Frequency maps analysis of tracking and experimental data for the SLS storage ring

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Abstract
Frequency Maps Analysis (FMA) has been widely used in beam dynamics in order to study dynamical aspects of the particles linear and non-linear motion, such as optics functions distortion, coupling, tune-shift and resonances. In this paper, FMA is employed to explore the dynamics of models of the Swiss Light Source (SLS) storage ring and compare them with measured turn by turn (TxT) position data. In particular, a method is proposed for estimating the momentum spread using synchrotron sidebands of the Fourier spectrum of the TxT data.

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Abstract

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INTRODUCTION

The SLS is a third generation light source which provides photon beams of high brilliance to 20 beam lines. Due to the requirement of producing ultra low equilibrium emittances, the SLS lattice is prone to non-linearities. In order to study the non-linear dynamics of the SLS ring, FMA [1, 2] is applied on the ideal lattice.

An additional aspect of the FMA, is the ability to measure optics properties from TxT experimental data [3]. In this paper, we focus on the estimation of the momentum spread, $\sigma_\delta$. In this respect, an analytical relationship of $\sigma_\delta$ and its dependence to synchrotron sidebands is deduced and is applied to the BPM TxT data measured in the SLS.

FREQUENCY MAPS ANALYSIS

Laskar’s FMA obtains from a set of numerically tracked particles the fundamental frequencies of motion. The tracking program PTC is employed for this purpose, for a total of 1056 turns, for on-momentum and for the off-momentum cases $\delta = \pm 1.5\%$. NAFF performs a refined Fourier analysis to the obtained trajectories, constructing the frequency map of the system. In addition, a diffusion coefficient can be derived by the change in transverse tunes as computed in two consecutive time spans. The random resonance lines in the frequency maps are shown with blue lines, systematic resonances with red and the dashed lines are resonances due to skew sextupoles.

In Fig.1, the left plots represent the on momentum case with magnet fringe fields taken into account by the tracking code, whereas the right plots are obtained with fringe fields switched off. In the first case, the dynamic aperture is clearly reduced showing higher diffusion rates than the case with fringe fields off. For particles at horizontal amplitude $x=5$ mm and above, there is a high diffusion zone for small vertical values. This corresponds in the frequency space to the crossing of a multitude of quite high order resonances. Lowering the tune may avoid these resonance crossings. In addition the $4^{th}$ order resonance seems to cause severe limitations for the particles survival. In the case with no fringe fields, the dynamic aperture is larger as expected and particles are less chaotic, although still high diffusion zones exist in smaller density, especially close to the crossing of the 4th and 5th order resonance.

For the off-momentum case, two separate tracking runs were performed for $\delta = \pm 1.5\%$ with fringe fields on and with model chromaticities $Q'_x = 4.92$, $Q'_y = 4.69$, so the working point is expected to move accordingly. Left plots of Fig.2 show the diffusion and frequency map for $\delta=-1.5\%$, while right plots show the $\delta=+1.5\%$ case. For the former case, the dynamic aperture is larger in the vertical amplitudes compared to the on-momentum case with fringe fields on. The main limitation seems to come from the 2nd order sum resonance (1,1) which distorts frequency space and increase diffusion rates for around 6 mm in both planes. For the positive momentum spread, the dynamic aperture is severely limited by the crossing of the 2nd order, half integer horizontal resonance, which limits the horizontal plane a 10 mm and the vertical one at 6 mm.

Figure 1: Diffusion (top) and frequency maps (bottom) for the on-momentum case, with (left) and without magnet fringe field. All points are coloured according to the frequency diffusion coefficient.
The following approximation can be used
Eq.(2) yields $A_q$ around the main frequency. The amplitudes influence of chromaticity exhibits synchrotron sidebands

column:Case with $\delta=1.5\%$. Top figure is the diffusion map and bottom figure is the frequency maps with various resonances shown. Again, top figure is the diffusion map and bottom figure is the frequency maps with various resonances shown. In all figures, a colormap was used with the values of the diffusion coefficient in logarithmic scale.

**ENERGY SPREAD MEASUREMENTS**

The Fourier spectrum of the TBT BPM data in the influence of chromaticity exhibits synchrotron sidebands around the main frequency. The amplitudes $A_q$ of the chromatic sidebands of order $q$, in the presence of non-linearities, is given by [4]:

$$A_q = e^{-s^2} \left| a I_0(s^2) + \frac{\Delta \beta_1}{4\beta} \sigma_q(is)[I_{q-1}(s^2) - I_{q+1}(s^2)] \right|$$

where $s = \frac{Q_s}{\sqrt{Q_s^2 - Q_x^2}}$, with $Q_s$ the horizontal chromaticity and $Q_x$ the synchrotron tune, $a$ the initial condition of the beam distribution, $\Delta \beta_1$ the chromatic beta beat and $I_q$ the modified Bessel function of the first kind. By using the recursion relationship $I_{q-1}(z) - I_{q+1}(z) = \frac{2}{\beta} I_q(z)$, for Bessel functions of order $q$ and argument $z$, Eq.(1) becomes

$$A_q = e^{-s^2} aI_0(s^2)(1 + \frac{\Delta \beta_1}{2\beta} \frac{Q_s}{Q_x} q)$$

(2)

This form can be used to derive relationships for $\sigma_\delta$ and $\Delta \beta_1$. For $q = 1$ and calculating the ratio $\frac{A_1 + A_{-1}}{A_0}$ from Eq.(2) yields

$$\frac{A_1 + A_{-1}}{A_0} = \frac{2I_1(s^2)}{I_0(s^2)}$$

(3)

The following approximation can be used $I_q(z) \approx \frac{\Gamma(q + 1)}{\Gamma(q + 1 + T)^2} z^q$, where $\Gamma(q + 1) \approx q!$ which is only valid if $0 < |z| < \sqrt{\frac{q+1}{q+T}}$. For the case of SLS, the theoretical momentum spread is $\sigma_{\delta_{th}} = 8.58 \times 10^{-4}$ and the synchrotron tune is $Q_s = 6.24 \times 10^{-3}$ and $Q_x = 4.92$, so the condition is met. By applying to Eq.(3), and solving for $\sigma_\delta$ yields:

$$\sigma_\delta = \pm \frac{Q_s}{Q_x} \sqrt{\frac{A_1 + A_{-1}}{A_0}}$$

(4)

Eq.(4) gives the momentum spread with respect to the Fourier amplitudes of the synchrotron sidebands and the ones of the main betatron tune. An expression of the chromatic beta beat can be also obtained as

$$\frac{\Delta \beta_1}{\beta} = \frac{2Q_s}{Q_x} \left( \frac{A_1 - A_{-1}}{A_1 + A_{-1}} \right)$$

The previous expressions are used to estimate the momentumspread from experimental TTX data. The bunches are kicked transversally and the TTX data exhibit oscillation under the influence of decoherence and recoherence. The period of the bunch centroid’s oscillation, $N_T$, is about 200 turns. The synchrotron tune is given by the inverse of $N_T$, i.e. $Q_s = 1/N_T = 5 \times 10^{-3}$ for the experimental data. A slow damping is also observed due to synchrotron radiation. Decoherence caused by chromaticity and finite tune spread modulates the bunch centroid’s motion and it is damped to zero at $N_T/2$ turns while it recoheres at $N_T$ turns. First, standard frequency analysis using the NAFF algorithm on the available set of 988 turns was concluded. Before 200 turns, synchrotron sidebands are not observed due to resolution of the measurements. For this reason, the analysis is done from the 200th turn up to the 988th with a constant turn window of 50 turns. The synchrotron tune for each BPM and for every turn window, was computed by the distance of the synchrotron side-bands in each BPMs with respect to the main tune. The analysis was done using both the positive and the negative harmonics of the frequency spectrum, in order to determine any significant contribution differences. The model value of $Q_s x$ was used. In the top part of Fig. 4, the average measured momentum spread $\langle \sigma_\delta \rangle$ is shown over all the BPMs versus the number of turns. In the bottom the corresponding
mean synchrotron tune measurements $\langle Q_x \rangle$ with bars representing one standard deviation is shown, in order to illustrate the dependence of the synchrotron tune on turns and its effect in momentum spread estimation. From 300 turns to 500 turns, the measurements of momentum spread are very close to the theoretical value with uncertainty of $10^{-4}$ and an offset of $10^{-5}$ from the theoretical value. After 500 turns uncertainty is at the order of $10^{-5}$ with an offset of $10^{-4}$ from the ideal value. Synchrotron sidebands resolution was better for the positive band while no significant differences were observed in measurements. The synchrotron tune for the corresponding turns, has an offset of $10^{-4}$ with uncertainty of $10^{-3}$ at every turn window except the bump on 400 turns, and after 550 turns, it converges to a value with an offset of $10^{-3}$ from the theoretical value.

Figure 4: Mean momentum deviation with standard deviation over BPMs vs consecutive turns $N$ for a constant window of 50 turns. In blue is measurements from the positive band, in magenta from negative band, and red dashed is the theoretical value for the SLS. Missing values correspond to no solution

The second stage of the analysis involves a similar procedure but this time a moving turn window of constant width is used over consecutive turns. This type of analysis will reveal the TxT dependence of the method’s parameters with the best possible resolution. In Fig. 3, average momentum spread and synchrotron tune measurements over the BPMs vs. their corresponding turn fractions are shown. For 100 turns window, the synchrotron tune oscillates around the theoretical value with $N_T/2$ period. Standard deviation is of the order of $10^{-5}$, approaching the theoretical value every $N_T/2$ at an offset of $10^{-4}$. Measurement of momentum spread approaches the theoretical value every $N_T/2$ turns, like the synchrotron tune, with an offset and uncertainty of $10^{-5}$ and $10^{-4}$ respectively. For 200 turns, the synchrotron tune approaches the theoretical value every $N_T$ turns with an offset of $10^{-4}$ and a standard deviation of $10^{-5}$. This works in favour of synchrotron sidebands observation and enhances the measurements for the momentum spread which shows an uncertainty of $10^{-5}$ and a varying offset between $10^{-4}$ and $10^{-5}$. For 300 turns, the synchrotron tune starts with an offset of $10^{-4}$ from the theoretical value and standard deviation around $10^{-3}$. Every $N_T/2$, it goes away from the theoretical value at about $10^{-3}$. Every $N_T$, the offset and uncertainty are the same with initial values and there is a slow damping of the measurements as turn fraction increases. Momentum spread has the least offset from the theoretical value every $N_T/2$ turns with a varying offset between $10^{-4}$ and $10^{-5}$. Turn dependence of the synchrotron tune affects significantly the accuracy of the momentum spread measurements. The window of 200 turns appeared to give better results.

In Fig. 5, the mean amplitudes of $A_0$ and $A_1 + A_{-1}$ and $A_0$ over all BPMs are shown, vs. their corresponding turn fractions for each window case. For all cases, $A_1 + A_{-1}$ oscillates twice faster than $A_0$ as expected. $A_0$ is larger than $A_1, A_{-1}$ and it dominates momentum spread calculation. For $A_1 + A_{-1}$ the case of 200 turns provides more data points which explains larger number of momentum spread and synchrotron tune measurements. For all cases, the trade off between $A_1, A_{-1}$ and the synchrotron tune turn dependence, define the efficiency of the method.

CONCLUSIONS

FMA technique was used on the ideal model of SLS, investigating the non-linear dynamics for the on and off-momentum case and revealing resonances that can lead to particle losses. The behavior of the ideal SLS lattice can be used for a comparison with a future experimental frequency maps analysis. In addition an equation for extracting the momentum spread directly from the Fourier spectrum of the TxT data was given and tested in real TxT data of the SLS giving promising results but yet needs to be understood more. A relationship for chromatic beta beating was derived in the same manner and an analysis is currently undergoing.

REFERENCES

[3] P. Zisopoulos, Y. Papaphilippou, A. Streun, V. Ziemann, Beam optics measurements through turn by turn position data in the SLS, IPAC 2013, WEPEA067