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**STRUCTURE OF THE SCALAR POTENTIAL IN
GENERAL $N=1$ HIGHER DERIVATIVE SUPERGRAVITY IN FOUR DIMENSIONS**

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ABSTRACT

We derive general properties of the scalar potential in matter-coupled supergravity theories with arbitrary higher-derivative interactions. The relevance of these results for the vacuum configurations of superstring theories is outlined.

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Recently, modified supergravity theories including higher-curvature terms and higher-derivative interactions have become popular in connection with the effective action for the massless modes of superstrings [1].

It is known that higher-curvature terms of any order appear in string theories as an effect of integrating out the massive modes with many $O(\alpha')$ [2]. This has to be contrasted with the Kaluza-Klein modes, which already exist in point-field theories, with the property of only modifying low-derivative interactions [3].

In theories [4] which undergo a compactification, preserving $N=1$ supersymmetry in $D=4$, the integration over the Kaluza-Klein modes is controlled by the functions appearing in the $N=1$ standard supergravity action [5]. These functions are the Kähler potential $J(z, \bar{z})$ (z denotes a set of complex scalar fields belonging to chiral $N=1$ supermultiplets), the superpotential $g(z)$ (analytic in z), and another chiral function $f_{AB}(z)$ appearing in gauge kinetic terms.

The standard supergravity form has been used to derive non-renormalization theorems for superstring theories, mainly using the fact that the superpotential is unrenormalized as a consequence of the axion symmetries of the superstring.

The standard supergravity scalar potential is [5]

$$V = e^G [G^{\alpha\bar{\beta}} G_{\alpha} G_{\bar{\beta}} - 3] + \frac{1}{2} (Re f_{AB})^{-1} D^A D^B \quad (1)$$

A supersymmetric extremum of V is always given by

$$g_{\alpha} = \partial g / \partial z^{\alpha} = 0, \quad g = 0 \quad (2)$$

irrespective of the form of the Kähler potential J , provided Eq. (2) admits a solution. Equation (2) corresponds to unbroken supersymmetry in Minkowski space. Supersymmetric extrema with $G_{\alpha} = 0$ but $g \neq 0$ (anti de Sitter vacuum) are ruled out in superstring theories.

However, as we show later, the form of the scalar potential given in Eq. (1) is only appropriate for standard supergravity, and is not retained if higher-derivative interactions, in the gravitational, the Yang-Mills, and the σ -model variables, are introduced.

As a consequence, non-renormalization theorems, in order to be properly discussed, must use the general form of the scalar potential when higher-curvature terms and their supersymmetric completion are included in the effective action of superstrings.

The purpose of this letter is to show that in spite of the modification of the scalar potential due to higher-order terms, one can reach exact conclusions by using the supersymmetry constraints on the full scalar potential.

First we will recall that Eq. (1) can be written in a model-independent form as follows [6]:

$$V(z, \bar{z}) = 2 C^I(z, \bar{z}) Z_I{}^J(z, \bar{z}) C_J(z, \bar{z}) - 3 m^2(z) \quad (3)$$

where C^I is the (constant) scalar field dependence in the spin- $1/2$ supersymmetry variations and $M(\phi)$ is the analogous term for the spin- $3/2$ gravitino field,

$$\delta \chi_L^I = C^I(z, \bar{z}) \epsilon_L, \quad (4)$$

$$\delta \psi_{\mu R} = \frac{1}{2} \gamma_\mu M(z, \bar{z}) \epsilon_L, \quad (5)$$

$$\mathcal{L}_{KF} = -\frac{1}{2} \bar{z}_I{}^J \bar{\chi}^I \not{\partial} \chi_J + h.c. \quad (6)$$

Equation (3) is true provided the Einstein and Rarita-Schwinger terms have canonical form and there are no mixed spin- $1/2$ -spin- $3/2$ kinetic terms in the Lagrangian. These terms can always be removed by appropriate redefinition of the gravitino field [6-8].

In the standard $N=1$ supergravity action with chiral fermions χ_L^α , gauginos λ_L^A , and gravitino $\psi_{\mu L}$, the previous general formulae specify as follows [5]:

$$C_0^\alpha = -\frac{1}{2} e^{G/2} G^{\alpha\bar{\beta}} G_{\bar{\beta}} \quad (7)$$

$$C_0^A = \frac{i}{2} \text{Re} f_{AB}^{-1} D^B \quad (8)$$

$$M_0 = e^{G/2} \quad (9)$$

$$\bar{z}_{0\alpha\bar{\beta}} = 2 G_{\alpha\bar{\beta}}, \quad \bar{z}_{0AB} = \text{Re} f_{AB} \quad (10)$$

where

$$G(z, \bar{z}) = J(z, \bar{z}) + \ln |g(z)|^2, \quad D = g_A G_\alpha T_A{}^\alpha{}_\beta z^\beta$$

In Eqs. (7) to (10) the subscript '0' refers to a Lagrangian with at most two derivatives on bosons and one derivative on fermions, i.e. to the standard supergravity Lagrangian. In global supersymmetry there are similar formulae with

$$M_0 = 0, \quad C_0^\alpha = \frac{1}{2} \bar{\Phi}^{\alpha\bar{\beta}} \frac{\partial g}{\partial \bar{z}^{\bar{\beta}}}, \quad C_0^A = \frac{i}{2} \text{Re} f_{AB}^{-1} D^B \quad (11)$$

where now Φ is the global Kähler potential and

$$D_A = g_A \Phi_{, \alpha} T^{A \alpha}_{\beta} Z^{\beta} + \mu_A$$

(μ_A is the Fayet-Iliopoulos term).

The global scalar potential is

$$2 C^I Z_I^J C_J. \quad (12)$$

We claim that the scalar potential in an arbitrary global or local supersymmetric theory, with arbitrary derivative interactions and curvature terms of any order, is still given by Eqs. (3) and (12), where now the functions C , M , and Z defined in Eqs. (4), (5), and (6) are no longer given by Eqs. (7) to (10) but rather are functions of these variables:

$$C_I = C_I(C_{0I}, Z_{0I}^J, M_0) \quad (13)$$

$$M = M(C_{0I}, Z_{0I}^J, M_0) \quad (14)$$

$$Z_I^J = Z_I^J(C_{0I}, Z_{0I}^J, M_0) \quad (15)$$

Moreover, the C , M , and Z functions have a formal power series expansion in the variables C_{0I} , Z_{0I}^J , M_0 of the form

$$C_I = C_{0I} + O_C(C_0, M_0, Z_0) \quad (16)$$

$$M = M_0 + O_M(C_0, M_0, Z_0) \quad (17)$$

$$Z_I^J = Z_{0I}^J + O_Z(C_0, M_0, Z_0) \quad (18)$$

where O_C and O_M are at least quadratic in C_0 and M_0 , and O_Z is at least quadratic in C_0 and M_0 .

Equations (3) and (12) for higher-derivative theories follow from a generalization of the same proof that was obtained in the case of the standard supergravity Lagrangian. It can be arrived at using properties of the supersymmetry current algebra [6], or by using Noether techniques [7] or by means of Ward identities [8]. Equations (13) to (18) are more subtle. They follow from the fact that the fermionic supersymmetry variations that are relevant for the scalar potential are related to the

'auxiliary fields' of matter and supergravity multiplets, and to the fact that higher-derivative interactions lead to equations of the form

$$f(A) + AX = 0 \quad (19)$$

where A is a set of auxiliary fields and X is a function of the physical scalar fields. Note that the function $f(A) = \sum_{n \geq 2} C_n A_n$ has coefficients which can depend on the physical scalar fields only through superinvariant functions [as, for instance, the kinetic matrices $f_{AB}(z)$ and the Kähler metric $G_{\alpha\bar{\beta}}$], but this does not affect our conclusion.

Equations (3) and (12) tell us that although the scalar potential may be a rather complicated function of the standard supergravity fermionic variations C_0 and M_0 (see the examples below), it is still under control.

In particular, we are in a position to make general statements about supersymmetric field configurations.

Suppose there exists a configuration for which

$$C_0 = M_0 = 0 \quad (20)$$

Then from Eqs. (16) to (18) the overall scalar potential will be

$$V(\phi) = V_0(\phi) + \delta V(C_0, M_0, Z_0) \quad (21)$$

with $\delta V(C_0, M_0, Z_0)$ at least cubic in the C_0 and M_0 variables. It therefore follows that

$$\frac{\partial V}{\partial Z} = 0, \quad V = 0 \quad (22)$$

is still solved by Eqs. (20). Moreover, since $\partial^2 \delta V / \partial z^\alpha \partial \bar{z}^\beta = 0$, all the particle masses are the same as in the standard theory. This is so because $Z_I{}^J = Z_{0I}{}^J$ when Eqs. (20) are fulfilled.

Let us apply these results to superstring theories. If the superpotential is not renormalized [9] it means that Eqs. (20) always have a solution, as given by the supergravity limit. Because of Eq. (21) it means that these solutions, the gauge symmetry-breaking, and the particle masses computed in the naïve approximation remain exact to all orders in α' . Because string-loop effects do not modify the superpotential, these solutions are unchanged.

The conclusion is that Eqs. (20) are supersymmetric field configurations to all orders of the σ -model as well as of the string-loop expansion. The general result previously obtained allows us, in principle, even to discuss situations with supersymmetry breaking.

Suppose, for instance, we have a naturally positive potential as in no-scale models [10].

This means that there is a chiral field (denoted by T) such that

$$Z_{T0}^{1/2} C_{0T} = \sqrt{3} m_0 \quad (23)$$

as a functional identity.

To oversimplify the situation, suppose that $Z_{Ti} = 0$ ($i \neq T$); then the supergravity potential is

$$V = 2 C_{0i}^i Z_{i0}^j C_{0j} \quad (24)$$

Equation (24) is solved by $C_{0i} = 0$ but C_{0T} undetermined.

Now consider $\delta V(C_{0i}, C_{0T}, Z_0)$. It may happen that δV is independent of C_{0T} , or rather vanishes for $C_{0i} = 0$ (it must be at least quadratic in C_{0i}). Then it would follow that the point $C_{0i} = 0$, C_{0T} arbitrary, is still a supersymmetry-breaking extremum with vanishing cosmological constant for the full theory.

This is, for instance, what would happen if we couple a standard supergravity with flat potential to higher-curvature terms of the gravitational and Yang-Mills multiplets. However, as our last example will show, flatness is not preserved if we consider general higher-derivative interactions for the matter fields.

Of course this fact is related to the problem of zero vacuum energy in the presence of supersymmetry breaking.

We now give two illustrative examples of the previous results in globally supersymmetric theories with high-derivative terms.

The first is an Abelian gauge theory coupled to charged fields with terms quartic in the field strength.

The Lagrangian is

$$\begin{aligned} \mathcal{L} = & \left(\int d^2\theta W^2 + \text{h.c.} \right) + \alpha \int d^4\theta W^2 \overline{W}^2 \\ & + \int d^4\theta \Sigma_\alpha S^\alpha e^{g_\alpha V} \overline{S}^\alpha - \mu \int d^4\theta V, \quad \alpha > 0 \end{aligned} \quad (25)$$

Since the auxiliary fields of the chiral multiplets vanish, the only relevant terms for the scalar potential are

$$\begin{aligned} -V = & \frac{1}{2} D^2 - DX + \frac{\alpha}{4} D^4 \\ X = & \Sigma_\alpha g_\alpha S^\alpha \overline{S}^\alpha + \mu \end{aligned} \quad (26)$$

where D is the auxiliary field of the vector multiplet 0 , and g_i is the gauge coupling of the chiral multiplet S_i .

The fermionic kinetic terms are

$$- \bar{\lambda}_L \not{D} \lambda_R \left(1 + \frac{3\alpha}{2} D^2\right) - \bar{\chi}_L^\alpha \not{D} \chi_R^\alpha \quad (27)$$

The equation of motion for D is

$$X = D \left(1 + \alpha D^2\right) + \text{Fermionic Terms} \quad (28)$$

which, inserted back into Eq. (26), gives

$$V = \frac{1}{2} D^2 \left(1 + \frac{3\alpha}{2} D^2\right). \quad (29)$$

The 'photino' normalization coefficient Z is defined in terms of the Lagrangian, after the elimination of the auxiliary fields $\hat{\mathcal{L}}$, as

$$\begin{aligned} Z &= \frac{\partial \hat{\mathcal{L}}}{\partial(-\bar{\lambda} \not{D} \lambda)} = \frac{\partial \mathcal{L}}{\partial D} \frac{\partial D}{\partial(-\bar{\lambda} \not{D} \lambda)} + \left(\frac{\partial \mathcal{L}}{\partial(-\bar{\lambda} \not{D} \lambda)} \right)_0 = \\ &= \left(\frac{\partial \mathcal{L}}{\partial(-\bar{\lambda} \not{D} \lambda)} \right)_0 = 1 + \frac{3\alpha}{2} D^2, \end{aligned} \quad (30)$$

where we used the equation of motion $\partial \mathcal{L} / \partial D = 0$.

Then we have

$$V = \frac{1}{2} D^2 \left(1 + \frac{3\alpha}{2} D^2\right) = \frac{1}{2} Z(D) D^2 \quad (31)$$

as predicted by the general arguments above [see Eq. (3)].

This kind of argument also gives information on the potential for more general situations, in which arbitrarily high powers of the field strength appear; in the end, we shall have a potential of the form

$$-V = f(D) - DX \quad (32)$$

and kinetic terms

$$- (\bar{\lambda}_L \not{D} \lambda_R) g(D). \quad (33)$$

Then, consistency with Eq. (3) gives the following condition for the functions $f(D)$, $g(D)$:

$$\frac{\partial}{\partial D} \left(\frac{f(D)}{D} \right) = \frac{1}{2} g(D). \quad (34)$$

Equation (28) can easily be solved if we put

$$\eta = \frac{1}{3} \sinh^{-1} \left(\frac{\sqrt{2\lambda}}{2} X \right). \quad (35)$$

Then

$$D = \frac{2}{\sqrt{3\alpha}} \sinh \eta, \quad (36)$$

$$Z = \cosh 2\eta \quad (37)$$

$$V = \frac{2}{3\alpha} \cosh 2\eta (\sinh \eta)^2. \quad (38)$$

From Eqs. (35) to (38) it follows that the new potential is a monotonically increasing function of $1/2 X^2$, which is the potential for the corresponding canonical theory without fourth-order terms (i.e. with $\alpha = 0$). Then the minima of the new potential are exactly in the same position as the old ones, even if we break supersymmetry using the Fayet-Iliopoulos term μ .

Our second example is a supersymmetric σ -model with four derivative couplings. In a standard σ -model,

$$\mathcal{L} = \int d^4\theta \, \Phi(z^\alpha, \bar{z}^\beta), \quad (39)$$

the ‘composite’ vector multiplet Φ possesses a kind of effective Abelian gauge invariance,

$$\Phi \longrightarrow \Phi + i(\Lambda - \bar{\Lambda}) \quad (40)$$

(where Λ is a chiral superfield), which is known as ‘Kähler invariance’. If we want to maintain this effective gauge-invariance in the higher-derivative terms as well, the new couplings must be constructed out of the superfield strength $W_\alpha(\Phi) = \bar{D}\bar{D}D_\alpha\Phi$, for the composite vector Φ .

Then the most general Kähler-invariant Lagrangian, up to four derivatives, is

$$\mathcal{L} = \int d^4\theta \, \Phi(z, \bar{z}) + \left(\int d^2\theta \, g(z) + \lambda \int d^2\theta \, W^2(\Phi) + h.c. \right) \quad (41)$$

$\lambda > 0$

where the holomorphic function $g(z)$ is the superpotential.

The terms in $\mathcal{L}$ that are relevant for the scalar potential are

$$-V = 2 \phi_{\alpha\bar{\beta}} h^\alpha h^{\bar{\beta}} + 4\lambda (\phi_{\alpha\bar{\beta}} h^\alpha h^{\bar{\beta}})^2 + g_\alpha h^\alpha + \bar{g}_{\bar{\alpha}} h^{\bar{\alpha}}, \quad (42)$$

and the fermionic kinetic terms are

$$- Z_{\alpha\bar{\beta}} \bar{\chi}^\alpha \overleftrightarrow{\not{\partial}} \chi^\beta, \quad (43)$$

where

$$Z_{\alpha\bar{\beta}} = 2 \left[\phi_{\alpha\bar{\beta}} (1 + 4\lambda \phi_{\sigma\bar{\rho}} h^\sigma h^{\bar{\rho}}) + 2\lambda \phi_{\alpha\bar{\sigma}} h^{\bar{\sigma}} \phi_{\bar{\rho}\beta} h^{\bar{\rho}} \right]. \quad (44)$$

The h^α equation of motion gives

$$g_\alpha = -2 \phi_{\alpha\bar{\beta}} h^{\bar{\beta}} (1 + 4\lambda \phi_{\sigma\bar{\rho}} h^\sigma h^{\bar{\rho}}), \quad (45)$$

which, inserted back into Eq. (42), gives

$$\begin{aligned} V &= 2 \phi_{\alpha\bar{\beta}} h^\alpha h^{\bar{\beta}} (1 + 6\lambda \phi_{\sigma\bar{\rho}} h^\sigma h^{\bar{\rho}}) \\ &= Z_{\alpha\bar{\beta}} h^\alpha h^{\bar{\beta}}, \end{aligned} \quad (46)$$

in agreement with the general argument [6] [see Eq. (3)].

Equation (45) can be solved by defining

$$\xi = \frac{1}{3} \cosh^{-1} \left(1 + \frac{27\lambda}{2} |dg|^2 \right), \quad (47)$$

where, as usual,

$$|dg|^2 = \phi^{\alpha\bar{\beta}} g_\alpha g_{\bar{\beta}}, \quad (48)$$

$$\phi^{\alpha\bar{\beta}} = (\phi^{-1})_{\alpha\bar{\beta}}. \quad (49)$$

The result is

$$h^\alpha = -\frac{3}{2} \frac{\phi^{\alpha\bar{\beta}} g_{\bar{\beta}}}{1 + 2 \cosh \xi} \quad (50)$$

$$Z_{\alpha\bar{\beta}} = \frac{2}{3} (1 + 2 \cosh \xi) \phi_{\alpha\bar{\beta}} + 9\lambda \frac{g_\alpha g_{\bar{\beta}}}{(1 + 2 \cosh \xi)^2}, \quad (51)$$

$$V = \frac{2}{3\lambda} \cosh \xi \left(\sinh \xi / 2 \right)^2 \quad (52)$$

Note that V is again a monotonically increasing function of $|dg|^2$; that is of the old scalar potential (i.e. the one for $\lambda = 0$). Hence, the minima of the potential are independent of λ and remain in the same positions as in the corresponding standard σ -model, given by Eq. (48). In the present case, this is true even if supersymmetry is broken by some O'Raifeartaigh-type mechanism.

Contrary to the standard case, $Z_{\alpha\bar{\beta}}$ is no longer a Kähler metric; indeed, its torsion is

$$\begin{aligned} T_{\alpha\bar{\beta}\bar{\gamma}} &= \partial_{[\alpha} Z_{\bar{\beta}\bar{\gamma}]} \\ &= \frac{4}{3} \sinh \xi \partial_{[\alpha} \xi \phi_{\bar{\beta}\bar{\gamma}]} - \frac{36 \lambda \sinh \xi}{(1 + \cosh \xi)^3} \partial_{[\alpha} \xi g_{\bar{\beta}\bar{\gamma}]} \bar{g}_{\bar{\gamma}} \end{aligned} \quad (53)$$

which, if the target has dimension larger than one, vanishes if—and only if— $\partial_{\alpha} \xi = 0$; that is, if $\bar{V} = \text{const.}$ Then we have 'Kähler invariance' with a Hermitian metric which is no longer Kählerian!

Our last example has local supersymmetry. It is just the same four-derivative σ -model as above, coupled to $N = 1$ supergravity. In superconformal notation [5], the Lagrangian reads ($\lambda > 0$),

$$\begin{aligned} \mathcal{L} = & \\ & - \frac{1}{2} [\Phi S_0 \bar{S}_0]_D + \left\{ [S_0^3 g]_F + \frac{\lambda}{4} [W^2(3 \ln \Phi)]_F + \text{h.c.} \right\} \end{aligned} \quad (54)$$

Note that now W_{α} is the field strength of the composite vector $3 \ln \Phi$ rather than Φ ; this is due to the fact that in the local case the Kähler invariance becomes multiplicative [11] on Φ , $\Phi \rightarrow \exp(\Lambda + \bar{\Lambda})\Phi$, and so the invariant field strength is $W_{\alpha}(3 \ln \Phi)$.

If, as usual,

$$G = -3 \ln (\Phi / |g|^{2/3}) \quad (55)$$

the Lagrangian has the following component form:

$$\begin{aligned} e^{-1} \mathcal{L} = & e^{-1} \mathcal{L}_{SG} - Z_{\alpha\bar{\beta}} (\bar{\chi}_L^{\alpha} \not{\partial} \chi_R^{\bar{\beta}} + \text{h.c.}) \\ & + G_{\alpha\bar{\beta}} h^{\alpha} \bar{h}^{\bar{\beta}} + \lambda (G_{\alpha\bar{\beta}} h^{\alpha} \bar{h}^{\bar{\beta}})^2 - \\ & - 3 |h_0|^2 |z_0|^{-2} + (3 z_0^2 h_0 + z_0^3 h^{\alpha} G_{\alpha} + \text{h.c.}) + \dots \end{aligned} \quad (56)$$

where z_0 is the compensator multiplet scalar field which—as a gauge choice [12] is put equal to $\exp(1/6 G)$, h_0 is the corresponding auxiliary field, and h^{α} are the auxiliary fields of the physical chiral multiplets. The matrix $Z_{\alpha\bar{\beta}}$ in Eq. (56) is

$$Z_{\alpha\bar{\beta}} = [1 + 2 \lambda G_{\rho\bar{\sigma}} h^{\rho} \bar{h}^{\bar{\sigma}}] G_{\alpha\bar{\beta}} + \lambda G_{\alpha\bar{\rho}} h^{\bar{\rho}} G_{\bar{\beta}\bar{\sigma}} h^{\sigma} \quad (57)$$

Solving the auxiliary field equations and inserting it back into the Lagrangian, we get

$$V = Z_{\alpha\bar{\beta}} h^\alpha h^{\bar{\beta}} - 3e^G, \quad (58)$$

which is just what we expected from the general arguments, since

$$\begin{aligned} \delta \chi^\alpha &= \frac{1}{2} h^\alpha \epsilon + \dots, \\ \delta \psi_\mu &= \frac{1}{2} e^{G/2} \gamma_\mu \epsilon + \dots. \end{aligned} \quad (59)$$

As before, we can get more explicit formulae by solving the auxiliary field equation of motion.

Let

$$\xi = \frac{1}{3} \cosh^{-1} (1 + 27\lambda e^G G^{\alpha\bar{\beta}} G_\alpha G_{\bar{\beta}}), \quad (60)$$

then

$$V(G^{\alpha\bar{\beta}} G_\alpha G_{\bar{\beta}}, e^G) = \frac{2}{3\lambda} \cosh \xi (\sinh \xi/2)^2 - 3e^G, \quad (61)$$

which, apart from the gravitino contribution^{*)} $-3e^G$, has exactly the same structure as in the global model, except that now $\phi^{\alpha\bar{\beta}} g_{\alpha\bar{\beta}}$ is replaced by $e^G G^{\alpha\bar{\beta}} G_\alpha G_{\bar{\beta}}$, as dictated by the exponential form of the Kähler invariance of supergravity. Note that the new fermionic shifts, Eq. (59), turn out to be functions of the old ones (and, of course, of $G_{\alpha\bar{\beta}}$), as they should.

Our results suggest an intriguing possibility, namely that a higher-derivative supersymmetric theory may have different dynamical realizations (phases), only one of which is continuously connected (as the couplings of the higher terms vanish) with the standard theory. This would happen if the relevant equation for the auxiliary fields has more than one solution satisfying the required reality conditions. This phenomenon is not present in the models we have studied above, where there is just *one* solution. Note, however, that positivity of the kinetic terms and therefore absence of negative-metric states put severe constraints on the acceptable exotic solutions. Then it is a non-trivial fact that a given supersymmetric theory has exotic realizations.

^{*)} We note that the gravitino mass is the same as in standard supergravity, because the W^2 term in Eq. (54) does not depend on the compensating multiplet S_0 . This is exactly what happens for a Yang-Mills coupling to supergravity.

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