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CHROMOMAGNETIC MONOPOLES*

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ABSTRACT

It is shown that localized configurations with chromomagnetic charge can be defined in QCD. The way how these configurations show up in lattice regularization and some of their properties are discussed. It is argued that these configurations are relevant in the confinement mechanism.

I. INTRODUCTION

The understanding of the confinement mechanism in QCD is an exciting problem of high energy physics. The usual nonperturbative analytic way to approach this question is the semiclassical expansion where one considers certain "relevant" regions in the function space of the path integral representation. However the contributions of other "random" configurations remain uncontrolled due to the lack of a small parameter to justify the expansion. The numerical methods of lattice field theory offer new directions to embark this problem. To indicate the difficulties of a possible lattice approach let us first consider the question of instanton configurations as an example.

There have been several geometrically motivated "pattern recognition" procedures suggested to define the winding number for lattice gauge field configurations.¹ By following the intuitive picture of the semiclassical expansion in continuum theory these constructions intend to locate the "relevant" region where a given lattice configuration belongs to. But there is no nontrivial saddle point structure in lattice gauge theory. These geometrical constructions succeed in partitioning the configuration space into non-overlapping sectors but the relation of this rearrangement to the dynamics remains unclear. Consequently properties of the winding number isolated in such a rather arbitrary way have no obvious connection with the original problem one started with. An alternative approach is to look for quantum mechanical rather than geometrical definition of the topological charge. The suggestion made in Ref. [2] is to consider the dynamics of the localized zero modes of the fermion matrix. Quantities defined in such a way prove to be useful if one can combine analytical and numerical results when using them to derive quantitative relations.

A very nice and conceptually simple mechanism for confinement in gauge theories is based on the dual Meissner effect.³ Problems to solve along this line are: (i) What kind of localized gluon states carrying chromomagnetic charges are preferred by the dynamics? (ii) How to justify the semiclassical approximation used in the dual effective theory? We address these questions in this work. We present a definition of chromomagnetic monopoles of QCD in Section II. The way how magnetic monopoles are recognized in lattice regularization will be discussed in Section III. The field operator corresponding to magnetic monopoles is introduced in Section IV. The relevance of such states in the confinement mechanism will be demonstrated in Section V.

II. DYNAMICAL HIGGS FIELD IN QCD

We shall show that the path integral expressions for the time evolution operator involve a space dependent field variable which transforms according to the adjoint representation of the gauge group. This field variable will be used as a matter field to construct the analogue of 't Hooft-Polyakov monopoles⁴ in pure $SU(3)$ Yang-Mills theories.

Consider Yang-Mills theory in temporal gauge

$$\hat{A}_0(\vec{x}, t) = 0 \quad (1)$$

We shall use Lie algebra valued field variables $\hat{A}_\mu(\vec{x}, t) = \hat{A}_\mu^a(x, t)\lambda^a/2i$, etc., in this paper. Matrix elements of the time evolution operator $\exp(-i\hat{H})$ between field diagonal states $\hat{A}_i(\vec{x})|A\rangle = A_i(\vec{x})|A\rangle$ can be written as

$$\begin{aligned} \langle \hat{A}^{(I)} | \hat{P} e^{-i\hat{H}} | \hat{A}^{(i)} \rangle &= \int \mathcal{D}[A_\mu(\vec{x}, t)] e^{iS} \\ \hat{A}(\vec{x}, 0) &= \hat{A}^{(i)}(\vec{x}) \\ \hat{A}(\vec{x}, t) &= \hat{A}^{(I)}(\vec{x}) \end{aligned} \quad (2)$$

Here, the operator $\hat{P}$ projects into the gauge invariant subspace to guarantee Gauss's Law and is represented by the integration over the auxiliary variable denoted by $A_0(\vec{x}, t)$. Matrix elements between the states characterized by the wave functionals $\psi[\hat{A}(\vec{x})]$ and $\psi'[\hat{A}(\vec{x})]$ are of the form

$$\begin{aligned} \langle \psi' | P e^{-i\hat{H}} | \psi \rangle \\ = \int \mathcal{D}[\hat{A}^{(i)}(\vec{x})] \mathcal{D}[\hat{A}^{(I)}(\vec{x})] \psi'^*[\hat{A}^{(I)}(\vec{x})] \psi[\hat{A}^{(i)}(\vec{x})] \langle \hat{A}^{(I)} | \hat{P} e^{-i\hat{H}} | \hat{A}^{(i)} \rangle \end{aligned} \quad (3)$$

There is no need of the projection operator $\hat{P}$ in the case of gauge invariant states $\psi[\hat{A}] = \psi[\hat{A}]$. $A_\mu^0 = g(\partial_\mu + A_\mu)g^\dagger$. In fact, it is easy to find the time dependent gauge transformation which eliminates A_0 and leaves the matrix element (3) unchanged.

It is advantageous to label the physical states by a non gauge invariant representative field configuration $|\hat{A}^{(i)}\rangle_{\text{phys}} = \hat{P}|\hat{A}^{(i)}\rangle$, when using path integrals. The transition matrix element corresponding to $|\hat{A}^{(i)}\rangle_{\text{phys}}$ and $|\hat{A}^{(I)}\rangle_{\text{phys}}$ can be written as

$$\begin{aligned} {}_{\text{phys}} \langle \hat{A}^{(I)} | e^{-i\hat{H}} | \hat{A}^{(i)} \rangle_{\text{phys}} &= \int \mathcal{D}[h(\vec{x})] \mathcal{D}[A_\mu(\vec{x}, t)] e^{iS} \\ \hat{A}(\vec{x}, 0) &= \hat{A}^{(i)h}(\vec{x}) \\ \hat{A}(\vec{x}, t) &= \hat{A}^{(I)h}(\vec{x}) \end{aligned} \quad (4)$$

We define a variable analogous to the Polyakov-line of the finite temperature formalism⁵ as $\Omega(\vec{x}) = P \exp \left\{ \int_0^1 d\tau A_0(\vec{x}, \tau) \right\}$, where P stands for path ordering. Ω transforms as $\Omega(\vec{x}) \rightarrow g(\vec{x})\Omega(\vec{x})g^\dagger(\vec{x})$ under local gauge transformations in the domain of integration defined by (4). The gauge $A_0(\vec{x}, t) = 0$ is not allowed here, instead we may use the static gauge where $\partial A_0(\vec{x}, t)/\partial t = 0$. Observe that the auxiliary variable $A_0(\vec{x}, t)$ appearing in the path integrals has no connection with the original operator $\hat{A}_0(\vec{x})$ eliminated by the gauge choice (1).

It is instructive to consider the partition function at finite temperature

$$Z = \text{tr } P \exp \left\{ -\beta \hat{H} \right\}.$$

The summation over different representatives in the trace operation and the integration over gauge copies in (4) amount to the integration over all possible gauge field configurations. We thus obtain the result $Z = \int \mathcal{D}[A_\mu(\vec{x}, t)] \exp \{-S_{\text{Euc}}\}$ with the boundary condition in time $A_\mu(\vec{x}, \beta) = A_\mu(\vec{x}, 0)$. The convention (4) leads to strict periodicity in the partition function instead of periodicity up to a gauge transformation. The periodicity for A_0 is imposed to simplify the boundary conditions. The transformation property of the Polyakov line is $\Omega(\vec{x}, t) = P \exp \left\{ \int_t^{t+\tau} d\tau A_0(\vec{x}, \tau) \right\} \rightarrow h(\vec{x}, t)\Omega(\vec{x}, t)h^\dagger(\vec{x}, t)$ in this case.

The boundary conditions become irrelevant when the corresponding size is large in the absence of long range interactions. So one expects A_0 to become an irrelevant quantity and to decouple from the dynamics in the limit of $t \rightarrow \infty$ or $\beta = 1/T \rightarrow \infty$ for QCD with nonvanishing gap. Nevertheless, $A_0(\vec{x})$ remains to be useful variable. Consider the expectation value of the Wilson loop $W[C]$ corresponding to the rectangular with corners at $(t, \vec{x}) = (0, 0)$, $(0, L)$, (t, L) , $(t, 0)$ computed by the path integral (4). It gives the energy of a heavy quark-antiquark pair with separation L , inserted in the states $|\vec{A}^{(i)}\rangle$ and $|\vec{A}^{(I)}\rangle$, $\exp \{-i\langle V(L) + \text{const.} \rangle\} = \langle \text{tr } W[C] \rangle$ for $t \gg L$. The correlation length of the variable $A_0(\vec{x})$ in three space is $\xi = 1/\langle \partial_\mu \rangle$ when $V(L) = L\sigma$. The way that $A_0(\vec{x})$ decouples in the long time limit contains important information about the dynamics. This example shows that the same information about $V(L)$ comes either from the spacelike or the timelike part of the Wilson loop $W[C]$ considered in temporal or axial gauge, respectively. The Gauss's Law is satisfied by the use of proper initial and final states in temporal gauge or is introduced during the time evolution by the nonvanishing $A_0(\vec{x}, t)$ in axial gauge. The rest of this paper discusses another example when the use of $A_0(\vec{x})$ leads to less obvious results.

The variable $A(\vec{x})$ raises several questions. The first is whether this field generates dynamical breakdown of the global gauge symmetry as in the case of Yang-Mills-Higgs systems.⁸ Unfortunately we know no signatures of such symmetry breaking in theories like QCD without small parameter. Another question is whether the variable $A_0(\vec{x})$ can be used to construct localized configurations which are relevant in the path integral. The obvious choice is to introduce magnetic monopole configurations⁴ using $A_0(\vec{x})$ in static gauge as an adjoint Higgs field.^{9,10} The "unbroken" electromagnetism is characterized by the Abelian field strength tensor⁴

$$\begin{aligned} F_{\mu\nu}^A(\vec{x}, t) &= \partial_\mu A_\nu^A(\vec{x}, t) - \partial_\nu A_\mu^A(\vec{x}, t) \\ &= \frac{1}{|A_0(\vec{x})|} \text{tr} \left\{ A_0(\vec{x}) \left(F_{\mu\nu}(\vec{x}, t) - \frac{1}{|A_0(\vec{x})|^2} [D_\mu A_0(\vec{x}), D_\nu A_0(\vec{x})] \right) \right\} \end{aligned} \quad (5)$$

in $SU(2)$ Yang-Mills theory, where $A_\mu^3(\vec{x}, t) = \frac{1}{\sqrt{A_0(\vec{x})}} \text{tr} (A_0(\vec{x}) A_\mu(\vec{x}, t))$ and $|A_0| = \det A_0$. The generalization of (5) to the case of $SU(3)$ gauge group has been given in.¹⁴ The gluon field configuration $A_\mu(\vec{x}, t)$ in (4) contains a chromomagnetic monopole at the location $\vec{x}_0, t_0$ if $F_{\mu\nu}^A$ displays a Dirac monopole¹³ at x_0, t_0

$$B_i(\vec{x}, t_0) = \frac{1}{2} \epsilon_{ijk} F_{jk}^A(\vec{x}, t_0) = \frac{1}{g} \frac{x_i - x_{0i}}{|\vec{x} - \vec{x}_0|^3} \quad (6)$$

where $x \sim x_0$. These monopole configurations are neither saddle points, nor the solutions of any physical equations.

The reason why these objects may nevertheless be relevant from the point of view of the confinement mechanism is the following: Consider QCD at finite temperature where the static confinement properties are described by the correlation functions of the Polyakov line. The three dimensional effective theory of the Polyakov lines or $A_0(\vec{x})$ has already been investigated in connection to the deconfining phase transition.¹⁰ Enlarge this effective theory to include the component of the complete gluon vector field in the color direction of $A_0(\vec{x})$. This is the effective theory of the gauge fields which are diagonal in the static "unitary" gauge where $\partial A_0(\vec{x}, t)/\partial t = 0$ and $A_0(\vec{x})$ is diagonal. This $U(1) \times U(1)$ gauge model has the same Polyakov line as the original $SU(3)$ theory. Consequently this Abelian reduction leaves the static confinement mechanism unattached. The confinement of the compact $U(1)$ gauge theory (compactness is the result of having nonabelian gauge group to start with) is believed to be generated by the condensation of Dirac monopoles.^{11,12} But the regular monopoles of the nonabelian system defined by the help of $A_0(\vec{x})$ are nothing else then Dirac monopoles [13] in this effective Abelian gauge model.

III. MAGNETIC MONOPOLES IN LATTICE REGULARIZATION

One needs nonperturbative tools to investigate the system of electric and magnetic charges. Our goal is thus to construct the operators in lattice regularization which create or annihilate the magnetic field (6) of the monopole. These operators act in the linear space corresponding to the temporal gauge Hamiltonian mentioned in the previous Section. In fact, the path integral (2) can be considered in the static gauge as an expression of amplitudes which correspond to the theory described by the canonical field variables $\hat{A}(\vec{x})$, $\hat{E}(\vec{x})$ and the Hamiltonian

$$\hat{H}_{\Lambda_0} = \int d^3x \left\{ \frac{g^2}{2} \hat{E}^2(\vec{x}) + \frac{1}{2g^2} \hat{B}^2(\vec{x}) + \frac{g^2}{2} (D_i A_0(\vec{x}))^2 + g^2 \hat{E}_i(\vec{x}) D_i A_0(\vec{x}) \right\}$$

with $A_0(\vec{x})$ as a random external field. The monopole creation and annihilation operators shift the eigenvalues of those color components of $\hat{A}_i(\vec{x})$ which commute with $A_0(\vec{x})$. These operators are not diagonal in the basis which is used in the path integral. Thus we discuss first the form of the diagonal operator $\hat{q}(\vec{x})$ which corresponds to the magnetic charge density of these monopoles.

The regular 't Hooft-Polyakov monopoles are extended objects, $\hat{q}(\vec{x})$ defined on elementary spacelike cubes of the lattice should give the total magnetic charge of monopoles corresponding to that cube. In order to avoid double counting we use the position of the singularity in (6) to locate monopoles. Thus, the eigenvalue of $\hat{q}(\vec{x})$, $q(\vec{x})$ is supposed to be the product of the elementary magnetic charge and the number of monopoles minus anti-monopoles (which have negative magnetic charge) whose core defined by (6) lies inside the elementary cube at $\vec{x}$. There are $N - 1$ different kind of regular 't Hooft-Polyakov monopoles in $SU(N)$ gauge theory. Consequently, $q(\vec{x})$ should be an $N - 1$ component quantity. Moreover the magnetic charges of the lattice monopoles located by $q(\vec{x})$ should agree with the corresponding charges of the continuum monopoles.

We briefly summarize the construction of $q(\vec{x})$ for $SU(3)$ lattice gauge theory. The basic difficulty of such a procedure is that the magnetic field is given in terms of the gauge fields which are multi-valued functions of the link variables. We shall use different modulo functions in extracting the gauge fields and the magnetic flux from the link and plaquette variables, respectively.

The first step is to transform the configuration in the gauge where U_0 is time independent and diagonal. This gauge choice leaves the diagonal $U(1) \times U(1)$ and a discrete gauge symmetry unfix. The latter corresponds to those nondiagonal gauge transformations which permute the eigenvectors of U_0 . The construction is manifestly invariant in the continuous $U(1) \times U(1)$ part. The discrete gauge invariance is fixed by requiring that the eigenvalues of U_0 be ordered in increasing phase. This choice is close to the continuum case where this discrete gauge symmetry appears as a global one only. Gauge transformations which are non-periodical in the time direction by a global Z_3 transformation act multiplicatively on Ω so they may change the order of the eigenvalues since their phase is obtained in $\text{mod } 2\pi$ only. Thus, the configurations are brought into the real Z_3 sector by applying such non-periodic Z_3 transformation which maximizes $\text{Re} \sum_{\vec{x}} \text{tr} \Omega(\vec{x})$.

The next step is to find the color components of the gauge fields which commute with U_0 . This is achieved by writing the spatial link variables as $U_i = D_i V_i$ where D_i is diagonal and $-i \log D_{ij} = -i \log U_{ij} - \phi$, $3\phi = \sum_{j=1}^3 -i \log U_{ij} \pmod{2\pi}$. This form is the decomposition of U_i into the product of two $SU(3)$ matrices which transform a covariant way under the $U(1) \times U(1)$ gauge transformations. The spatial vectors D_i together with the diagonal $D_0 = U_0$ form the link variables of the $U(1) \times U(1)$ gauge model mentioned in the Introduction.

Finally, we have to locate Dirac monopoles in the $U(1) \times U(1)$ lattice gauge model described by D_μ . We start this process by obtaining two independent gauge fields A_μ^3 and A_μ^8 from the three diagonal elements of D_μ , $A_\mu^8 = i/2 \log(D_\mu)_{33}$, $A_\mu^3 = -i [\log(D_\mu)_{11} + \frac{1}{2} \log(D_\mu)_{33}] \pmod{2\pi}$. The naive magnetic fields $\vec{B}_n^3(\vec{x})$ are defined as $\vec{B}_n^3(\vec{x}) = \vec{\nabla} \times \vec{A}^3(\vec{x})$, $\alpha = 3$ or 8. These variables may jump by certain units when acting by $U(1) \times U(1)$ gauge transformations. The gauge invariant part is written as $\vec{B}^\alpha = \text{mod}(\vec{B}_n^3, \vec{B}_n^8)$. The two dimensional "modulo function" $y^8 = \text{mod}^3(x^3, x^8)$ and $y^8 = \text{mod}^8(x^3, x^8)$ gives that pair of numbers (y^3, y^8) which have the smallest absolute magnitude and satisfies

$$e^{ix^3 y^3 + iz^8 y^8} = e^{iy^3 y^3 + iy^8 y^8} \quad (7)$$

where $\mu^3 = \text{diag}(1, -1, 0)$ and $\mu^8 = \text{diag}(1, 1, -2)$. The magnetic charge density $q^a(\vec{x})$ is finally given as $q^a(\vec{x}) = \vec{\nabla} \cdot \vec{B}^a(\vec{x})$. The div operation here involves $\vec{B}^a(\vec{x})$ taken on the plaquettes of the elementary cube. It is easy to verify that $q^a(\vec{x})$ is an integer multiple of a basic unit. Due to (7) this basic unit satisfies the quantization conditions of the continuum theory. The double modulus function $\text{mod}(\vec{x}^3, \vec{x}^8)$ removes the non-physical Dirac strings and guarantees the proper charge quantization in the same time.⁸

IV. CREATION AND ANNIHILATION OPERATORS

Consider the case of compact $U(1)$ gauge theory for simplicity. The generalization of the construction to non-Abelian models is straightforward. The expression of the magnetic charge density in the compact $U(1)$ gauge theory is $q(\vec{x}) = \vec{\nabla} \cdot \vec{B}(\vec{x})$. $\vec{B}(\vec{x}) = \vec{\nabla} \times \vec{A}(\vec{x}) \pmod{2\pi}$, where $\vec{A}(\vec{x})$ is the phase of the link variables in spatial directions. The simple prescription¹⁵ that the creation and annihilation operators shift the eigenvalues of the gauge field operators by a lattice version of the magnetic Coulomb field is not satisfactory. In fact, this procedure may change the magnetic charge density at any location due to the function $\text{mod } 2\pi$ appearing in the definition of $q(\vec{x})$. A simple way to cure this defect is to define a field dependent shift $\delta A[\vec{x}, \vec{A}]$. The shift should be such that (i) it causes $q(\vec{x})$ to decrease by one unit at the chosen location if it was positive there; and, (ii) the values of $\vec{A}(\vec{x}) + \delta \vec{A}[\vec{x}, \vec{A}]$ sweep through every element of the zero monopole sector ones and only ones as $\vec{A}(\vec{x})$ goes through all fields of the one monopole sector. By the help of such a shift we define the annihilation operator for monopoles $a(\vec{x}_0)$ and creation operator for anti-monopoles $b^\dagger(\vec{x}_0)$ as

$$\begin{aligned} a(\vec{x}_0) &= \sum_{\vec{n}(\vec{x}_0)} \sqrt{n(\vec{x}_0) + 1} \sum_{\vec{n}(\vec{x}), \vec{x} \neq \vec{x}_0} \prod_{\vec{x}} \hat{P}(n(\vec{x}), \vec{x}) e^{-i \sum_{\vec{x}} \vec{B}(\vec{x}) \cdot \vec{A}[\vec{x}, \vec{A}]} \times \\ &\quad \times \prod_{\vec{x} \neq \vec{x}_0} \hat{P}(n(\vec{x}), \vec{x}) \hat{P}(n(\vec{x}_0) + 1, \vec{x}_0) \\ b^\dagger(\vec{x}_0) &= \sum_{\vec{n}(\vec{x}_0)} \sqrt{n(\vec{x}_0) + 1} \hat{P}(-n(\vec{x}_0) - 1, \vec{x}_0) \sum_{\vec{n}(\vec{x}), \vec{x} \neq \vec{x}_0} \times \\ &\quad \times \prod_{\vec{x} \neq \vec{x}_0} P(-n(\vec{x}), \vec{x}) e^{-i \sum_{\vec{x}} \vec{B}(\vec{x}) \cdot \vec{A}[\vec{x}, \vec{A}]} \prod_{\vec{x}} P(-n(\vec{x}), \vec{x}) \end{aligned} \quad (8)$$

where $\vec{B}$ is the canonically conjugated operator to $\vec{A}$ and $\hat{P}(n, \vec{x})$ is the projection operator projecting into the space where $q(\vec{x})$ has eigenvalue $2n\pi$. The summations at $\vec{x} = \vec{x}_0$ and $\vec{x} \neq \vec{x}_0$ extend over non-negative and all integers, respectively. The shift $\delta \vec{A}[\vec{x}, \vec{A}]$ is chosen to decrease the magnetic charge density by one unit in the cube at $\vec{x} = \vec{x}_0$.

What is left to find is the actual form of the shift $\delta \vec{A}[\vec{x}, \vec{A}]$. We shall give an algorithm to obtain such shift for a wide class of configurations. Configurations where there is a cube with at least three faces which have large fluxes $\cos(\vec{\nabla} \cdot \vec{A}_j - \vec{\nabla}_j \cdot \vec{A}_i) < 0$ require additional considerations. But nothing we are going to say about the properties

of magnetic monopoles depends on the particular choice of $\delta \vec{A}$. The shift $\delta \vec{A}$ will be gauge invariant by definition. Its value for gauge configurations satisfying unique gauge fixing conditions is obtained in terms of the shift of the magnetic flux $\vec{B}_i = \frac{1}{2} \epsilon_{ijk} \vec{\nabla}_j \cdot \vec{A}_k$ in the following way: First we order the elementary cubes in such a way that the first cube is the one at $\vec{x} = \vec{x}_0$ and every cube has common face with at least one of the preceding ones. The shift will be computed in the order of the cubes. Since the cube at $\vec{x}_0$ contains a monopole the change of the fluxes on the plaquettes of that cube $\vec{B} \rightarrow (\vec{B} + \pi) [1 - 2\pi / (6\pi + q(\vec{x}_0))] - \pi$ decreases the monopole number exactly by one unit. The shift of the magnetic flux at the new, uncommon plaquettes of the next cube is $\vec{B} \rightarrow (\vec{B} + \pi) [1 + \phi_0 / (6\pi + q(\vec{x}_0))] - \pi$ where ϕ_0 is the flux corresponding to the plaquettes common with earlier cubes. This latter procedure is to be repeated for each cube.

It is easy to see that $q(\vec{x})/2\pi = a^\dagger(\vec{x})a(\vec{x}) - b^\dagger(\vec{x})b(\vec{x})$. The verification of the canonical commutation relation leads to a surprise: magnetic monopoles obey parastatistics. In fact, since the absolute magnitude of the magnetic charge in an elementary cube is bounded $|q(\vec{x})| \leq 12\pi$, sufficiently high power of the creation or annihilation operators vanish e.g. $(a(\vec{x}))^n = 0$ for $n > 24$. The actual bound on the magnetic charge density depends on the lattice used but it is finite for any other lattice geometry (The case of random lattice requires special investigations. We believe that different moments of the distribution of the upper bound remains finite in this case). We suspect that this circumstance is not a lattice artifact but rather reflects the physical properties of magnetic monopoles. What is important for this simple result is the presence of the ultraviolet cutoff (ask no question about the distribution of the magnetic charge inside of the elementary cube) and the existence of the compact phase factor (contribution of each elementary surface to the magnetic flux is bounded). It would be very illuminating to find the decomposition of a and b into the finite sum of fermionic operators.

V. CHROMOMAGNETIC MONOPOLES AT FINITE TEMPERATURE

It was mentioned in the Introduction that the elimination of nondiagonal gluons in unitary gauge leads to an Abelian model with the same static confinement mechanism. We shall now present a strong coupling picture of the condensation of magnetic monopoles and the mass generation for dual gluons in the low temperature phase. These phenomena lead to confinement via dual Meissner effect.³ Such description of the confinement mechanism is less direct as the straightforward lattice strong coupling expansion and contains few assumptions. But we hope that those assumptions can ultimately be tested by numerical computations and the approach to the continuum will be simpler in this scenario.

First, we show that the states dominating the partition function at strong coupling have indefinite number of magnetic monopoles. Instead of computing $\langle a(\vec{x}_0) \rangle$ or $\langle b(\vec{x}_0) \rangle$, we consider a simpler quantity $\langle a'(\vec{x}_0) \rangle$.¹⁶ $a'(\vec{x}_0) = \exp \left\{ i \sum_{\vec{x}} \vec{B}(\vec{x}) \cdot \vec{A}(\vec{x}) \right\}$ where $\vec{A}(\vec{x})$ is the lattice analogue of the field of the continuum Dirac monopole located at $\vec{x} = \vec{x}_0$. All what we need is that the operator $a'(\vec{x}_0)$ changes the number of

monopoles (it may change at $x \neq x_0$ as well). It is easy to see that $\langle a'(\vec{x}_0) \rangle = 1 + e/g^2$ with finite c . In fact, strong coupling in the nonabelian theory allows local, strong coupling expansion in the effective Abelian model as well. The first correction in $1/g^2$ is finite because it is proportional to $\sum_{\vec{x}} (\vec{\nabla} \times \vec{A})^2$ and the point singularity of the magnetic Coulomb field is smoothed out by the regularization.

Numerical calculation of $\rho = \langle a'(\vec{x}_0) a(\vec{x}_0) \rangle = \langle b'(\vec{x}_0) b(\vec{x}_0) \rangle$ at finite temperature⁹ shows a cusp at $T = T_{dec}$. Moreover, the ultraviolet finite, temperature dependent part of ρ is nonzero for $T \neq T_{dec}$. Assuming the absence of bulk phase transitions as $1/g^2$ leaves the realm of the strong coupling expansion we may expect that the monopole condensate is present for $T < T_{dec}$. In the analogous case of the singular Z_3 monopoles in the mixed fundamental-adjoint action gauge models⁶ one finds a first order bulk phase transition at the crossover region where the condensate of these monopoles disappears.

The actual form of the effective $U(1) \times U(1)$ gauge model is not known but it should be local in the strong coupling limit. A simplest choice for the effective action would be

$$S = \frac{1}{e^2} \sum \text{Re tr } P[D] \quad (9)$$

$$= \frac{1}{e^2} \sum [\cos(F_{\mu\nu}^3 + F_{\mu\nu}^8) + \cos(F_{\mu\nu}^3 - F_{\mu\nu}^8) + \cos(2F_{\mu\nu}^8)]$$

where $P[D]$ is the plaquette formed by the diagonal matrices D_μ and $F_{\mu\nu}^3$ and $F_{\mu\nu}^8$ are the field strength tensors corresponding to A_μ^3 and A_μ^8 and e^2 is proportional to g^2 . It has already been noted that the condensation of Dirac monopoles leads to area law for Wilson loops in compact $U(1)$ gauge theory.^{11,12} The analytical strong coupling description of this phenomenon¹¹ uses auxiliary variables to define the magnetic current. To make contact with the monopole configurations defined above we replace (9) by

$$S = \frac{1}{e^2} \sum [(G_{\mu\nu}^3)^2 + 3(G_{\mu\nu}^8)^2] \quad (10)$$

with $G_{\mu\nu}^\alpha = \text{mod}^\alpha(F_{\mu\nu}^3, F_{\mu\nu}^8)$. In order to simplify the notation we shall consider the case of $SU(2)$ theory where there is only one Abelian field A_μ^3 . The generating functional in Minkowski space-time (which is necessary to read off masses directly) can be written as

$$Z[j] = \int_{-\infty}^{\infty} \mathcal{D}[A_\mu] e^{i \sum [F_{\mu\nu} - \frac{2\pi}{e} n(F_{\mu\nu})]^2 + i \int j_\mu F_{\mu\nu}} \quad (11)$$

where $A_\mu(x) = A_\mu^3(x)/e$, $f_{\mu\nu} = \nabla_\mu A_\nu - \nabla_\nu A_\mu$, the integer valued function $n(f_{\mu\nu})$ is introduced to take care of the mod 2π prescription $|f - 2\pi n(f)/e| < \pi/e$ and $j_{\mu\nu}$ is the external current. It is advantageous to introduce the dual variables

$$\tilde{f}_{\mu\nu} = \frac{1}{2} \epsilon_{\mu\nu\rho\sigma} f_{\rho\sigma}, \quad \tilde{j}_{\mu\nu} = \frac{1}{2} \epsilon_{\mu\nu\rho\sigma} j_{\rho\sigma} \quad (12)$$

and $\tilde{A}_\mu$ which satisfies the conditions $\tilde{f}_{\mu\nu} = \nabla_\mu \tilde{A}_\nu - \nabla_\nu \tilde{A}_\mu$ and $\nabla_\mu \tilde{A}_\mu = 0$. The generator functional (11) reads as

$$Z = \int \mathcal{D}[\tilde{A}_\mu] \prod_{\vec{x}} \delta(\nabla_\mu \tilde{A}_\mu) \times \quad (13)$$

$$\times e^{i \sum (\tilde{f}_{\mu\nu} + e \tilde{j}_{\mu\nu})^2 - i \frac{e}{2} \sum \epsilon_{\mu\nu\rho\sigma} \tilde{A}_\mu + i \frac{e^2}{2} \sum n^2(\tilde{f}_{\mu\nu}) - i \frac{e^2}{2} \sum j_{\mu\nu}^2}$$

in terms of the dual variables and $\ell_\mu = \epsilon_{\mu\nu\rho\sigma} \nabla_\nu n(f_{\rho\sigma})$. This latter integer describes the current of the magnetic monopoles defined in the previous Sections.

We show finally in the strong coupling expansion that the dual gluon field $\tilde{A}_\mu$ acquires mass in the presence of monopole condensate. Set $j_{\mu\nu} = 0$ temporally and expand the exponential function in $1/e$. The leading order nonlocal contribution to the dual gluon self energy $\pi(\vec{x}, t)$ comes from the insertion of the second term of the exponent in (13). It contains the correlation functions of the operators $a'(\vec{x}, t) a(\vec{x}, t)$, $a'(\vec{0}, 0) a(\vec{0}, 0)$ and $b'(\vec{x}, t) b(\vec{x}, t)$, $b'(\vec{0}, 0) b(\vec{0}, 0)$, respectively. The time coordinate of the operators denotes the time slice where they shift the magnetic field. Since the creation and annihilation operators do not cluster for small $1/e^2$, arbitrarily large monopole loops are present in the path integral so $\pi(\vec{k}, \omega)$ is nonzero in the infrared limit. Static chromoelectric charges which show up through the current $j_{\mu\nu}$ as magnetic charges in this dual model experience linear potential due to the Meissner effect.

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