**Heavy Flavour Production in Large Transverse
Momentum Proton-Antiproton Collisions**

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ABSTRACT

We discuss the question of heavy flavour production in hard proton-antiproton collisions at the CERN Sp $\bar{p}$ S Collider. Our emphasis in this paper is mainly on the inclusive muon and dimuon cross-sections recently measured by the UA1 collaboration. We estimate contributions to these final states from both strong and weak production processes using perturbative quantum chromodynamics, QCD. We find that large p_t production of heavy flavours at the CERN collider energies is quantitatively well described by QCD. We present a detailed calculation to estimate the ratio of same-sign to opposite-sign dimuon production rate $R(\pm\pm/\mp\mp)$ at the CERN $\bar{p}p$ collider, making use of the extensive data available on the production and decays of bottom and charmed hadrons in e^+e^- annihilation experiments. The calculated ratio $R(\pm\pm/\mp\mp)$ is found to be significantly lower than the UA1 measurements. We argue that this ratio is theoretically well reproduced if one assumes substantial $B_s^0 - \bar{B}_s^0$ mixing. We determine the 2σ lower limit on the mixing probability $r_s \equiv \Gamma(B_s^0 \rightarrow l^+ \nu_l X) / \Gamma(\bar{B}_s^0 \rightarrow l^+ \nu_l X)$ to be bigger than 0.14. In the standard model this gives a lower bound on the Kobayashi-Maskawa matrix element V_{ts} which we estimate. Flavour correlations confirming $B_s^0 - \bar{B}_s^0$ mixing in future experiments are emphasized.

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1. Introduction

The subject of heavy flavour production in hadronic collisions has received considerable attention in the past [1]. Attempts to describe charm production rates at the FNAL-ISR energies in terms of perturbative quantum chromodynamics, QCD, contributions are known not to work. This failure probably can be ascribed both to the relatively low mass of the charm quark and the low center of mass FNAL-ISR energies. On the other hand, the experimental situation at these energies is still being clarified.

The situation at the CERN proton-antiproton collider is qualitatively different. With most data obtained at $\sqrt{s} = 546$ GeV and $\sqrt{s} = 630$ GeV, there is sufficient energy at the CERN collider to produce charm, bottom, and top quarks. Since $m_b \approx 5$ GeV/c² and $m_t > 23$ GeV/c², one expects perturbative QCD to work quantitatively down to low p_t in the production of bottom and top quarks. On the other hand heavy flavour triggers in use at the UA1/UA2 detectors are sensitive to the central and/or large p_t components of the cross-sections only. Thus, it is not unreasonable to expect that the large p_t charm production at the CERN collider energies will also be well described by perturbative QCD.

It is generally recognized that there are two important contributions to large p_t heavy flavour production processes at the CERN collider and higher energies. The first and probably the better calibrated contribution is due to the production and decays of $W^\pm$ and Z^0 bosons. In this case phase space factors and leading order QCD corrections [2] for the processes $p\bar{p} \rightarrow W^\pm \rightarrow qq'^1$, $p\bar{p} \rightarrow Z^0 \rightarrow q\bar{q}$ can be used together with the observed cross-sections $\sigma(p\bar{p} \rightarrow W^\pm \rightarrow l^+\nu_l)$ and $\sigma(p\bar{p} \rightarrow Z^0 \rightarrow l^+l^-)$ to estimate the corresponding heavy flavour cross-sections $\sigma(p\bar{p} \rightarrow W^\pm, Z^0 \rightarrow (t, b, c) X)$. This source of heavy flavour production, however, is not very important except for the top quark [3]. The second and dominant contribution to heavy flavour production is due to perturbative QCD processes, in particular the gluon fusion mechanism $gg \rightarrow Q\bar{Q}$ [4]. It has been pointed out that higher order corrections from the processes $gg \rightarrow gg^*$, $g^* \rightarrow Q\bar{Q}$ may give large contributions. This is particularly so for the case of the charm quark [5], where the virtuality of g^* is not very large. A completely quantitative calculation of the inclusive heavy flavour production cross-section $\sigma(p\bar{p} \rightarrow Q X)$ to $O(\alpha_s^3)$ is not yet at hand. Pending such a calculation, the best that one can do is use perturbative QCD calculations in the Born approximation in regions of phase space which are far from infra-red and collinear divergences. Such a semi-quantitative approach is still quite instructive. It has a close analogy with the present perturbative QCD treatment of inclusive jet production, where also the complete $O(\alpha_s^3)$ calculations have not yet been applied in the analysis of the collider data. However, the existing theoretical machinery based on Born terms provides a rather satisfactory description of the data, certainly within a factor 2 [6]. This is the approach that we will follow in discussing heavy flavour production in this paper.

Theoretical calculations for heavy flavour production based on the $2 \rightarrow 2$ and $2 \rightarrow 3$ QCD processes already exist in literature [7]. These predictions can now be sharpened by taking into account the constraints on the choice of the structure functions and the scale Q^2 in the argument of the effective coupling constant, $\alpha_s(Q^2)$, obtained from experimental jet studies [8]. We shall make use of this information in estimating rates and distributions for heavy flavour production. Since the measured final states in the study of heavy flavours involve in most cases leptons, a trustworthy comparison of theoretical expectations with data

¹ For convenience we denote $W^\pm \rightarrow q\bar{q}'$, $\bar{q}q'$ by $W^\pm \rightarrow qq'$ throughout the paper

necessarily demands the construction of a detailed model for heavy quark fragmentation and decay. Fortunately, a very large amount of experimental information is available concerning fragmentation and decays of the charm and bottom quarks [9]. We have made extensive use of this information to constrain the parameters of our model. This eliminates many uncertainties in estimating the cross-sections and the distributions of experimental interest at the CERN collider energies.

The calculations reported here are part of a general purpose Monte Carlo program, called EUROJET [10], developed to study hard hadronic collisions in the framework of perturbative QCD. Though the Monte Carlo has a large area of applicability, we shall concentrate here on the inclusive muon, dimuon and trimuon production processes only.

This paper is organized as follows. In section 2 we present theoretical cross-sections for the processes $p\bar{p} \rightarrow Q X$ ($Q = c, b, t$) calculated to $O(\alpha_s^3)$ and for $p\bar{p} \rightarrow (W^\pm, Z^0) \rightarrow Q X$, calculated to $O(\alpha_s)$. In both cases phase space effects due to finite quark masses are properly taken into account. The relative importance of the leading order QCD corrections to the lowest order contributions is presented and emphasized. In section 3, we discuss the details of our model for heavy quark fragmentation and decays. There we make use of the available e^+e^- data. In particular, all known decays of the charmed hadrons $D^0, D^\pm, F^\pm, \Lambda_c$ and the bottom hadrons $B_d^0, B_u^\pm$ are implemented. The semileptonic decays of the charmed hadrons are described in terms of exclusive decays [11], like for example $D \rightarrow (K, K^*) l \nu_l$, with appropriate form factors and matrix elements taken into account [12].

Likewise for the production and decays of the bottom hadrons, we have implemented all known aspects including some recent results concerning the decays $B \rightarrow J/\psi X$ and $B \rightarrow F^\pm X$, which have a direct bearing on the question of inclusive muon and dimuon production. For the decay of top hadrons we use the free quark decay model with the standard V-A matrix elements [13]. Thus, top decays $t \rightarrow b l \nu_l$ and $t \rightarrow b q \bar{q}'$ give rise to, respectively, one and three jets (in general). The complete cascade of $t \rightarrow b \rightarrow c \rightarrow s$ is generated with the appropriate decays of the bottom and charmed hadrons as described above. The fragmentation of the heavy quarks into heavy hadrons is assumed to be described by the so-called Peterson fragmentation function [14]. We have made use of the analysis in e^+e^- annihilation experiments [15] in determining the fragmentation parameters. In this section we also present a detailed comparison of our model calculations with available leptonic and hadronic distributions, mostly from e^+e^- experiments.

In section 4, we compare our calculations with the data obtained by the UA1 collaboration. We find that the perturbative QCD calculations well reproduce the experimental cross-sections and p_t distributions in the processes $p\bar{p} \rightarrow \mu^\pm X, \mu^\pm \mu^\pm X', \mu^+ \mu^- X'$. Concentrating on the dimuon data, we show comparisons with the dimuon azimuthal angle distribution $d\sigma/d\Delta\phi \mu\mu$ which is well described by perturbative QCD calculations. However, inclusion of $2 \rightarrow 3$ processes is essential in explaining the tail of the $\Delta\phi \mu\mu$ distribution. We find that the so-called non-isolated dimuon data sample [16] is dominated by the bottom quark pair production ($> 75\%$). The shape of the dimuon invariant mass distribution is well described by the bottom and charmed hadron decays.

Next, we discuss the dimuon ratio $R(\pm\pm/\pm-)$ measured by the UA1 collaboration in the process $p\bar{p} \rightarrow \mu^\pm \mu^\pm X, \mu^+ \mu^- X$. The experimental number [17] for this quantity is: $R = 0.42 \pm 0.07 \pm 0.03$. We find that bottom hadron cascades and the processes $p\bar{p} \rightarrow W^\pm, Z^0 \rightarrow \mu^\pm \mu^\pm X, \mu^+ \mu^- X$ and $p\bar{p} \rightarrow t\bar{t} \rightarrow \mu^\pm \mu^\pm X, \mu^+ \mu^- X$ together contribute significantly and give $R = 0.24 \pm 0.05$. This is approximately a factor 2 smaller compared to the UA1 central value. We present our estimates of

theoretical errors on R , the dominant components of which are due to the imprecise determination of the charm quark contribution to the dimuon cross-section as well as from the top quark. Since one expects substantial mixing in the $B_s^0-\bar{B}_s^0$ sector [18], we investigate the consequences of such a mixing for the dimuon data. We find that the ratio $R(\pm\pm/\pm-)$ is better described if one assumes mixing, though the actual number depends on the amount of mixing, r_s , and the relative production rate of the B_s^0 meson, P_s . For maximum $B_s^0-\bar{B}_s^0$ mixing (i.e. $r_s = 1.0$) and $P_s = 0.20$, we find $R \approx 0.38$, in good agreement with the UA1 measurements.

In section 5 we discuss the implications of the UA1 result for the mixing parameters in the standard model. We use the upper limits on the $B_d^0-\bar{B}_d^0$ mixing probability, r_d , from e^+e^- data and use $R(\pm\pm/\pm-)$ from the UA1 measurement to determine a *lower limit* on the mixing probability, r_s . Within our theoretical calculations, we find $r_s > 0.14$ (2σ). In the standard model of flavour mixing [19], this can be interpreted in terms of a *lower bound* on the Kobayashi-Maskawa matrix element, V_{ts} [20]. The actual bound depends on the top quark mass and the value of the pseudoscalar meson coupling constant, f_{B_s} . With the assumption $m_t = 40 \text{ GeV}/c^2$ and $\mathcal{B} f_{B_s}^2 = (150 \text{ MeV})^2$ (where $\mathcal{B}$ is the Bag constant, $\mathcal{B} \approx 1$ for the vacuum insertion approximation) we find $|V_{ts}/V_{bc}| > 0.5$, which together with the value $|V_{bc}| \approx 0.05$ obtained from the bottom lifetime measurements [11] gives $|V_{ts}| > 0.030$. This gives a first indication of the value of the KM matrix element $|V_{ts}|$.

We conclude in section 6 with a discussion of flavour correlations that $B_s^0-\bar{B}_s^0$ mixing entails and propose tests to consolidate mixing in the $B_s^0-\bar{B}_s^0$ sector in future experiments.

2. Heavy Flavour Production In Hard Hadronic Collisions

The lowest order cross-sections for heavy flavour production in hadronic collisions, involving $2 \rightarrow 2$ processes in perturbative QCD shown in fig. 1, were calculated by Combridge in 1979 [4]. The cross-sections for the subprocesses

$$\begin{aligned} q + \bar{q} &\rightarrow Q + \bar{Q} \\ g + g &\rightarrow Q + \bar{Q} \end{aligned} \quad (2.1)$$

are completely integrable in $O(\alpha_s^2)$ giving

$$\begin{aligned} \sigma(q + \bar{q} \rightarrow Q + \bar{Q}) &= \frac{8\pi \alpha_s^2(Q^2)}{27\hat{s}^2} (\hat{s} + 2m_Q^2) \beta \\ \sigma(g + g \rightarrow Q + \bar{Q}) &= \frac{\pi \alpha_s^2(Q^2)}{3\hat{s}} \left[-(7 + 31m_Q^2/\hat{s}) \beta / 4 + (1 + 4m_Q^2/\hat{s} + m_Q^4/\hat{s}^2) \ln \frac{1+\beta}{1-\beta} \right] \end{aligned}$$

$$\begin{aligned} \text{where } \hat{s} &= x_1 x_2 s, \quad s = E_{cm}^2 \\ \beta &= (1 - 4m_Q^2/\hat{s})^{1/2} \end{aligned} \quad (2.2)$$

with x_1 and x_2 having the usual definition of being the fractional momenta carried by the partons $x_i = 2E_i/\sqrt{s}$.

The observed cross-section can be obtained by convoluting the cross-section σ given by (2.2) with the appropriate structure functions

$$\sigma(s) = \int dx_1 dx_2 [F_{i/p}(x_1, Q^2) F_{j/p}(x_2, Q^2) + i \leftrightarrow j] \sigma(x_1, x_2, s) \quad (2.3)$$

The cross-section $\sigma(s)$ depends on structure functions, centre-of-mass energy, quark masses and the value of $\alpha_s(Q^2)$. To get an idea of heavy flavour cross-sections we calculated $\sigma(s)$, using the EHLQ structure function parametrization with $\Lambda = 0.2$ GeV [21], at $\sqrt{s} = 630$ GeV. With $m_c = 1.75$ GeV/c², $m_b = 5.1$ GeV/c² and $m_t = 40$ GeV/c² we find the following cross-sections ($O(\alpha_s^2)$, $\sqrt{s} > 10$ GeV)

$$\left. \begin{aligned} \sigma(p\bar{p} \rightarrow c X) &\cong 58 \text{ } \mu\text{b} \\ \sigma(p\bar{p} \rightarrow b X) &\cong 13 \text{ } \mu\text{b} \\ \sigma(p\bar{p} \rightarrow t X) &\cong 1.4 \text{ nb} \end{aligned} \right\} |\eta^Q| < 3.5 \quad (2.4)$$

Thus, heavy flavour cross-sections at the CERN collider are expected to be very large. Of course, due to the various heavy flavour triggers involving transverse momentum cuts and the limited geometrical acceptance of the detectors, only small fractions of the cross-sections (2.4) are actually measurable. We shall discuss them in detail later but first we would like to discuss the contributions of higher order processes to heavy flavour production at collider energies.

In next-to-leading order in α_s , there are two types of contributions to $\sigma(p\bar{p} \rightarrow Q X)$. The $2 \rightarrow 3$ Born diagrams are shown in fig. 2. The virtual corrections to $2 \rightarrow 2$ diagrams of fig. 1 are not yet available. Since these also contribute in $O(\alpha_s^3)$, there remains an overall uncertainty in the estimates of the cross-sections for $2 \rightarrow 2$ processes in $O(\alpha_s^3)$. The dominant contribution in $O(\alpha_s^3)$ is due to the process [5]

$$g + g \rightarrow g + g^* \quad (2.5)$$

$\searrow$
 $Q\bar{Q}$

since it really is an $O(\alpha_s)$ correction to the gluon-gluon fusion process $gg \rightarrow gg$ leading to 3-jet production. Usual QCD estimates give $\sigma(gg \rightarrow gg)/\sigma(gg \rightarrow Q\bar{Q}) \sim O(100)$ and thus one expects the contribution of (2.5) to overwhelm the contribution from the process (2.1) in the cross-section $\sigma(p\bar{p} \rightarrow Q X)$ by a large factor. However, one has to realize that this large enhancement holds for on-shell gluons but the gluon g^* in (2.5) is virtual. This virtuality is obviously different for charm, bottom and top quarks and hence (2.5) leads to very different enhancements for these flavours. Likewise, the importance of these processes depends on the actual measurements in an experiment. For example, for experimental cuts used by the UA1 collaboration in the analysis of the process $p\bar{p} \rightarrow \mu^+ \mu^- X$, the cut on the invariant dimuon mass, namely $M^{\mu^+ \mu^-} > 6$ GeV/c² would greatly reduce the contribution of (2.5) with $g^* \rightarrow c\bar{c} \rightarrow \mu^+ \mu^- X$. In contrast, the contribution of (2.5) to the inclusive muon cross-section $\sigma(p\bar{p} \rightarrow \mu^\pm X)$, or to the cross-section $\sigma(p\bar{p} \rightarrow Q X)$ with the heavy quark recoiling against a jet [22] (like for example in the $\sigma(p\bar{p} \rightarrow D^{*\pm} X)$ measurements), is expected to be significant. It is obvious that a blanket statement about the importance of higher order corrections in heavy flavour production cannot be made. We shall show detailed

comparisons of the $2 \rightarrow 2$ and $2 \rightarrow 3$ contributions for the stated theoretical and/or experimental configurations.

The $2 \rightarrow 3$ processes, shown in fig. 2, belong to the following three general classes:

$$\begin{aligned} q + \bar{q} &\rightarrow g + Q + \bar{Q} \\ g + g &\rightarrow g + Q + \bar{Q} \\ q + g &\rightarrow q + Q + \bar{Q} \end{aligned} \quad (2.6)$$

where we have only included the contributions of light quarks in the initial state. Since we include the gluon splitting $g \rightarrow Q\bar{Q}$ as part of our $2 \rightarrow 3$ QCD processes, we do not admit any heavy quark component in the structure functions. Including both the processes (2.6) and non-zero structure functions for heavy quarks (charm, bottom, top) in the (anti-) proton would amount to double counting. A corollary of this is that we *do not* have the so-called flavour excitation contributions [23]

$$g + Q \rightarrow g + Q \quad (2.7)$$

added on top of (2.1) and (2.6).

We have used the $2 \rightarrow 3$ matrix elements with $m_Q \neq 0$ derived by Kunszt and Pietarinen [7], which have been recently cast in a very compact form in ref. [24]. Thus, the correct kinematics of the production process is properly taken into account. The processes (2.6) have one difference compared to the $2 \rightarrow 2$ processes of (2.1), namely that the cross-sections based on (2.6) are not completely integrable by themselves. Like their counterparts $gg \rightarrow g q\bar{q}$ which contribute to the (light) jet production, the processes (2.6) need an infra-red cut-off (to avoid $E_q, E_g \rightarrow 0$ singularity) as well as a collinearity cut-off (to avoid singularities from the initial state). To control this singularity we have used a cut-off $p_{t,q,g} > p_t^{\text{cut}}$ for the additional gluon / light quark recoiling against a heavy quark. We use $p_t^{\text{cut}} = 5 \text{ GeV}/c$ as a reasonable value for this cut-off. Experimentally, it turns out to be very difficult to reconstruct jets with energies less than $\sim 5 \text{ GeV}$. Theoretically, the $2 \rightarrow 3$ cross-section should be integrated for $p_{t,q,g} < p_t^{\text{cut}}$ and added to the cross-section for $2 \rightarrow 2$ processes to define the effective $2 \rightarrow 2$ heavy quark pair production cross-section in $O(\alpha_s^3)$. Like in the case of the $p\bar{p} \rightarrow W + \text{jet} + X$ cross-section [25], one would have to exponentiate the leading terms to get a reasonable behaviour for the low $p_{t,q,g}$ region. This part of the calculation has not yet been attempted and hence there remains an overall p_t^{cut} dependence. Although we have fixed $p_t^{\text{cut}} = 5 \text{ GeV}/c$ for our detailed analysis, we will show the dependence of the measured cross-sections on the parameter p_t^{cut} for a reasonable range as an indication of our systematic errors.

The QCD calculations presented in this report are based on the choice of the EHLQ parametrization for the structure functions with $\Lambda = 0.2 \text{ GeV}$. In addition, we have fixed the argument of $\alpha_s(Q^2)$ at $Q^2 = E_t^2 = p_t^2 + m_Q^2$, which is motivated by the UA1 analyses of inclusive jet production at the CERN collider: $p\bar{p} \rightarrow \text{jet} + X$. For all processes involving the weak bosons $W^\pm$ or Z^0 , we have chosen the Duke and Owens [26] parametrization with $\Lambda = 0.2 \text{ GeV}$, fixing Q^2 at $Q^2 = M_W^2, M_Z^2$. The $O(\alpha_s)$ cross-sections are regulated using the same cut-off, $p_{t,q,g} > p_t^{\text{cut}}$ with $p_t^{\text{cut}} = 5 \text{ GeV}/c$, for the jets recoiling against the $W^\pm$ or Z^0 bosons. This choice reproduces the observed cross-sections for the leptonic decays of the $W^\pm$ and Z^0 by UA1 and UA2 rather well [26]. Also the number of jets produced in association with the $W^\pm$ and Z^0 is in good agreement with experimental observations. The

heavy quark production cross-sections via $W^\pm$ and Z^0 boson decays are thus fixed, provided we take into account the correct phase-space suppression factors.

In fig. 3 we show the inclusive- p_t^Q distribution $d\sigma(p\bar{p} \rightarrow Q X)/dp_t^Q$ for $Q = c, b$ and t quarks, for $|\eta^Q| < 3.5$ and two values of m_t ; 23 GeV/c² and 40 GeV/c². Corresponding distributions for the heavy hadrons are shown in fig. 4; $d\sigma(p\bar{p} \rightarrow H X)/dp_t^H$ for $H =$ charmed, bottom and top hadrons, satisfying $|\eta^H| < 3.5$. The relative importance of the $2 \rightarrow 3$ contribution can be seen in fig. 5 where we show the pseudo-rapidity distribution $d\sigma/d\eta^Q$ for $2 \rightarrow 2$ and $2 \rightarrow 3$ processes. In order to improve our efficiency for the calculation of lepton spectra, we introduce a cut on $\sqrt{\hat{s}}$, $\sqrt{\hat{s}} > 10$ GeV for $2 \rightarrow 2$ processes and $\sqrt{\hat{s}} > 15$ GeV for $2 \rightarrow 3$ processes. This clearly only affects the charm-quark cross-sections, whereas leptonic cross-sections from charmed particle decays are not affected at all, due to the rather stringent cuts on the data. The integrated cross-sections are shown in table 1, where $|\eta^Q| < 3.5$. We note that if we relax the cut on $2 \rightarrow 3$ processes to $\sqrt{\hat{s}} > 10$ GeV our choice of quark masses and parameters leads to effective K-factors that lie between ~ 2.0 for the charm quark and ~ 1.6 for the top quark. The effective K-factor for the charm quark-case with the p_t cut-off value, $p_t^{\text{cut}} = 5$ GeV/c is in good agreement with a recent leading log. calculation of the same quantity by Mueller and Nason [27]. This assures us that our choice of the p_t cut-off provides us with a rate estimate for $2 \rightarrow 3$ processes which is in close agreement with leading log. results. Thus there are no spurious or questionable enhancements of $\sigma(p\bar{p} \rightarrow Q X)$, neither have we cut out perfectly finite cross-sections unnecessarily. Of course, we expect that the detailed distributions involving quarks and leptons will be better described by a calculation which also takes into account the non-leading terms, as is the case with the one described here.

Next, we briefly discuss processes involving the weak bosons $W^\pm$ and Z^0 . There is a simple way of calibrating the cross-sections of the following processes

$$\begin{aligned} p\bar{p} &\rightarrow W^\pm X \rightarrow QQ'X \\ p\bar{p} &\rightarrow Z^0 X \rightarrow Q\bar{Q}X \end{aligned} \quad (2.8)$$

in terms of the experimentally measured cross-sections of the corresponding processes involving leptons

$$\begin{aligned} p\bar{p} &\rightarrow W^\pm X \rightarrow l^\pm \nu_l X \\ p\bar{p} &\rightarrow Z^0 X \rightarrow l^+ l^- X \end{aligned} \quad (2.9)$$

The phase-space factors in the decays $W^\pm \rightarrow QQ'$ and $Z^0 \rightarrow Q\bar{Q}$ both in zeroth order and in $O(\alpha_s)$ are well known [2]. The $O(\alpha_s)$ corrections have also been calculated [28]. A good approximation to estimate the cross-sections for the processes (2.8) is to use the following expressions

$$\begin{aligned} \sigma.B(p\bar{p} \rightarrow W^\pm \rightarrow tb) &\cong \sigma.B(p\bar{p} \rightarrow W^\pm \rightarrow l^\pm \nu_l) \Gamma(W^\pm \rightarrow tb) / \Gamma(W^\pm \rightarrow l^\pm \nu_l) \\ \sigma.B(p\bar{p} \rightarrow Z^0 \rightarrow t\bar{t}) &\cong \sigma.B(p\bar{p} \rightarrow Z^0 \rightarrow l^+ l^-) \Gamma(Z^0 \rightarrow t\bar{t}) / \Gamma(Z^0 \rightarrow l^+ l^-) \end{aligned} \quad (2.10)$$

Experimental numbers for both $\sigma.B(p\bar{p} \rightarrow W^\pm \rightarrow l^\pm \nu_l)$ and $\sigma.B(p\bar{p} \rightarrow Z^0 \rightarrow l^+ l^-)$ are measured by the UA1 and UA2 collaborations [26]. The results of the independent UA1 and UA2 measurements are summarized below for $\sqrt{s} = 630$ GeV

$$\begin{aligned}
\sigma.B(p\bar{p} \rightarrow W^\pm \rightarrow e^\pm \nu_e) &= 0.60 \pm 0.05 \pm 0.09 \text{ nb} & (\text{UA1}) \\
\sigma.B(p\bar{p} \rightarrow W^\pm \rightarrow e^\pm \nu_e) &= 0.61 \pm 0.05 \pm 0.07 \text{ nb} & (\text{UA2}) \\
\sigma.B(p\bar{p} \rightarrow Z^0 \rightarrow e^+ e^-) &= 73 \pm 14 \pm 11 \text{ pb} & (\text{UA1}) \\
\sigma.B(p\bar{p} \rightarrow Z^0 \rightarrow e^+ e^-) &= 69 \pm 13 \pm 6 \text{ pb} & (\text{UA2})
\end{aligned}$$

in addition UA1 has measured the following cross-sections involving μ and τ leptons,

$$\begin{aligned}
\sigma.B(p\bar{p} \rightarrow W^\pm \rightarrow \mu^\pm \nu_\mu) &= 0.67 \pm 0.08 \pm 0.14 \text{ nb} \\
\sigma.B(p\bar{p} \rightarrow W^\pm \rightarrow \tau^\pm \nu_\tau) &= 0.61 \pm 0.12 \pm 0.11 \text{ nb} \\
\sigma.B(p\bar{p} \rightarrow Z^0 \rightarrow \mu^+ \mu^-) &= 73 \pm 19 \pm 13 \text{ pb}
\end{aligned} \tag{2.11}$$

Our theoretical calculations based on the work reported in ref. [28] give good agreement with both the data and other calculations [25]. We estimate

$$\begin{aligned}
\sigma.B(p\bar{p} \rightarrow W^\pm \rightarrow l^\pm \nu_l) &= 0.580 \text{ nb} \\
&\quad +0.05 \\
&\quad -0.13 \\
\sigma.B(p\bar{p} \rightarrow Z^0 \rightarrow l^+ l^-) &= 68 \text{ pb} \\
&\quad +3 \\
&\quad -16
\end{aligned} \tag{2.12}$$

Table 1 contains the cross-sections for hadronic decays of W and Z calculated up to $O(\alpha_s)$. There are two different entries for the top quark mass ($m_t = 23 \text{ GeV}/c^2$ and $m_t = 40 \text{ GeV}/c^2$). Fig. 6 shows the top quark mass dependence of the cross-sections (2.10) explicitly for both $\sqrt{s} = 546 \text{ GeV}$ and $\sqrt{s} = 630 \text{ GeV}$ in lowest order and $O(\alpha_s)$. These cross-sections are to be compared with the top quark cross-section from QCD processes. Leaving the top quark mass again as a free parameter, fig. 7 shows the relative importance of the QCD and weak production processes of top quarks at $\sqrt{s} = 630 \text{ GeV}$. QCD clearly dominates over $p\bar{p} \rightarrow W^\pm \rightarrow t\bar{b}$ for $m_t < 40 \text{ GeV}/c^2$, whereas top production via Z^0 decay is down by an order of magnitude. In fact, including the QCD contribution to the one from the process $p\bar{p} \rightarrow W^\pm \rightarrow t\bar{b}$ will reduce the semileptonic branching ratio $t \rightarrow l^+ X$, compared to what has been reported by the UA1 collaboration [29], and would bring it closer to the standard model expectations [13]. Some issues involving top quark production and decays including the effects of the $W^\pm$ polarization will be discussed in a forthcoming paper [30].

3. Fragmentation and Weak Decays of Heavy Quarks

We assume that quarks and gluons fragment independently. This assumption is in broad agreement with data on jets, in particular on leading particles inside jets [31]. It is less satisfactory for describing the distribution of very soft particles, in particular the ones which are found at very large angles from the jet direction [32]. We are not very much interested here in the parametrization of the very soft edge of a fragmenting jet in the presence of a very large hadronic background due to minimum bias events, but rather in having a faithful description of the inclusive hadron and lepton spectra and of the weak decay

characteristics of the top, bottom and charm quarks. For the bottom and charm quark fragmentation and decays, there exist vast amounts of data, mostly from e^+e^- experiments, which we have incorporated in our model.

For the fragmentation of heavy quarks we use the parametrization of Peterson et al. [14] defined as

$$f(z) = \frac{z(1-z)^2}{[z(1-z) - (1-z) - \epsilon_Q z]^2} \quad (3.1)$$

with $z = (E + p_{\parallel})^H / (E + p_{\parallel})^Q$; the parameter ϵ_Q depends on the heavy quark fragmenting into the heavy hadron H ; we consider both heavy mesons and heavy baryons.

The heavy quark $\rightarrow$ heavy hadron fragmentation $\bar{b} \rightarrow B X$ and $c \rightarrow (D, D^*)$ have been measured in e^+e^- experiments [15]. Extracting the Peterson et al. parameters ϵ_c and ϵ_b from data is, however, a non-trivial matter. In particular, the softening effects in the fragmentation functions due to perturbative QCD gluon radiation and initial state photon radiation have to be taken into account. These processes contribute differently in proton-antiproton collisions and since we have taken the QCD corrections in pp collisions explicitly into account, both in the initial and final state, one should extract ϵ_c and ϵ_b from e^+e^- data by correcting the data for QCD and QED effects. This has been done in the analysis of the e^+e^- data for the charm quark fragmentation by Bethke [15], resulting in $\langle z_c \rangle = 0.71 \pm 0.014 \pm 0.03$. This value of $\langle z_c \rangle$ is found to be consistent with the interpretation of the data in the c.m. energy region 10 GeV-34 GeV being scale invariant. Since the average c.m. energy in the parton-parton system, $\sqrt{\hat{s}}$, in $p\bar{p}$ collisions at the CERN collider is marginally larger than the energy range in e^+e^- experiments at PEP and PETRA, we expect that the value of $\langle z_c \rangle$ should not be very different in $p\bar{p}$ collisions. We use $\epsilon_c = 0.05$ giving $\langle z_c \rangle = 0.68$, which is in agreement with the analysis of Bethke. Likewise, we use $\epsilon_b = 0.015$ leading to $\langle z_b \rangle = 0.77$ and $\epsilon_t = 0.0002$ giving $\langle z_t \rangle = 0.93$. The normalized distributions $1/N dN/dz$ for charm, bottom and top quark fragmentation are shown in fig. 8.

We would like to point out here that a correct incorporation of the fragmentation of charm and bottom quarks is crucial in estimating not only the overall rate of inclusive μ and $\mu\mu$ production in pp collisions but also in estimating the ratio $R(\pm\pm/\pm-)$, the quantity we are principally interested in here. A harder $\bar{b} \rightarrow B X$ fragmentation leads to more energetic primary and secondary leptons and this would in turn increase the ratio $R(\pm\pm/\pm-)$. A harder $c \rightarrow D X$ fragmentation, on the other hand, decreases $R(\pm\pm/\pm-)$, since the process $p\bar{p} \rightarrow c\bar{c} X \rightarrow \mu\mu X$ contributes to the $\mu^+\mu^-$ final state only. Determining the rate and shape of the charmed hadron energy distributions by an independent measurement, like for example by the process $p\bar{p} \rightarrow D^* X$ [22], would be an important cross-check on the inclusive charmed hadron production.

Next we discuss the implementation of weak decays in our calculations. We have implemented all the measured branching ratios of the charmed hadrons D^0 (amounting to $\sim 93\%$ of the total) and D^+ ($\sim 80\%$ of the total) [11]. Likewise, all known decay modes of the D_s^+ (F^+) mesons [33] are included. The rest are filled using isospin and phase-space to estimate the individual contributions. For the charmed hadron semileptonic decays, like for example $D \rightarrow (K^*, K, \dots) l^+ \nu_l$, we have incorporated the relevant form factors [12] in the calculation of the decay spectra. A comparison of the EUROJET Monte Carlo distributions for the lepton energy spectrum in charm decays with the DELCO data [34] is shown in fig. 9. The

agreement is quite satisfactory. The semileptonic branching ratios are given in table 2.

The decays of the bottom hadrons B_u^- , B_d^0 are taken into account as far as they are known, otherwise we calculate the bottom quark decays based on the free quark decay model

$$\begin{aligned} b &\rightarrow c \bar{u} d \\ b &\rightarrow c \bar{c} s \\ b &\rightarrow c l \bar{\nu}_l \quad (l = e, \mu, \tau) \end{aligned} \quad (3.2)$$

The semileptonic branching ratios for all bottom hadrons are assumed to be equal and are fixed at their experimental value, $BR(\bar{B} \rightarrow l^+ X) \cong 12\%$ [32]. A comparison of the EUROJET Monte Carlo predictions for the inclusive electron and muon spectra from the process $e^+e^- \rightarrow \Upsilon(4s) \rightarrow B\bar{B} \rightarrow e^\pm X, \mu^\pm X$ with the CLEO data [35] is shown in fig. 10. The theoretical curves take into account the Lorentz boost of the B meson ($p \sim 500$ MeV). The agreement is quite reasonable. The charged particle multiplicity distribution of our model in the process $e^+e^- \rightarrow \Upsilon(4s) \rightarrow B\bar{B}$ is shown in fig. 11 and compared with the DELCO data at $\Upsilon(4s)$. Again, the model describes the data rather well.

In fig. 12 we compare our model predictions for the $D^{*\pm}$ momentum distribution with the CLEO data for $B \rightarrow D^{*\pm} X$. Fig. 13 shows the comparison for the D^0 momentum distribution in $B \rightarrow D^0 X$ again with the CLEO data [36]. The agreement in both cases is very good which leads us to conclude that the importance of the two-body modes, like $B \rightarrow D\pi, D\rho$ etc., have been adequately taken into account.

The decay mode $b \rightarrow c \bar{c} s$ has received quite a bit of attention both theoretically [37] and experimentally [38]. The measurements of the decays $B \rightarrow J/\psi K X$ and $B \rightarrow F^\pm X$ have been experimentally established. The decay modes $B \rightarrow J/\psi X$, with the J/ψ inside a jet, provide another independent check of the bottom quark production cross-section at the $p\bar{p}$ -collider. Since the decay $J/\psi \rightarrow l^+l^-$ gives rise to dileptons, the reaction [38]

$$\begin{aligned} p\bar{p} &\rightarrow B\bar{B} X \rightarrow J/\psi l^\pm X \\ &\searrow \\ &l^+l^- \end{aligned} \quad (3.3)$$

would contribute both to $l^\pm l^\pm X$ and $l^+l^- X$. It is therefore important to have their contributions properly included. We use a branching ratio $BR(B \rightarrow J/\psi X) = 1\%$, of which 20% is in the form of two-body decays, $B \rightarrow J/\psi K$ etc.. The resulting inclusive J/ψ momentum spectrum in the process $e^+e^- \rightarrow \Upsilon(4s) \rightarrow B\bar{B} \rightarrow J/\psi X$ is shown in fig. 14 and compared with the data from CLEO and ARGUS [38]. The shape of the $p_{J/\psi}$ distribution is very well reproduced.

For the inclusive modes $B \rightarrow F^\pm X$ we use a branching ratio $BR(B \rightarrow F X) = 9.6\%$, with 60% in two-body decays. This branching ratio is in agreement with the recent measurements [39] and in fact has been motivated by the CLEO data. A comparison of the $p_{F^\pm}$ spectrum in the decays $B \rightarrow F^\pm X$ with the CLEO data is shown in fig. 15. The agreement is again very satisfactory. The importance of two-body decays in $B \rightarrow F^\pm X$ and its relevance to the ratio $R(\pm\pm/\mp\mp)$ has also been recently emphasized in ref. [40]. The comparisons here prove that we have properly taken into account the hardness of the $F^\pm$ -spectrum in B decays.

For the decays of top quarks very little data are available. Since $m_t > 23$ GeV/c², due to negative searches at PETRA [41], the description of the top quark

decays in terms of the free-quark decay model should turn out to be a very good description. In fact if the UA1-topology $p\bar{p} \rightarrow e^\pm + \text{jet} + \text{jet} + \cancel{E}_T$ is due to top quark decays, then all exotic decays of the top quark, like for example $t \rightarrow b H^\pm$, involving a charged Higgs, are forbidden. We assume here that the top quark decays the way it is expected in the standard (3-family) model. We have modelled the dominant top quark decays as follows

$$\begin{aligned} t &\rightarrow b u \bar{d} \\ t &\rightarrow b c \bar{s} \\ t &\rightarrow b l^+ \nu_l \quad (l = e, \mu, \tau) \end{aligned} \quad (3.4)$$

where the decays are governed by the V-A matrix elements [12]. All the quarks in (3.4) lead to jets. In some rare cases when the energy of the quark in (3.4) is not sufficient, we combine it with the antiquark and form a hadron which is on-shell. Energy, momentum and all flavour quantum numbers are conserved in the decay procedure using a scheme from ref. [42]. We have checked that this energy-momentum conservation is indeed a soft process and that it does not disturb the distributions, characteristics of a three-body V-A decay of a spin-1/2 object. Since there are two dominant sources of top quark production in $p\bar{p}$ collisions which we have discussed in the previous section and since the $W^\pm$ are known to be produced highly polarized, we have taken into account the effects of $W^\pm$ polarization in the decay chain $W^\pm \rightarrow tb X$, using the methods of ref. [43]. However, the top quark contribution to the process $p\bar{p} \rightarrow \mu \mu X$ is small and we shall not go into quantitative details of such effects here.

4. Estimates of Multimuon Production at the CERN $p\bar{p}$ Collider and Comparisons with the UA1 Data

Having stated our theoretical framework and phenomenological input from experiments in e^+e^- annihilation, we present our numerical results for the following three final states in pp collisions at $\sqrt{s} = 630$ GeV

$$\begin{aligned} p\bar{p} &\rightarrow QX \rightarrow \mu^\pm X \\ p\bar{p} &\rightarrow QX \rightarrow \mu^+\mu^- X, \mu^\pm\mu^\pm X \\ p\bar{p} &\rightarrow QX \rightarrow \mu\mu\mu X \end{aligned} \quad (4.1)$$

We emphasize that in the estimates presented here only contributions from heavy quark production are taken into account, and include both QCD production and the processes $p\bar{p} \rightarrow W^\pm, Z^0 \rightarrow Q\bar{Q}', Q\bar{Q} \rightarrow \mu X, 2\mu X, 3\mu X$; all are shown separately. The cross-sections for the final states (4.1) are calculated with a cut-off on p_T^μ and on the pseudo-rapidity of the muon, η^μ . Since the triggers used by the UA1 collaboration in measuring the cross-sections (4.1) differ for each of the final states shown, we shall always include these conditions while comparing our results with their data. We also give theoretical cross-sections for somewhat larger μ -acceptance in η^μ in view of various improvements of the UA1 detector under way.

In figs. 16, 19 and 22 we show the differential cross-section for the heavy flavour induced inclusive muon production $d\sigma(p_T^\mu > 3\text{GeV}/c)/d\eta^\mu$ for the QCD, $W^\pm$ and Z^0 production processes respectively, where contributions from $2 \rightarrow 2$ and 2

→ 3 processes are shown separately. The integrated inclusive muon cross-sections

$$\int \frac{d^2\sigma(p\bar{p} \rightarrow Q\bar{Q} X \rightarrow \mu^\pm X)}{dp_t^\mu d\eta^\mu} dp_t^\mu d\eta^\mu \quad (4.2)$$

$p_t^\mu > 3 \text{ GeV/c}$

from the charm, bottom and top quarks are shown in tables 3 and 4 for the two values of η^μ , (i) $|\eta^\mu| < 2.5$ and (ii) $|\eta^\mu| < 1.5$. As can be seen from fig. 16, tables 3 and 4, the charm and bottom cross-sections in the QCD process $p\bar{p} \rightarrow Q\bar{Q} X \rightarrow \mu^\pm X$ are comparable.

The inclusive dimuon cross-sections $d\sigma(p\bar{p} \rightarrow Q\bar{Q} X \rightarrow \mu^+\mu^- X, \mu^\pm\mu^\pm X)/d\eta^\mu$ for the QCD, $W^\pm$ and Z^0 production processes, are shown in figs. 17, 20 and 23 for $|\eta^\mu| < 2.0$ and $p_{t1}^\mu, p_{t2}^\mu > 3 \text{ GeV/c}$. The integrated cross-sections

$$\int \frac{d^2\sigma(p\bar{p} \rightarrow Q\bar{Q} X \rightarrow \mu^+\mu^- X, \mu^\pm\mu^\pm X)}{dp_{t1}^\mu d\eta^\mu} dp_{t1}^\mu d\eta^\mu \quad (4.3)$$

$p_{t1}^\mu, p_{t2}^\mu > 3 \text{ GeV/c}$
 $|\eta^\mu| < 2.0$

are given in tables 3 and 4. Here the dominant cross-section is due to the bottom pair production process $p\bar{p} \rightarrow b\bar{b} X \rightarrow \mu\mu X$, contributing $\sim 75\%$ of the total. This result is in fair agreement with earlier estimates [44]. As can be seen in fig. 17, the contribution to the dimuon final state from $2 \rightarrow 3$ bottom pair production processes is substantial. Another way to see this contribution is in the prediction of the azimuthal angle distribution of the dimuons, $d\sigma/d\Delta\phi^{\mu\mu}$, where the $2 \rightarrow 3$ induced radiative tail should be clearly discernible in contrast to the almost back-to-back dimuons expected from the $2 \rightarrow 2$ $b\bar{b}$ process. We shall return to this point shortly when we compare our results with the UA1 data.

The inclusive trimuon cross-sections from $p\bar{p} \rightarrow Q\bar{Q} X \rightarrow 3\mu X$, for the QCD, $W^\pm$ and Z^0 production processes respectively, with $p_t^\mu > 3 \text{ GeV/c}$ and $|\eta^\mu| < 2.0$ are shown in figs. 18, 21 and 24. Note that up to $O(\alpha_s^3)$, the trimuon final states receive contributions only from the bottom and the top quark intermediate states. Again the contributions from $2 \rightarrow 2$ and $2 \rightarrow 3$ processes are shown separately. The integrated trimuon cross-sections

$$\int \frac{d^2\sigma(p\bar{p} \rightarrow Q\bar{Q} X \rightarrow 3\mu X)}{dp_t^\mu d\eta^\mu} dp_t^\mu d\eta^\mu \quad (4.4)$$

$p_t^\mu > 3 \text{ GeV/c}$
 $|\eta^\mu| < 2.0$

are also presented in tables 3 and 4. Here again the QCD process $p\bar{p} \rightarrow b\bar{b} X \rightarrow 3\mu X$ dominates for $m_t > 30 \text{ GeV/c}^2$, though for smaller values of m_t , the $t\bar{t}$ state also gives a significant contribution. For example, for $m_t = 23 \text{ GeV/c}^2$ the $t\bar{t} \rightarrow 3\mu X$ cross-section becomes larger than the corresponding $b\bar{b}$ cross-section.

In figs. 25-27, we show the p_t^μ distributions for $p\bar{p} \rightarrow Q\bar{Q} X \rightarrow \mu X$, $\mu\mu X$ and $\mu\mu\mu X$ final states respectively, where now we have summed the $2 \rightarrow 2$ and $2 \rightarrow 3$ processes. Note that the pseudo-rapidity interval is now different for all final states. The η^μ values have been chosen so as to conform to the UA1 triggering conditions. In addition, for the dimuon final state, there is an additional cut-off used to calculate $d\sigma/dp_t^\mu$ namely $M^{\mu\mu} > 6 \text{ GeV/c}^2$. The distributions (25-27) can be directly compared to the UA1 data, once the data have been corrected for

efficiency. For completeness we show in figs. 28-30 the inclusive p_t^μ distributions from the production of $W^\pm$ bosons and their decays involving heavy quarks for the processes $p\bar{p} \rightarrow W^\pm X \rightarrow (cs, tb) X \rightarrow \mu^\pm X$, $\mu\mu X$ and $\mu\mu\mu X$ respectively, with the indicated experimental cuts. Figs. 31-33 show the corresponding distributions from the Z^0 intermediate state, $p\bar{p} \rightarrow Z^0 X \rightarrow (c\bar{c}, b\bar{b}, t\bar{t}) X \rightarrow \mu^\pm X$, $\mu\mu X$ and $\mu\mu\mu X$. All top quark related distributions are calculated for two values of m_t , namely $m_t = 23 \text{ GeV}/c^2$ and $m_t = 40 \text{ GeV}/c^2$ and include the $O(\alpha_s)$ corrections.

There are a number of comments that we would like to make about the p_t^μ distributions shown in figs. 25-33. Based on $d\sigma/dp_t^\mu$ distributions alone, it would be very difficult to detect the presence of the top quark. As shown in fig. 25, the inclusive p_t^μ distribution is dominated by the $p\bar{p} \rightarrow c\bar{c} X$, $b\bar{b} X \rightarrow \mu^\pm X$ states, unless m_t is close to its present experimental lower bound. In the case of dimuons, the cross-section $d\sigma/dp_t^\mu$ is dominated by the $b\bar{b}$ intermediate state for $p_t^\mu < 10 \text{ GeV}/c$ ($20 \text{ GeV}/c$) for $m_t \sim 23 \text{ GeV}/c^2$ ($40 \text{ GeV}/c^2$). However, the dimuon cross-section for large p_t^μ ($p_t^{\mu 1}, p_t^{\mu 2} > 10 \text{ GeV}/c$) is small so that a lot more data are needed before one could use the dimuon events to establish the top quark. With $m_t = 40 \text{ GeV}/c^2$, the estimates of fig. 26 and adding the processes $p\bar{p} \rightarrow W^\pm, Z^0 \rightarrow tb, t\bar{t} \rightarrow \mu\mu X$, we estimate from figs. 29, 32 and the present UA1 luminosity that $O(10)$ dimuon pairs could possibly come from the top decays. This number would be substantially larger if m_t is smaller. For $m_t = 23 \text{ GeV}/c^2$, the cross-sections $d\sigma(p\bar{p} \rightarrow t\bar{t} \rightarrow \mu\mu X)/dp_t^\mu$ is also shown. Our interest in the dimuon distribution is two-fold. First, it could signal the presence of the top quark in a precision p_t^μ measurement, combined with other kinematic distributions. Secondly, and for the discussion in this paper more importantly, the processes $p\bar{p} \rightarrow t\bar{t} \rightarrow \mu\mu X$, and $p\bar{p} \rightarrow W^\pm, Z^0 \rightarrow tb, t\bar{t} \rightarrow \mu\mu X$ give rise to both $\mu^+\mu^- X$ and $\mu^\pm\mu^\pm X$ states. In fact with the UA1 cuts, the ratio $R(\pm\pm/\pm-)$ from the $t\bar{t}$ state alone could reach as high a value as 0.5. Since m_t is not known, one has to include possible contributions from this source. Of course, one expects that the azimuthal angle distribution $d\sigma/d\Delta\phi_{\mu\mu}$ from the massive top quark decays (and for that matter from the decays of any other massive object decaying weakly into leptons) would be quite different from the corresponding distributions from the processes $p\bar{p} \rightarrow c\bar{c} X, b\bar{b} X \rightarrow \mu\mu X$. This, in turn can be used to disentangle the two sources.

The trimuon distributions $p\bar{p} \rightarrow b\bar{b} X, t\bar{t} X \rightarrow 3\mu X$ shown in fig. 27 are rather instructive. In particular they show that the trimuon p_t distributions could be dominated by the $t\bar{t}$ state for $p_t^\mu > 6 \text{ GeV}/c$, if m_t is close to $23 \text{ GeV}/c^2$. The $t\bar{t}/b\bar{b}$ cross-over is shifted to higher p_t^μ values with increasing m_t . For $m_t = 40 \text{ GeV}/c^2$, the cross-over is around $p_t^\mu \sim 14 \text{ GeV}/c$. The trimuon events can also reveal the presence of the top quark, particularly if one applies a cut-off on the dimuon invariant mass of all three dimuon pairs. The point here is rather simple; in the process $p\bar{p} \rightarrow b\bar{b} X \rightarrow 3\mu X$, one of the dimuon pairs ($\mu^+\mu^-$) must emerge from the decay of the *same* bottom hadron. Thus, applying an invariant mass cut $M_{\mu\mu} > 6 \text{ GeV}/c^2$ will practically remove all trimuons originating from the $b\bar{b}$ decays. We advocate the trimuon final state in establishing the existence of the top quark. With the cross-sections shown in figs. 27, 30, 33, tables 3 and 4, we do expect $\sim 5-10$ trimuon events assuming $m_t = 40 \text{ GeV}/c^2$, using the present UA1 luminosity and μ detection efficiency. This number is expected to be larger for smaller values of m_t , as shown in these figures.

Before presenting a comparison of our calculations with the UA1 data, we would like to come back to the theoretical uncertainties in our calculations. We

have already indicated the uncertainty on the fragmentation parameters ϵ_c and ϵ_b . In addition, the cross-sections depend on the value of the QCD scale parameter and on the structure functions. The question related to the QCD scale is intimately connected with the virtual corrections in the process $p\bar{p} \rightarrow Q\bar{Q} X$, which we do not have at hand yet. Instead, we fix the QCD scale and the structure functions from jet data [8]. Since we use a cut-off on the transverse energy of the light (quark or gluon) jet in $2 \rightarrow 3$ processes, produced in association with the heavy $Q\bar{Q}$ pair, the cross-sections for the inclusive muon and dimuon final states depend on the precise value of the cut-off parameter, p_t^{cut} , as we have discussed earlier. The dependence of $\sigma(p\bar{p} \rightarrow c\bar{c} X, b\bar{b} X \rightarrow \mu^\pm X)$ and $\sigma(p\bar{p} \rightarrow c\bar{c} X, b\bar{b} X \rightarrow \mu\mu X)$ on p_t^{cut} with the UA1 trigger conditions, is shown in figs. 34 and 35, respectively, in the range $3 \text{ GeV}/c < p_t^{\text{cut}} < 7 \text{ GeV}/c$. The cross-sections differ by a factor $\sim 1.5 - 2$ in this range. This dependence can be considerably reduced if one incorporates a physical damping factor for small p_t^{jet} as done, for example, in the theoretical estimates for $d\sigma/dp_t^W$ [25]. We instead use the value $p_t^{\text{cut}} = 5 \text{ GeV}/c$, motivated by the K-factor that one obtains from higher orders for the charm pair production cross-section in leading log. approximation [27].

In fig. 36 we show a comparison of our theoretical calculations with the UA1 data for the processes $p\bar{p} \rightarrow \mu^\pm X$ and $p\bar{p} \rightarrow \mu^\pm \mu^\pm X, \mu^+ \mu^- X$. The dimuon data belong to the so-called "non-isolated μ -sample", and are expected to be due to heavy flavours. The single muon data contain both isolated and non-isolated muons, however the contribution from isolated muons is expected to be small (Drell-Yan etc.). The theoretical curves include only the contributions from the processes $p\bar{p} \rightarrow c\bar{c} X, b\bar{b} X$ and we have put in the UA1 muon-detection efficiency. The agreement between theory and experiment is rather satisfactory, noting that the overall normalizations for both $p\bar{p} \rightarrow \mu^\pm X$ and $p\bar{p} \rightarrow \mu\mu X$ are predictions and not fits. Despite the fair to good fits shown in fig. 36, there is some room for a possible top quark contribution particularly for large p_t^μ . Going back to figs. 25-27, we recall that it is the large p_t^μ tail, where the effect of the top quark is expected to be significant. Based on this observation we now include also the contributions from the processes involving the top quark, namely $p\bar{p} \rightarrow t\bar{t} X$ and $p\bar{p} \rightarrow W^\pm X \rightarrow t\bar{b} X', p\bar{p} \rightarrow Z^0 X \rightarrow t\bar{t} X'$, again for the two representative values $m_t = 23 \text{ GeV}/c^2$ and $m_t = 40 \text{ GeV}/c^2$. The resulting distributions for the inclusive muon spectra for $p\bar{p} \rightarrow \mu X, 2\mu X, 3\mu X$ from both QCD and weak processes are shown in fig. 37. The inclusion of the top quark contribution brings the inclusive μ data and theory in remarkable agreement, including the large p_t^μ tail. This line of argument, however, assumes that the contributions from the processes $p\bar{p} \rightarrow c\bar{c}, b\bar{b} X \rightarrow \mu^\pm X'$ are precisely calculable. Unfortunately, with the present technology QCD predictions are reliable only to within a factor 2. Hence, the inclusive muon differential cross-section, $d\sigma/dp_t^\mu$, measured with respect to the beam axis is too indirect a method to search for the top quark. A lot more confident statement about the presence and the mass of the top quark would be possible if p_t^μ at the $p\bar{p}$ collider could be measured with respect to the jet axis. In the absence of such information we would like to see a detailed analysis of the multimMuon data, in particular of the large p_t^μ tail to get more definitive results about the top quark mass. Such searches have already been advocated by other authors [45]. Our detailed analysis presented here could also be indicating the presence of top-like quark objects in the inclusive muon and dimuon cross-sections though it is difficult to make a more quantitative statement.

In the rest of this paper we shall concentrate on the dimuon data sample. In fig. 38 we show a comparison of our model with the dimuon azimuthal angle distribution $d\sigma/d\Delta\phi_{\mu\mu}$ obtained by the UA1 collaboration. Note the large tail in

the $\Delta\phi_{\mu\mu}$ distribution, which receives contributions both from the QCD radiative corrections ($2 \rightarrow 3$ and higher order processes) as well as from the top quark decays due to the large top mass, $m_t \geq 23 \text{ GeV}/c^2$, giving rise to dimuons at all angles $\Delta\phi_{\mu\mu}$. Again, the comparison of our model calculations with the UA1 data is quite satisfactory. Fig. 38 indicates that forcing the two muons to lie in the same hemisphere $\Delta\phi_{\mu\mu} < 90^\circ$ and demanding large $p_{t\mu}$ for the two muons, say $p_{t\mu} > 6 \text{ GeV}/c$ would reduce the $p\bar{p} \rightarrow c\bar{c} X, b\bar{b} X \rightarrow \mu\mu X'$ background drastically. Detailed kinematic fits can then be attempted for the remaining $\mu\mu$ -data sample to determine the possible top quark content.

In fig. 39 we show the distribution $d\sigma/dp_{t\mu\mu}$ for the dimuon data, as measured by UA1, together with our model calculations. A comparison of the invariant dimuon mass distribution $d\sigma/dM_{\mu\mu}$ of the same data with our calculations is shown in fig. 40; the agreement is rather good. Based on the comparisons shown in figs. 38-40, it is fair to conclude that our model, which is based on perturbative QCD, provides a successful description of heavy flavour production at the CERN $p\bar{p}$ Collider. Both the absolute cross-section and various distributions measured by the UA1 collaboration are well reproduced. Thus, it is fair to say that also the heavy flavour production aspect of the collider data is under the quantitative control of perturbative QCD.

Having convinced ourselves that we have been able to provide a quantitative description of heavy flavours, we turn to estimates of the same sign to opposite sign dimuon ratio, $R(\pm\pm/\pm-)$. In estimating $R(\pm\pm/\pm-)$, we shall not consider Drell-Yan type processes $p\bar{p} \rightarrow \gamma^*, Z^0 \rightarrow \mu\mu X'$ and direct QCD bound $Q\bar{Q}$ production $p\bar{p} \rightarrow J/\psi, \Upsilon \rightarrow \mu\mu X'$. These measurements have been analysed by others and have been found to be in good agreement with estimates based on perturbative QCD. Concentrating on the "non-isolated" dimuon data and fixing the sign of the muons, we recall that the following states contribute :

$$\begin{aligned}
 p\bar{p} &\rightarrow c\bar{c} X \rightarrow \mu^+\mu^- X' \\
 p\bar{p} &\rightarrow b\bar{b} X \rightarrow \mu^+\mu^- X', \mu^\pm\mu^\pm X' \\
 p\bar{p} &\rightarrow t\bar{t} X \rightarrow \mu^+\mu^- X', \mu^\pm\mu^\pm X' \\
 p\bar{p} &\rightarrow W^\pm X \rightarrow tb X' \rightarrow \mu^+\mu^- X'', \mu^\pm\mu^\pm X'' \\
 p\bar{p} &\rightarrow Z^0 X \rightarrow (c\bar{c}, b\bar{b}, t\bar{t}) X' \rightarrow \mu^+\mu^- X'', \mu^\pm\mu^\pm X''
 \end{aligned} \tag{4.5}$$

only taking into account production processes in the leading order and in the next-to-leading order. In these orders, the intermediate state $c\bar{c} X$ contributes only to the $\mu^+\mu^- X'$ final state. In $O(\alpha_s^4)$, one has in addition the process :

$$p\bar{p} \rightarrow c\bar{c} c\bar{c} X \rightarrow \mu^+\mu^- X', \mu^\pm\mu^\pm X' \tag{4.6}$$

giving rise to both same- and opposite-sign dimuons. Theoretical calculations for the 4-parton final state processes have already been done [46] and we have used them to estimate the cross-section for the process (4.6). Since the results numerically are not very significant for $R(\pm\pm/\pm-)$, we defer their discussion to a separate publication [47].

In table 5 we present the results for $R(\pm\pm/\pm-)$ based on all the five processes (4.5), together with their relative contributions corresponding to the UA1 dimuon cuts. The numbers also indicate the effect of $B^0-\bar{B}^0$ mixing on the ratio $R(\pm\pm/\pm-)$, with the assumed value of the B_s^0 production fraction ($P_s \sim 16\% - 20\%$). The ratio $R(\pm\pm/\pm-)$ can then be computed by the relation

$$R(\pm\pm/\pm-) = \sum_{i=1}^5 R_i(\pm\pm/\pm-) BR_i \quad (4.7)$$

where the sum $i=1, 5$ is over the five processes shown in (4.5). BR_i has the obvious definition

$$BR_i = \frac{\sigma(p\bar{p} \rightarrow i \rightarrow \mu^+ \mu^- X)}{\sum_i \sigma(p\bar{p} \rightarrow i \rightarrow \mu^+ \mu^- X)} \quad (4.8)$$

With our choice of the branching ratios, quark masses and model parameters, we find

$$R(\pm\pm/\pm-) = 0.20 - 0.24, \quad m_t = 40 - 23 \text{ GeV}/c^2, \text{ no mixing}$$

The values of $R_i(\pm\pm/\pm-)$ are shown in table 5. Compared with the UA1 result $R(\pm\pm/\pm-) = 0.42 \pm 0.07 \pm 0.03$ (1.64 σ lower bound: $R(\pm\pm/\pm-) > 0.295$) all the values for $R(\pm\pm/\pm-)$ with no mixing fail to reproduce the data. The values with complete $B_s^0 - \bar{B}_s^0$ mixing are, on the other hand, in better agreement with the UA1 data (table 6)

$$R(\pm\pm/\pm-) = 0.38, \quad P_s = 0.20, \quad \text{complete } B_s^0 - \bar{B}_s^0 \text{ mixing.}$$

We shall discuss the mixing case in detail in the next section, but here we would like to comment on possible errors in $R(\pm\pm/\pm-)$ assuming *no* $B^0 - \bar{B}^0$ mixing.

The biggest uncertainties on $R(\pm\pm/\pm-)$ are probably due to both the contribution from the process $p\bar{p} \rightarrow c\bar{c} X \rightarrow \mu^+ \mu^- X'$ and the fraction of secondary muons from the bottom cascade decay. The p_t^μ spectrum, measured with respect to the jet axis can be used to determine the $c\bar{c}/(c\bar{c} + b\bar{b})$ content. The UA1 measurements [17] lead to an estimate of the $c\bar{c}$ contribution to the dimuon cross-section giving $15 \pm 7\%$. The numbers for R_i quoted in table 5 are based on the semileptonic charm branching ratios shown in table 2 and 15% cc contribution. The charm cross-sections presented in tables 1, 3 and 4 represent the upper limit of the UA1 measurement, namely, 22% of the dimuon cross-section is due to $c\bar{c}$. For a softer charmed hadron spectrum, the cross-section $\sigma(p\bar{p} \rightarrow c\bar{c} X \rightarrow \mu^+ \mu^- X')$ with the UA1 cuts decreases, consequently increasing $R(\pm\pm/\pm-)$. The largest value of $R(\pm\pm/\pm-)$ is then reached with the smallest allowed $c\bar{c}$ content. We estimate this uncertainty on $R(\pm\pm/\pm-)$ to be 0.04. The errors $\Delta R(\pm\pm/\pm-)$, due to various branching ratios, D^*/D values, spread in ϵ_b and semileptonic decay spectra are estimated to be of the same order. Thus, for the choice of parameters, which maximizes $R(\pm\pm/\pm-)$, we find that it is safe to quote a limit based on $b\bar{b}$ and $c\bar{c}$ production

$$R(\pm\pm/\pm-) < 0.27 \quad (4.9)$$

for the case *without* $B^0 - \bar{B}^0$ mixing.

The entries in table 5 indicate that the top quark may increase $R(\pm\pm/\pm-)$ particularly for lower values of m_t . However, estimates based on perturbative QCD show that even including the top quark contribution makes it difficult to push $R(\pm\pm/\pm-)$ above 0.29. Thus, with our present knowledge of the quark masses, branching ratios and fragmentation functions, we find that the quantity $R(\pm\pm/\pm-)$

without $B^0-\bar{B}^0$ mixing is in the range of $0.19 \leq R(\pm\pm/\pm-) \leq 0.29$. The ratio $R(\pm\pm/\pm-)$ with $B^0-\bar{B}^0$ mixing considering all the uncertainties mentioned above lies in the range $0.31 \leq R(\pm\pm/\pm-) \leq 0.42$.

5. Implications of $R(\pm\pm/\pm-)$ for the Parameters in the Standard Model

We shall pursue in this section the consequences of interpreting the UA1 data on $R(\pm\pm/\pm-)$ in terms of $B^0-\bar{B}^0$ mixing. Both experiment and the standard model predict no substantial $D^0-\bar{D}^0$ mixing [11], whereas the total charm contribution to the dimuon cross-section is expected to be relatively small. Since we have just described an analysis for $R(\pm\pm/\pm-)$ in $p\bar{p}$ collisions, we would like to review here the situation in e^+e^- annihilation experiments. The object of interest is the quantity "r" defined as the ratio of the "wrong-sign" semileptonic fraction to the "right-sign" [48]

$$r_i \equiv \frac{\Gamma(B_i^0 \rightarrow l^+ \nu_l X^-)}{\Gamma(B_i^0 \rightarrow l^- \nu_l X^+)} \quad (i = d, s) \quad (5.1)$$

with the well-known relationship

$$r = \eta^2 \frac{x^2 + y^2}{2 + x^2 - y^2} \quad (5.2)$$

$$\bar{r} = \eta^{-2} \frac{x^2 + y^2}{2 + x^2 - y^2}$$

where $x = \Delta M/\Gamma$, $y = \Delta \Gamma/2\Gamma$ with $\Gamma = (\Gamma_H + \Gamma_L)/2$ and η is a measure of CP-violation defined as

$$\langle B_H | B_L \rangle = \frac{1 - \eta^2}{1 + \eta^2} \quad (5.3)$$

ΔM and $\Delta \Gamma$ are, respectively, the mass and width differences between eigenstates having a definite mass and lifetime. In what follows we shall assume CP-conservation and set $\eta = 1$. It is also common to use the quantity χ_i defined as

$$\chi_i \equiv \frac{\Gamma(B_i^0 \rightarrow l^+ \nu_l X^-)}{\Gamma(B_i^0 \rightarrow l^\pm \nu_l X^-)} \quad (i = d, s) \quad (5.4)$$

with the obvious relationship $\chi_i = r_i/(1 + r_i)$. In the continuum measurements one has the relation

$$R_b^p(\pm\pm/\pm-) = \frac{2\chi(1-\chi)}{(1-\chi)^2 + \chi^2} \quad (5.5)$$

where R_b^p denotes contributions due to $B\bar{B}$ production, keeping only the primary decay leptons. The quantity χ is related to χ_d and χ_s via the relationship

$$\chi = \frac{[(BR)_d P_d \chi_d + (BR)_s P_s \chi_s]}{\langle BR \rangle} \quad (5.6)$$

where P_d (P_s) is the fraction of B_d^0 (B_s^0) meson production in the fragmentation $\bar{b} \rightarrow B_d^0 X$ ($\bar{b} \rightarrow B_s^0 X$); $(BR)_d$, $(BR)_s$ and $\langle BR \rangle$ are respectively the semileptonic branching ratios for B_d^0 , B_s^0 and average semileptonic branching ratios of the bottom hadrons. The best present limit on the quantity χ from e^+e^- experiments is [49]

$$\chi(e^+e^-) < 0.12 \quad (90\% \text{ C.L.}) \quad (5.7)$$

This can be converted into limits on the two mixing probabilities r_d and r_s assuming some reasonable values for P_d , P_s , $(BR)_d/\langle BR \rangle$ and $(BR)_s/\langle BR \rangle$. With the assumption $(BR)_d = (BR)_s = \langle BR \rangle$, the contours corresponding to three choices (a) $P_d = 0.35$, $P_s = 0.10$, (b) $P_d = 0.375$, $P_s = 0.15$, (c) $P_d = 0.4$, $P_s = 0.2$ are shown in fig. 41. Another limit on $\chi(e^+e^-)$ has been obtained by the measurement of the electroweak forward-backward asymmetry in the reaction $e^+e^- \rightarrow b\bar{b} \rightarrow l\bar{l}X$, which is expected to deviate from its standard value due to B - $\bar{B}$ mixing [50]. This method gives $\chi < 0.13$ [51], a somewhat less stringent bound than the MARK II limit, (5.7).

Two other measurements of $R_b^p(\pm\pm/\pm-)$ in e^+e^- annihilation have been attempted so far at $\Upsilon(4s)$ [52]. Since the decay $\Upsilon(4s) \rightarrow B\bar{B}$ is a p-wave, one has an extra suppression factor for $R(\pm\pm/\pm-)$ due to the Bose statistics [53]. Calling $R_b^p(\pm\pm/\pm-)$ at $\Upsilon(4s)$ R_{odd} , one has (for $r_d \ll 1$)

$$R_{\text{odd}} \equiv R_b^p(\pm\pm/\pm-) \Big|_{\Upsilon(4s)} = \frac{\chi(\Upsilon(4s))}{1 - \chi(\Upsilon(4s))} \quad (5.8)$$

where

$$\chi(\Upsilon(4s)) = \frac{(BR)_d P_d(\Upsilon(4s)) \chi_d}{\langle BR \rangle} \approx P_d(\Upsilon(4s)) \chi_d \quad (5.9)$$

and $P_d(\Upsilon(4s))$ is the probability $(\Upsilon(4s) \rightarrow B_d\bar{B}_d)/(\Upsilon(4s) \rightarrow B\bar{B})$. With $P_d = 0.4$ and $(BR)_d = (BR)_u = \langle BR \rangle$, the present limits on R_{odd} are as follows [52]

$$\begin{aligned}
 R_{\text{odd}} &< 0.12 && (90 \% \text{ C.L.}) \text{ ARGUS} \\
 &< 0.18 && (90 \% \text{ C.L.}) \text{ CLEO}
 \end{aligned}
 \tag{5.10}$$

The more stringent of the two bounds is shown in fig. 41 (curve d); it gives

$$\chi(Y(4s)) < 0.11 \tag{5.11}$$

which, on assuming $y_d^2/x_d^2 \ll 1$, an excellent approximation, gives

$$x_d < 0.52 \tag{5.12}$$

Thus, experiments in e^+e^- annihilation rule out significant mixing in the $B_d-\bar{B}_d$ sector.

In $p\bar{p}$ collisions, the UA1 collaboration has measured the ratio $R(\pm\pm/\pm-)$ to be (we emphasize that $R_{p\bar{p}} \neq R$)

$$R(\pm\pm/\pm-) = 0.42 \pm 0.07 \pm 0.03 \tag{5.13}$$

Thus, adding statistical and systematical errors in quadrature

$$R(\pm\pm/\pm-) > 0.295 \quad (1.64 \sigma) \tag{5.14}$$

The estimates for $R(\pm\pm/\pm-)$ without mixings were presented in the preceding section, where we found that there is considerable amount of uncertainty on this quantity. With $r_d = r_s = 0$, i.e. no $B-\bar{B}$ mixing, we found

$$R(\pm\pm/\pm-) = 0.24 \pm 0.05 \tag{5.15}$$

Thus, in our opinion the UA1 measurement (5.13) constitutes a definite excess of same-sign dimuons compared to the expectations (5.15). The UA1 measurements together with the background estimate (4.9) give

$$\chi(p\bar{p}) = 0.125 \pm 0.05 \tag{5.16}$$

This can be converted into a limit for r_d and r_s . The 1σ contour for r_d and r_s is shown in fig 41 with the choice $P_d = 0.4$, $P_s = 0.2$. This *together* with the ARGUS limit $r_d < 0.12$ gives

$$r_s > 0.18 \quad (1 \sigma) \tag{5.17}$$

For smaller values of P_s the lower bound on r_s is larger than (5.17). In the rest of this section we would like to briefly review the Standard Model predictions for r_d and r_s and extract a bound on the CKM matrix element V_{ts} from the UA1 lower bound on r_s .

In the standard model of flavour mixing, ΔM is calculated by the well-known box diagram involving the exchange of W^+ , W^- bosons. This gives [54]

$$\Delta M(B_d^0) = \frac{G_F^2 f_{B_d}^2 m_{B_d}}{6 \pi^2} M_W^2 \mathfrak{B} F_d(m_b, m_t, \lambda)$$

$$\Delta M(B_s^0) = \frac{G_F^2 f_{Bs}^2 m_{Bs}}{6 \pi^2} M_W^2 \mathfrak{B} F_s(m_b, m_t, \lambda) \quad (5.18)$$

Here $\mathfrak{B}$ is the so-called bag constant (expected to be ~ 1) for the bottom hadrons, f_{Bd} (f_{Bs}) are the pseudo-scalar coupling constants and the functions F_d and F_s are given by

$$\begin{aligned} F_d(m_b, m_t, \lambda) &= (\lambda_{cd})^2 U_1 + (\lambda_{td})^2 U_2 + 2 (\lambda_{td})(\lambda_{cd}) U_3 \\ F_s(m_b, m_t, \lambda) &= (\lambda_{cs})^2 U_1 + (\lambda_{ts})^2 U_2 + 2 (\lambda_{ts})(\lambda_{cs}) U_3 \end{aligned} \quad (5.19)$$

The CKM-angle-dependent factors λ_{ij} have the following values in the Wolfenstein parametrization [55]

$$\begin{aligned} |\lambda_{cd}| &\equiv |V_{cb}^* V_{cd}| = \lambda^3 \\ |\lambda_{td}| &\equiv |V_{tb}^* V_{td}| = \lambda^3 \sqrt{(1 - \rho)^2 + \eta^2} \leq \lambda^3 \\ |\lambda_{cs}| &\equiv |V_{cb}^* V_{cs}| = \lambda^2 \\ |\lambda_{ts}| &\equiv |V_{tb}^* V_{ts}| = \lambda^2 \end{aligned} \quad (5.20)$$

where $\lambda = \sin \theta_c \approx 0.23$. The quark-mass-dependent factors U_i have been calculated by Buras et al. [54] including QCD corrections to which we refer for details. The leading contributions to U_i are given by

$$\begin{aligned} U_1 &\sim m_c^2/M_W^2 \\ U_2 &\sim m_t^2/M_W^2 \\ U_3 &\sim m_c m_t/M_W^2 \end{aligned} \quad (5.21)$$

Thus, the dominant contribution for both $\Delta M(B_d)$ and $\Delta M(B_s)$ is due to the U_2 term. It is clear from (5.20), the experimental bound on $R \equiv \Gamma(b \rightarrow u l \nu_l)/\Gamma(b \rightarrow c l \nu_l)$ and the measurements of $\langle \tau_B \rangle$ [11] that

$$\begin{aligned} x_d &= \Delta M/\Gamma(B_d) \sim O(\lambda^2) \Rightarrow r_d \sim O(\lambda^4) \\ x_s &= \Delta M/\Gamma(B_s) \sim O(1) \Rightarrow r_s \sim O(1) \end{aligned} \quad (5.22)$$

The precise values of x_d and x_s depend on m_t , f_{Bd} and $\mathfrak{B}$. The expected value of r_d and r_s are shown in fig. 41 for $110 \text{ MeV} \leq f_{Bd} \leq 150 \text{ MeV}$, $150 \text{ MeV} \leq f_{Bs} \leq 200 \text{ MeV}$, $\lambda_{td} < 0.015$, $\lambda_{ts} = 0.05$ and $30 \text{ GeV}/c^2 \leq m_t \leq 60 \text{ GeV}/c^2$. Thus, we estimate $r_d \leq 0.01$ and r_s to lie in the range $0.3 < r_s < 1.0$.

In view of the small value of r_d in the standard model just calculated, we set $r_d \approx 0$ and reanalyse the UA1 measurement (5.13). Now, in the standard model this gives a lower bound on χ_s

$$\chi \approx \chi_s P_s > 0.025 \quad (2 \sigma) \quad (5.23)$$

which for $P_s = 0.2$ gives the bound on r_s but now at 2σ due to the assumption $r_d = 0$. Thus, in the standard model, the UA1 data give the following limit

$$r_s > 0.14 \quad (2 \sigma) \quad (5.24)$$

This in turn gives

$$x_s > 0.57 \quad (2 \sigma) \quad (5.25)$$

The limit on x_s can be immediately converted into a lower bound for the CKM-angle-dependent λ_{ts} . Putting in the numbers one has for $m_t \leq M_W$ [20]

$$x_s = 1.75 \frac{\Re f_{Bs}^2}{(150 \text{ MeV})^2} \left[\frac{m_t}{40 \text{ GeV}/c^2} \right]^2 \left[\frac{\lambda_{ts}}{\lambda^2} \right]^2 \quad (5.26)$$

which, for $x_s > 0.57$ and the indicated values of m_t and $\Re f_{Bs}^2$, gives

$$\lambda_{ts} > 0.57 \lambda^2 \cong 0.030 \quad (5.27)$$

Since in the standard 3-family model one expects $V_{tb} \cong 1$, (5.27) would imply a lower bound on the CKM matrix element $|V_{ts}|$, namely $V_{ts} > 0.030$. Clearly, there are quite a few unknowns in (5.26) but hopefully they will be determined much better in due course of time. In any case, we find the result (5.27) remarkable in the sense that it is the first direct experimental evidence in the standard model that $V_{ts} \neq 0$.

The experimental limit (5.12) on x_d and the lower bound (5.25) on x_s are in agreement with the expectations of the standard model, as expected from the Cabibbo suppressed and allowed patterns indicated in eq. (5.22). More precise tests of the standard model of flavour mixing in the bottom meson sector are expected to emerge from ongoing and forthcoming experiments through measurements of x_d and x_s .

6. Summary and Conclusions

We have addressed the problem of heavy flavour production at the CERN $p\bar{p}$ collider in the context of perturbative QCD. The calculations concerning the production and decays presented here are part of a general purpose Monte Carlo program to study hard hadronic collisions, called EUROJET [10]. We have modelled the decays and fragmentation of the charm and bottom quarks by using data from e^+e^- experiments. All known aspects of the heavy quark data are painstakingly incorporated. The decay properties of the top quark are incorporated in the context of the free quark model with the standard V-A interactions.

There are two main sources of heavy flavour production, namely strong production via perturbative QCD processes and in the production and decays of $W^\pm$ and Z^0 bosons. The latter mechanism is important for the top quark only. The calculations presented here are based on leading order and next-to-leading order in the strong interaction coupling constant, $\alpha_s(Q^2)$. This means $2 \rightarrow 2$ and $2 \rightarrow 3$ processes for the strong production, and $W^\pm$, Z^0 production processes with and without an additional jet. In both cases heavy quark masses in calculating the production and decay processes are taken into account. The calculations are based on the use of Born diagrams only. This leaves an overall dependence on the infra-red cut-off, p_t^{cut} . We have used $p_t^{\text{cut}} = 5 \text{ GeV}/c$ for theoretical calculations. This choice gives us the same effective K-factor as the one obtained in the leading log. approximation. The data clearly need $2 \rightarrow 3$ and higher orders. This is obvious

from the comparison of data with estimates based on $2 \rightarrow 2$ processes, with the structure functions fixed from inclusive-jet analysis at the CERN collider. Both the normalization and shape of the distributions require higher orders to be explained in the context of QCD.

Though the model described here has a wide range of applicability and predictions, we have concentrated on the inclusive muon, dimuon and trimuon final states because of the availability of data. Our comparison here with the UA1 data shows that perturbative QCD successfully explains both the heavy flavour cross-sections and topologies of the experimental data. Based on this detailed comparison it is fair to say that large- p_T and/or central (rapidity) heavy flavour production at the CERN collider is quantitatively under the control of QCD. It would be worthwhile to reanalyse the data on charm and bottom quark production at the ISR/FNAL energies when the experimental situation is consolidated.

There is one exception to the list of successful predictions of perturbative QCD in the study of the $p\bar{p}$ collider physics, namely the ratio of same-sign to opposite-sign dimuon production $R(\pm\pm/+-)$, which is not explained by QCD alone. The UA1 measurement $R(\pm\pm/+-) = 0.42 \pm 0.07 \pm 0.03$ is typically 2.4σ or a factor 1.8 too high, though values of $R(\pm\pm/+-) = 0.30$ could perhaps be accommodated by stretching the errors on the various underlying parameters. Since the dimuon data are dominated by the process $p\bar{p} \rightarrow b\bar{b} X \rightarrow \mu\mu X$ ($> 75\%$) and all distributions are in agreement with this being the dominant source, it is very likely that the explanation of the larger $R(\pm\pm/+-)$ value lies in the hypothesis of $B^0-\bar{B}^0$ oscillations. With the probability of $B^0_d-\bar{B}^0_d$ mixing, r_d , already bounded by e^+e^- experiments to lie below 0.12, only significant $B^0_s-\bar{B}^0_s$ mixing is conceivable. In the standard model $x_d \approx O(10^{-2})$ and $x_s > 1.5$. Neglecting x_d and using $P_s = 0.20$ we find that at 2σ , UA1 data give a lower bound on r_s , $r_s > 0.14$. This gives a lower bound on the ratio $x_s = \Delta M/\Gamma(B^0_s-\bar{B}^0_s)$, $x_s > 0.57$. In the standard model this constitutes the first direct measurement that the CKM matrix element V_{ts} is non-zero. Assuming $m_t = 40 \text{ GeV}/c^2$ and $\Re f_{B_s^2} = (150 \text{ MeV})^2$, the limit $x_s > 0.57$ gives $V_{ts} > 0.030$. With complete $B^0_s-\bar{B}^0_s$ mixing i.e. $r_s = 1$ and $P_s = 0.20$, we get $R(\pm\pm/+-) = 0.38 - 0.42$, in good agreement with the UA1 data.

The evidence for $B^0_s-\bar{B}^0_s$ mixing through measurements of $R(\pm\pm/+-)$ is at best indirect only. More data are needed to consolidate this result and specific flavour correlations, which are definite signatures of $B^0_s-\bar{B}^0_s$ mixing, have to be measured before definitive and final conclusions can be drawn. Some of these correlations have already been discussed in the literature [56]. We would like to propose a number of measurements that would help to consolidate the $B^0_s-\bar{B}^0_s$ mixing interpretation of the $R(\pm\pm/+-)$ measurements by UA1.

(i) Measurement of the p_T^μ spectrum with respect to the jet axis. This would enable the enrichment of the $\mu^\pm$ from the bottom primary decay $b \rightarrow \mu^\pm X$, if a large p_T^μ cut-off is used. The cascade decays of the bottom quark which provide the largest background to $R(\pm\pm/+-)$ would then be eliminated. Clearly a larger p_T^μ cut-off will also reduce the primary $b \rightarrow \mu^\pm X$ events and hence bottom enrichment techniques require a larger dilepton sample. This would hopefully be available from the new collider runs at CERN, Fermilab and in e^+e^- experiments.

(ii) Since the semileptonic decays of the bottom hadrons are expected to be dominated by the decays $B_d^0 \rightarrow l^+\nu_l (D^+, D^{*+}, \dots)$, $B_u^- \rightarrow l^-\nu_l (D^0, D^{*0}, \dots)$, $B_s^0 \rightarrow l^+\nu_l (F^+, F^{*+}, \dots)$, observation of the final states $l^+l^-F^\pm X$, $l^+l^-F^\pm X$, with $l^\pm$ definitely tagged as the primary $b \rightarrow l^\pm X$ leptons, would confirm mixing in the $B^0_s-\bar{B}^0_s$

sector. Hence a good $F^\pm$ detection efficiency is at a premium. In this context it might be useful to point out that the decays $D^0, D^+ \rightarrow K^+ X$ are both Cabibbo suppressed and are found to be small experimentally. The decays $F^+ \rightarrow K^\pm X$ are on the other hand Cabibbo allowed. Thus a good strategy to enhance the $F^\pm$ signal is to look for the decays $F^+ \rightarrow K^+ X, F^- \rightarrow K^+ X$. Likewise, final states $l^+ l^+ \Lambda^0$ and $l^+ l^- \bar{\Lambda}^0$ in e^+e^- annihilation and $p\bar{p}$ collisions would be signatures of $B^0_s - \bar{B}^0_s$ mixing [56]. Similarly, the measurements of the processes $e^+e^-, p\bar{p} \rightarrow b\bar{b}$, with $b \rightarrow l^+ K^- K^- X$, where the lepton and kaons belong to the same jet are a sign of $B^0_s - \bar{B}^0_s$ mixing.

(iii) Of particular importance in this context are experiments in the Υ -region. The bound $x_d < 0.52$ has been set from measurements on $e^+e^- \rightarrow \Upsilon(4s) \rightarrow b\bar{b} \rightarrow l^\pm l^\pm X$ experiments [38]. It would be nice to search for resonances which decay preferentially into $B_s^0 X$. The absence of a signal in $R(\pm\pm/+-)$ at $\Upsilon(4s)$ but observation of a corresponding signal at a higher resonance, which would allow for production of B_s^0 mesons would be a very nice confirmation of the standard model predictions, $x_d \ll 1, x_s > 1$. We urge the bottom physics community to build high luminosity bottom factories in the Υ -region and elsewhere to test the standard theory of heavy flavour production with much greater precision than has been achieved by the present UA1 data.

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Table 1: The heavy flavour quark inclusive cross-sections in the processes $p\bar{p} \rightarrow Q X$ ($Q = c, b, t$) from the QCD and weak production ($W^\pm, Z^0$) mechanisms at $\sqrt{s} = 630$ GeV. For $2 \rightarrow 3$ we have used a cut-off, $p_t^{\text{cut}} = 5$ GeV/c. In addition, QCD contributions are calculated with $\sqrt{\hat{s}} > 10, 15$ GeV for $2 \rightarrow 2$ and $2 \rightarrow 3$, respectively.

QCD Processes	2 $\rightarrow$ 2		2 $\rightarrow$ 3		sum	
$c\bar{c}$	58	μb	63	μb	120	μb
$b\bar{b}$	13	μb	6.2	μb	19	μb
$t\bar{t}$ ($m_t = 23$ GeV/c ²)	32	nb	20	nb	52	nb
$t\bar{t}$ ($m_t = 40$ GeV/c ²)	1.4	nb	0.81	nb	2.2	nb
Weak processes	Drell-Yan		$O(\alpha_s)$		sum	
$W^\pm \rightarrow cs$	1.2	nb	0.45	nb	1.7	nb
$W^\pm \rightarrow tb$ ($m_t = 23$ GeV/c ²)	1.1	nb	0.41	nb	1.5	nb
$W^\pm \rightarrow tb$ ($m_t = 40$ GeV/c ²)	0.76	nb	0.29	nb	1.1	nb
$Z^0 \rightarrow c\bar{c}$	0.30	nb	0.12	nb	0.42	nb
$Z^0 \rightarrow b\bar{b}$	0.38	nb	0.14	nb	0.52	nb
$Z^0 \rightarrow t\bar{t}$ ($m_t = 23$ GeV/c ²)	0.22	nb	0.085	nb	0.31	nb
$Z^0 \rightarrow t\bar{t}$ ($m_t = 40$ GeV/c ²)	0.062	nb	0.024	nb	0.086	nb

Table 2: Semileptonic branching ratios of charmed, bottom and top hadrons.

$D^0 \rightarrow \mu^+ \nu_\mu X$ (7.5 %)			
$K^- \rightarrow \mu^+ \nu_\mu$	3.25 %	$K^{*-} \rightarrow \mu^+ \nu_\mu$	3.25 %
$\pi^- \rightarrow \mu^+ \nu_\mu$	0.50 %	$\rho^- \rightarrow \mu^+ \nu_\mu$	0.50 %
$D^+ \rightarrow \mu^- \bar{\nu}_\mu X$ (18 %)			
$\bar{K}^0 \rightarrow \mu^- \bar{\nu}_\mu$	8.50 %	$\bar{K}^{*0} \rightarrow \mu^- \bar{\nu}_\mu$	8.50 %
$\pi^0 \rightarrow \mu^- \bar{\nu}_\mu$	0.50 %	$\rho^0 \rightarrow \mu^- \bar{\nu}_\mu$	0.50 %
$F^+ \rightarrow \mu^- \bar{\nu}_\mu X$ (8.5 %)			
$\eta \rightarrow \mu^- \bar{\nu}_\mu$	3.40 %	$\eta' \rightarrow \mu^- \bar{\nu}_\mu$	1.70 %
$\phi \rightarrow \mu^- \bar{\nu}_\mu$	3.40 %		
$\Lambda_c \rightarrow \mu^+ \nu_\mu X$ (8.5 %)			
$b\text{-hadrons} \rightarrow \mu^+ \nu_\mu X$ (12 %)			
$(b\text{-hadrons} \rightarrow J/\psi X$ (1 %), $J/\psi \rightarrow \mu^+ \mu^- X$ (7.4 %))			
$t\text{-hadrons} \rightarrow \mu^+ \nu_\mu X$ (11 %)			

Table 3: Inclusive muon, dimuon and trimuon cross-sections for the weak production processes. The inclusive muon cross-sections are obtained using $p_t^\mu > 3$ GeV/c and two values of $|\eta^\mu|$, $|\eta^\mu| < 2.5$ and $|\eta^\mu| < 1.5$. The inclusive dimuon cross-sections are obtained with $p_t^{\mu_1}, p_t^{\mu_2} > 3$ GeV/c, $|\eta^{\mu_1}| < 1.3$, $|\eta^{\mu_2}| < 2.0$ and $M^{\mu\mu} > 6$ GeV/c². Unlike-sign and like-sign dimuon cross-sections are shown separately. The cuts for the trimuon final state are $p_t^{\mu_i} > 3$ GeV/c, $|\eta^{\mu_i}| < 2.0$ with $i = 1, 2, 3$.

	Drell-Yan		$O(\alpha_s)$		sum.	
$\sigma(p\bar{p} \rightarrow W^\pm, Z^0 X \rightarrow (cs, tb), (c\bar{c}, b\bar{b}, t\bar{t}) X' \rightarrow \mu^\pm X''), \eta^\mu < 2.5$						
$W^\pm \rightarrow cs$	68	pb	27	pb	95	pb
$W^\pm \rightarrow tb$ ($m_t = 23 \text{ GeV}/c^2$)	0.29	nb	0.11	nb	0.40	nb
$W^\pm \rightarrow tb$ ($m_t = 40 \text{ GeV}/c^2$)	0.22	nb	0.084	nb	0.30	nb
$Z^0 \rightarrow c\bar{c}$	18	pb	6.7	pb	25	pb
$Z^0 \rightarrow b\bar{b}$	47	pb	18	pb	65	pb
$Z^0 \rightarrow t\bar{t}$ ($m_t = 23 \text{ GeV}/c^2$)	37	pb	15	pb	52	pb
$Z^0 \rightarrow t\bar{t}$ ($m_t = 40 \text{ GeV}/c^2$)	12	pb	4.6	pb	17	pb
$\sigma(p\bar{p} \rightarrow W^\pm, Z^0 X \rightarrow (cs, tb), (c\bar{c}, b\bar{b}, t\bar{t}) X' \rightarrow \mu^\pm X''), \eta^\mu < 1.5$						
$W^\pm \rightarrow cs$	60	pb	24	pb	84	pb
$W^\pm \rightarrow tb$ ($m_t = 23 \text{ GeV}/c^2$)	0.25	nb	0.093	nb	0.34	nb
$W^\pm \rightarrow tb$ ($m_t = 40 \text{ GeV}/c^2$)	0.19	nb	0.075	nb	0.27	nb
$Z^0 \rightarrow c\bar{c}$	16	pb	5.9	pb	22	pb
$Z^0 \rightarrow b\bar{b}$	41	pb	16	pb	57	pb
$Z^0 \rightarrow t\bar{t}$ ($m_t = 23 \text{ GeV}/c^2$)	33	pb	14	pb	47	pb
$Z^0 \rightarrow t\bar{t}$ ($m_t = 40 \text{ GeV}/c^2$)	11	pb	4.2	pb	15	pb
$\sigma(p\bar{p} \rightarrow W^\pm, Z^0 X \rightarrow (cs, tb), (c\bar{c}, b\bar{b}, t\bar{t}) X' \rightarrow \mu^+ \mu^- X'')$						
$W^\pm \rightarrow tb$ ($m_t = 23 \text{ GeV}/c^2$)	19	pb	7.1	pb	26	pb
$W^\pm \rightarrow tb$ ($m_t = 40 \text{ GeV}/c^2$)	18	pb	6.4	pb	24	pb
$Z^0 \rightarrow c\bar{c}$	1.1	pb	0.40	pb	1.5	pb
$Z^0 \rightarrow b\bar{b}$	3.4	pb	1.0	pb	4.4	pb
$Z^0 \rightarrow t\bar{t}$ ($m_t = 23 \text{ GeV}/c^2$)	4.1	pb	1.5	pb	5.6	pb
$Z^0 \rightarrow t\bar{t}$ ($m_t = 40 \text{ GeV}/c^2$)	1.6	pb	0.55	pb	2.2	pb
$\sigma(p\bar{p} \rightarrow W^\pm, Z^0 X \rightarrow (cs, tb), (c\bar{c}, b\bar{b}, t\bar{t}) X' \rightarrow \mu^+ \mu^+ \mu^- \mu^- X'')$						
$W^\pm \rightarrow tb$ ($m_t = 23 \text{ GeV}/c^2$)	20	pb	7.3	pb	27	pb
$W^\pm \rightarrow tb$ ($m_t = 40 \text{ GeV}/c^2$)	15	pb	6.0	pb	21	pb
$Z^0 \rightarrow b\bar{b}$	2.3	pb	0.78	pb	3.1	pb
$Z^0 \rightarrow t\bar{t}$ ($m_t = 23 \text{ GeV}/c^2$)	2.6	pb	1.3	pb	3.9	pb
$Z^0 \rightarrow t\bar{t}$ ($m_t = 40 \text{ GeV}/c^2$)	0.81	pb	0.35	pb	1.2	pb
$\sigma(p\bar{p} \rightarrow W^\pm, Z^0 X \rightarrow (cs, tb), (c\bar{c}, b\bar{b}, t\bar{t}) X' \rightarrow 3\mu X'')$						
$W^\pm \rightarrow tb$ ($m_t = 23 \text{ GeV}/c^2$)	4.2	pb	1.4	pb	5.6	pb
$W^\pm \rightarrow tb$ ($m_t = 40 \text{ GeV}/c^2$)	3.1	pb	1.5	pb	4.6	pb
$Z^0 \rightarrow b\bar{b}$	0.54	pb	0.17	pb	0.71	pb
$Z^0 \rightarrow t\bar{t}$ ($m_t = 23 \text{ GeV}/c^2$)	0.81	pb	0.42	pb	1.2	pb
$Z^0 \rightarrow t\bar{t}$ ($m_t = 40 \text{ GeV}/c^2$)	0.28	pb	0.10	pb	0.38	pb

Table 4: The same as table 3 for the QCD production processes.

	2 → 2	2 → 3	sum
$\sigma(p\bar{p} \rightarrow (c\bar{c}, b\bar{b}, t\bar{t}) X' \rightarrow \mu^\pm X''), \eta^\mu < 2.5.$			
$c\bar{c}$	0.075 μb	0.13 μb	0.21 μb
$b\bar{b}$	0.14 μb	0.099 μb	0.24 μb
$t\bar{t}$ ($m_t = 23 \text{ GeV}/c^2$)	4.4 nb	2.7 nb	7.1 nb
$t\bar{t}$ ($m_t = 40 \text{ GeV}/c^2$)	0.28 nb	0.16 nb	0.44 nb
$\sigma(p\bar{p} \rightarrow (c\bar{c}, b\bar{b}, t\bar{t}) X' \rightarrow \mu^\pm X''), \eta^\mu < 1.5.$			
$c\bar{c}$	0.056 μb	0.10 μb	0.16 μb
$b\bar{b}$	0.10 μb	0.076 μb	0.18 μb
$t\bar{t}$ ($m_t = 23 \text{ GeV}/c^2$)	3.8 nb	2.3 nb	6.1 nb
$t\bar{t}$ ($m_t = 40 \text{ GeV}/c^2$)	0.25 nb	0.14 nb	0.39 nb
$\sigma(p\bar{p} \rightarrow (c\bar{c}, b\bar{b}, t\bar{t}) X' \rightarrow \mu^+ \mu^- X'')$			
$c\bar{c}$	0.75 nb	0.51 nb	1.3 nb
$b\bar{b}$	2.4 nb	1.2 nb	3.6 nb
$t\bar{t}$ ($m_t = 23 \text{ GeV}/c^2$)	0.40 nb	0.26 nb	0.66 nb
$t\bar{t}$ ($m_t = 40 \text{ GeV}/c^2$)	39 pb	21 pb	60 pb
$\sigma(p\bar{p} \rightarrow (b\bar{b}, t\bar{t}) X' \rightarrow \mu^+ \mu^+ \mu^- \mu^- X'')$			
$b\bar{b}$	0.57 nb	0.28 nb	0.85 nb
$t\bar{t}$ ($m_t = 23 \text{ GeV}/c^2$)	0.19 nb	0.12 nb	0.31 nb
$t\bar{t}$ ($m_t = 40 \text{ GeV}/c^2$)	21 pb	12 pb	33 pb
$\sigma(p\bar{p} \rightarrow (b\bar{b}, t\bar{t}) X' \rightarrow 3\mu X'')$			
$b\bar{b}$	48 pb	41 pb	89 pb
$t\bar{t}$ ($m_t = 23 \text{ GeV}/c^2$)	48 pb	30 pb	78 pb
$t\bar{t}$ ($m_t = 40 \text{ GeV}/c^2$)	7.7 pb	3.9 pb	12 pb

Table 5: The ratio of same-sign to opposite-sign dimuons (R_i) and their relative contributions (BR_i) to $R(\pm\pm/+-)$ for QCD and weak production processes as a function of the top quark mass m_t (in GeV/c^2). The relative contributions have been derived assuming that 15% of the dimuon cross-section arises from $c\bar{c}$ intermediate states. $B^0-\bar{B}^0$ mixing is not included. The numbers are obtained using the cross-sections from tables 3 and 4.

QCD processes	R_i	$BR_i (m_t = 23)$	$BR_i (m_t = 40)$
$c\bar{c}$	0.0	0.184	0.180
$b\bar{b}$	0.24	0.683	0.799
$t\bar{t} (m_t = 23)$	0.47	0.125	-
$t\bar{t} (m_t = 40)$	0.55	-	0.013
Weak processes	R_i	$BR_i (m_t = 23)$	$BR_i (m_t = 40)$
$W^\pm \rightarrow tb (m_t = 23)$	1.04	0.005	-
$W^\pm \rightarrow tb (m_t = 40)$	0.88	-	0.005
$Z^0 \rightarrow c\bar{c}$	0.0	0.001	0.001
$Z^0 \rightarrow b\bar{b}$	0.70	0.001	0.001
$Z^0 \rightarrow t\bar{t} (m_t = 23)$	0.70	0.001	-
$Z^0 \rightarrow t\bar{t} (m_t = 40)$	0.55	-	0.001

Table 6: Neglecting the small contributions from the weak processes, we summarize the estimates for $R(\pm\pm/+-)$ as a function of m_t and P_s :

$m_t (\text{GeV}/c^2)$	P_s	$R(\pm\pm/+-)$
23 (no $B_s^0-\bar{B}_s^0$ mixing)	0.16	0.23
40 (no $B_s^0-\bar{B}_s^0$ mixing)	0.16	0.20
40 (complete $B_s^0-\bar{B}_s^0$ mixing)	0.16	0.35
40 (complete $B_s^0-\bar{B}_s^0$ mixing)	0.20	0.38

Figure Captions

- Fig. 1 Lowest-order perturbative QCD processes for heavy flavour production in hadron-hadron collisions.
- Fig. 2 Next-to-leading order perturbative QCD processes at the Born level for heavy flavour production in hadron-hadron collisions.
- Fig. 3 Inclusive heavy quark transverse momentum distribution, $d\sigma/dp_T^Q$, for $Q = c, b$ and t quarks at $\sqrt{s} = 630$ GeV with a pseudo-rapidity cut-off $|\eta^Q| < 3.5$. The distributions are based on the diagrams shown in figs. 1 and 2, the EHLQ structure functions with $\Lambda = 0.2$ GeV, $Q^2 = p_T^2 + m_Q^2$, the heavy quark masses $m_c = 1.75$ GeV/c², $m_b = 5.1$ GeV/c² and two representative values for m_t , 23 GeV/c² and 40 GeV/c².
- Fig. 4 Same as fig. 3 for the heavy hadron distribution $d\sigma/dp_T^H$ with $|\eta^H| < 3.5$, and the Peterson et al. fragmentation function with $\epsilon_c = 0.05$, $\epsilon_b = 0.015$, $\epsilon_t = 0.0002$.
- Fig. 5 Inclusive heavy quark pseudo-rapidity distribution, $d\sigma/d\eta^Q$, in the (QCD) processes $p\bar{p} \rightarrow Q\bar{Q}X$, with $Q = c, b, t$ at $\sqrt{s} = 630$ GeV and a pseudo-rapidity cut-off at $|\eta^Q| < 3.5$. The calculations are based on leading order $2 \rightarrow 2$ and $2 \rightarrow 3$ processes; for the $2 \rightarrow 3$ processes we have applied a cut-off $p_T^{\text{cut}} = 5$ GeV/c to regulate the cross-sections.
- Fig. 6 Top quark production cross-sections from the processes $p\bar{p} \rightarrow W^\pm X \rightarrow t\bar{b} X'$ and $p\bar{p} \rightarrow Z^0 X \rightarrow t\bar{t} X'$ at $\sqrt{s} = 546$ GeV and $\sqrt{s} = 630$ GeV. The contributions from the Born term Drell-Yan processes and leading order QCD are shown separately as a function of the top quark mass, m_t . The distributions are based on the Duke & Owens structure functions [26] with $\Lambda = 0.2$ GeV.
- Fig. 7 Inclusive top quark production cross-section in $p\bar{p}$ collisions at $\sqrt{s} = 630$ GeV as a function of m_t . Contributions from the QCD process $p\bar{p} \rightarrow t\bar{t} X$ and from the $W^\pm$ and Z^0 production processes $p\bar{p} \rightarrow W^\pm X \rightarrow t\bar{b} X'$ and $p\bar{p} \rightarrow Z^0 X \rightarrow t\bar{t} X'$ are shown separately. For each process the leading QCD corrections are included as explained in the text.
- Fig. 8 Heavy quark fragmentation functions $1/N dN/dz$ for the top, bottom and charm quarks used in the analysis of $p\bar{p}$ data with the indicated values of the Peterson et al. parameters [14] and the average values for $\langle z_Q \rangle$, $Q = t, b, c$.
- Fig. 9 A comparison of the lepton momentum spectrum in charm hadron semileptonic decays from the EUROJET MC with DELCO data [34].
- Fig.10 A comparison of the lepton momentum spectrum in bottom hadron decays for the process $e^+e^- \rightarrow \Upsilon(4s) \rightarrow B\bar{B} \rightarrow \ell^\pm X$ from the EUROJET MC with the CLEO data [35].
- Fig.11 A comparison of the charged particle multiplicity distribution per $B\bar{B}$ event at $\Upsilon(4s)$ from the EUROJET MC with the CLEO data [36].
- Fig.12 A comparison of the $D^{*\pm}$ momentum spectrum in the decays $B \rightarrow D^{*\pm} X$ from the EUROJET MC (drawn line) with the CLEO data at $\Upsilon(4s)$ [36]. Also shown is the $p_{D^{*\pm}}$ spectrum in the semi-leptonic decays $B \rightarrow D^{*\pm} e \nu_e$ (dashed line).
- Fig.13 A comparison of the p_{D^0} spectrum from the EUROJET MC with the CLEO data [36] as in fig. 12.
- Fig.14 A comparison of the J/ψ -momentum spectrum in the decays $B \rightarrow J/\psi X$ at $\Upsilon(4s)$ from the EUROJET MC with the data of ARGUS and CLEO [38].
- Fig.15 A comparison of the $F^\pm$ -momentum spectrum in the decays $B \rightarrow F^\pm X$ at $\Upsilon(4s)$ from the EUROJET MC with the CLEO data [39].
- Fig.16 Inclusive muon pseudo-rapidity distribution, $d\sigma/d\eta^\mu$, for $|\eta^\mu| < 2.5$ and $p_T^\mu > 3$ GeV/c from the QCD heavy flavour production processes $p\bar{p} \rightarrow Q\bar{Q}X \rightarrow \mu^\pm X'$, $Q = c, b, t$ at $\sqrt{s} = 630$ GeV, for the two values of m_t ; $m_t = 23$ GeV/c² and $m_t = 40$ GeV/c². The contributions of $2 \rightarrow 2$ and $2 \rightarrow 3$ processes are shown separately.

- Fig.17 Inclusive muon pseudo-rapidity distribution, $d\sigma/d\eta^\mu$, for the heavy flavour processes $p\bar{p} \rightarrow Q\bar{Q} X \rightarrow \mu\mu X'$, $Q = c, b, t$ at $\sqrt{s} = 630$ GeV with the cut-offs $p_{t\mu_1}, \mu_2 > 3$ GeV/c, $|\eta^{\mu_1}, \mu_2| < 2.0$.
- Fig.18 Inclusive muon pseudo-rapidity distribution, $d\sigma/d\eta^\mu$, from the QCD heavy flavour processes $p\bar{p} \rightarrow Q\bar{Q} X \rightarrow 3\mu X'$, $Q = b, t$ with the cuts $p_{t\mu_i} > 3$ GeV/c, $|\eta^{\mu_i}| < 2.0$, $i = 1, 2, 3$.
- Fig.19 Inclusive muon pseudo-rapidity distribution, $d\sigma/d\eta^\mu$, for $|\eta^\mu| < 2.5$ and $p_{t\mu} > 3$ GeV/c from the weak boson production processes $p\bar{p} \rightarrow W^\pm X \rightarrow \mu^\pm X'$, at $\sqrt{s} = 630$ GeV. The top quark contribution has been calculated for the two values $m_t = 23$ GeV/c² and $m_t = 40$ GeV/c².
- Fig.20 Inclusive muon pseudo-rapidity distribution, $d\sigma/d\eta^\mu$, from the weak boson production processes $p\bar{p} \rightarrow W^\pm X \rightarrow \mu\mu X'$, at $\sqrt{s} = 630$ GeV with the cut-offs $p_{t\mu_1}, \mu_2 > 3$ GeV/c, $|\eta^{\mu_1}, \mu_2| < 2.0$.
- Fig.21 Inclusive muon pseudo-rapidity distribution, $d\sigma/d\eta^\mu$, from the weak boson production processes $p\bar{p} \rightarrow W^\pm X \rightarrow 3\mu X'$, at $\sqrt{s} = 630$ GeV with $p_{t\mu_i} > 3$ GeV/c, $|\eta^{\mu_i}| < 2.0$, $i = 1, 2, 3$.
- Fig.22 Inclusive muon pseudo-rapidity distribution, $d\sigma/d\eta^\mu$, with $p_{t\mu} > 3$ GeV/c and $|\eta^\mu| < 2.5$ from the weak processes $p\bar{p} \rightarrow Z^0 X \rightarrow (c\bar{c}, b\bar{b}, t\bar{t}) X' \rightarrow \mu^\pm X''$ at $\sqrt{s} = 630$ GeV. The top quark contribution has been calculated for the two values $m_t = 23$ GeV/c² and $m_t = 40$ GeV/c². Theoretical calculations include Drell-Yan production processes together with the $O(\alpha_s)$ corrections as explained in the text.
- Fig.23 Inclusive muon pseudo-rapidity distribution, $d\sigma/d\eta^\mu$, for the weak processes $p\bar{p} \rightarrow Z^0 X \rightarrow (c\bar{c}, b\bar{b}, t\bar{t}) X' \rightarrow \mu\mu X''$ at $\sqrt{s} = 630$ GeV with the cut-offs $p_{t\mu_1}, \mu_2 > 3$ GeV/c, $|\eta^{\mu_1}, \mu_2| < 2.0$.
- Fig.24 Inclusive muon pseudo-rapidity distribution $d\sigma/d\eta^\mu$ from the weak processes $p\bar{p} \rightarrow Z^0 X \rightarrow (b\bar{b}, t\bar{t}) X' \rightarrow 3\mu X''$, with the cuts $p_{t\mu_i} > 3$ GeV/c, $|\eta^{\mu_i}| < 2.0$, $i = 1, 2, 3$.
- Fig.25 Inclusive $p_{t\mu}$ distribution from the QCD heavy flavour production processes $p\bar{p} \rightarrow Q\bar{Q} X \rightarrow \mu X'$, $Q = c, b, t$ at $\sqrt{s} = 630$ GeV with $|\eta^\mu| < 1.5$. We have summed over the $2 \rightarrow 2$ and $2 \rightarrow 3$ processes.
- Fig.26 Inclusive $p_{t\mu}$ distribution from the QCD heavy flavour production processes $p\bar{p} \rightarrow Q\bar{Q} X \rightarrow \mu\mu X'$, $Q = c, b, t$ at $\sqrt{s} = 630$ GeV and the indicated cut-offs.
- Fig.27 Inclusive $p_{t\mu}$ distribution from the QCD heavy flavour production processes $p\bar{p} \rightarrow Q\bar{Q} X \rightarrow 3\mu X'$, $Q = b, t$ at $\sqrt{s} = 630$ GeV and $|\eta^\mu| < 2.0$.
- Fig.28 Inclusive $p_{t\mu}$ distributions from the weak processes $p\bar{p} \rightarrow W^\pm X \rightarrow (tb, cs) X' \rightarrow \mu^\pm X''$ with $|\eta^\mu| < 1.5$ for two values of m_t at $\sqrt{s} = 630$ GeV. Theoretical calculations include the Drell-Yan production process together with the $O(\alpha_s)$ corrections as explained in the text.
- Fig.29 Inclusive $p_{t\mu}$ distributions for the dimuon final state from the weak processes $p\bar{p} \rightarrow W^\pm X \rightarrow tb, X' \rightarrow \mu\mu X''$ at $\sqrt{s} = 630$ GeV with the indicated cuts.
- Fig.30 Inclusive $p_{t\mu}$ distributions for the trimuon final state from the weak processes $p\bar{p} \rightarrow W^\pm X \rightarrow tb X' \rightarrow 3\mu X''$ at $\sqrt{s} = 630$ GeV, $p_{t\mu_i} > 3$ GeV/c and $|\eta^{\mu_i}| < 2.0$, $i = 1, 2, 3$.
- Fig.31 The same as fig. 28 for the weak processes $p\bar{p} \rightarrow Z^0 X \rightarrow Q\bar{Q} X' \rightarrow \mu X''$, $Q = t, b, c$.
- Fig.32 The same as fig. 29 for the weak processes $p\bar{p} \rightarrow Z^0 X \rightarrow Q\bar{Q} X' \rightarrow 2\mu X''$, $Q = t, b, c$.
- Fig.33 The same as fig. 30 for the weak processes $p\bar{p} \rightarrow Z^0 X \rightarrow Q\bar{Q} X' \rightarrow 3\mu X''$, $Q = t, b$.

- Fig.34 Dependence of the inclusive muon cross-section from the perturbative QCD $2 \rightarrow 3$ processes on the infra-red cut-off p_t^{cut} , shown for the process $p\bar{p} \rightarrow (c\bar{c}, b\bar{b}) X \rightarrow \mu^\pm X'$ at $\sqrt{s} = 630$ GeV.
- Fig.35 Dependence of the inclusive dimuon cross-section from the perturbative QCD $2 \rightarrow 3$ processes on the infra-red cut-off p_t^{cut} , shown for the process $p\bar{p} \rightarrow (c\bar{c}, b\bar{b}) X \rightarrow \mu \mu X'$ at $\sqrt{s} = 630$ GeV.
- Fig.36 A comparison of the inclusive p_t^μ -distributions from the perturbative QCD $2 \rightarrow 2$ and $2 \rightarrow 3$ processes $p\bar{p} \rightarrow (c\bar{c}, b\bar{b}) X \rightarrow \mu^\pm X', \mu \mu X''$ with the UA1 data [17]. The dimuon data are the so-called non-isolated dimuon sample. The p_t^μ -distributions from the trimuon process $p\bar{p} \rightarrow b\bar{b} X \rightarrow 3\mu X'$, with $p_t^{\mu_i} > 3$ GeV/c, $|\eta^{\mu_i}| < 2.0$, $i = 1, 2, 3$ at $\sqrt{s} = 630$ GeV are also shown.
- Fig.37 A comparison of the inclusive p_t^μ -distributions from the heavy flavour processes $p\bar{p} \rightarrow (c\bar{c}, b\bar{b}, t\bar{t}) X \rightarrow \mu^\pm X', \mu \mu X'', 3\mu X'''$ and $p\bar{p} \rightarrow W^\pm X, Z^0 X \rightarrow \mu^\pm X', \mu \mu X'', 3\mu X'''$ with the UA1 data [17]. The dimuon data are the so-called non-isolated dimuon sample. The calculations are based on the leading-order and next-to-leading order contributions from all these processes for the two values of m_t .
- Fig.38 A comparison of the EUROJET MC prediction for the azimuthal angle distribution $d\sigma/\Delta\phi^{\mu\mu}$ from the processes $p\bar{p} \rightarrow (c\bar{c}, b\bar{b}, t\bar{t}) X \rightarrow \mu \mu X'$ and $p\bar{p} \rightarrow W^\pm, Z^0 X \rightarrow \mu \mu X'$ ($m_t = 40$ GeV/c²) with the UA1 non-isolated dimuon data [17]. Note the $2 \rightarrow 3$ contribution (dashed line) for small values of $\Delta\phi^{\mu\mu}$.
- Fig.39 A comparison of the EUROJET MC prediction for the distribution $d\sigma/p_t^{\mu\mu}$ from the processes $p\bar{p} \rightarrow (c\bar{c}, b\bar{b}, t\bar{t}) X \rightarrow \mu \mu X'$ and $p\bar{p} \rightarrow W^\pm, Z^0 X \rightarrow \mu \mu X'$ with the UA1 non-isolated dimuon data [17].
- Fig.40 A comparison of the EUROJET MC prediction for the distribution $d\sigma/M^{\mu\mu}$ from the processes $p\bar{p} \rightarrow (c\bar{c}, b\bar{b}, t\bar{t}) X \rightarrow \mu \mu X'$ and $p\bar{p} \rightarrow W^\pm, Z^0 X \rightarrow \mu \mu X'$ with the UA1 non-isolated dimuon data [17].
- Fig.41 Upper limits on the $B_d^0\text{-}\bar{B}_d^0$ mixing probability, r_d , and the $B_s^0\text{-}\bar{B}_s^0$ mixing probability, r_s , from e^+e^- experiments, standard model expectations and the lower limit from the UA1 measurements of the dimuon ratio $R(\pm\pm/\pm\mp)$,
 (a)-(c) MARK II (90 % C.L.) limits [49],
 (a) $P_d = 0.35$, $P_s = 0.10$,
 (b) $P_d = 0.375$, $P_s = 0.15$,
 (c) $P_d = 0.40$, $P_s = 0.20$,
 (d) ARGUS (90 % C.L.) limits from $\Upsilon(4s) \rightarrow B\bar{B}$ [52],
 (e) standard model estimates [18],
 (f) Lower limit (1σ) from UA1 [17].

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