CERN-TH.4614/86
BRX TH-218

ANTI-HIGGS EFFECT IN TWO-DIMENSIONAL GAUGE THEORIES

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ABSTRACT

Two-dimensional gravitational and vector gauge theories obtained by reduction of $D=3$ topologically massive ones exhibit an "anti-Higgs" effect: the gauge fields are absorbed by their massless scalar or spinor partners which thereby acquire a mass. The analysis of the supersymmetric vector model is also carried out in superspace, where it is shown that it can be correspondingly formulated in terms of pseudo-scalar (rather than spinor) superfield strengths. Its first order formulation is used to derive the self-interaction by a Noether procedure from the Abelian action directly in superspace. The scalar superfield is also used to construct other σ -model like non-linear actions.

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I. Introduction

Topologically massive vector gauge and gravitational theories in three spacetime dimensions [1,2] as well as their supersymmetric completions [3,4] exhibit a number of unusual features, particularly the coexistence of gauge invariance and mass. Odd spacetime dimensionality was essential for the construction of the Chern-Simons characteristics which are specific to these models; they are obtainable by descent from the familiar D=4 Pontryagin invariants. We will see here that a further reduction to D=2 brings out a new set of amusing phenomena. The most interesting of these is that the massless "Kaluza-Klein" scalars and spinors of the reduced theories absorb the gauge fields, thereby acquiring a mass; hence our title. The number of degrees of freedom is of course preserved in the process, since all D=2 gauge fields are non-dynamical. The origin of a Yukawa coupling peculiar to D=2 SYM [5] is also traceable to the dimensional reduction from D=3.

In a parallel analysis we will treat the supersymmetric vector system in superspace. In addition to recovering the same results, we will see that there exists (in addition to the usual spinorial one) a pseudo-scalar superfield version of this theory which is peculiar to D=2 and is in harmony with the fact that the bosonic excitations are scalars rather than vectors. This new scalar superfield description may be useful for more general D=2 σ -models.

II. Vector Gauge Models

The two-dimensional vector model we consider has the action

$$I_2^{(1)} = \int d^2x \left[+\frac{1}{2} G^2 + \mu G\phi - \frac{1}{2} (D_1\phi)^2 \right], \quad G \equiv \epsilon^{ij} F_{ij} \equiv 2F_{01} \quad (1)$$

where $i,j = 0,1$, F_{ij} is the Yang-Mills field strength and D_1 the usual gauge covariant derivative of the scalar field ϕ (the coupling constant g has been set to 1 and internal indices omitted). This action is easily seen to be the naive dimensional reduction (i.e., neglecting all dependence on the third coordinate) of the topologically massive action [1,2]

$$I_3^{(1)} = \frac{1}{4} \int d^3x \left[-F_{\mu\nu}^2 + \mu \epsilon^{\alpha\beta\gamma} (F_{\alpha\beta} v_\gamma + \frac{1}{3} v_\alpha v_\beta v_\gamma) \right] ; \quad (2)$$

A_2 has been renamed ϕ in (1) to emphasize its scalar character in $D=2$. Since the anti-Higgs effect we want to exhibit is a kinematical one, we need consider only the free, quadratic parts of (1). Inclusion of interactions will lead to a more complicated σ -model-like form for the effective scalar action we obtain below. Regrouping the abelian part of (1) as

$$I_2^{(1)} = \int d^2x \left[\frac{1}{2} (G + \mu\phi)^2 - \frac{1}{2} \mu^2 \phi^2 - \frac{1}{2} (\partial\phi)^2 \right] \quad (3)$$

clearly shows that $G + \mu\phi$ vanishes, since its variation with respect to v_i implies that $\epsilon^{ij} \partial_i (G + \mu\phi) = 0$; this leaves an action describing a massive scalar field. [The analysis can also be performed in the Hamiltonian form of the theory, and indeed even more simply in the Hamiltonian form of the $D=3$ theory [2], followed by dimensional reduction.] If an external source of v_i is included, its effective self-interaction is just the attractive finite range form $\int j^i ([\Box - \mu^2]^{-1} j_i) d^2x$, to confirm the massive scalar (rather than massless vector) excitation content (conservation of j^i implies that $j^i = \epsilon^{ij} \partial_j \rho$). The degree of freedom count is of course the same in $D=3$ (one parity violating massive photon excitation) as in $D=2$ (where the vector has no excitations and the scalar has one). The reduction of the fermionic partner of the $D=3$ massive photon is of course trivial: it begins as a massive spinor and remains one in $D=2$, since the number of its components is unchanged. The supersymmetric case will be reconsidered in superspace formalism below. We shall see that for spin $3/2$ the picture is more like that for gauge bosons.

III. Supergravity

We turn now to the topologically massive gravity and supergravity theories and show that both the $D=2$ gravitons and their partners are absorbed by their accompanying Kaluza-Klein scalars and spinors, in a two-step process involving also the Kaluza-Klein vector fields. Again we deal only with the quadratic, kinematic parts. Consider first the spin $3/2$ sector. Here [4]

$$I_3^{(3/2)} = -\frac{i}{2} \int d^3x \left[\bar{\psi}_\mu f^\mu + \frac{1}{2\mu} \bar{f}^\mu \gamma_\nu \gamma_\mu f^\nu \right], \quad f^\mu \equiv \epsilon^{\mu\alpha\beta} \partial_\alpha \psi_\beta, \quad (4)$$

which reduces dimensionally to

$$I_2^{(3/2)} = -i \int d^2x \left[\bar{H}\lambda + \frac{1}{4\mu} (H - \bar{\partial}\lambda)^2 \right], \quad H \equiv f^2 \equiv \epsilon^{ij} \partial_i \psi_j, \quad \lambda \equiv \psi_2 \quad (5)$$

The ψ_1 variation of (5) leads to the first order constraint equation $H = (\partial - 2\mu)\lambda$ in our second derivative model. Inserting it into (5) leaves the massive λ -spinor action

$$I_2^{(3/2)} = -i \int d^2x \bar{\lambda} (\partial - \mu)\lambda \quad (6)$$

so the gauge spinor field has been swallowed by its Higgs particle.

The gravity sector is the most striking example of the scalar field's voraciousness. The topologically massive action describing a single graviton excitation reads [2]

$$I_3^{(2)} = \frac{1}{2} \int d^3x \left[\sqrt{-g} R + \mu^{-1} \frac{1}{8} \epsilon^{\alpha\beta\mu} (R_{\alpha\beta ab} \omega_\mu^{ab} + \frac{1}{3} \omega_\alpha^{ab} \omega_\beta^{bc} \omega_\mu^{ca}) \right]. \quad (7)$$

The linearized Einstein part in $h_{\mu\nu} \equiv g_{\mu\nu} - \eta_{\mu\nu}$ reduces as usual to

$$\frac{1}{2} \int \sqrt{-g} R d^3x \longrightarrow \frac{1}{2} \int d^2x \left[\phi^{(2)} R_L - 2F^2 \right] \quad (8)$$

where $\phi \equiv h_{22}$, $A_i \equiv h_{2i}$, $F \equiv \epsilon^{ij} \partial_i A_j$ and $^{(2)}R_L$ is the linearized two-curvature scalar $^{(2)}R_L$. The Chern-Simons term reduces to

$$\mu^{-1} \int d^2x \left[\frac{1}{2} F^{(2)} R_L - F[[]\phi] \right] \quad (9)$$

so that the sum of (8) and (9) becomes

$$I_2^{(2)} = \frac{1}{2} \int d^2x \left[^{(2)}R_L \frac{1}{2} (\phi + \mu^{-1}F) - F^2 - \mu^{-1}F[[]\phi] \right]. \quad (10)$$

At this level, the gravitational field h_{ij} only appears in $^{(2)}R_L \sim (\partial_{ij}^2 - \delta_{ij} \nabla^2) h_{ij}$, so its variation generates the constraint $\phi + \mu^{-1}F = 0$, whose insertion in (10) yields the massive scalar action

$$I_2^{(2)} = \frac{1}{2} \int d^2x \phi([\] - \mu^2)\phi . \quad (11)$$

This time the scalar has absorbed both the graviton and the photon; the degree of freedom count is unchanged from the single massive graviton in $D=3$.

A shortcut derivation of the same result is obtained by considering the $D=3$ action with all constraints solved, and then dimensionally reducing; this removes some of the glamour of the previous manipulations as well. In [2], it was shown that

$$I_3^{(2)} = \int d^3x \frac{1}{2} \phi([\] - m^2)\phi \quad (12)$$

where ϕ was the following gauge invariant transverse component of the spatial ($m, n = 1, 2$) metric g_{mn}

$$g_{mn} = (\delta_{mn} - \nabla^{-2} \partial_m \partial_n) \phi + (\partial_m \xi_n + \partial_n \xi_m) . \quad (13)$$

Our reduction assumes $\partial_2 = 0$, so $\nabla^2 = \partial_1^2$; it follows that $\phi = h_{22}$, while $h_{11} = 0 = h_{12}$, i.e., there are no gravitational or photon fields in $D=2$, but only the massive scalar ϕ .

For supergravity then, the anti-Higgs transmuted actions remain supersymmetric, as both the scalar ϕ and 2-spinor λ acquire the same mass, just as in the SYM system.

There is another formulation of linearized topologically massive gravity [6] (as there is of the corresponding vector case [7]), namely, a self-dual one. It is

$$-I_3^{(2)} = \frac{\mu}{2} \int d^2x \{ \epsilon^{\mu\nu\alpha} w_{\mu\beta} \partial_\nu w_{\alpha\beta} + \mu (w_{\mu\alpha} w_{\alpha\mu} - w_{\mu\mu}^2) \} \quad (14)$$

in terms of a nonsymmetric tensor field $w_{\mu\alpha}$. Its $D=2$ reduction is

$$-I_2^{(2)} = \mu \int d^2x \{ \epsilon^{ij} (v_\ell \partial_i w_{j\ell} + \phi \partial_i u_j) + \frac{\mu}{2} (w_{ij} w_{ji} - w_{\ell\ell}^2 + 2v_i u_i - 2\phi w_{\ell\ell}) \} \quad (15)$$

where $u_i \equiv w_{i2}$, $v_i \equiv w_{2i}$, $\phi \equiv w_{22}$. The two vector field equations determine (u, v) according to

$$\mu v_i = -\varepsilon_{ij} \partial_j \phi, \quad \mu u_i = -\varepsilon^{j\ell} \partial_j w_{\ell i}. \quad (16)$$

When (u,v) are eliminated, there remains a second order action for (ϕ, w_{ij}) ,

$$-I_2^{(2)} = \int d^2x \{ \phi([\delta_{ij} - \partial_{ij}^2] w_{ij} + \frac{1}{2} \mu^2 (w_{ij} w_{ji} - w_{\ell\ell}^2 - 2\phi w_{\ell\ell})) \}. \quad (17)$$

The antisymmetric part of w_{ij} is clearly an auxiliary field which vanishes.

Decomposing w_{ij} and $\phi_{,ij}$ into their traceless parts and trace,

$$w_{ij} = \bar{w}_{ij} + \frac{1}{2} \eta_{ij} w_{\ell\ell}, \quad \bar{\phi}_{,ij} = (\partial_{ij}^2 - \frac{1}{2} \eta_{ij} []) \phi$$

we may regroup (17) in the form

$$\begin{aligned} -I_2^{(2)} = \int d^2x \{ & \frac{1}{2} \mu^2 (\bar{w}_{ij} - \frac{1}{\mu^2} \bar{\phi}_{,ij})^2 - \frac{1}{4} \mu^2 [w - \frac{1}{\mu^2} ([] - 2\mu^2)\phi]^2 \\ & - \frac{1}{2} \phi ([] - \mu^2) \phi \}. \end{aligned} \quad (18)$$

Obviously, $(\bar{w}_{ij} - \mu^{-2} \bar{\phi}_{,ij})$ and $w - \mu^{-2} ([] - 2\mu^2)\phi$ are both vanishing auxiliary fields, leaving as the reduced action the last term, i.e., the same massive scalar action (11) as in the conventional formulation.

Finally we note that the N=2 topologically massive supergravity [8] is just the sum, at the linearized level, of N=1 supergravity and a spin $(3/2, 1)$ multiplet. Upon reduction to D=2 the two spin 3/2 fields then each become the companions of the massive scalars to which gravity and the photon reduce. In the next sections we shall trace some of the implications for superspace formulations of the scalar character of the D=2 excitations.

IV. Vector Gauge Theory: Superspace Formulation

Because spinors have the same number of components in D=2 and D=3, it is easy to perform the reduction to D=2 from the D=3 superspace formulation [9] of the topologically massive vector model; in particular, the super Chern-Simons term will retain its form. What is especially interesting from the superspace point of view is that there emerges a natural formulation in terms of a scalar superfield, in parallel with the fact that the bosonic part reduced to a scalar excitation.

In D=2, the prepotential V is a Lie-algebra valued Majorana superspinor [5]:

$$V^\alpha = \eta^\alpha + \theta_\beta \left(\frac{1}{2} v^\beta \epsilon^{\beta\alpha} + v^i \gamma_i^{\beta\alpha} + \frac{1}{2} v_2 \gamma_2^{\beta\alpha} \right) + \frac{i}{2} \theta^2 \psi^\alpha. \quad (19a)$$

It transforms according to

$$\delta V^\alpha = -i D^\alpha \omega + i [V^\alpha, \omega] \equiv -i \nabla^\alpha \omega \quad (19b)$$

under an infinitesimal gauge transformation ω . This also defines the gauge-invariant spinor derivative ∇ . Our internal matrices obey $[T_a, T_b] = i c_{abc} T_c$, $T_a = T_a^+$ and we have set $g=1$; internal traces are understood throughout. Our time-space signature is $(0,1) = (-,+)$, $\epsilon^{01} = +1$, and the two γ -matrices (γ^0, γ^1) (as well as γ_2 , which plays the role of γ_5) are real, $\gamma^i \gamma^j = \eta^{ij} - \epsilon^{ij} \gamma_2$. The spinorial metric is $\epsilon^{\alpha\beta} = \epsilon_{\alpha\beta}$, $\epsilon_{12} = 1$. Spinorial derivatives are defined by $D \equiv \partial/\partial\theta + i\partial/\partial\bar{\theta}$, and obey the supersymmetry relations $\{D, \bar{D}\} = 2\not{p}$. [In our calculations we have found it useful to introduce the supersymmetric momentum $q_2 \equiv \frac{1}{2} D^2$, and write the commutators as $[D_\alpha, D_\beta] = 2\epsilon_{\alpha\beta} q_2$; note that $Dq_2 = \not{p}D = -q_2 D$ and that $q_2^2 = p^2$.]

The D=2 field strength, obtained from that of D=3, is the Majorana superspinor W^α

$$W^\alpha \equiv i D_\beta D^\alpha V^\beta - i [V_\beta, D^\beta V^\alpha] + \frac{i}{3} [V_\beta (V^\alpha, V^\beta)], \quad (20)$$

which transforms homogeneously under gauge variations.

The usual SYM action is then

$$I_0 \equiv \frac{1}{8} \int d^2x d^2\theta \bar{W} W \quad (21)$$

where gauge invariance implies the Bianchi identity $\nabla_\alpha W^\alpha \equiv 0$, while the D=2 "Chern-Simons" term is generated by

$$I_\mu \equiv \frac{1}{4} \mu \int d^2x d^2\theta V_\alpha (W^\alpha + \frac{i}{3} [V_\beta, D^\beta V^\alpha] - \frac{i}{6} [V_\beta (V^\alpha, V^\beta)]), \quad (22)$$

it has the same structure as the original D=3 one, but with D=2 spinorial derivatives.

We shall now see how the ostensibly vectorial action $I_0 + I_\mu$ describes a self-interacting (pseudo) scalar system. In Wess-Zumino gauge, $\eta^\alpha = 0 = v$, so the field strength and massive action become

$$W^\alpha = \psi^\alpha + \epsilon^{ij} D_i v_2 \theta_\beta \gamma_j^{\beta\alpha} - 2f(v_1) \theta_\beta \gamma_2^{\beta\alpha} + \frac{1}{2} \theta^2 [\not{D}\psi - \frac{1}{2} \gamma_2 [v_2, \psi]]^\alpha, \quad (23a)$$

$$I_0 + I_\mu = \int d^2x \{ 4f^2(v_1) - (D_i v_2)^2 - i\psi [\not{D}\psi - i\gamma_2 [v_2, \psi]] - \mu(i\psi\psi + 4v_2 f) \}, \quad (23b)$$

where D is the usual covariant gauge derivative and $f \equiv 1/2 \epsilon^{ij} f_{ij}$, $f_{ij} \equiv \partial_i v_j - \partial_j v_i + i[v_i, v_j]$. We have already encountered all these terms* in our earlier component treatment; there we saw that the true variables are the scalar $\phi \equiv v_2$ and the spinor ψ , both having acquired the same mass. More precisely, it is the left-handed component $\frac{1}{2} (1 - \gamma_2) \psi$ which is the scalar's partner. This can be checked explicitly for example in light-front coordinates and gauge ($v_- = 0$), where the action reads

$$I_0 + I_\mu = \int d^2x \left\{ \frac{1}{2} v_2' \dot{v}_2 - \frac{1}{4} v_+ [v_2, v_2'] - \frac{1}{4\sqrt{2}} v_+ [\psi_2, \psi_2] \right. \\ \left. + \frac{1}{2\sqrt{2}} \psi_2 \dot{\psi}_2 + \frac{1}{8} \psi_1 [v_2, \psi_2] + \frac{\mu}{2} v_2 v_+' + \frac{i\mu}{4} \psi_2 \psi_2 \right\} \quad (24a)$$

where $x^\pm \equiv 2^{-1/2}(x^0 \mp x^1)$ and primes and dots denote (x^-, x^+) derivatives. In (24a) v_+ and ψ_1 are functions of (v_2, ψ_2) determined by solving the bosonic and fermionic constraints

$$C_B \equiv v_+' + \frac{1}{4} [v_2, v_2'] + \frac{1}{4\sqrt{2}} [\psi_2, \psi_2] + \mu v_2' = 0, \quad (24b)$$

$$C_F \equiv \sqrt{2} \psi_1' - \frac{i}{2} [v_2, \psi_2] - \mu \psi_2 = 0. \quad (24c)$$

This confirms the (v_2, ψ_2) set to be the dynamical one for the full system.

*Except for the Yukawa term $\sim \psi \gamma_2 \psi v_2$, which was omitted in the abelian discussion of Sec. II. Note that this unusual D=2 coupling has a natural origin in the D=3 model upon reduction.

A superfield understanding of this transformation of the vector to a scalar, specific to $D=2$, can be achieved as follows. From the $D=3$ relation $-i\partial_2 = \frac{1}{2} \bar{D}\gamma_2 D$ it follows that in reducing to $D=2$, $0 = 1/2 \bar{D}\gamma_2 D$. This permits a supersymmetric transverse-longitudinal decomposition of Majorana superspinors V^α in our flat superspace, according to

$$2iV = (\gamma_2 D) V_2^T + DV^L \quad (25)$$

where $i[\gamma_2 V_2^T] = q_2 \bar{D}\gamma_2 V$, $i[\gamma_2 V^L] = q_2 \bar{D}V$. The spinorial divergence D_α ($\text{curl } \bar{D}\gamma_2$) of the first (second) term on the right of eq. (25) vanishes identically. Thus, since at the abelian level $D_\alpha W^\alpha = 0$, W^α is "transverse" and must be the curl of some (pseudo) scalar superfield W_2 . Returning to the full non-abelian theory we define [5]

$$W_2 \equiv \bar{D}\gamma_2 V - \frac{1}{2} \{\bar{V}, \gamma_2 V\} , \quad (26a)$$

which also transforms homogeneously under gauge variations. Obviously, $\nabla^\alpha W_2 = i(\gamma_2 W)^\alpha$; in W - Z gauge it is

$$W_2 = v_2 + i\bar{\theta}\gamma_2\psi + i\theta^2 f(v_1) . \quad (26b)$$

Note that for any scalar superfield Φ transforming like an isovector under gauge transformations, the identity $\bar{D}\gamma_2 D \equiv 0$ becomes $\bar{D}\gamma_2 \nabla\Phi = -[W_2, \Phi]$.

We can now introduce two non linear actions

$$I_\sigma \equiv \frac{1}{8} \int d^2x d^2\theta g_{ab}(W_2) \bar{\nabla}W_2^a \nabla W_2^b ; \quad (27)$$

$$I_\tau \equiv \int d^2x d^2\theta F(W_2) , \quad (28)$$

in terms of the scalar superfield $g_{ab}(W_2)$ transforming as a symmetric product in the adjoint representation of the internal group G and F is a G invariant. The first action describes a (real) σ -model with coordinates v_2^a and metric $g_{ab}(v_2)$ coupled to the gauge fields v_1^a and the spinor ψ^a .

This is manifest in the component form of (27),

$$\begin{aligned}
I_\sigma = \int d^2x \{ & g_{ab}(v_2) \left[-\frac{1}{2} D_1 v_2^a \cdot D_1 v_2^b + 2f^a f^b - \frac{i}{2} \bar{\psi}^a D \psi^b \right. \\
& - \frac{i}{2} \bar{\psi}^a \gamma_2 [v_2, \psi]^b \left. \right] + i \Gamma_{ab,c} [\bar{\psi}^a \psi^b f^c + D_1 v_2^a \bar{\psi}^b \gamma_1 \psi^c] \\
& - \frac{1}{12} R_{acbd}^L \bar{\psi}^a \psi^b \bar{\psi}^c \psi^d \left. \right\}, \quad (29)
\end{aligned}$$

R_{acbd}^L is the linearized part of the Riemann tensor. When there is no gauge field coupling to v_2 or ψ , and the f is treated as an independent variable, its elimination reduces (29) to the usual supersymmetric σ -model [10]. Thus our non-conformally invariant model (because $D=2$ Yang-Mills is not invariant) reduces to the usual conformal σ -model in this limit.

Another σ -model is defined by adding (28) to the pure gauge field action $I_0(W_2)$; the component form of (28) is

$$I_\tau = 4 \int d^2x \left\{ f^a \frac{\partial}{\partial v_2^a} + i \bar{\psi}^a \psi^b \frac{\partial^2}{\partial v_2^a \partial v_2^b} \right\} F(v_2). \quad (30a)$$

With the particular choice $F(W_2) = -\frac{1}{2} \mu (W_2^a)^2$, (30a) reduces to

$$I_\tau = -\mu \int d^2x \{ 4v_2 f + i \bar{\psi} \psi \} \quad (30b)$$

which is exactly the $D=2$ Chern-Simons term (22) obtained from $D=3$ (see (23b)).

The full action (21) + (22) then has the simple form

$$I_0 + I_\mu = \frac{i}{8} \int d^2x d^2\theta \{ \bar{\nabla} W_2^a \nabla W_2^a + 2i \mu (W_2^a)^2 \}. \quad (31a)$$

Note that the original kinetic action (21) in terms of the spinor field W^α has become a typical scalar field action in terms of W_2 because of the relation $\nabla W_2 = i\gamma_2 W$. Variation of the independent field V from which W_2 is constructed in (26a), i.e., $\delta W_2 = \bar{\nabla} \gamma_2 \delta V$, leads to the simple equation

$$\nabla_\beta \left(\frac{i}{2} \bar{\nabla} \nabla + \mu \right) W_2 = 0. \quad (31b)$$

The particularly simple formulation of σ -models in terms of the scalar superfield may be of use also in string-related investigations.

Since the definition (26a) of the D=2 field strength involves only one spinorial derivative, one can also give a first order (in this derivative) version of this system by introducing conjugate momenta (π_α, P) in the usual way

$$I_0 + I_\mu = \int d^2x d^2\theta \left\{ -\frac{1}{8} \bar{\pi} \pi + \frac{1}{4} \bar{\pi} \nabla W_2 - \frac{\mu}{4} W_2^2 + P[W_2 - \bar{D}\gamma_2 V + \frac{1}{2} (\bar{V}, \gamma_2 V)] \right\}, \quad (32)$$

so that now W_2 is independent of V .

We now use this first order form to obtain the full action (32) from its abelian limit by the standard Noether self-interaction mechanism [11] directly in superspace, i.e., starting from the uncoupled system. For concreteness, we sketch the process for SU(2), keeping internal indices explicit. In the abelian limit, the action reduces to

$$I_0^L + I_\mu^L = \int d^2x d^2\theta \left\{ -\frac{i}{8} \bar{\pi}^a \pi^a - \frac{\mu}{4} W_2^a W_2^a + P^a W_2^a + \frac{i}{4} \bar{\pi}^a D W_2^a + P^a \gamma_{2\alpha\beta} D^\alpha V^{\beta a} \right\} \quad (33a)$$

which describes a triplet of uncoupled (pseudo) scalar superfields W_2^a . Any type of superfield Φ_I^a transforms as

$$\delta \Phi_I^a = [\Phi_I, x\omega]^a \quad (34)$$

under a global rotation. The associated Noether current is

$$J_{\alpha a} = \frac{\delta [I_0^L + I_\mu^L]}{\delta D^\alpha \Phi_I^b} \frac{\delta \Phi_I^b}{\delta \omega^a} = \frac{i}{4} (\pi_\alpha \times W_2)^a + (P \times \gamma_{2\alpha\beta} V^\beta)^a. \quad (35)$$

We now seek an interaction term ΔI whose functional derivative with respect to $V^{\alpha a}$ is $J_{\alpha a}$. It can be checked that

$$\Delta I = \int d^2x d^2\theta \left\{ \frac{i}{4} (\bar{\pi} \cdot W_2 \times V) + \frac{1}{2} (P \cdot (\overline{\gamma_2 V}) \times V) \right\}. \quad (33b)$$

Furthermore, there is no next step in this series, since ΔI has no explicit derivatives and hence generates no further Noether term. The sum of the initial abelian action (33a) together with the interaction (33b) is precisely

the total action (32); this also explains the factor $\frac{1}{2}$ in the last, nonlinear, term there.

Finally, we would like to mention that the V_α gauge superfield can be used to formulate (classically) conformal invariant σ -models on coset spaces G/H , V_α being an H -Lie algebra valued auxiliary superfield. A known example is provided by CP^n σ -models which have been extensively studied over the past years [12] .

Acknowledgments

The major parts of this work were completed in December 1982 at Silarg 4, U.S.B., Caracas, whose hospitality S. D. and S. F. enjoyed. Likewise, C. A. thanks Brandeis University for hospitality during its completion.

Support for C.A. and S. D. was provided in part by NSF grant 82-01094, and for S. F. by DOE contract DE-AA03-765F00034.

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