COMPTON SCATTERING AND PION NUMBER IN NUCLEI

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ABSTRACT

We explore the relation between the spin independent Compton amplitude and the pion excess number per nucleon n_π in nuclear matter and for the deuteron. We show that the enhancement factor K of the photonuclear sum rule, which appears as a meson exchange correction to this amplitude, is related to n_π by $K \approx (16/3) n_\pi M \langle \omega_\pi^{-1} \rangle$ where $\langle \omega_\pi^{-1} \rangle$ is the mean inverse pion energy. This relation is justified by the consideration of the chiral limit where the pion mass vanishes.

The spin independent part of the forward Compton amplitude on a composite object is subject to two constraints:

- at low energy it is given by the Thomson limit $A_{LE} = -(Ze)^2/M_{tot}$ where Ze is the charge and M_{tot} the total mass of the system
- at high energy, well above the system internal excitations, and in the absence of exchange forces, it is the incoherent sum of the Thomson amplitude on the charged constituents.

In the nuclear case these two limits are $A_{LE} = -(Ze)^2/AM$ and $A_{HE} = -Ze^2/M$, where M is the nucleon mass. However the fact that the nuclear interaction is mediated by the exchange of charged particles modifies the high energy result and in fact A_{HE} has to be written as [1]:

$$A_{HE} = -e^2Z/M - e^2(ZN/AM)K \quad (1)$$

where K can be extracted from a subtracted dispersion relation for the forward Compton amplitude [2,3].

In ref. [1] it was suggested that the correction factor should be interpreted as the Thomson scattering on the virtual exchanged charged pions, the last contribution being enhanced by the small value of the pion energy as compared to the nucleon one. So $(ZN/AM)K \cong (A/4M)K$ was identified with $A(2/3)n_\pi \langle \omega_\pi^{-1} \rangle$ where $(2/3)n_\pi$ is the charged pion excess number per nucleon and $\langle \omega_\pi^{-1} \rangle$ is the average inverse pion energy, leading to:

$$n_\pi = 3/8 K / (\langle \omega_\pi^{-1} \rangle M) \quad (2)$$

Actually such a relation would provide an evidence for the pion excess and also a possibility of estimation, as K is based on experimental quantities and the magnitude of $\langle \omega_\pi^{-1} \rangle$ is under control. However Eq. (2) is an oversimplified guess which we want to explore in detail in the present work.

Let us start by the structure of the enhancement factor K . In the one pion

exchange (OPE) potential model its value, K_π , is related to the OPE potential V_{OPE} (in r or q space) by

$$K_\pi = -1/3 \langle 0 | r^2 V_{\text{OPE}}(r) | 0 \rangle = 1/3 \langle 0 | (\partial/\partial \mathbf{q})^2 V_{\text{OPE}}(q) | 0 \rangle \quad (3)$$

It is the sum of the nuclear average of the four amplitudes [4,5] represented diagrammatically in fig.s 1a,b,c,d. These diagrams evaluated for photon momentum $\mathbf{k}=0$, lead respectively to the following contributions:

$$K_1 = g^2/[\pi^2 M] \int_0^\infty S_C(q) q^2/(q^2+\mu^2) dq \quad (4a)$$

$$K_2 = g^2/[\pi^2 M] \int_0^\infty -4/3 [S_C(q)+S_T(q)] q^4/(q^2+\mu^2)^2 dq \quad (4b)$$

$$K_3 = g^2/[\pi^2 M] \int_0^\infty 4/3 [S_C(q)+S_T(q)] q^6/(q^2+\mu^2)^3 dq \quad (4c)$$

$$K_4 = g^2/[\pi^2 M] \int_0^\infty -[S_C(q)+S_T(q)] q^4/(q^2+\mu^2)^2 dq \quad (4d)$$

where g is the pseudoscalar coupling constant $g=13.4$ and $S_C(q)$ and $S_T(q)$ are the spin central and tensor correlation functions defined as follows:

$$S_C(q) = 1/A \sum_{i \neq j} \langle 0 | 1/9 (\underline{\sigma}_i \cdot \underline{\sigma}_j) (\underline{\tau}_i \cdot \underline{\tau}_j) \exp(i\mathbf{q} \cdot \mathbf{r}_{ij}) | 0 \rangle \quad (5a)$$

$$S_T(q) = 1/A \sum_{i \neq j} \langle 0 | 1/9 [3(\underline{\sigma}_i \cdot \hat{\mathbf{r}}_{ij})(\underline{\sigma}_j \cdot \hat{\mathbf{r}}_{ij}) - (\underline{\sigma}_i \cdot \underline{\sigma}_j)] (\underline{\tau}_i \cdot \underline{\tau}_j) \exp(i\mathbf{q} \cdot \mathbf{r}_{ij}) | 0 \rangle \quad (5b)$$

Here $\underline{\sigma}$ and $\underline{\tau}$ are the spin and isospin Pauli matrices and μ is the pion mass. In Eq.s (5a,b) the dependence on the direction of $\mathbf{q}$ disappears for unpolarized nuclei. On the other hand the isospin factor $(\underline{\tau}_i \cdot \underline{\tau}_j) - \tau_i^3 \tau_j^3$ which appears in Compton scattering has been substituted by $2/3 (\underline{\tau}_i \cdot \underline{\tau}_j)$, which is valid for $N=Z$ nuclei.

These four elementary amplitudes naturally appear in the Feynman diagram expansion of the Compton amplitude. However there is a natural grouping between them which follows from chiral symmetry. In the limit $\mu \rightarrow 0$ exact cancellations occur: the central contributions which individually are large sum to zero, and the tensor parts of K_2 and K_3 cancel, leaving the tensor part of K_4 as the only

contribution. In this limit $K_\pi = (K_\pi)_{\text{tensor}} = (K_4)_{\text{tensor}}$. Notice that similar cancellations are known to occur in the pion exchange corrections to the electromagnetic current [6].

In the physical situation one then expects that K_π should have a small central piece and that its tensor part should be essentially determined by that of K_4 . This will be confirmed later by a numerical analysis. Now, as K_4 given by the diagram 1d is the seagull term on the exchanged pion, an approximate relation between K_π and the pion excess number n_π should exist.

Indeed the pion excess number which is

$$n_\pi = -3g^2/[16\pi^2 M^2] \int_0^\infty [S_C(q) + S_T(q)] q^4 / (q^2 + \mu^2)^{3/2} dq \quad (6)$$

obeys the following relation, a first step towards our goal to relate n_π and K_π , obtained by comparing Eq. (4d) with eq. (6)

$$n_\pi = (3/16) K_4 / (\langle \omega_\pi^{-1} \rangle M) \quad (7)$$

where the average inverse pion energy $\langle \omega_\pi^{-1} \rangle = \langle (\mu^2 + q^2)^{-1/2} \rangle$ is taken over the pion density distribution.

We notice the factor 3/16 which appears in Eq. (7) instead of 3/8 of the naive guess of Eq. (2). Actually since pions are virtual three processes contribute to the Compton amplitude: the Thomson scattering on the exchanged pions (fig. 2a), considered in ref.[1]. In addition there appears also the two pion production by two photons through the contact term (fig. 2b) and the corresponding two pions absorption (fig. 2c). The corresponding expressions are given by:

$$K_{4(a)} = g^2/[2\pi^2 M] \int_0^\infty [S_C(q) + S_T(q)] q^4 / (q^2 + \mu^2)^2 dq \quad (8a)$$

$$K_{4(b,c)} = g^2/[2\pi^2 M] \int_0^\infty 1/2 [S_C(q) + S_T(q)] q^4 / (q^2 + \mu^2)^2 dq \quad (8b,c)$$

The last two processes invalidate the simple guess of ref. [1]. They double the contribution to the Compton amplitude arising from the only Thomson amplitude

on the exchanged pions.

Relation (7) applies as well to the central and tensor part with different values of $\langle \omega_\pi^{-1} \rangle$. To transform it into a useful relation, namely one between n_π and K_π , it is enough to show that both n_π and K_π are dominated by their tensor pieces with $(K_\pi)_{\text{tensor}} = (K_4)_{\text{tensor}}$. For K_π this dominance has previously Eq.s (5a,b) the discussed. As for that of n_π it has been pointed out by Friman et al. [7]. It is fact due to the influence of short range correlations as discussed below.

Moreover the smallness of the central piece of n_π as compared to that of K_4 , in spite of the fact that the two quantities differ only, in momentum space, by one power of $\omega_q = \sqrt{\mu^2 + q^2}$, also deserves some comments. For this it is convenient to visualize their differences in r-space comparing also the OPE central potential. We introduce the quantity $V_\pi(r)$, $n_\pi(r)$ and $K_4(r)$ related as follows to the central parts (denoted with the index C) of the pion interaction energy E_π , of n_π and of K_4 , respectively:

$$E_{\pi C} = \rho \int S_C(r) V_\pi(r) d^3r = 1/6 f^2 \rho \int S_C(r) [-4\pi \delta^3(r)/\mu^2 + e^{-z}/r] d^3r \quad (9a)$$

$$n_{\pi C} = \rho \int S_C(r) n_\pi(r) d^3r = -3/(4\pi) f^2 \rho \int S_C(r) [K_1(z)/z - K_0(z)] d^3r \quad (9b)$$

$$K_{4C} = \rho \int S_C(r) K_4(r) d^3r = -f^2 M/\mu^2 \rho \int S_C(r) [(2-z) e^{-z}/r] d^3r \quad (9c)$$

where $f^2 = g^2/4\pi$ ($\mu^2/4M^2$) = 0.078, $z = \mu r$, ρ is the nuclear matter density and $S_C(r)$ is the central correlation function in r-space. In the free Fermi gas model $\rho S_C(r)$ is given by $(6/\pi^2) p_F^3 [J_1(p_F r)/(p_F r)]^2$.

The short range correlations in $S_C(r)$ suppress the $\delta^3(r)$ contribution to $E_{\pi C}$ with a resulting considerable modification: the central part of E_π turns into an attractive quantity, from 18 MeV without short range correlations to -5 MeV when they are introduced. A similar phenomenon, although less pronounced, occurs for the central part of the pion number. Here the equivalent of the $\delta^3(r)$ singularity is a

$1/r^2$ behaviour on the origin and the change in sign, which occurs at $r=0$ in $V(r)$, here occurs at $r = 1.9$ fm., as shown in fig.3. The presence of $1/r^2$ singularity makes the pion number very sensitive to short range correlations: when these are introduced the central part of n_π is appreciably reduced, from -0.06 to -0.02. As for the quantity K_4 , it is less singular with a $1/r$ behaviour at the origin. (fig. 3). The short range correlations therefore have a small influence on the central part of K_4 which is large.

Neglecting the central parts we can thus transform Eq.(7) into the following relation between K_π and n_π :

$$n_\pi = (3/16) K_\pi / (\langle \omega_\pi^{-1} \rangle M) \quad (10)$$

where $\langle \omega_\pi^{-1} \rangle$ now refers only to the tensor value.

Before giving the numerical values, we will discuss the effect of the introduction of a form factor at the πNN vertex $\Gamma(q^2)$. This introduction has to preserve gauge invariance. To achieve this result we have replaced, in the expression (3) of K_π , $V_{OPE}(q)$ by $V_{OPE}(q) \Gamma^2(q)$ with the monopole form factor $\Gamma(q) = (\Lambda^2 - \mu^2) / (\Lambda^2 + q^2)$ with $\Lambda = 1$ GeV. The integrands of K_i are then multiplied by the following different combinations $R_i(q)$ of the form factor $\Gamma(q)$ and its q^2 derivatives:

$$R_1(q^2) = \Gamma^2(q^2) \quad (11a)$$

$$R_2(q^2) = R_4(q^2) = \Gamma^2(q^2) - 2(\mu^2 + q^2) \Gamma'(q^2) \Gamma(q^2) \quad (11b)$$

$$R_3(q^2) = R_2(q^2) + (\mu^2 + q^2)^2 [\Gamma''(q^2) \Gamma(q^2) + \Gamma'^2(q^2)] \quad (11c)$$

The overall effect of the form factor is small, it only reduces the central part of K_π . The numerical values of the different quantities are given in Table I. They have been obtained with the nuclear matter correlation functions $S(q)$ calculated by Fantoni [8] with a V14 and using the FHNC/SOC method [9]. The parentheses refer to the results including the monopole form factor with $\Lambda = 1$ GeV.

The same calculation has been done for the deuteron. Here there is a stronger

suppression of the short range contributions. So the results are rather insensitive to the introduction of a form factor and are only given in Table II for the pure OPE potential .

TABLE I : *Values of the central and tensor contributions to the K_i , K_π (Eq.'s 4a,b,c,d) and n_π (Eq. 6) for the correlated nuclear matter. The averaged values are given for pure OPE and, in parenthesis, for OPE with a monopole form factor with cut-off $\Lambda=1000$ MeV (Eq. 11a,b,c).*

	Central	Tensor	Total
K_1	1.27 (1.17)	0	1.27 (1.17)
K_2	-.94 (-.92)	1.15 (1.13)	.21 (.21)
K_3	.57 (.56)	-1.02 (-1.01)	-.46 (-.45)
K_4	-.70 (-.68)	.86 (0.84)	.16 (.16)
K_π	.20 (.13)	1.00 (.95)	1.20 (1.09)
n_π	-.019 (-.017)	.079 (.070)	.060 (.053)

TABLE II : *Values of the central and tensor contributions to the K_i , K_π (Eq.'s 4a,b,c,d) and n_π (eq. 6) for the deuteron (pure OPE).*

	Central	Tensor	Total
K_1	.279	0.	.279
K_2	-.181	.525	.344
K_3	.100	-.446	-.346
K_4	-.136	.393	.257
K_π	.062	.472	.534
n_π	-.0035	.032	.029

The numbers in these tables confirm the theoretical considerations based on the chiral limit $\mu \rightarrow 0$. They show that the main features of this limit are preserved, namely:

- i) in the central parts, although the individual values of the K_i are large, they sum to a small number, in such a way that K_π is dominated by its tensor part.
- ii) in the tensor parts, those of K_2 and K_4 approximately cancel, as expected, leaving $(K_4)_{\text{tensor}}$ as the main term.

On the other hand the tables also confirm that the central part of n_π is small. Therefore the relation (10) between n_π and K_π holds, with an accuracy of the order of 30%.

It now remains to discuss the contribution of the pion exchange potential K_π to the quantity K obtained with the full potential. The pion contribution dominates: in the calculation of ref. [10] $K_\pi = 1.2$ versus a total value $K \approx 1.5-1.7$.

In particular the contribution from the region below the pion threshold is experimentally determined to be 0.76 [11] and is substantially pionic. Since there are also contributions above pion threshold we can state that K_π is actually measured in photonuclear reaction and that its value is of order of 1.

We can now use this "measured" value to estimate the pion excess in nuclear matter. With an average value $1/\langle\omega_\pi^{-1}\rangle \approx 0.7 \langle\omega_\pi\rangle$ with $\langle\omega_\pi\rangle = 400$ MeV [12] we obtain from Eq. (12) $n_\pi \approx 0.07$, in agreement with the purely theoretical estimation of table I.

This value is sizeably smaller than the one of ref. [7] $n_\pi = 0.18$. The origin of this discrepancy is due to the omission in our work of the Δ contribution which also participates substantially in the pion excess. Our analysis thus provides a lower limit for the pion excess number.

In conclusion we have discussed the pion excess number in nuclei. We have

linked it to the enhancement factor of the dipole sum rule, exploiting the fact that both quantities are essentially governed by tensor correlations. We have shown that the relation between the two quantities differs by a factor two from the one obtained by identifying the factor K with the Compton amplitude on the exchanged pions.

The relation that we use to derive the pion number has an accuracy of about 30% . Moreover by ignoring the Δ contribution we underestimated this quantity. The interest of this study is to evidentiate the existence of a pion excess by linking it to a physical quantity, the photoproduction cross-section. The concept of pion excess in nuclei is an important one which has been applied for instance in the interpretation of EMC effect. A display of its existence and an estimate, even at a qualitative level, is therefore of interest.

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FIGURE CAPTIONS

Fig. 1a,b,c,d

Feynman diagrams of the one pion exchange contributions to the Compton amplitude.

Fig. 2a,b,c

Time ordered contribution to the contact term given in fig. 1d. a) is the Thomson amplitude on the pion in flight, b) the two pion production and c) the two pion annihilation.

Fig. 3.

Plot in the r -space of the integrand of the central part of the pion number (eq.9b) and of the contact term K_4 (eq. 9c). The two function are multiplied by $10r^2$ and by r^2 respectively to allow a better analysis of the intermediate range behaviour.

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a

b

c

d

FIG 1

FIG 2

FIG 3