CERN-TH.4599/86

UCLA-86-TEP-47

FLAT POTENTIALS IN HIGHER DERIVATIVE SUPERGRAVITY

S. Cecotti

CERN - Geneva

S. Ferrara^{*)}

CERN - Geneva

and

Department of Physics
University of California, Los Angeles

and

L. Girardello^{**)}

Dipartimento di Fisica, Università di Milano
and INFN, Sezione di Milano

ABSTRACT

In $N = 1$ supergravity, the addition of higher derivative couplings drastically modifies the structure of the scalar potential. Despite this, the corresponding flat potential models are closely related to the standard flat models: they still have a characterization in terms of complex Monge-Ampère equations. As a result, there exists a map $\phi: R \rightarrow R$ such that, if G_0 is the Kähler-invariant function of a standard flat model, $G = \phi \circ G_0$ is the corresponding function for the given higher derivative flat potential model. We explicitly construct the map by ϕ by quadratures. A specific example is also discussed.

^{*)} This work has been supported in part by the US Department of Energy, Contract No. DE-AA03-765F00034.

^{**)} Partially supported by the Ministry of Public Instruction (MPI), Italy.

As it is well known in a standard supergravity theory^{*)}, the scalar potential reads¹⁾

$$V = e^G (G^{\alpha\bar{\beta}} G_{\alpha} G_{\bar{\beta}} - 3) \quad (1)$$

where G is the Kähler-invariant function specifying the geometry of the scalar field manifold.

Recently^{2),3)}, it was realized that Eq. (1) is no longer valid when higher derivative couplings are added to the Lagrangian. In particular, if we start with a function G corresponding to a flat potential model^{4),5)} (i.e., V is identically vanishing), and then we add higher derivative terms, the resulting theory will be no longer flat. Indeed, in typical situations, there is simply no field configurations for which V vanishes.

In this paper we discuss the conditions under which an arbitrary Kählerian higher derivative supergravity is flat, i.e., has an identically vanishing scalar potential. In the case of standard supergravity, it was shown in Ref. 5) that the model is flat if and only if

$$(i \partial \bar{\partial} e^{-\frac{1}{3}G})^n = 0 \quad (2)$$

where n is the (complex) dimension of the Kähler manifold. We shall see that in general, a characterization analogous to Eq. (2) is still valid, where now $\exp[-1/3G]$ is replaced by another function of G , $\Omega(G)$, whose specific form depends on the details of the higher curvature terms present in the Lagrangian. This fact has two important consequences: i) the complex manifolds underlying the flat potential models have the same local structure as in ordinary supergravity; ii) the problem of finding explicit flat models in higher derivative supergravity is equivalent, up to quadratures, to that of finding standard flat models.

Since our results apply to the special class of Kähler-invariant theories⁶⁾, we begin by recalling briefly their definition [see Ref. 6) for more details]. The standard supergravity Lagrangian [in conformal notation⁷⁾] is¹⁾

*) We consider theories with chiral supermultiplets only.

$$\mathcal{L}_0 = -[\Phi(z^\alpha, \bar{z}^{\bar{\alpha}}) S_0 \bar{S}_0]_D + ([g(z^\alpha) S_0^3]_F + h.c.) \quad (3)$$

where z^α , $\alpha = 1, \dots, n$, are the physical chiral multiplets, S_0 is the compensator chiral multiplet, and the holomorphic function $g(z^\alpha)$ is the chiral superpotential. Equation (3) has the following invariance (Kähler-invariance)

$$S_0 \rightarrow e^{-i\lambda/3} S_0, \quad \Phi \rightarrow e^{i(\lambda - \bar{\lambda})/3} \Phi, \quad g \rightarrow e^{i\lambda} g \quad (4)$$

(λ is a chiral superfield), which acts on the composite vector multiplet $3\ln \Phi$ as an Abelian gauge invariance

$$3 \ln \Phi \rightarrow 3 \ln \Phi + i(\lambda - \bar{\lambda}). \quad (5)$$

Requiring Eq. (4) to be an invariance of the full theory puts severe restrictions on the higher derivative terms⁶⁾. Indeed, the chiral fields z^α can appear in the superconformal Lagrangian only through the basic invariant chiral multiplets $\Sigma[\Phi S_0 \bar{S}_0]$, $g S_0^3$ and $W^2(3\ln \Phi)$, the square of the field strength of the composite vector (5). We can write down a general Lagrangian using the compact notation of Ref. 6). Starting from $\Sigma_{(1,0,0)} = \Sigma[\Phi S_0 \bar{S}_0]$, $\Sigma_{(0,1,0)} = W^2(3\ln \Phi)$ and $\Sigma_{(0,0,1)} = S_0^3 g$, we define recursively the following chiral multiplets⁶⁾

$$\Sigma_{\vec{m} + i\vec{m}^*} = \Sigma \left[\Sigma_{\vec{m}} \bar{\Sigma}_{\vec{m}^*} (\Phi S_0 \bar{S}_0) \right] \quad (6)$$

where $\vec{m}$ is a three-vector of complex integers (i.e., Gauss integers). A general Lagrangian of a Kählerian higher derivative supergravity is

$$\mathcal{L} = \frac{1}{2} \mathcal{L}_0 + \sum_{\vec{m} \in \mathbb{Z}[i]^3} \lambda_{\vec{m}} [\Sigma_{\vec{m}}]_F + h.c. \quad (7)$$

where $\lambda_{\vec{m}}$ are some numerical coefficients. As discussed in Ref. 6), we can also add couplings to higher gravitational terms in which case the indices in Eqs. (6) and (7) take values on higher rank $\mathbb{Z}[i]$ modules.

From the explicit form of the components of the above basic invariant multiplets, it is obvious that, after the conformal gauge-fixing and elimination of the auxiliary fields, the scalar potential has the form

$$V = V(G^{\alpha\bar{\beta}} G_{\alpha} G_{\bar{\beta}}, e^G) \quad (8)$$

where

$$G = -3 \ln(\mathbb{I}/|g|^{2/3}) \quad (9)$$

is the Kähler-invariant function and $G^{\alpha\bar{\beta}}$ is the inverse of the matrix $G_{\alpha\bar{\beta}} = \partial_{\alpha} \partial_{\bar{\beta}} G$. Of course, V depends also on the coupling parameters λ_m .

Now, a flat potential model is, by definition, specified by a function G such that

$$V = V(G^{\alpha\bar{\beta}} G_{\alpha} G_{\bar{\beta}}, e^G) \equiv 0. \quad (10)$$

The condition $V(x,y) = 0$ is the implicit equation of some plane curve that may be made explicit by solving Eq. (10) for the first argument in function of the second^{*)}. Then, we can rewrite Eq. (10) as

$$G^{\alpha\bar{\beta}} G_{\alpha} G_{\bar{\beta}} = \frac{1}{H(G)} \quad (11)$$

where $H(G)$ depends parametrically on the λ_m 's. Below we show that we can solve Eq. (11), provided $H(G)$ is positive, (as it is in all known cases).

Let us study for the moment the $n = 1$ case. Then Eq. (11) may be rewritten as

$$G_{z\bar{z}} = H(G) G_z G_{\bar{z}}. \quad (12)$$

In order to solve this equation, let us define

$$\Omega(G) = \int^G dx e^{-\int_0^x H(s) ds}. \quad (13)$$

Then,

*) This is always possible by the local inversion theorem, the general identity for the potential [see Ref. 3)] and Kähler covariance.

$$\partial_z \Omega(G) = e^{-\int_0^G H(s) ds} G_z \quad (14)$$

and

$$\begin{aligned} \partial_{\bar{z}} \partial_z \Omega(G) &= \\ &= e^{-\int_0^G H(s) ds} [G_{\bar{z}z} - H(G) G_{\bar{z}} G_z] \equiv 0 \end{aligned} \quad (15)$$

because of Eq. (12).

Then, $\Omega(G)$ is harmonic and, up to irrelevant field redefinitions, the general solution to Eq. (15) is

$$\Omega(G) = -3(z + \bar{z}). \quad (16)$$

Let $\phi(x)$ be the inverse function of Ω^* . Then, from Eq. (16), we have that a flat model is specified by the function

$$G = \phi[-3(z + \bar{z})] \quad (17)$$

In the standard case, $H(G) = 1/3$, $\Omega(G) = 3 - 3\exp[-G/3]$ and $G = -3\ln[z + \bar{z}]$, as it is well known⁴⁾. We see that for $n = 1$, the new flat potential, Eq. (17) is a function of the standard one G_0 , $G = \Omega^{-1}[3 - 3\exp(-G_0/3)]$. As we shall see in a moment, this turns out to be the general case.

For $n > 1$, let us define

$$\Omega_n(G) = \int_0^G dx e^{-n \int_0^x H(s) ds} \quad (18)$$

Then,

$$\partial_\alpha \Omega_n(G) = e^{-n \int_0^G H(s) ds} G_\alpha \quad (19)$$

and

^{*}) ϕ exists globally since $\Omega(G)$ is a strictly monotonically increasing function.

$$\begin{aligned} G^{\alpha\bar{\beta}} \partial_{\alpha} \partial_{\bar{\beta}} \Omega(G) &= \\ &= n H(G) e^{-n \int_0^G H(s) ds} [G^{\alpha\bar{\beta}} G_{\alpha} G_{\bar{\beta}} - H(G)^{-1}] \end{aligned} \quad (20)$$

Comparing Eqs. (11) and (20), we see that the Kähler function G corresponds to a flat potential if and only if

$$\square \Omega_n(G) = 0 \quad (21)$$

where $\square = \bar{\partial} \partial + \partial \bar{\partial}$ is the Laplace-Beltrami operator computed with the Kähler metric $G_{\alpha\bar{\beta}}$. For instance, in the standard case, $\Omega_n(G) = (3/n) - (3/n) \exp(-nG/3)$, and Eq. (21) corresponds to $\square \phi^n = 0$, see Refs. 4) and 5).

Now we are ready to give a general characterization of flat potential models, which in the standard case, reduces to that of Ref. 5), $(i\partial\bar{\partial}\phi)^n = 0$, where $\phi = -3\exp(-G/3)$.

From Eq. (13), we have

$$i \partial \bar{\partial} \Omega(G) = e^{-\int_0^G H(s) ds} [L - H(G) i \partial G \wedge \bar{\partial} G] \quad (22)$$

where $L = i\partial\bar{\partial}G$ is the Kähler form. Then,

$$\begin{aligned} [i \partial \bar{\partial} \Omega(G)]^n &= \\ &= e^{-n \int_0^G H(s) ds} L^n - n L^{n-1} e^{-n \int_0^G H(s) ds} H(G) i \partial G \wedge \bar{\partial} G = \\ &= e^{-n \int_0^G H(s) ds} L^n + i L^{n-1} \partial \left(e^{-n \int_0^G H(s) ds} \right) \wedge \bar{\partial} G = \\ &= e^{-n \int_0^G H(s) ds} L^n + i L^{n-1} \left[\partial \left(e^{-n \int_0^G H(s) ds} \bar{\partial} G \right) - \right. \\ &\quad \left. - e^{-n \int_0^G H(s) ds} \partial \bar{\partial} G \right] = i L^{n-1} \partial \bar{\partial} \Omega_n(G). \end{aligned} \quad (23)$$

and⁸⁾,

$$\begin{aligned}
 i L^{n-1} \partial \bar{\partial} \Omega_n(G) &= \frac{i}{n} [\Lambda, L^n] \partial \bar{\partial} \Omega_n(G) = \\
 &= \frac{i}{n} L^n \Lambda \partial \bar{\partial} \Omega_n(G) = \frac{i}{n} L^n [\Lambda, \partial] \bar{\partial} \Omega_n(G) = \\
 &= -\frac{1}{n} L^n \partial \bar{\partial} \Omega_n(G) = -\frac{1}{n} L^n \square \Omega_n(G).
 \end{aligned} \tag{24}$$

That is

$$[i \partial \bar{\partial} \Omega(G)]^n = -\frac{1}{n} L^n \square \Omega_n(G). \tag{25}$$

Comparing Eq. (25) with Eq. (21), we have our main result: G gives a flat potential if and only if,

$$[i \partial \bar{\partial} \Omega(G)]^n = 0 \tag{26}$$

in particular, the result of Ref. 5) is obtained putting in Eq. (29) the function Ω corresponding to the standard supergravity

$$\Omega_{ST}(G) = 3[1 - \exp(-G/3)] \tag{27}$$

Equation (26) has important consequences, both from a purely mathematical and practical point of view. First of all, from Eqs. (22) and (26) we see that $\Omega(G)$ has exactly the same properties as $\Omega_{ST}(G)$ in the standard supergravity case. Then if G_0 is the Kähler-invariant function specifying a standard flat model, a solution to Eq. (10) is given by a function G such that

$$\Omega(G) = \alpha \Omega_{ST}(G_0) \tag{28}$$

where $\alpha > 0$ is a proportionality coefficient.

In other words, if Ω^{-1} denotes the inverse of the function Ω^*),

$$G = \Omega^{-1} [3\alpha(1 - \exp(-G_0/3))] \tag{29}$$

Note that Eqs. (22), (26) and (28) and $H(G) > 0$ imply that the Kähler metric $G_{\alpha\bar{\beta}}$

*) Ω^{-1} exists globally, see the previous footnote.

- with G given by Eq. (29) - is positive definite, so all relevant conditions are met.

Equation (29) is the map between the flat potentials of standard supergravity and of the higher derivative theory we are looking for; of course, it depends parametrically on the coefficients λ_m of Eq. (7). Since the map $\Omega^{-1}(\cdot)$ can be constructed by quadratures - see Eq. (13) - the problem of finding flat potential models in higher derivative supergravity is equivalent to the standard one (up to quadratures).

Unfortunately, in general the map (29) does not transform complete Kähler metrics into complete ones. To see this, we need to discuss the behaviour of the map (28) on the boundary of the physical domain, i.e., as $G \rightarrow \pm\infty^*$. In the limit $G \rightarrow -\infty$, supersymmetry is asymptotically restored; since supersymmetric configurations are not "renormalized" by higher derivative couplings³⁾, in this limit we find $G \sim G_0 - 3\ln \alpha$, and hence the component of the boundary in which $G \rightarrow -\infty$ is the same as in the standard case. The situation is different for the other component of the boundary where $G \rightarrow +\infty$. Indeed, the right-hand side of Eq. (28) is bounded above by 3α in the physical domain for the standard model G_0 , whereas - unless for large positive G we have

$$\int_0^G H(s) ds \gtrsim (1+\epsilon) \ln G. \quad (30)$$

the left-hand side of Eq. (28) diverges as $G \rightarrow \infty$. Then, the physical domain M_0 of the original model G_0 is mapped by Eq. (29) into a subdomain of the new physical domain M , in which

$$G < \Omega^{-1}(3\alpha) < +\infty.$$

Stated in more physical terms, this means that, unless Eq. (30) holds, the physical domain (i.e., the domain in which the kinetic terms are positive) gets enlarged by the higher derivative interactions. The function G continues to be expressed by Eq. (28) but now $\Omega_{ST}(G_0)$ must be continued outside the physical domain of standard supergravity, M . One important consequence is that the group of holomorphic automorphisms of the Kähler manifold gets modified. For instance,

*) In general the boundary has three components: 1) $G \rightarrow +\infty$, 2) $G \rightarrow -\infty$, 3) $(\partial\bar{\partial}G)^n = 0$. If G is real analytic the last one can be ignored.

for $n = 1$ we pass from the $SU(1,1)$ group of the standard case⁴⁾ to the affine group

$$A = \{z \mapsto az+b; a, b \in \mathbb{C}, a \neq 0\},$$

however, the isometries of the Kähler function, which are a subgroup of the isometries of the Kähler metric, remain intact [a non-compact $U(1)$, in the $n = 1$ case]. We shall discuss these phenomena in an explicit model below.

From the above discussion, we see that the new flat potential models are characterized by the same local geometrical structure of the standard case, i.e., by Monge-Ampère foliations [see Ref. 9) for a discussion], but are globally very different Kähler manifolds (in general, at least).

In most particular cases, the above general procedure of finding flat functions G by quadratures turns out to be rather unpractical, and we need some more manageable, approximate technique to solve Eq. (26). We shall discuss some possibilities in the context of a specific illustrative example.

A four-derivative example

As an example, we consider the four-derivative Kähler σ -model coupled to $N = 1$ supergravity discussed in Ref. 3). In superconformal notation, its Lagrangian reads

$$\mathcal{L} = -\frac{1}{4} [\Phi S_0 \bar{S}_0]_D + [g S_0^3]_F + \frac{\lambda}{4} [W^2 (3 \ln \Phi)] + h.c. \quad (31)$$

where, for definiteness, we assume $\lambda > 0$. Defining the two functions,

$$G = -3 \ln(\Phi / |g|^{2/3}) \quad (32)$$

$$\xi = \frac{1}{3} \cosh^{-1} (1 + 27 \lambda e^G G^{\alpha\bar{\beta}} G_{\alpha} G_{\bar{\beta}}) \quad (33)$$

the scalar potential is³⁾

$$V = \frac{2}{3\lambda} \cosh \xi (\sinh \xi / 2)^2 - 3 e^G \quad (34)$$

which is of the form in Eq. (8). $V \equiv 0$ implies

$$(\cosh \xi)^2 - \cosh \xi - 9 \lambda e^G = 0 \quad (35)$$

that is

$$\cosh \xi = \frac{1}{2} [1 + (1 + 36\lambda e^G)^{1/2}]. \quad (36)$$

Since,

$$\cosh 3\xi = 4(\cosh \xi)^3 - 3\cosh \xi, \quad (37)$$

we get

$$G^{\alpha\bar{\beta}} G_{\alpha} G_{\bar{\beta}} = 2 + \frac{e^{-G}}{54\lambda} [(1 + 36\lambda e^G)^{3/2} - 1]. \quad (38)$$

Note that the right-hand side of Eq. (38) goes to 3 as $\lambda \rightarrow 0$ (which gives back the standard theory), as it should.

Comparing Eqs. (11) and (38), we have

$$H(G) = \frac{54\lambda e^G}{108\lambda e^G - 1 + (1 + 36\lambda e^G)^{3/2}} \quad (39)$$

notice that Eq. (39) depends only on the particular combination λe^G ; so $G \rightarrow -\infty$ is equivalent to $\lambda \rightarrow 0$. This explicitly shows our statement above, that approaching (asymptotically) a supersymmetric configuration (i.e., as $G \rightarrow -\infty$) we get the same function $H(G)$ as in the standard theory, $\lambda = 0$.

The first quadrature may be performed in a closed form yielding

$$\begin{aligned} \int_0^G H(s) ds &= \frac{1}{9} \ln \frac{(1 + 36\lambda e^G)^{1/2} - 1}{(1 + 36\lambda e^G)^{1/2} + 2} - \\ &- \frac{2}{3} \frac{1}{[2 + (1 + 36\lambda e^G)^{1/2}]} - \ln \rho \end{aligned} \quad (40)$$

[here ρ is a constant, determined in such a way that the right-hand side of Eq. (40) vanishes for $G = 0$].

Then, $\Omega(G)$ is given by

$$\rho \int_0^G dG \left[\frac{(1 + 36\lambda e^G)^{1/2} + 2}{(1 + 36\lambda e^G)^{1/2} - 1} \right] e^{\frac{2}{3} \frac{1}{[2 + (1 + 36\lambda e^G)^{1/2} + 2]}} \quad (41)$$

Equivalently, we have that the new flat potential Kähler function G is expressed in terms of the standard one G_0 as

$$G = G [3\alpha (1 - \exp - G_0/3)] \quad (42)$$

where $G(x)$ is the (unique) solution to the Cauchy problem

$$\frac{dG}{dx} = \exp \left[\int_0^G H(s) ds \right] \quad (43)$$

$$G(0) = 0 \quad (44)$$

and the argument of the exponential in Eq. (43) is given explicitly by Eq. (40).

As $G \rightarrow +\infty$

$$H(G) \simeq \frac{e^{-\frac{1}{2}G}}{4\sqrt{\lambda}} \quad (45)$$

i.e.,

$$\Omega(G) \simeq A G e^{-2H(G)} [1 + o(e^{-\frac{1}{2}G})] \quad (46)$$

where A is some constant. So, for large positive G (and $n = 1$ for concreteness) Eq. (28) becomes

$$AG \simeq 3\alpha [1 - (z + \bar{z})] \quad (47)$$

so we see that $\text{Re} z$ now takes in the physical domain M arbitrarily large negative values. Since $\text{Re} z \rightarrow +\infty$ corresponds to $G \rightarrow -\infty$, and hence to the standard boundary, we see that the physical domain ($n = 1$) is now the entire complex plane $\mathbb{C}$ (or, perhaps, the infinite cylinder) whereas in the standard case the physical domain is the half-plane $\Pi_+ = \{z \in \mathbb{C} : \text{Re} z > 0\}$ (or the semi-infinite cylinder). The automorphism groups are correspondingly changed.

Having reduced the problem to quadratures, even when the relevant integrals cannot be computed in closed form, there exists a number of approximate techniques available to compute the flat potential models with higher derivatives.

One of the most efficient approximation technique is given by a combination of adiabatic and series expansion; eventually, it becomes exact (at least in a

certain subdomain of the Kähler manifold M) when the number of chiral multiplets $n \rightarrow \infty$; indeed, Eq. (18) can be evaluated by saddle point methods in this limit.

Let

$$G_* = \ln m_{gr}^2|_{\text{vacuum}} \quad (48)$$

where $m_{gr}|_{\text{vacuum}}$ is the gravitino-mass computed in the vacuum configuration. If the derivative of the function $H(G)$ at G_* is small, the right-hand side of Eq. (11) can be considered to be a constant $H_*^{-1} = H(G_*)^{-1}$, at least for not too large fluctuations of the scalar fields from their vacuum value. But if the right-hand side is a constant, Eq. (11) can be reduced to the standard equation for flat potentials simply by rescaling the Kähler function G . Then, in this adiabatic approximation, the answer is simply

$$G - G_* = \frac{1}{3H_*} G_0 \quad (49)$$

where the function G_0 corresponds to a standard flat potential and it is normalized so that it vanishes on the given vacuum field configuration. Equation (49) shows that, to the leading order in the above "adiabatic" approximation, the Kähler metric (in a neighbourhood of the vacuum configuration) of the new flat potential model is just a rescaling of an old one. [Notice, however, that in presence of higher derivatives and supersymmetry braking, the scalar σ -model metric is not the same as the Kähler metric $G_{\alpha\bar{\beta}}$. See Ref. 3) for a discussion.]

We can use the result in Eq. (49) as a starting point for a systematic power series approximation.

Let

$$\Omega_*(G) = \int_{G_*}^G dx e^{-\int_{G_*}^x H(s) ds} \quad (50)$$

Then we have

$$\begin{aligned} \Omega_*(G) = & (G - G_*) - \frac{1}{2} H_* (G - G_*)^2 + \frac{1}{6} (H_*^2 - H_*') (G - G_*)^3 - \\ & - \frac{1}{24} (H_*^3 - 3H_* H_*' - H_*'') (G - G_*)^4 + O[(G - G_*)^5] \end{aligned} \quad (51)$$

where $H_*', H_*'', \dots$ are the derivatives of $H(G)$ computed for $G = G_*$.

Let G_0 be a standard flat potential Kähler function normalized so that $G_{0*} = 0$ (i.e., G_0 vanishes for the vacuum field configuration).

The desired function G is then (locally!) a solution to the equation

$$3 H_* \Omega_*(G) = 3 (1 - \exp -G_0/3) \quad (52)$$

which, using Eq. (51) yields

$$G - G_* = G_0 + \frac{1}{2} \frac{H'_*}{H_*^2} G_0^3 + O(G_0^4). \quad (53)$$

REFERENCES

- 1) E. Cremmer, B. Julia, J. Scherk, S. Ferrara, L. Girardello and P. van Nieuwenhuizen, Nucl. Phys. B147 (1979) 105;
E. Cremmer, S. Ferrara, L. Girardello and A. van Proeyen, Phys. Lett. 116B (1982) 23; Nucl. Phys. B212 (1983) 413;
E. Witten and I. Bagger, Phys. Lett. 115B (1982) 202.
- 2) S. Cecotti, S. Ferrara, L. Girardello, M. Porrati and A. Pasquinucci, Phys. Rev. D33 (1986) 2504.
- 3) S. Cecotti, S. Ferrara and L. Girardello, CERN preprint TH.4595/86 (1986).
- 4) E. Cremmer, S. Ferrara, C. Kounnas and D.V. Nanopoulos, Phys. Lett. 133B (1983) 61;
J. Ellis, C. Kounnas and D.V. Nanopoulos, Nucl. Phys. B241 (1984) 406; Nucl. Phys. B247 (1984) 373.
- 5) R. Barbieri, E. Cremmer and S. Ferrara, Phys. Lett. 163B (1985) 143.
- 6) S. Cecotti, S. Ferrara, L. Girardello and M. Porrati, CERN preprint TH.4572/86 (1986) and UCLA-86-TEP-40 (1986), to appear in Phys. Lett. B.
- 7) E. Cremmer, S. Ferrara, L. Girardello and A. van Proeyen, Nucl. Phys. B212 (1983) 413;
T. Kugo and S. Uehara, Nucl. Phys. B222 (1983) 125, Nucl. Phys. B228 (1983) 49.
- 8) S. Goldberg, "Curvature and Homology", Dover 1982.
- 9) E. Bedford and M. Kalka, Comm. Pure and Appl. Math. XXX (1977) 543;
E. Bedford and D. Burns, Ann. Scuola Norm. Sup. Pisa 6 (1979) 381;
W. Stoll, Ann. Scuola Norm. Sup. IV, V.VII, No.1 (1980) 87;
D. Burns, Ann. of Math. 115 (1982) 349.