ENERGY AND LUMINOSITY REQUIREMENTS FOR THE NEXT
GENERATION OF LINEAR COLLIDERS*

U. Amaldi

CERN, Geneva, Switzerland

1. THEORETICAL FRAMEWORK: THE STANDARD MODEL

The standard model of the electroweak interaction is very successful in explaining a host of data and, at the same time, very unsatisfactory. This is true not only because it contains about twenty free parameters, but also because it needs an ad hoc mechanism, not yet confirmed experimentally, to justify the rest energies of the intermediate bosons $W^+$, $W^-$ and $Z^0$. At the fundamental level these bosons (which I shall call 'asthenons') are very similar to photons. A photon has zero rest mass and thus, once emitted by a particle in a virtual process, can fly far away from its source (Fig. 1a). Since a mediator of zero mass gives rise to a force of infinite range, according to the well known Heisenberg relation $R \propto \hbar/mc^2$, the Coulomb electric force has infinite range. Why the range of the weak force is instead $\sim 10^{-15}$cm, i.e. why the masses of the asthenons are $\sim 100$ GeV?

The answer theorists give is: Because of the Higgs mechanism. According to the prevailing point of view, all space is filled by a boson field which behaves as the collection of Cooper pairs in a superconductor. The macroscopic wave function of such a collection repels out of the metal the virtual photons of an external magnetic field, so that they do not penetrate in the conductor (Meissner effect). The Higgs field has a similar effect on the asthenons, but the other way around: since it fills all space, it pushes back the asthenons radiated by leptons and quarks, so that they can be felt only at less than $10^{-15}$cm from their source (Fig. 1b,c). The Higgs field fills the physical vacuum because its self-interaction is so strong that the state of minimum energy is reached when the average field is non-zero.

As always in quantum theory, quanta are associated to the Higgs field. They are called 'Higgs particles' or simply 'Higgs'. As the Cooper pairs of normal superconductivity, they carry no intrinsic angular momentum -- in other words they are bosons of spin $J=0$. In the simplest model there is only one neutral Higgs field, but it is well possible that charged Higgs fields are also present. In the following we consider only the simplest possibility, because neutral Higgs particles are the most difficult to detect in electron-positron collisions.
The above arguments do not fix the mass of the neutral Higgs quantum. One of the main arguments in this direction comes from considering W-W scattering at very high energies (Fig. 2a). Without the intervention of Higgs-bosons in the intermediate state, the computed cross-section increases as the square of the center of mass energy ($\sigma \sim E_{\text{cm}}^2$) and violates probability conservation (unitarity, in theoretical parlance) when $E_{\text{cm}} \geq 1800$ GeV. If a scalar ($J = 0$) Higgs particle exists with a mass smaller than about 1 TeV, it will contribute to the virtual processes: calculations show that in this case there is no unitarity violation. This argument gives an upper limit for the mass of the Higgs, but one may wonder if the reasoning is sound enough.

We can find some confirmation going back to past experience and recalling that already once unitarity guided us to a correct mass scale. In fact, Fermi theory for the scattering neutrino-quark depitch in Fig. 2b gives a cross-section which is proportional to $E_{\text{cm}}^2$, so that unitarity is violated when the energy is larger than about 300 GeV. This was a fundamental problem already thirty years ago, and theorists were brave enough to conclude that, to cure this disease, something had to happen at energies smaller than about 300 GeV. We now know that asthenons have masses smaller than 100 GeV, even less than what it is absolutely needed to avoid the feared violation of probability conservation. Experience is thus telling us that unitary arguments are very powerful and the indication of violation in W-W scattering should not be lightly dismissed. By taking at face value the factor of three appearing in the neutrino-quark case (300 GeV for the violation energy, while the asthenon masses are around 100 GeV) we could even be tempted to conclude that the new healing phenomenon should take place around 600 GeV.

2. ABOVE THE STANDARD MODEL

Higgs particles could be bona fide Cooper pairs, so that they can be broken into two fermions at energies of the order of 1 TeV. For this reason some theorists contemplate composite models, in which the known 'elementary' particles are made of more fundamental fermions. However, many more pursue
today the idea of a fundamental Higgs, and here they meet another difficulty: while propagating through space an Higgs particle H emits and reabsors various other particles (Fig. 3) and through their interaction picks up very easily weight. This is a general property of spin zero particles since there is nothing to protect them from becoming massive. Theorists are then faced with the problem of cancelling the many different contributions to the mass increase. As it appears from Fig. 3, the contributions due to virtual bosons (W, Z, H) are positive, while the contributions of virtual fermion pairs (ff) are negative. It can be shown that a miraculous cancellation occurs if each known boson has a related fermion (and vice versa) and their couplings are in a well defined relation.

**Supersymmetry** gives exactly the needed pattern: for each particle of integer (half integer) spin there is a sparticle of half integer (integer) spin (Table 1).

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<th>Table 1: Particle and sparticles</th>
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If the cancellation is to be effective, the masses of the sparticles should be not much larger than 1 TeV, the known limit on the Higgs mass. Other arguments also point at the existence of sparticles, but we cannot discuss them here.

To conclude we will make an even bolder step into the unknown and consider the today fashionable superstring theories, in which fundamental particles are not point-like objects but strings about $10^{-33}$ cm long. Superstring theory gives rise to a consistent treatment of quantum gravity and to supersymmetry, but it is not unique. Some theorists think that in the cold world in which we live it even reduces naturally to the standard model, but with some interesting additions. A particular fashionable model foresees the existence
of an additional neutral asthenon Z', and a set of other 'normal' particles (as a neutrino, a neutral lepton and a quark of charge $-1/3$). These last predictions are very definite but not all experts agree on their necessity. However, they may be useful in trying to indicate reasonable energies and luminosities for the electron-positron linear colliders to be built after SLC and LEP 200.

In summary, theorists indicate to machine builders and experimentalists the following 'discovery targets':

(i) Higgs particle(s) (or a phenomenon which has the same power in explaining the asthenon masses);
(ii) sparticles with masses less than about 1000 GeV;
(iii) a new neutral asthenon $Z'$ of unknown mass;
(iv) various new normal particles, and among them a new quark and a new neutral lepton.

In this list each prediction is less reliable than the previous one.

3. HIGGS PARTICLES

A neutral higgs decays into the heaviest particle-antiparticle pair which is kinematically possible. Monte Carlo studies show that $\geq 10^6$ real Z's decays are needed to collect enough events, if the Higgs mass is smaller than about 50 GeV. At LEP 200 the limit will be pushed to about 80 GeV. If the Higgs mass is larger than 160 GeV, the very characteristic decay $H \to W^+ + W^-$ can probably be seen, even among the many other products of a proton-antiproton or proton-proton collision. This leaves a mass window $80 \leq m_H \leq 160$ GeV which is very difficult to cover with a hadron collider; here electron-positron collisions play an essential role. The c.m. energy needed for such a search is $E > (m_H + 100$ GeV), so that at a linear collider having an energy per beam $E \simeq 1000$ GeV and a luminosity $L \geq 10^{33} \text{cm}^{-2}\text{s}^{-1}$ the reach is $m_H \leq 900$ GeV, very close to the upper limit coming from the arguments given in the first Section.
4. SPARTICLES

Sparticle production can be computed as a function of their assumed masses. Of course virtual sparticles intervene also in the intermediate states and any particular cross-section may depend on many parameters. Detailed calculations have been recently done by C. Dionisi and M. Dittmar in the framework of the ongoing activity on a possible Cern Linear Collider (CLIC). I have summarized some of the results in Fig. 4.

The continuous curve of Fig. 4 represents all what is known today in electron-positron physics. It is based on experimental data up to energies $2E \approx 45$ GeV and on predictions of the standard model above $45$ GeV. The dotted areas represent the ranges of cross-section predicted for reasonable values of the masses of the sparticles. Since we consider pair production (i.e. $e^+ e^-, \tilde{W}^+ \tilde{W}^-, \tilde{\mu}^+ \tilde{\mu}^-$) the mass of each of the final sparticles is just equal to the beam energy $E$. The figure shows that winos are produced with a cross-section which is about five times smaller than the selectrons, while smuon-pairs are created with a rate which is another factor five smaller. (This is true when the photino is light.)

However, rate is not enough: the events have also to have a good signature. Heavy sparticles decay into lighter ones, and the chain ends with the emission of the lightest sparticle. In the previous paragraph I have assumed the neutral photino to be such a sparticle, and since its interaction with matter is weak, the interesting events are expected to show an overall unbalance both in energy and in momentum. But this property is general, even if the photino is not the lightest sparticle.

The signatures and the cross-sections of Fig. 4 are such that one needs a luminosity $L \approx 10^{32}$cm$^{-2}$s$^{-1}$ to collect few good $\tilde{W}^+ \tilde{W}^-$ events per year at a beam energy $E = 500$ GeV, i.e. for a mass $M_{\tilde{W}} = 500$ GeV. To reach larger sparticles masses the luminosity has to follow the usual scaling law, i.e. $L \approx (2E/\text{TeV})^2 10^{32}$cm$^{-2}$s$^{-1}$. Such a luminosity would allow the study of the most abundant channels, if opened. A reasonably complete study of the of the less copious ones requires $L \geq (2E/\text{TeV})^2 10^{33}$cm$^{-2}$s$^{-1}$. 
It is important to remark that the value of the average energy spread $\langle \varepsilon \rangle$ in the bunch-bunch collision is not relevant as far as the event rate is concerned, but may be important for the signature of sparticles events. Beamstrahlung radiation produces an unbalance in the longitudinal momentum which cannot be distinguished from the unbalance caused by the emission of neutrino-like particles in the forward and backward cones. If $\langle \varepsilon \rangle$ is large (let us say $\langle \varepsilon \rangle > 10\%$), the momentum loss in the longitudinal direction is of the same order of the momentum carried away by photinos, and event reconstruction at an electron-positron linear collider may become a problem, even if it is less severe than at hadron-hadron colliders. Of course, conservation of transverse momentum will always be an essential tool in analysing sparticle events.

To put the above arguments in a more general perspective, it is enough to remind that at present it is experimentally known that the masses of the sparticles, most abundantly produced in proton-antiproton collisions, are larger than about 60 GeV. In the next years the study of proton-antiproton collisions at Fermilab should push this limit up to about 150 GeV. It is seen that, in this particular case, an electron-positron collider of (300 + 300) GeV opens a reasonable window in this search, even if theorists indicate a much wider mass range.

5. NEW NEUTRAL ASTHENONS

In the minimal and not unique superstring model, the cross-section for the production of the asthenon $Z'$ of mass $M$ is $\approx 10^{-34} \text{cm}^2/(\text{M/TeV})^2$. The relative width of such a particle is expected to be $\approx 3\%$. The $Z'$ peak of Fig. 4, drawn for a mass $M = 1$ TeV, is very apparent and a luminosity of the order of $10^{34} \text{cm}^{-2}\text{s}^{-1}$ is enough to find it. Of course the average energy spread $\langle \varepsilon \rangle$ has to be smaller than 0.03, otherwise the rate will be reduced and a larger luminosity is required. Ten times more luminosity will be needed for a detailed study of the decay properties of such a particle.

Which are the present limits on the existence of a $Z'$? It is already known that its mass has to be larger than about 200 GeV and it is expected that the proton-antiproton data collected in the next years at
Fermilab will push the limit to about 350 GeV. Muon asymmetry measurements at LEP 200 will see a standard Z' if the mass is $M \leq 600$ GeV. The conclusion is that one needs a linear collider of energy definitely larger than $(300 + 300)$ GeV to cover an unexplored territory.

6. SUMMARY AND PLEA FOR LOW ENERGY LINEAR COLLIDERS

In summary, the discovery of new 'standard' neutral asthenons requires little luminosity [$L \geq 10^{31}$ $(M/\text{TeV})^2 \text{cm}^{-2}\text{s}^{-1}$] but a large energy span, since in few years the limit from other accelerators should be as high as $M \geq 600$ GeV. A luminosity $L \simeq (2E/\text{TeV})^2 \times 10^{32} \text{cm}^{-2}\text{s}^{-1}$ should allow the study of the most abundant sparticle channels, but a study of the less abundant ones requires about a factor of 10 more intensity. For distinguishing the various possible channels it would be better not to have a large fraction of the initial energy radiated by beamstrahlung.

All these arguments do not apply to the search and discovery of the "unknown", here large luminosities are of course a bonus, but the quantification is impossible. Still we can take the example of the measurement of muon asymmetry, which may point to the existence of a Z' having a mass outside the energy range kinematically available. Fig. 4 shows that a luminosity $L \simeq (2E/\text{TeV})^2 \times 10^{34} \text{cm}^{-2}\text{s}^{-1}$ gives about $10^4 \mu^+\mu^-$ events per year; this will allow to explore Z' masses up to $\simeq 4E$, a very interesting possibility indeed.

Since such large luminosities are difficult to achieve, summing-up all arguments, at present most experimentalists consider a luminosity $L \simeq (2E/\text{TeV})^2 \times 10^{33} \text{cm}^{-2}\text{s}^{-1}$ as a reasonable compromise to be given as target to machine builders. Many would feel more comfortable with a factor 3 more rate, i.e. $L \simeq 10^{34} \text{cm}^{-2}\text{s}^{-1}$ at (1+1) TeV. For others a downgrade by a factor of 3 with respect to $(2E/\text{TeV})^2 \times 10^{33}$ may not be disastrous, but, given the small cross-section of many interesting processes, it is clear to everybody that in doing so we are moving on thin ice.
Before closing I would like to underline another use of linear colliders, whose importance has been fully realized only during the last year. In October 1985, at the CERN Accelerator School held in Oxford, I proposed a fully superconducting linear collider to be used as 'toponium factory'. Onium factories must have the possibility of running with a very small energy spread, definitely smaller than the width of the states one wants to produce, and certainly superconducting linacs have this property. During this year the UA1 Collaboration has found evidence of $B\bar{B}$ oscillations, and this has triggered a new interest in beauty factories. A session of this Symposium will be devoted to present ideas. I want here to stress the importance of these new developments, even if somebody may think that they are not really connected with the high energy frontier we are pursuing. From a physics point of view low energy-high luminosity linear colliders offer unique possibilities. On top they will allow the study of some of the fundamental problems facing us when the very costly TeV colliders will have to be built, as for instance the construction of low emittance damping rings and of high gradient structures. Moreover they could bring back good physics and accelerator research to laboratories of moderate dimensions, thus widening again the base of our field of research. I am convinced that our community should take this unique possibility very seriously.

I am very grateful to J.Ellis for useful discussions on the content of this contribution.
Fig. 1  Leptons (and quarks) radiate virtual photons (γ) and virtual asthenos (W, Z), but the Higgs field obliges the asthenons to remain close to the source so that the range of the weak force is $R \approx 10^{-15}$ cm.

Fig. 2 (a) WW scattering and (b) neutrino quark scattering have cross-sections which increase with the center-of-mass energy.

Fig. 3  A Higgs particle (H) gets mass by virtual emission and absorption of (a) asthenons, (b) Higgs and fermion-antifermion pairs.
Fig. 4 The electron positron cross-section plotted versus the c.m. energy $2E$. (From U.Amaldi, Nucl.Inst.Meth. A243, 312, 1986.) The shaded area represent the ranges of the computed sparticle cross-section for the three channels $\bar{W}W$, $\mu\mu$ and $ee$. The dotted peak correspond to a $Z'$ having a mass $M = 1$ TeV.