FCC-ee/CepC BEAM-BEAM SIMULATIONS WITH BEAMSTRAHLUNG

K. Ohmi, KEK, Tsukuba, Japan, and F. Zimmermann, CERN, Geneva, Switzerland

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Abstract

Beamstrahlung, namely synchrotron radiation emitted during the beam-beam collision \[1\], can be an important effect for circular high-energy lepton colliders such as FCCee (TLEP) \[2\] and CepC \[3\]. In this paper we study beambeam effects in the presence of energy spreading and bunch lengthening due to beamstrahlung.

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Abstract

Beamstrahlung, namely synchrotron radiation emitted during the beam-beam collision [1], can be an important effect for circular high-energy lepton colliders such as FCC-ee (TLEP) [2] and CepC [3]. In this paper we study beam-beam effects in the presence of energy spreading and bunch lengthening due to beamstrahlung.

BEAMSTRAHLUNG

Beamstrahlung (BS) introduces an additional source of steady-state energy spread, which lengthens the bunches [5]. The strength of the beamstrahlung is characterized by the parameter $\Upsilon \equiv B/B_c$, with $B_c = m_e^2c^2/(e\hbar) \approx 4.4$ GT the Schwinger critical field. The average value of $\Upsilon$ during the collision of Gaussian beams is $[6, 7] \Upsilon \approx (5/6)r_e^2\gamma N_b/(\alpha x_1^2 + \alpha y_1^2)$, where $\alpha$ denotes the fine structure constant ($\alpha \approx 1/137$). For all proposed circular colliders $\Upsilon$ is much smaller than 1. Then we can approximate the number of photons per collision as $[7] n_\gamma \approx 2.1 \alpha x_1 N_b/(\sigma_x + \sigma_y)$, the average relative energy loss as $\delta_B \approx 0.86 r_e^2\gamma N_b^2/(\sigma_x(\sigma_x + \sigma_y)^2)$, and the standard deviation of the energy loss as $[6]$

$$\sigma_{\delta,B} \approx \delta_B \left( 0.333 + \frac{4.583}{n_\gamma} \right)^{1/2}. \quad (1)$$

The additional steady-state energy spread due to beamstrahlung (added in quadrature) can be estimated from [5]

$$\Delta \sigma_{\delta,B} \approx \frac{1}{2} \left( \frac{\tau_{\text{damp}} n_{\text{int}}}{T_0} \right) \sigma_{\delta,B} \equiv A \sigma_z, \quad (2)$$

with $\tau_z$ the damping time, $T_0$ the revolution period, $n_{\text{int}}$ the number of interaction points, and, in the last step, we have singled out the dependence on $\sigma_z$. Adding the natural rms energy spread from synchrotron radiation, $\sigma_{z,SR}$, yields the total relative energy spread of

$$\sigma_\delta \approx \sqrt{\left( \Delta \sigma_{\delta,B} \right)^2 + \sigma_{z,SR}^2}. \quad (3)$$

Using $\sigma_{z,tot} = \sigma_{z,\text{tot}} \sigma_{z,SR}/\sigma_{z,SR}$, self-consistency requires

$$\sigma_{z,\text{tot}}^2 - \sigma_{z,SR}^2 = \left( \frac{\sigma_{z,SR}}{\sigma_{z,\text{tot}}} \right)^2, \quad (4)$$

where the subindex “SR” refers to the bunch length or energy spread computed with arc synchrotron radiation only. In the explicit solution for the total energy spread is

$$\sigma_{\delta,\text{tot}} = \left( \frac{1}{2} \sigma_{\delta,SR}^2 + \frac{1}{4} \sigma_{\delta,SR}^2 + \frac{1}{2} \left( \frac{\sigma_{z,SR}^2}{\sigma_{z,SR}} \right)^{1/2} \right)^{1/2}. \quad (5)$$

SIMULATION APPROACHES

The upgraded weak-strong and strong-strong beam-beam codes BBWS and BBS take into account the combined effect of both standard synchrotron radiation and beamstrahlung in a semi- or fully self-consistent manner. Both codes were used to simulate the beam-beam behavior for the various proposed running modes of FCC-ee and CepC, considering the beam and machine parameters of Ref. [4].

Figure 1 illustrates the recipe employed for modeling the beamstrahlung. The collision is divided into many small steps. Individually tracked particles randomly emit synchrotron radiation according to their local bending radius $1/\rho = |\Delta \sqrt{x'^2 + y'^2}/\Delta s|$. The probability of the emission of a photon is proportional to $\Delta s/\rho$.

Two models for the random emission were implemented. The first represents the photon emission as a Gaussian fluctuation with the correct rms value (including the average energy loss). The second model generates the exact full photon spectrum as described by the $K_{5/3}$ Bessel function, by inverting a pre-computed table for $N_\gamma(\omega)$. These two approaches yield about the same simulated luminosity and bunch length (see Figs. 7 and 8), whereas the beam lifetime is sensitive to the detailed photon spectrum.

In case of the weak-strong simulation the bunch length of the strong bunch is regularly updated (every 100 turns) so as to correspond to the bunch length of the weak beam, which is evolving under the influence of the beamstrahlung. When simulating the beam lifetime, similar self-consistent updates are applied to the horizontal beam size.

Figure 1: Schematic view of beamstrahlung simulation.

Equilibrium values are quickly reached for the bunch lengths, the luminosity, and the transverse beam sizes, as is illustrated in Fig. 2, which shows the result of a weak-strong simulation for CepC without and with beamstrahlung (including the self-consistent bunch length).

In the strong-strong simulation with BBSS the bunches of both beams are divided into 15–20 slides. Each slide contains many macroparticles (of order 10³). The collision is calculated slice by slice. Using a 3D symplectic integrator the beam potential $\phi$ is computed on each slice boundary $z_i$, and then interpolated longitudinally for the next tracking step of the macroparticles. The interpolation is important. The macroparticles also suffer energy

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† kazuhito.ohmi@kek.jp
changes in proportion to $\partial \varphi / \partial z$. The calculation procedure is repeated several times during a collision, until all slices of two bunches have passed through each other, at each step updating the trajectories and the potentials.

**LUMINOSITY PERFORMANCE**

Figure 3 presents weak-strong simulation results for FCC-ee at four different collision energies. The simulated luminosities are close to the analytically expected values, as is illustrated in Table 1, which also compares calculated and simulated equilibrium bunch lengths. The simulations reveal extended vertical beam tails (Fig. 4).

**Figure 3: Weak-strong simulation of luminosity for FCC-ee at four different c.m. energies.**

Figure 5 displays weak-strong simulation results for the special FCC-ee low-emittance crab-waist scenario at the Z pole [8]. The bunch length is almost tripled due to the beamstrahlung, both with and without the crab waist. However, switching on the crab waist reduces the vertical beam size by a factor of 5 and the horizontal one by a factor of 2; most importantly, it increases the luminosity about 5-fold.

**Figure 5: Weak-strong simulation for FCC-Z low-emittance parameters with (blue) and without crab waist (red): luminosity (top left), vertical beam size (top right), horizontal beam size (bottom left), and bunch length (bottom right). Green dashes indicate beam sizes without BS.**

The simulated performance, in terms of luminosity and beam size, varies with the betatron tune, while the bunch length is nearly independent of the working point. Figure 6 presents the results of a horizontal tune scan for fixed vertical tune, revealing synchro-betatron resonances close to the half integer.

**Figure 6: Luminosity (left) and horizontal beam size (right) vs $Q_x$ at $Q_y = 0.61$ (tunes/IP), for FCC-Z crab waist scenario, from a weak-strong simulation ($Q_s = 0.062$).**

Strong-strong results for FCC-H and -t are shown in Figs. 7 and 8, which also illustrate the difference between a simple Gaussian fluctuation and the exact photon spectrum. The much weaker radiation damping for FCC-W and Z would render the corresponding strong-strong computations more difficult. Computing demands are further aggravated for the FCC-Z crab-waist scheme, the proper modeling of which would require a significantly larger number of slices.

**BEAM LIFETIME**

The beam lifetime due to beamstrahlung can be calculated in a number of ways. Two alternative analytical formulae were proposed in Refs. [8] and [9]. Several methods...
are also available for inferring the beam lifetime from the simulations. One approach is to directly compute the particles lost by exceeding a limiting momentum acceptance of, e.g., 1.5%, or a vertical aperture limit, taken to be $40\sigma_y$, i.e., $\tau_{BS,1} = T_{\text{sim}} N_{\text{tot}}/(\Delta N)_{\text{lost}}$, with $T_{\text{sim}}$ the simulated time interval, $N_{\text{tot}}$ the total number of macroparticles, and $(\Delta N)_{\text{lost}}$ the number of lost macroparticles. A second approach is to calculate the incoming flux due to radiation damping at the limiting amplitude (longitudinally $\sqrt{2J_z}$, where $J_z$ denotes the action variable, or $y$ transversely) from the equilibrium beam-tail distribution simulated without acceptance limit, in close analogy to the lifetime calculation for conventional synchrotron radiation [10]. E.g., for the longitudinal plane one has $\tau_{BS,2} = \tau_z/(2\xi\rho(\xi))$, with $\rho(\xi)$ the density, and $\xi \equiv J_z/\epsilon_z \equiv \delta_{\text{max}}^2/(2\sigma_y^2)$ the normalized acceptance.

Table 2 compares simulated beam lifetimes, as computed by the aforementioned two approaches, with predictions from the analytical formulae of Refs. [9] or [8]. Reassuringly, the direct beam loss simulations and the calculation from the equilibrium distribution at the acceptance limit (Figs. 9 and 10) yield consistent results. However, at a given value of $\delta_{\text{max}}$ the simulated lifetimes are a factor 10–20 shorter than the analytical estimates. They are dominated by the longitudinal plane, with at most a few per cent contribution from the vertical. The lifetime varies strongly with $\delta_{\text{max}}$ (Fig. 10), but it is almost independent of the value of $\beta^*_y$ [11].

**Table 2: Expected and simulated BS lifetime.**

<table>
<thead>
<tr>
<th>$\tau_{BS}$ [min]</th>
<th>FCC-H</th>
<th>FCC-t</th>
<th>CepC</th>
</tr>
</thead>
<tbody>
<tr>
<td>analytical [9]</td>
<td>310</td>
<td>3.6</td>
<td>113</td>
</tr>
<tr>
<td>analytical [8]</td>
<td>1400</td>
<td>3.3</td>
<td>619</td>
</tr>
<tr>
<td>weak-strong (loss)</td>
<td>26</td>
<td>0.3</td>
<td>5.5</td>
</tr>
<tr>
<td>weak-strong (distr.)</td>
<td>33</td>
<td>0.3</td>
<td>—</td>
</tr>
</tbody>
</table>

Figure 9: Equilibrium distribution for FCC-H (left) and FCC-t (right) from tracking 100 particles over $10^8$ turns.

Figure 10: Lifetime vs. momentum acceptance inferred from the equilibrium distributions in Fig. 9 for FCC-H (left) and FCC-t (right).

**CONCLUSIONS**

Both weak-strong and strong-strong simulations confirm the analytically expected luminosities for FCC-ee and CepC. The formula (5) is consistent with the steady-state bunch length obtained in strong-strong simulations, with differences at the few per cent level. The beam-lifetime values predicted from the simulated losses or from the equilibrium distributions are considerably shorter than those predicted by the available analytical expressions. Similar discrepancies were reported previously [8, 12].
REFERENCES