CERN-TH.4617/86

UCLA 86-TEP-49

SUPERSYMMETRIC BORN-INFELD LAGRANGIANS

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ABSTRACT

The supersymmetric completion of four-dimensional Born-Infeld-type actions is established by means of superspace techniques for the construction of higher-curvature invariants. The coupling of these new actions to supergravity is also obtained. These results are relevant for the understanding of the full effective action of superstring theories.

^{*)} This work has been supported in part by the US Department of Energy, Contract No. DE-AA03-76SF00034.

In four-dimensional space-time the standard Born-Infeld Lagrangian [1] describes a non-linear version of Maxwell theory given by the Lagrangian^{*}

$$\mathcal{L}_B = \det(\delta_{\mu\nu} + F_{\mu\nu})^{1/2} - 1 \quad (1)$$

This Lagrangian has the nice property of becoming a density when coupled to gravity, in which case $\delta_{\mu\nu}$ is replaced by $g_{\mu\nu}$, the metric tensor field.

The Lagrangian given by Eq. (1) describes self-interactions of the electromagnetic field, which are of arbitrary degree in the curvature tensor $F_{\mu\nu}$. The Maxwell term proportional to $F_{\mu\nu}^2$ is obtained by expanding $\mathcal{L}_B$ as a power series of the basic invariants $F_{\mu\nu}^2$ and $F_{\mu\nu}\tilde{F}_{\mu\nu}$.

The important point is that the Lagrangian given by Eq. (1) describes the full effective self-interaction of vector fields (in the Abelian limit) in open bosonic strings [2, 3], in the limit of constant curvature and metric $g_{\mu\nu} = \delta_{\mu\nu}$.

In a recent paper [4] it has been shown that the analogous Lagrangian, in the case of superstrings, has a drastically different form, and it becomes

$$\mathcal{L}_{SS} = \frac{1}{4} F_{\mu\nu}^2 + \mathcal{L}_S, \quad (2)$$

where

$$\mathcal{L}_S = \frac{2}{3} \det(\delta_{\mu\nu} + F_{\mu\nu})^{1/2} \det(1 + \sigma^{\mu\nu} F_{\mu\nu})^{-1} - \frac{2}{3} \quad (3)$$

The basic difference between Eqs. (1) and (2) is that the non-linear modification $\mathcal{L}_S$ contains terms at least of order $F_{\mu\nu}^4$, but most importantly it vanishes for self-dual (or antiself-dual) configurations of the Maxwell curvature.

Note that the above formulae refer to the Euclidean Lagrangian; the Minkowski Lagrangian has an overall minus sign with respect to Eqs. (2) and (3).

In contrast, in Eq. (1) both the F^2 and higher-order terms come from the expansion of the square root of the determinant.

The quartic terms of the two Lagrangians happen to coincide, so the non-linear modifications are different starting from F^6 terms.

It was already suggested in Ref. [4] that the two terms in Eq. (2) correspond to an F- and D-type action, if a supersymmetric completion of Eq. (2) exists. The result of the present note is to prove that the bosonic Lagrangian given by Eq. (2) has a *natural* supersymmetric extension in terms of higher-curvature invariants, which are given in terms of the field strength multiplet of an Abelian gauge theory.

^{*}) In this paper $F_{\mu\nu}$ stands for the dimensionless curvature $2\pi\alpha'F_{\mu\nu}$. An overall normalization factor $(2\pi\alpha'g)^{-2}$ for the action is also understood; α' is a parameter of dimension $[m]^{-2}$ which in the case of strings coincides with the Regge slope.

Interestingly enough we will also show that if the vector field is coupled to charged matter, then there is a modification of the scalar potential which just happens to be the same as in the case of a polynomial theory, including terms that are quartic in the curvatures [5].

This potential has recently been derived and its properties explored [5]. The crucial ingredient in the supersymmetric completion of the Born-Infeld action is the curvature supermultiplet [6]

$$W_\alpha = \bar{D}\bar{D} D_\alpha V \quad (4)$$

(V is a gauge vector superfield), with the property that the chiral multiplet W^2 contains the term $(F^-)^2 - D^2$ in its last (F) component. Here $F^\pm$ are the (complex) self-dual and antiself-dual combinations of the Maxwell curvature,

$$F^\pm = \frac{F \pm \tilde{F}}{2}, \quad (F^\pm)^2 = \frac{1}{2}(F^2 \pm F\tilde{F}). \quad (5)$$

(Note that in our combinations $\tilde{F}^* = -\tilde{F}$ in Minkowski space.) Consider now the quartic Maxwell invariant

$$W_\alpha^2 \bar{W}_{\dot{\alpha}}^2. \quad (6)$$

It has the property that its last (D) component contains the term

$$(F^+)^2 (F^-)^2 = \frac{1}{4}[(F^2)^2 - (F\tilde{F})^2] \quad (7)$$

as well as the terms

$$D^4 - D^2 F^2. \quad (8)$$

Consider now the chiral superfield

$$T(W^2) = \bar{D}\bar{D} W^2 \quad \text{and its complex conjugate } \bar{T}. \quad (9)$$

These superfields have, as their first component, precisely the complex combinations

$$(F^\pm)^2 - D^2. \quad (10)$$

Then we can construct a general invariant of the form^{*)}

^{*)} It can be shown that the invariant given by Eq. (11) is nothing but the explicit solution of the recurrence formula derived in Ref. [7].

$$\Psi(T, \bar{T}) W^2 \bar{W}^2 \quad (11)$$

which will give a contribution to the component Lagrangian of the type

$$\Psi(F^{+2}, F^{-2}) F^{+2} F^{-2} \quad (12)$$

as far as the pure Maxwell curvature terms are concerned. It is now obvious that we can fix the Ψ function from the form of $\mathcal{L}_S$ as given in Eq. (3).

In fact, from the explicit expression of the determinants [4]

$$\det(\delta_{\mu\nu} + F_{\mu\nu}) = \left(1 + \frac{1}{4} F^2\right)^2 - \frac{1}{16} [(F^2)^2 - (F\tilde{F})^2] \quad (13)$$

$$\det(1 + \sigma \cdot F) = 1 + \frac{1}{4} F^2 + \frac{1}{4} \frac{(F^2)^2 - (F\tilde{F})^2}{16}, \quad (14)$$

we get

$$\mathcal{L}_S = A(X, Y) \frac{(F^2)^2 - (F\tilde{F})^2}{16} \quad (15)$$

where

$$A(X, Y) = \frac{2}{3} \frac{[(X^2 - Y)^{1/2} - (X + \frac{1}{4}Y)]}{Y(X + \frac{1}{4}Y)} \quad (16)$$

$$= -\frac{2}{3} \frac{1 + \frac{1}{16}Y + \frac{1}{2}X}{(X + \frac{1}{4}Y)[(X^2 - Y)^{1/2} + (X + \frac{1}{4}Y)]}$$

and

$$X = 1 + \frac{1}{4} F^2, \quad Y = \frac{1}{16} [(F^2)^2 - (F\tilde{F})^2]. \quad (17)$$

In terms of the T and $\bar{T}$ multiplets, we have

$$X = 1 - \frac{1}{2} (T + \bar{T}), \quad Y = T\bar{T} \quad (18)$$

so that

$$A(X, Y) = \Psi(T, \bar{T}). \quad (19)$$

Note that $A(Y = 0, X = 1) = -1/2$, so the leading term in $\mathcal{L}_S$ is simply given by

$$-\frac{1}{32}[(F^2)^2 - (F\tilde{F})^2] . \quad (20)$$

The term given by Eq. (20) is nothing but the square of the Maxwell energy momentum tensor, owing to the fact that the superfield $W^2\bar{W}^2$ is the square of the supercurrent multiplet [8] $V_{\alpha\dot{\alpha}} = W_\alpha\bar{W}_{\dot{\alpha}}$.

The full supersymmetric Born-Infeld action is therefore given by^{*)}

$$\int d^2\theta W^2 + \int d^4\theta \Psi(T, \bar{T}) W^2 \bar{W}^2 . \quad (21)$$

The supersymmetric completion of $\mathcal{L}_S$ simply amounts to having a non-trivial Ψ function in front of the quartic Yang-Mills invariant.

Incidentally, we note at this point that $\mathcal{L}_B$ given by Eq. (1) also admits a supersymmetric completion, even if it will probably not correspond to any superstring theory.

The completion looks less natural than in the superstring case, but it can be carried out because $\mathcal{L}_B$, although non-vanishing for self-dual configurations, reduces to the Maxwell term $F_{\mu\nu}^2$. This means that the non-polynomial corrections to F^2 vanish for self-dual F , and we can envisage a supersymmetric completion as for Eq. (3). In fact this completion has the same general form as that of Eq. (21) but with a different Ψ function. To compute Ψ in this case we just note that Eq. (1) can be rewritten as

$$\begin{aligned} \mathcal{L}_B &= X - 1 + Y \frac{(X^2 - Y)^{1/2} - X}{Y} = \\ &= \frac{1}{4} F^2 - \frac{1}{16} [(F^2)^2 - (F\tilde{F})^2] B(X, Y) , \end{aligned} \quad (22)$$

with

$$B(X, Y) = \frac{1}{(X^2 - Y)^{1/2} + X} = -\Psi_B(T, \bar{T}) . \quad (23)$$

Let us now discuss the uniqueness of these Lagrangians. It is obvious that they are *not* unique since they are defined up to terms with derivatives of the curvatures and up to terms proportional to the equations of motion of the free Maxwell theory^{**) .} In superspace the latter terms are of the form

^{*)} The component expression corresponding to Eq. (21) will be given elsewhere.

^{**) There is another sort of non-uniqueness (present also in the standard two-derivative case [10]) which is due to the fact that one can add superinvariants which vanish when the fermions are set to zero.}

$D^\alpha W_\alpha$. Since the $D^\alpha W_\alpha$ multiplet has the D auxiliary field of the Maxwell supermultiplet as its first component, the square $(D^\alpha W_\alpha)^2$ will have D^2 as its first component. Note that $D^\alpha W_\alpha$ is real because of the Bianchi identity $D^\alpha W_\alpha = \bar{D}_{\dot{\alpha}} \bar{W}_{\dot{\alpha}}$. We may therefore modify the chiral multiplets T and $\bar{T}$ into non-chiral ones, V and $\bar{V}$, by adding terms proportional to $(D^\alpha W_\alpha)^2$, such that the first component of V is F^2_+ instead of $F^2_+ - D^2$. In particular, we may use the multiplet^{*)} $D_{(\alpha} W_{\beta)} = F_{\alpha\beta}$, whose square differs from T by terms of the form $(D^\alpha W_\alpha)^2$ and $W_\beta D^\beta D^\alpha W_\alpha$ which vanish when $D^\alpha W_\alpha = 0$. The multiplet $F_{\alpha\beta}$ allows us to rewrite the second term in Eq. (21) as a superspace determinant. In fact in Eq. (3) we may replace $F_{\mu\nu}$ with the corresponding superfield $\text{Re}(\sigma_{\mu\nu}^{\alpha\beta} F_{\alpha\beta})$. Then the bosonic determinant is the first component of the superfield

$$\det [\delta_{\mu\nu} + \text{Re}(\sigma_{\mu\nu}^{\alpha\beta} F_{\alpha\beta})] ,$$

whilst the fermionic determinant is the first component of the superfield

$$\det(\epsilon_{\alpha\beta} + F_{\alpha\beta}) \det(\epsilon_{\dot{\alpha}\dot{\beta}} + F_{\dot{\alpha}\dot{\beta}}) ,$$

It then follows that the $\mathcal{L}_S$ term is given by the superfield corresponding to $\mathcal{L}_S$ integrated over the new measure $d^4\theta(W^2 \bar{W}^2/Y)$. Note that this measure is non-singular on superfields which vanish at least like Y , i.e. on self-dual configurations. This corresponds to the fact that the Ψ function is non-singular for $Y \rightarrow 0$.

All these expressions can be generalized to local supersymmetry, using the arguments given below. None of these different choices of multiplets for inserting in the Ψ function would affect the pure curvature part of the action; they would affect only terms involving fermions. However, these different choices would make a computational difference when the Abelian field is coupled to charged supermatter, since in this case the D field no longer vanishes on bosonic configurations.

If, instead of the T multiplet, we choose the V multiplet alluded to before, then the D equations of motion are not affected by the Ψ function, which can be just set equal to $\Psi(Y = 0, X = 1) = \text{const.}$ So the scalar potential will be the same as for a theory with only W^2 and $W^2 \bar{W}^2$ terms. Such a theory and the corresponding scalar potential have recently been constructed [5]. The scalar potential has the standard form in terms of the photino variation $\delta\lambda = [D(R)\epsilon]/2$ and the kinetic metric $Z[D(R)]$,

$$V = \frac{1}{2} Z D^2 , \quad (24)$$

^{*)} This multiplet naturally appears in the constrained formulation of super Yang-Mills theories [9].

where now D and Z are rather complicated functions of the naïve D term of the standard Maxwell theory,

$$R = q_i C_i C_i^* , \quad (25)$$

C_i being a set of charged fields with charges q_i .

The explicit forms of D and Z are [5]

$$D = 2 \sqrt{\frac{2}{3}} \sinh \eta , \quad Z = \cosh \eta \quad (26)$$

and

$$\eta = \frac{1}{3} \sinh^{-1} \left(\frac{1}{2} \sqrt{\frac{2}{3}} R \right) . \quad (27)$$

The actions we have been writing here are ‘globally’ supersymmetric actions. However, our Lagrangian can easily be coupled to supergravity using superconformal techniques [11].

Let us first discuss the ‘non-minimal’ formulation [analogous to Eq. (21)] of the global case, and postpone the discussion of the field-redefinition ambiguities and minimality to the end.

Obviously, in order to couple the above Lagrangians to supergravity in a manifestly supercovariant way, the Weyl and chiral weights of W^2 and of the T -operation in Eq. (9) need to be compensated. The dependence of the resulting conformal multiplets on the chiral compensator field S_0 [12], induces — through the superconformal gauge conditions — a coupling of the above theories to the chiral scalar σ -model (if chiral multiplets are present at all). Then we have a potential ambiguity in the precise form of the coupling with the scalar’s σ -model. However, there is a unique geometrically natural way of resolving this ambiguity, requiring the coupling to be super-Kähler-invariant [7].

The above amounts to defining a Kähler-covariant T -operation, denoted by T_K , as follows:

$$T_K(W^2) = \Sigma [(S_0 \bar{S}_0 \Phi)^{-2} \bar{W}^2] , \quad (28)$$

where Σ is the superconformal chiral projection, S_0 is the chiral compensator superfield ($w = n = 1$), and^{*)}

$$\Phi = \exp \left[- \frac{1}{3} J(z, \bar{z}) \right] ,$$

where $J(z, \bar{z})$ is the Kähler potential of the standard $N = 1$ supergravity form [11]. The $T_K(W^2)$ so defined is a chiral superconformal multiplet with $w = n = 0$, and its Kähler properties are discussed in Ref. [7].

^{*)} For future convenience we have defined Φ to be $1/3$ of the usual one given in Ref. [11].

Moreover, the explicit factor $W^2 \bar{W}^2$ in Eq. (11) must be replaced by its Kähler-invariant superconformal counterpart, having the weights $w = 2, n = 0$, which is

$$W^2 \bar{W}^2 (S_0 \bar{S}_0 \Phi)^{-2}. \quad (29)$$

Then Eqs. (18) and (21) in the local case become

$$\begin{aligned} \mathcal{L} = & -3 [\Phi S_0 \bar{S}_0]_D + \left(\frac{1}{2} [W^2]_F + h.c. \right) + \text{superpotential} \\ & + [\Psi(T_K, \bar{T}_K) W^2 \bar{W}^2 (S_0 \bar{S}_0 \Phi)^{-2}]_D, \end{aligned} \quad (30)$$

where Ψ is Ψ_A (Ψ_B) for the superstring (bosonic string) case, $[\dots]_F$ and $[\dots]_D$ are the superconformal chiral and vector densities [11, 12], and in Eq. (30) the first line corresponds to the standard terms of supergravity.

Note that in the improved conformal gauge [12], $(S_0 \bar{S}_0 \Phi)$ has, as its first component, just 1; so the new term, which makes the T-operation in Eq. (28) covariant, does not spoil our previous analysis of the bosonic components of Eq. (30), at least if the chiral auxiliary fields vanish.

We now turn to the discussion of the ‘minimal scheme’ [4]. The global supermultiplets $D^\alpha W_\alpha$, $D_{(\alpha} W_{\beta)}$, and $W_\beta D^\beta D^\alpha W_\alpha$ all have a superconformal and super-Kähler-invariant counterpart. In particular, $D^\alpha W_\alpha$ is by itself a superconformal multiplet with the correct weights $w = n = 0$; then $(D^\alpha W_\alpha)^2$ is a multiplet which does not contain the compensator S_0 , has $w = n = 0$, and its first component is just D^2 . Defining

$$\mathcal{F} = T_K(W^2) - (D^\alpha W_\alpha)^2, \quad (31)$$

a ‘minimal’ Lagrangian is again given by Eq. (21) with the substitution

$$T_K \longrightarrow \mathcal{F}. \quad (32)$$

Now the scalar potential has the universality property discussed above for the global case.

Finally, we would like to make some speculation on the possible generalization of the Lagrangian as given by Eq. (21) for the full superstring action (neglecting terms with derivatives in the curvatures).

It is tempting to argue that the general Lagrangian, beyond the standard supergravity terms, has the same structural form as given by Eq. (21), where now the W refers to a set of Yang–Mills, gravitational, and σ -model curvatures, and the Ψ superfield is a matrix function of suitable chiral superfields present in the theory.

In order to obtain a realistic four-dimensional effective Lagrangian, one should also generalize the Born-Infeld coupled supergravity system to other multiplets present in superstring theories containing the dilaton and antisymmetric tensor fields.

These couplings naturally arise if one would consider a consistent truncation or a compactification from a higher-dimensional theory [3].

Recently, partial results on the global supersymmetric completion of quartic Yang-Mills curvature terms in $D = 6$ and $D = 10$ have been obtained [13]. However, the Born-Infeld-type Lagrangians involve infinitely many terms in the curvatures, and Noether coupling techniques seem inadequate for obtaining the full non-linear theory. Other methods such as superspace [14] or the group manifold approach may prove more efficient [15].

Acknowledgements

We would like to thank P. Di Vecchia, M. Duff and L. Girardello for useful discussions.

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