathbf{r}^i, \mathbf{p}^j] &= i\delta^{ij} \end{aligned} \quad (26)$$

that

$$\begin{aligned} U(\mathbf{v})\mathbf{r}U^{-1}(\mathbf{v}) &= \mathbf{r} + \mathbf{v}t \\ U(\mathbf{v})\mathbf{p}U^{-1}(\mathbf{v}) &= \mathbf{p} + m\mathbf{v} \end{aligned} \quad (27)$$

and that (22) is satisfied, but $U(\mathbf{v})$ acts on wave functions $\Psi(\mathbf{r})$ as in (22) with

$$2\pi\omega_1(\mathbf{v}; \mathbf{v}) = m\mathbf{v} \cdot \mathbf{r} + \frac{1}{2}m\mathbf{v}^2t \quad (28)$$

which satisfies (24).

B. 2-COCYCLE

In the next generalization, a phase is introduced into the composition law (22).

$$U(g_1)U(g_2) = e^{-i2\pi\omega_2(g_1g_2)}U(g_{12}) \quad (29)$$

A consistency condition on ω_2 follows from the assumed associativity of the composition law. If

$$(U(g_1)U(g_2))U(g_3) = U(g_1)(U(g_2)U(g_3)) \quad (30)$$

then

$$\omega_2(q^{\theta_1}; g_2, g_3) - \omega_2(g; g_{12}, g_3) + \omega_2(g; g_1, g_{23}) - \omega_2(g; g_1, g_3) = 0 \pmod{\text{integer}} \quad (31)$$

When a quantity depends on two group elements, g_1 and g_2 , and possibly on q , and also satisfies (31), it is called a $\mathcal{B}$ -cocycle. Representations that make use of 2-cocycles are called *projective* or *ray* representations and they occur frequently in quantum mechanics.

Indeed, the Heisenberg commutator algebra (26) indicates that translations on phase space: $\mathbf{r} \rightarrow \mathbf{r} + \mathbf{a}$, $\mathbf{p} \rightarrow \mathbf{p} + \mathbf{b}$ are represented by the operator

$$U(\mathbf{a}, \mathbf{b}) = e^{i(\mathbf{a} \cdot \mathbf{p} - \mathbf{b} \cdot \mathbf{r})} \quad (32)$$

i.e.

$$\begin{aligned} U(\mathbf{a}, \mathbf{b}) \mathbf{r} U^{-1}(\mathbf{a}, \mathbf{b}) &= \mathbf{r} + \mathbf{a} \\ U(\mathbf{a}, \mathbf{b}) \mathbf{p} U^{-1}(\mathbf{a}, \mathbf{b}) &= \mathbf{p} + \mathbf{b} \end{aligned} \quad (33)$$

$U(\mathbf{a}, \mathbf{b})$ composes according to (29) with

$$2\pi\omega_2(\mathbf{r}; \mathbf{a}_1 \mathbf{b}_1, \mathbf{a}_2 \mathbf{b}_2) = \frac{1}{2}(\mathbf{a}_1 \cdot \mathbf{b}_2 - \mathbf{a}_2 \cdot \mathbf{b}_1) \quad (34)$$

The U 's associate and ω_2 obeys the consistency condition (31). [Coordinate and momentum shifts form a symmetry operation for a free particle: Galilean transformations supplemented by spatial translations are realized in this way.] So, following Weyl and Bargmann, one may accurately say that the essence of quantum mechanics is the 2-cocycle, leading to ray or projective representation of the Abelian translation group.

When dealing with a continuous or Lie group of transformations, the discussion may be carried out in infinitesimal terms. Corresponding to the finite group element g , there is the infinitesimal quantity θ , $g = e^\theta$, and the composition law (17) is reflected in a Lie algebra.

$$[\theta_1, \theta_2] = \theta_{12} \quad (35)$$

Suppose further that we are dealing with dynamical variables q , governed by Lagrangian L . The infinitesimal action of the transformation on q is given by an infinitesimal version of (19).

$$q \rightarrow q + \delta_\theta q \quad (36)$$

When no cocycles occur, the representative of θ , that is, the infinitesimal generator G , where $U(g) = e^{iG}$, is given by $(\delta_\theta q) \partial L / \partial \dot{q} = (\delta_\theta q) p$, so that its action on wave functions is realized by $(\delta_\theta q)(1/i) \partial / \partial q$. Moreover, the generators satisfy the Lie algebra of the group (35).

$$[iG_1, iG_2] = iG_{12} \quad (37)$$

which is a consequence of (17) for infinitesimal quantities.

In the presence of a 1-cocycle, the generator becomes

$$G = (\delta_\theta q) p - \delta\omega_1 \quad (38)$$

where $\delta\omega_1$ is [proportional to] the infinitesimal part of the 1-cocycle; compare with (25) and (28). The 1-cocycle condition (24), in infinitesimal form, ensures that the modified generators (38) continue to satisfy the Lie algebra of the group (35).

A 2-cocycle indicates that the generators' Lie algebra acquires an extension.

$$[iG_1, iG_2] = iG_{12} - \delta\omega_2 \quad (39)$$

Here $\delta\omega_2$ is [proportional to] the infinitesimal portion of the 2-cocycle, whose consistency condition (31) ensures that (39) does not contradict the Jacobi identity.

$$[G_1, [G_2, G_3]] + [G_2, [G_3, G_1]] + [G_3, [G_1, G_2]] = 0 \quad (40)$$

This is the infinitesimal version of (30). The non-vanishing Heisenberg commutator (26) is an example of an extended Lie algebra (39).

C. 3-COCYCLE

Not unexpectedly, the 3-cocycle involves abandoning associativity (30), and for infinitesimal generators, the Jacobi identity (40) fails. It is important to appreciate that non-associating quantities cannot be represented by well-defined linear operators acting on a vector or Hilbert space since, by definition, operations on vectors necessarily associate. Hence, the quantities discussed below are abstract, algebraic objects obeying formal relations.

To encounter a 3-cocycle, we replace (30) by a formula which includes a phase.

$$\left(U(g_1) U(g_2) \right) U(g_3) = e^{-i\mathcal{B}(g_1, g_2, g_3)} U(g_1) \left(U(g_2) U(g_3) \right) \quad (41)$$

By considering fourfold products, and associating in different ways, it is established that consistency requires that ω_3 satisfy the $\mathcal{B}$ -cocycle condition.

$$\begin{aligned} \omega_3(q^{\theta_1}; g_2, g_3, g_4) - \omega_3(g; g_{12}, g_3, g_4) + \omega_3(g; g_1, g_{23}, g_4) \\ - \omega_3(g; g_1, g_2, g_{34}) + \omega_3(g; g_1, g_2, g_3) = 0 \pmod{\text{integer}} \end{aligned} \quad (42)$$

This arcane structure in fact arises in elementary Hamiltonian dynamics. Consider a particle with mass m and charge e , moving at $\mathbf{r}$ in an external magnetic field $\mathbf{B}(\mathbf{r})$. The dynamical equations express the Lorentz force law.

$$\begin{aligned} m\dot{\mathbf{v}} &= e\mathbf{v} \times \mathbf{B} \\ \dot{\mathbf{r}} &= \mathbf{v} \end{aligned} \quad (43)$$

The velocity of light is scaled to unity. We do not assume that $\mathbf{B}$ is necessarily divergence free, i.e., there may be magnetic sources. Since a magnetic field does no work, the energy is purely kinetic, and the Hamiltonian does not see $\mathbf{B}$.

$$H = \frac{1}{2} m v^2 \quad (44)$$

Fig. 2: Triangle at r , defined by two translations a_1 and a_2 , through which the flux Φ is calculated.

In order that the dynamical equations (43) emerge as canonical Hamiltonian equations, we postulate the following commutation relations for r and v . [In the classical theory, these would be Poisson brackets.]

$$[r^i, r^j] = 0 \quad (45a)$$

$$[r^i, mv^j] = i\delta^{ij} \quad (45b)$$

$$[mv^i, mv^j] = i\epsilon^{ijk} B^k \quad (45c)$$

Thus, the magnetic field, which is invisible in (44), reappears in (45c) so that (43) may be regained as

$$\dot{r} = i[H, r] = \dot{v} \quad (46)$$

$$m\dot{v} = i[H, mv] = cv \times B$$

If now (45) is subjected to the test of the Jacobi identity, we find that triple velocity commutators sum to

$$[mv^1, [mv^2, mv^3]] + [mv^2, [mv^3, mv^1]] + [mv^3, [mv^1, mv^2]] = e\nabla \cdot B \quad (47)$$

When $\nabla \cdot B$ is zero, we are dealing with an associative Lie algebra, and mv may be represented by $(1/i)\nabla - eA$, $B = \nabla \times A$. However, for $\nabla \cdot B \neq 0$ a 3-cocycle is present.

To understand the non-associativity, let us consider the finite quantities

$$U(a) = e^{ia \cdot mv} \quad (48)$$

These represent translations of r , in that $U(a)rU^{-1}(a) = r + a$. However, the representation is not faithful, because from (45c) it follows that

$$U(a_1)U(a_2) = e^{-ie\Phi}U(a_1 + a_2) \quad (49)$$

Fig. 3: Tetrahedron at point r , defined by three translations a_1 , a_2 and a_3 . The 3-cocycle is proportional to the flux out of the tetrahedron.

where Φ is the flux through the triangle at r formed from a_1 and a_2 ; see Fig. 2. Moreover, by considering the triple product $U(a_1)U(a_2)U(a_3)$, associated in the two different ways, one finds

$$\left(U(a_1)U(a_2) \right) U(a_3) = e^{-i2\pi\omega_3} U(a_1) \left(U(a_2)U(a_3) \right) \quad (50)$$

where the 3-cocycle $2\pi\omega_3$ is e times the flux out of the tetrahedron at r formed from a_1 , a_2 and a_3 ; see Fig. 3.

When $\nabla \cdot B = 0$, no flux emanates from a closed surface; the 3-cocycle vanishes and associativity is regained. When there are sources, $\nabla \cdot B \neq 0$, the flux is nonzero, but associativity will prevail if ω_3 is an integer, since then $e^{-i2\pi\omega_3} = 1$. This requirement forces (1) $\nabla \cdot B$ to consist of delta functions so that the total flux not vary continuously when the a_i change, i.e., sources must be monopoles; (2) since a monopole of strength g gives rise to flux $4\pi g$, hence, produces the cocycle $2eg$, eg must satisfy Dirac's quantization condition, i.e., it must be an integral multiple of $1/2$. In this way, the removal of the 3-cocycle, which is necessary for conventional quantum mechanics with associative operators on Hilbert space, limits magnetic sources to quantized Dirac monopoles. Other magnetic sources lead to a non-associative algebra.

Finally, note that even for quantized monopoles, where the finite 3-cocycle is invisible since $e^{-i2\pi\omega_3} = 1$, there is a remnant of non-associativity in the non-vanishing of the right-hand side of (47), which for a monopole at r_0 , reads $4\pi eg\delta(r - r_0) = 2\pi\omega_3(r - r_0)$. This does not interfere with an operator formulation on a space with one point — the location of the monopole — excluded.

Further generalization is possible; one may define 4- and higher cocycles, but thus far there has been no role found for these in physics.

D. BEYOND QUANTUM MECHANICS

The above concerns quantum mechanics. In quantum field theory, cocycles arise in the description of anomalies, and we saw in (15) the role that a 2-cocycle plays. The 1-cocycle also is relevant: it measures the gauge non-invariance of the fermionic determinant (5). Define a quantity $\omega_1(A;g)$ by

$$\mathcal{D}(A^g) = e^{-2\pi i \omega_1(A;g)} \mathcal{D}(A) \quad (51)$$

From the gauge group composition law, $g_1 g_2 = g_{12}$, it follows that $\mathcal{D}((A^{g_1})^{g_2}) = \mathcal{D}(A^{g_{12}})$. Hence $\omega_1(A;g)$ must satisfy (24), i.e., it is a 1-cocycle. We further recognize that the haphazard-looking formula (12) must coincide with the infinitesimal, first order in θ , contribution to $\omega_1(A;g)$, because according to (5) and (11) the anomalous divergence is an infinitesimal 1-cocycle. Moreover, from the 1-cocycle condition we see that the quantity $2\pi\omega_1(A;h)$ transforms under a gauge transformation $A \rightarrow A^\theta$, $h \rightarrow g^{-1}h$ exactly the same way as $W(A)$ in (4).

$$W(A^\theta) = W(A) - 2\pi\omega_1(A;g) \quad (52)$$

$$2\pi\omega_1(A^\theta;g^{-1}h) = 2\pi\omega_1(A;h) - 2\pi\omega_1(A;g)$$

Therefore, one may view the 1-cocycle as an effective action which transforms under gauge transformations in the same anomalous way as the effective action of chiral fermions. Necessarily there is present a field other than the gauge field A , viz. the "chiral" field h . When the 1-cocycle is used in this way it is called the *Wess-Zumino term*.¹

Please note that in (51) and (52) the cocycle functions in a way somewhat different from the representation theory examples discussed earlier: unlike in (23), ω_1 goes not arise from the action of an operator on a vector.

What about field theoretic 3-cocycles? There is evidence of Jacobi identity violation for commutators of spatial components of fermionic currents even in a non-interacting theory.^{1,5} However, the structure of the commutators is ambiguous and fraught with divergences. Moreover, spatial components are involved, but they are not generators of symmetry transformations. Hence, no coherent algebraic framework has been given for this pathology.

Another commutator anomaly, hinting at an anomaly-related 3-cocycle, has been encountered in the course of verifying (15).⁶ When the commutator of two electric fields is computed in perturbation theory, one finds a non-vanishing result.

$$[E_a^i(\mathbf{r}), E_b^j(\mathbf{r}')] = \frac{i}{48\pi^2} \epsilon^{ijk} d_{abc} A_c^k \delta(\mathbf{r} - \mathbf{r}') \quad (53)$$

If we view $\mathbb{E}$ as a "velocity" in field space — an interpretation which is appropriate since $\mathbb{E} = -\dot{A}$ in the Weyl, $A^0 = 0$, gauge, then the right-hand-side of (53) defines a $U(1)$ curvature in field space — a kind of functional "magnetic field" — compare (45c). [Commutators with A — a "coordinate" in field space — show no anomalies.] Moreover, the triple $\mathbb{E}$ commutator fails to satisfy the Jacobi identity.

Finally, we note that in string theory, violations of the Jacobi identity — potential 3-cocycles — have been found.⁷

However, none of these fascinating structures have been coherently described by a non-associative algebra. It is interesting to recall that already many years ago in a well-known body of work, P. Jordan investigated non-associative algebras for quantum mechanics. Also, P. Dirac predicted a physical role for non-associativity [1969 Rouse Ball lecture in Cambridge University, as communicated to the author by H. Gottlieb].

DISCUSSION

I. Schmidt (Valparaíso):

When you introduce a 3-cocycle, as in (50), do you still assume that a 2-cocycle arises in the composition, as in (29)?

R.J.J.:

In general, nothing is said about the composition law (29). But it is easy to show that if a phase is present there and also in (50), then (31), which is a consequence of associativity, is modified by the occurrence of the 3-cocycle on the right-hand side. Hence in that case the phase in composition law should not be called a cocycle.

M. Henneaux (Brussels):

Are the velocities which fail to satisfy the Jacobi identity linear operators?

R.J.J.:

No, they are not: linear operators acting on vectors necessarily associate by definition.

D. Gross (Princeton):

Is it that $m\mathbf{v}$ cannot be $\mathbf{p} - e\mathbf{A}$ because $\mathbf{A}$ is not well-defined?

R.J.J.:

$\mathbf{A}$ has not been introduced. I am giving a gauge-invariant discussion, without $\mathbf{A}$. Indeed, we cannot have a vector potential in the region that $\nabla \cdot \mathbf{B} \neq 0$.

Gross:

What goes wrong if you don't have associativity?

R.J.J.:

It is not clear. Certainly, you cannot have conventional quantum mechanics with linear operators on a Hilbert space. Perhaps you can do something with density matrices, but I don't know how far you can go.

Henneaux:

But even when the Dirac quantization condition for a monopole holds, you cannot realize the velocities as linear operators.

R.J.J.:

Since the failure of the Jacobi identity for a monopole is located at one point — the position of the monopole — one can realize $m\mathbf{v}$ as $\mathbf{p} - e\mathbf{A}$ away from that point. However, as is well-known, $\mathbf{A}$ is not globally defined.

van Nieuwenhuizen (Stony Brook):

If your formulas are not about linear operators, to what do they refer?

R.J.J.:

They are algebraic relations between quantities that I merely write on the blackboard. Whether they have some realization [representation] remains to be determined.

IV. APPROACHES TO QUANTIZING AN ANOMALOUS THEORY

Since the gauge transformation generator G_a coincides with the time component of $-D_\mu F^{\mu\nu} + J^\nu$, the field equation motion (2) requires that G_a vanish — this is Gauss's law. Moreover, if the spatial components of the field equation vanish, so must the time component, as a consequence of Lorentz invariance.

In a Hamiltonian, classical canonical or quantum mechanical, context $G_a = p_a - (D \cdot E)_a$ involves only canonical variables and no time derivatives [$-E_a$ is the canonical momentum in the Weyl gauge], hence its vanishing must be viewed as a constraint rather than an equation of motion.

Constraints are conveniently divided into first class, when they close under Poisson bracketing and second class otherwise. For first class constraints, the quantization procedure is standard, except that a further condition is imposed: physical states must be annihilated by the constraint. This is how a non-anomalous gauge theory is quantized, and the requirement $G_a|\Psi\rangle = 0$ insures that states are invariant against homotopically trivial gauge transformations.

Second class constraints cannot annihilate states, because their commutators do not close and a modification of the quantization rules is required to implement them.

Therefore, it is natural to suggest that an anomalous gauge theory can be consistently quantized, provided that Gauss's law is recognized as a second class constraint and that the quantization procedure is appropriately altered.⁸ Moreover, it has been conjectured that this procedure produces an effective action containing the Wess-Zumino term [1-cocycle], in addition to the gauge field and chiral fermion contributions.⁹ In this way, the total action would regain gauge invariance: the anomalous response to gauge transformations of the chiral fermions would be cancelled by the explicit gauge non-invariance of the Wess-Zumino term. The chiral field of the Wess-Zumino term signals the emergence of new degrees of freedom, which are seen in this scenario as a consequence of the anomaly.

This program of modified canonical quantization has thus far not been realized. The principal difficulty is that the second class nature of the constraint algebra emerges as a quantization anomaly only after the theory has already been quantized by conventional methods.

Nevertheless, various indicators are encouraging.

A. DECOUPLING ANOMALY-CANCELLING FERMIONS

Consider a potentially anomalous theory with spontaneous gauge symmetry breaking, effected by the Higgs mechanism, and fermion content properly adjusted to cancel the anomaly. By setting various parameters in the Higgs sector to infinity one may decouple some of the chiral fermions and for the remaining ones the anomaly no longer cancels. It is found that the decoupling fermions leave behind the Wess-Zumino term, so that gauge invariance is retained. [The chiral field here is the Higgs field].¹⁰

The above may be viewed as a method for quantizing an anomalous gauge theory. Indeed there is precedent for effecting difficult canonical quantization by first enlarging the theory in a way so that quantization is straightforward, and then removing the addition by sending parameters to infinity.¹⁰ Note however, that here the chiral field [Higgs field] retains a kinetic term, which is not needed for gauge invariance and may upset renormalizability.

B. INTEGRATING OVER ALL GAUGE POTENTIALS

It has been observed that an effective action for chiral fermions which includes the Wess-Zumino term can be obtained from the functional integral, supplemented by the Faddeev-Popov trick. One begins with a formal functional integral over *all* configurations of the gauge potential, including gauge copies.¹¹

$$Z = \int D A \exp i [I(A) + W(A)] \quad (54)$$

[The chiral fermion fields have been integrated to yield $W(A)$.] Next unity is resolved in the Faddeev-Popov way,

$$1 = \Delta_f(A) \int D g \delta \left(f \left(A^{g^{-1}} \right) \right) \quad (55)$$

Here $f(A) = 0$ is a gauge fixing, and $\Delta_f(A)$ is the gauge invariant Faddeev-Popov determinant which renders (55) true. Upon multiplying (54) by unity, as presented in (55), changing orders of functional integration and changing the integration variable from A to A^g , one is left with the announced result.

$$\begin{aligned} Z &= \int D A D g \Delta_f(A) \delta(f(A)) \exp i [I(A^g) + W(A^g)] \\ &= \int D A D g \Delta_f(A) \delta(f(A)) \exp i [I(A) + W(A) - 2\pi\omega_1(A^g)] \end{aligned} \quad (56)$$

However, one cannot view this as a true derivation from quantum mechanical first principles. The Faddeev-Popov trick is just that — a trick which short-circuits a lengthy canonical analysis of constraints. It is known to be a valid trick in conventional gauge theories, but misses the mark in more subtly constrained situations. In the present context it is not at all clear why one should start, as in (54), with an integral over *all* gauge potentials. One may worry that integrating over all gauge configurations will destroy positivity and/or unitarity, owing to the indefinite metric. Indeed, a soluble example, which we now discuss, shows that unitarity can be lost in this way.

V. CHIRAL SCHWINGER MODEL

In view of the uncertainties mentioned above with the general program, it is very satisfying that in some simple models one can analyze the situation completely.¹² Especially useful is the chiral Schwinger model: two dimensional electrodynamics with massless Weyl fermions of one chirality interacting with a $U(1)$ gauge field. The Lagrangian is

$$\begin{aligned} \mathcal{L} &= -\frac{1}{4} F^{\mu\nu} F_{\mu\nu} + \bar{\psi} \gamma^\mu \left(i \partial_\mu + e A_\mu \frac{(1 + i\gamma_5)}{2} \right) \psi \\ \gamma_5 &= i\gamma^0 \gamma^1 \end{aligned} \quad (57)$$

Owing to the well-known simplifications which hold in two dimensions, the chiral fermion determinant may be evaluated. The effective action is

$$I_{\text{eff}} = I(A) + \frac{e^2}{8\pi} \int A_\mu \left[a g^{\mu\nu} - (g^{\mu\alpha} + \epsilon^{\mu\alpha}) \frac{\partial_\alpha \partial_\beta}{\square} (g^{\beta\nu} - \epsilon^{\beta\nu}) \right] A_\nu \quad (58)$$

where " a " is a constant, left undetermined in the evaluation of the determinant. For no value of a is the answer gauge invariant — that is why the theory is anomalous. Since a cannot be determined, the strength of the contact term $\int A^2$ is arbitrary. In a gauge invariant theory contact terms are fixed by the requirement of gauge invariance; here we have no such principle.

An equivalent description is in terms of a bosonized action.

$$I(A, \varphi) = I(A) + \frac{1}{2} \int d^2x \left[\partial_\mu \varphi \partial^\mu \varphi + \frac{e}{\sqrt{\pi}} (g^{\mu\nu} - \epsilon^{\mu\nu}) \partial_\mu \varphi A_\nu + \frac{ae^2}{4\pi} A_\mu A^\mu \right] \quad (59)$$

where the a term now arises from ambiguities in the bosonization procedure. Functionally integrating over φ in (59) yields (58).

The theory described by (58) or (59) may be solved. One finds for $a \geq 1$ a unitary, positive definite model. The vector meson acquires mass m .

$$m^2 = \frac{e^2}{4\pi} \frac{a^2}{a-1} \quad (60)$$

This lies anywhere between $\frac{e^2}{\pi}$ and infinity, in contrast to the conventional Schwinger model, where gauge invariance fixes the corresponding constant a and the gauge boson mass.

It is noteworthy that a consistent, but not completely determined theory has emerged. [The one-parameter ambiguity at first caused confusion,¹³ but now has been re-derived by many different methods.¹⁴] In a sense, one can view the above calculation, with its loss of gauge invariance, as a calculation within the gauge invariant chiral action with Wess-Zumino term (56), in the "unitary" gauge $g = 1$, where the Wess-Zumino term vanishes. Alternatively, one can regard the result as coming from an anomaly-free theory, where additional anomaly-cancelling fermions have been decoupled, by sending parameters to infinity. However, neither of these alternative viewpoints explains why only a range of the undetermined parameter [$a \geq 1$] gives a unitary theory, while unitarity is lost outside this range [$a < 1$].

The bosonized action explicitly incorporates the anomaly, and canonical quantizing (59) does indeed involve second class constraints. Carrying out the details of this quantization reproduces the results obtained by the action integral.¹⁵

In all of these derivations, one fact remains: a consistent, Lorentz invariant,¹⁶ theory of a massive vector meson emerges from an anomalous gauge theory. Similar results can be established for the non-Abelian generalization, though the analysis is more formal and less explicit, since the model cannot be solved completely.¹⁷

Unfortunately nothing is learned from the 2-dimensional exercises about renormalizability: these 2-dimensional theories are finite. Thus it is not known whether 4-dimensional anomalous theories are renormalizable, but one may expect that anomalies do not spoil unitarity and positivity. This expectation is borne out by a formal canonical analysis of a bosonized action that describes the low energy dynamics of an anomalous 4-dimensional gauge theory,¹⁸ and by a formal BRST analysis of the complete gauge field plus fermionic action.¹⁹ In these investigations the gauge symmetry is broken, and the gauge field is massive.

DISCUSSION

Gross:

How can you see that the constraint in second class before you quantize?

R.J.:

As I mentioned, that is precisely the obstacle to carrying out the constrained quantization program with anomalies — they arise only after the theory has been quantized, but then they change the way that the theory should be quantized.

Teitelboim:

To get started you have to put in $\hbar$ classically.

R.J.:

Something like that. You can write an effective [bosonized] action, as in (59), which reproduces the anomalies by explicit [classical] reasoning. Then you can quantize, and you find you are dealing with second class constraints.

VI. GRAVITATIONAL THEORIES

In 2 mod 4 dimensions, chiral fermions induce gravitational anomalies.¹ This is to be expected since gravity theory may be formulated as a gauge theory, invariant against local Lorentz transformations. The current, which now is the symmetric energy momentum tensor $\theta_{\mu\nu}$, is not [covariantly] conserved in the presence of chiral fermions. Just as for gauge anomalies, which arise in 2 mod 2 dimensions, the 2-dimensional case is the most important one, but gravity in two dimensions has its own peculiarities, which have nothing to do with anomalies.

The usual equation governing gravity, involving the Einstein tensor, $G_{\mu\nu}$,

$$G_{\mu\nu} - \Lambda g_{\mu\nu} = G \theta_{\mu\nu} \quad (61)$$

$$G_{\mu\nu} \equiv R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} g^{\alpha\beta} R_{\alpha\beta}$$

where $R_{\mu\nu}$ is the Ricci curvature, Λ a cosmological constant, $g_{\mu\nu}$ the metric and G the gravitational constant, requires conservation of $\theta_{\mu\nu}$, since the left-hand side possesses this property. However, in 2-dimensions (61) cannot be postulated, since $G_{\mu\nu}$ vanishes identically. Correspondingly, the quantity $\int d^2x \sqrt{-g} R$, $g \equiv \det g_{\mu\nu}$, $R \equiv g^{\mu\nu} R_{\mu\nu}$, whose variation gives $G_{\mu\nu}$ in dimension greater than two, is a surface term in two dimensions, with vanishing variation.

Therefore, 2-dimensional gravity must be based on an equation different from (61), and the anomaly problem for 2-dimensional gravity becomes more subtle than a mere mismatch between the conservation properties of two sides of Einstein's equation, as in (61).

Nevertheless, the essence of the problem remains the same: the chiral fermion determinant [which may be computed exactly²⁰] loses local Lorentz invariance [general coordinate invariance can always be maintained]. Also, it possesses an arbitrariness, analogous to that parameterized by " a " in the gauge theory case.

The non-invariant chiral fermion effective action may be combined with an invariant action for 2-dimensional gravity whose form has been derived from the 3-dimensional Einstein-Hilbert action by dimensional reduction.²¹

$$I(g) = \int d^2x \sqrt{-g} (R + 2\Lambda) \quad (62)$$

Here N is a scalar Lagrange multiplier. The analysis of the complete theory cannot be carried out completely, since the model is non-trivial. But again, formal indications support unitarity with massless excitations for a range of the arbitrary parameter.²²

VII. CONCLUSIONS AND OUTLOOK

It should not be construed that my present pursuit of anomalous theories reproduces the anomaly cancellation principle which D. Gross and I enunciated in response to the Weinberg-Salam model in the 1970's. In its original formulation, that model is anomalous, yet a conventional quantization was proposed; this is inconsistent. The remedy we put forward cancels the anomaly with a quark-lepton balance, which is verified but not completely established experimentally: the top quark has not been discovered and the τ neutrino need not be different from the e and μ neutrinos. Of course, absence of the sixth quark can be accommodated at any given time if its mass is beyond experimental range. However, should ν_τ prove not to be a new particle, anomalies cannot cancel, and new ideas — perhaps those presented here — will be necessary.

Of course, renormalizability of a 4-dimensional anomalous theory remains an open problem, which is under consideration.²³ The hope is that infinities arising from the anomaly cancel against those from the Wess-Zumino term. Verification of this is complicated in the absence of a kinetic term for the Wess-Zumino chiral field.²⁴ Should a positive result hold, it could very well be coupled with the emergence of a mass for the gauge fields, as is seen in all the examples studied thus far. Gauge symmetry breaking in electroweak theories would then appear in a new light: the weak forces are mediated by massive vector mesons whose mass breaks the gauge symmetry owing to chiral anomalies; these are absent in the chirally symmetric electromagnetic channel with its massless photon.

While uncertainties remain about 4-dimensional anomalous theories, the complete success in two dimensions counts as an important result, in view of the many physical applications and the central role for the string program that 2-dimensional models enjoy.

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